Trajectory tracking control of a bionic robotic fish based on iterative learning

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Abstract A bionic robotic fish has great potential application prospect. High maneuverability swimming control of a bionic robotic fish has been one of the research hotspots in the robotic fish field. In this paper, an iterative learning method has been proposed to solve the trajectory tracking control problem of robotic fish swimming. First, a dynamic model of the multi-joint bionic robotic fish is established. By considering a three-joint robotic fish as an example, the unified expression of the dynamic equation of the three-joint bionic robotic fish is obtained by Lagrange method. Second, the iterative learning controller for controlling the bionic robotic fish is designed. Then the convergence of the iterative learning controller is proved. Finally, the trajectory tracking control simulation experiment based on iterative learning is conducted. The simulation results show that the trajectory tracking control method based on iterative learning for a bionic robotic fish is effective.

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1 Introduction

After hundreds of millions of years of evolution, fish has extremely high swimming skills with high propulsive efficiency and maneuverability [1–3]. Inspired by the fish, a robotic fish has become an important research focus in bionic robotics. Namely, robotic fish combines fish propulsion mechanisms with the robotics, providing new ideas for the development of underwater vehicles [4, 5]. Compared with the traditional underwater propellers, a robotic fish has the characteristics of small size, high efficiency, good maneuverability and low noise so that it has broad potential application prospects [6–8]. In many countries worldwide, such as the United States, Japan, the United Kingdom, and China, the bionic robotic fish has been researched since the end of the 20th century. The research includes a wide range of domains, such as bionic shape design, drag reduction mechanisms, prototype development, dynamics modeling, motion control, target tracking, and multi-robotic fish coordination, and others [1].

Fish in nature, such as tuna and pike, is propelled by the back of the body and tail fins when swimming. There are body waves propagating from their head to tail and their bodies regularly oscillate from side to side. Consequently, fish swims forward against the reaction of water. A bionic robotic fish should be bionic not only in shape, but also in propulsion mechanism. Therefore, a bionic robotic fish uses body wave to propel, which makes its swimming closer to that of a natural fish [9–13]. Coene [14] applied the

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slender body theory to flexible fish-like bodies. These slender bodies had a constant forward velocity perpendicular to the wave crest, and they traveled at a constant depth. Yu et al. [15] presented a novel formula describing the traveling body wave in fish swimming, offering an enhanced understanding of the locomotion and body shape by the body and/or caudal fin (BCF) modes. Following the fish body wave function proposed by Lighthill, we design a new central pattern generation (CPG) controller. This controller can reduce the number of parameters and obtain a fish body wave curve.

Trajectory tracking and velocity tracking are key issues of robotic fish to accomplish complex tasks and adapt to new environments. There are many researches on the speed trajectory tracking of a robotic fish [16–21]. Yu et al. [16] used an open-loop control method to modify the gait parameters so as to achieve the tangential speed tracking of robotic fish. Sproewitz et al. [17] combined the gradient-free online optimization algorithm with the CPG model to track the speed of robotic fish. Li et al. [18, 19] applied an iterative learning control method to a two-link bionic robotic fish to achieve accurate real-time speed tracking. Iterative learning control is an important type of learning control and it has great potential to be used in systems with repetitive motion [20]. In [21], a novel data-driven method was used to establish the nonlinear mapping between the tail angular motion and thrust for a multi-link bionic robotic fish to achieve accurate velocity tracking.

In the aspect of trajectory tracking control, Morgansen et al. [22] proposed a trajectory tracking algorithm of bionic robotic fish based on the geometric mechanics, and they realized the open-loop and closed-loop swimming of bionic robotic fish. In [23], an adaptive switching learning control method, called the adaptive switching learning PD control, was proposed. It was used for the trajectory tracking of robots in the iterative operation mode. The convergence speed of this method was faster. Zou et al. [24] realized a cooperative control of the multi-bionic robotic fish trajectory tracking based on the neural network sliding mode control method. In [25], an adaptive neural network switching control strategy was proposed. The proposed system included an adaptive switching neural controller and a robust compensation control law. The trajectory tracking of the robot was realized. Further a reinforcement learning method was combined with the behavior-based control structure in [26]. The global trajectory performance was simulated and verified. Liu et al. [27] proposed a control approach for a dolphin robot to follow a predefined path, which involved a line-of-sight (LOS)-based planner, a sliding mode controller, and a fuzzy strategy. Most of the existing robotics cannot freely adjust their three-axis attitude, which has an adverse effect on the free-swimming propulsion and plausible applications [28]. Therefore, it is necessary to design a control system for robots.

To avoid invalid spontaneous behavior and ensure high learning efficiency, learning algorithms rely on relevant knowledge or laws. Ji et al. [29] determined the controllability directly from the topology structures of communication graphs. They proposed the concept of destructive controllability nodes, which indicated that the difficulty in graphical characterization turns out to be the identification of topology structures of destructive controllability nodes. In iterative learning, it is usually to use a method that can obtain the knowledge in learning processes to improve the speed of subsequent learning. For instance, a learning law is designed with forgetting factors and feedback configuration. In addition, the tracking performance of a system under interference should be considered.

The remainder of this paper is organized as follows. A fish body wave model for multi-joint robotic fish is analyzed in Section 2. In Section 3, dynamics modeling based on the Lagrangian method is presented. An iterative learning controller for the bionic robotic fish designed to realize trajectory tracking of fish body wave is introduced in Section 4. In Section 5, simulations and experiments are provided. Finally, a summary is given in Section 6.

2 Robotic fish body wave equation

According to the research on fish behavior, there is a traveling wave from the back neck of the fish body to the tail during fish propulsion. Namely, fish transforms the reaction force of water to the fish body into forwarding the propulsion force and lateral force through body fluctuation. The lateral components
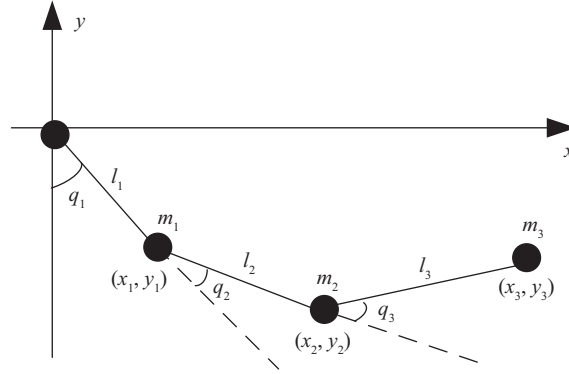
Figure 1 Three-joint bionic robot fish multi-link model.

of the fish body wave are reduced to each other. Therefore the fish gains forward propulsion. The fish body wave model was proposed by Lighthill based on the slender body theory [30], and it is expressed as follows:

$$y(x, t) = (c_1 x + c_2 x^2)\sin(kx + \omega t), \tag{1}$$

where y denotes the deviation of the fish body from the body axis, which is a function of the axial position x and time t of the fish body; c_1 and c_2 denote the envelope coefficient; k is the body wave number; and ω represents the fish body wave frequency. The fish body wave is a continuous wave, which is synthesized by the fish body envelope and sine wave. However, the input signal to the robotic fish steering gear should be a discrete periodic signal. Therefore, the fish body wave equation is discretized to obtain the control signal as follows:

$$\Theta_j = A_j \sin\left(2\pi \frac{i}{N} + \psi_j\right), \tag{2}$$

where Θ_j, A_j and ψ_j denote the swing position, amplitude and initial phase of the joint j of a bionic robotic fish, respectively; N is the number of movements of the bionic robotic fish in one cycle, and $i = 1, 2, \ldots, N$ is the i-th action of the bionic robotic fish in one action cycle.

3 Dynamics modeling of robotic fish

To achieve high maneuverability and high-efficiency swimming of a bionic robotic fish, it is necessary to establish a dynamic model of the bionic robotic fish to obtain the relationship among the moment, inertia and angular velocity. Because this paper does not study the movement of robotic fish, only the two-dimensional plane is considered in the fish model building process. A multi-joint bionic robotic fish can be regarded as a multi-link robot. By considering a three-joint bionic robotic fish developed in our lab, a dynamic model based on the Lagrange method is established. The three-joint robotic fish can be regarded as an oscillating multi-link structure. By assuming that the point mass at the end of each link represents the mass of the whole link, a multi-joint model of the robotic fish is established and it is shown in Figure 1.

Because a bionic robotic fish swims in the water, the gravity and buoyancy are approximately balanced. Thus, the effects of gravity and buoyancy can be ignored. Set up a Lagrangian function $L = K - P$, where $K = \frac{1}{2}mv^2$, and $P = mgh$. P denotes the potential energy, that is $P = 0$. Further, the dynamic equation of the system can be expressed as $F_i = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i}$, where F_i represents the force or moment, $\dot{q}_i$ denotes the angular displacement, and q_i is the angular velocity. The K value of each joint of a bionic robotic fish is determined below.

For joint 1,

$$\begin{cases} x_1 = l_1 \sin q_1, \\ y_1 = -l_1 \cos q_1, \end{cases} \tag{3}$$

$$\begin{cases} \dot{x}_1 = l_1 \cos q_1 \dot{q}_1, \\ \dot{y}_1 = -l_1 \sin q_1 \dot{q}_1. \end{cases} \tag{4}$$

Then,

$$K_1 = \frac{1}{2} m_1 l_1^2 \dot{q}_1^2. \tag{5}$$

For joint 2,

$$\begin{cases} x_2 = l_1 \sin q_1 + l_2 \sin(q_1 + q_2), \\ y_2 = -l_1 \cos q_1 - l_2 \cos(q_1 + q_2), \end{cases} \tag{6}$$

$$\begin{cases} \dot{x}_2 = l_1 \cos q_1 \dot{q}_1 + l_2 \cos(q_1 + q_2)(\dot{q}_1 + \dot{q}_2), \\ \dot{y}_2 = l_1 \sin q_1 \dot{q}_1 + l_2 \sin(q_1 + q_2)(\dot{q}_1 + \dot{q}_2). \end{cases} \tag{7}$$

Then,

$$K_2 = \frac{1}{2} m_2 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) + m_2 l_1 l_2 \cos q_2 (\dot{q}_1^2 + \dot{q}_1 \dot{q}_2). \tag{8}$$

For joint 3,

$$\begin{cases} x_3 = l_1 \sin q_1 + l_2 \sin(q_1 + q_2) + l_3 \sin(q_1 + q_2 + q_3), \\ y_3 = -l_1 \cos q_1 - l_2 \cos(q_1 + q_2) - l_3 \cos(q_1 + q_2 + q_3), \end{cases} \tag{9}$$

$$\begin{cases} \dot{x}_3 = l_1 \cos q_1 \dot{q}_1 + l_2 \cos(q_1 + q_2)(\dot{q}_1 + \dot{q}_2) + l_3 \cos(q_1 + q_2 + q_3)(\dot{q}_1 + \dot{q}_2 + \dot{q}_3), \\ \dot{y}_3 = l_1 \sin q_1 \dot{q}_1 + l_2 \sin(q_1 + q_2)(\dot{q}_1 + \dot{q}_2) + l_3 \sin(q_1 + q_2 + q_3)(\dot{q}_1 + \dot{q}_2 + \dot{q}_3). \end{cases} \tag{10}$$

Then,

$$\begin{aligned} K_3 = {}& \frac{1}{2} m_3 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_3 l_2^2 (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) + \frac{1}{2} m_3 l_3^2 (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2 + 2\dot{q}_1 \dot{q}_2 + 2\dot{q}_1 \dot{q}_3 + 2\dot{q}_2 \dot{q}_3) \\ & + m_3 l_1 l_2 \cos q_2 (\dot{q}_1^2 + \dot{q}_1 \dot{q}_2) + m_3 l_1 l_3 \cos(q_2 + q_3)(\dot{q}_1^2 + \dot{q}_1 \dot{q}_2 + \dot{q}_1 \dot{q}_3) \\ & + m_3 l_2 l_3 \cos q_3 (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 + \dot{q}_1 \dot{q}_3 + \dot{q}_2 \dot{q}_3 + \dot{q}_2^2). \end{aligned} \tag{11}$$

Therefore, the kinetic energy of the whole bionic robotic fish is given by

$$\begin{aligned} K = {}& K_1 + K_2 + K_3 \\ = {}& \frac{1}{2} (m_1 + m_2 + m_3) l_1^2 \dot{q}_1^2 + \frac{1}{2} (m_2 + m_3) l_2^2 (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) \\ & + \frac{1}{2} m_3 l_3^2 (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2 + 2\dot{q}_1 \dot{q}_2 + 2\dot{q}_1 \dot{q}_3 + 2\dot{q}_2 \dot{q}_3) \\ & + (m_2 + m_3) l_1 l_2 \cos q_2 (\dot{q}_1^2 + \dot{q}_1 \dot{q}_2) + m_3 l_1 l_3 \cos(q_2 + q_3)(\dot{q}_1^2 + \dot{q}_1 \dot{q}_2 + \dot{q}_1 \dot{q}_3) \\ & + m_3 l_2 l_3 \cos q_3 (\dot{q}_1^2 + 2\dot{q}_1 \dot{q}_2 + \dot{q}_1 \dot{q}_3 + \dot{q}_2 \dot{q}_3 + \dot{q}_2^2). \end{aligned} \tag{12}$$

Therefore, the Lagrangian function $L = K - P = K$. Finally, according to the dynamic equation $F_i = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i}, i = 1, 2, \ldots, n$, the dynamic equation of the robotic fish can be expressed as

$$\begin{aligned} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = {}& \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} D_{111} & D_{121} & D_{131} \\ D_{211} & D_{221} & D_{231} \\ D_{311} & D_{321} & D_{331} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} \\ & + \begin{bmatrix} D_{112} & D_{122} & D_{132} \\ D_{212} & D_{222} & D_{232} \\ D_{312} & D_{322} & D_{332} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \dot{q}_2 \\ \dot{q}_2 \dot{q}_3 \\ \dot{q}_1 \dot{q}_3 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}. \end{aligned} \tag{13}$$

That is, for the multi-link robotic fish, its dynamic equation can be written as

$$D(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + G(q(t), \dot{q}(t)) + F_a(t) = F(t). \tag{14}$$

As mentioned previously, the influence of gravity is ignored; therefore, we have $G(q(t), \dot{q}(t)) = 0$. To facilitate the simulation, the dynamic equation of the j-th iteration of the bionic robotic fish is given as

$$D(q^j(t))\ddot{q}^j(t) + C(q^j(t), \dot{q}^j(t))\dot{q}^j(t) + G(q^j(t), \dot{q}^j(t)) + F_a(t) = F^j(t), \tag{15}$$

where j denotes the number of iterations, $t \in [0, t_f]$, $D(q^j(t)) \in \mathbb{R}^{n \times n}$ is the inertia terms, $C(q^j(t), \dot{q}^j(t)) \in \mathbb{R}^n$ means the centrifugal and Coriolis force, $G(q^j(t), \dot{q}^j(t)) \in \mathbb{R}^n$ denotes the gravity term, $F_a(t) \in \mathbb{R}^n$ is the repeatable unknown interference, and lastly, $F^j(t) \in \mathbb{R}^n$ denotes the control input.

4 Iterative learning controller design

The PID type iterative learning control law is widely used, especially for trajectory tracking, which is why the PD control law is selected to be used in this study. For the dynamic equation of a bionic robotic fish given by (14), the iterative learning control law is as follows:

$$F^j(t) = K_p^j e^j(t) + K_d^j \dot{e}^j(t) + F^{j-1}(t), \quad j = 0, 1, \ldots, N. \tag{16}$$

The gain switching rule in the control law is shown as

$$K_p^j = \beta(j)K_p^0, \quad K_d^j = \beta(j)K_d^0, \quad \beta(j+1) > \beta(j), \tag{17}$$

where $F^{-1}(t) = 0$, $e^j(t) = q_d(t) - q^j(t)$, $\dot{e}^j(t) = \dot{q}_d(t) - \dot{q}^j(t)$, K_p^0, K_d^0 are the initial diagonal gain arrays in the PD type control and they are all positive and definite, and $\beta(j)$ is the control gain and it satisfies the condition $\beta(j) > 1$.

It is assumed that Eq. (15) satisfies the following conditions: the desired trajectory $q_d(t)$ is third-orderly steerable in the interval $t \in [0, t_f]$; the iterative process satisfies the initial condition of $q_d(0) - q^j(0) = 0, \dot{q}_d(0) - \dot{q}^j(0) = 0, \forall j \in \mathbb{N}$; and $D(q^j(t))$ is a symmetrically positive bounded matrix, while $\dot{D}(q^j(t)) - 2C(q^j(t), \dot{q}^j(t))$ is a diagonally symmetric matrix, so $x^{\mathrm{T}}(\dot{D}(q^j(t)) - 2C(q^j(t), \dot{q}^j(t)))x = 0$, where T stands for transpose operation. The iterative learning control law given by (16) is used to implement

$$q^j(t) \overset{j \to \infty}{\to} q_d(t), \quad \dot{q}^j(t) \overset{j \to \infty}{\to} \dot{q}_d(t).$$

Next, the convergence of the iterative learning controller given by (16) is analyzed. The Lyapunov method is adopted to analyze the convergence of controllers. This method has been widely used in the design and analysis of dynamic system controllers.

The Lyapunov function can be defined as follows:

$$V^j = \int_0^t \exp(-\rho\tau) {y^j}^{\mathrm{T}} K_d^0 y^j \mathrm{d}\tau \geqslant 0, \tag{18}$$

where $K_d^0 > 0$ denotes the initial gain of the D control term in the PD control and ρ is a positive real number. By analyzing whether the difference between V^{j+1} and V^j satisfies the condition of $V^{j+1} - V^j \leqslant 0$, the tracking error of the i-th iteration tends to be zero, which proves that the controller converges.

Define $y^j(t) = \dot{e}^j(t) + \Lambda e^j(t)$, and owing to $V^{j+1} = \int_0^t \exp(-\rho\tau)(y^{j+1})^{\mathrm{T}} K_d^0 y^{j+1} \mathrm{d}\tau$, we obtain

$$\begin{aligned}
\Delta V^j &= V^{j+1} - V^j \\
&= \int_0^t \exp(-\rho\tau)(\delta {y^j}^{\mathrm{T}} + y^j)^{\mathrm{T}} K_d^0 (\delta {y^j}^{\mathrm{T}} + y^j)\mathrm{d}\tau - \int_0^t \exp(-\rho\tau){y^j}^{\mathrm{T}} K_d^0 y^j \mathrm{d}\tau \\
&= \int_0^t \exp(-\rho\tau)(\delta {y^j}^{\mathrm{T}} K_d^0 \delta y^j + 2\delta {y^j}^{\mathrm{T}} K_d^0 y^j)\mathrm{d}\tau \\
&= \frac{1}{\beta(j+1)}\left\{ \int_0^t \exp(-\rho\tau)\delta {y^j}^{\mathrm{T}} K_d^{j+1} \delta y^j \mathrm{d}\tau - 2\int_0^t \exp(-\rho\tau)\delta {y^j}^{\mathrm{T}} D\delta \dot{y}^j \mathrm{d}\tau \right. \\
&\quad \left. -2\int_0^t \exp(-\rho\tau)\delta {y^j}^{\mathrm{T}} ((C(t) + C_1(t) \right.
\end{aligned}$$

$$-\Lambda D + K_d^{j+1})\delta y^j + (F - \Lambda(C(t) + C_1(t) - \Lambda D))\delta e^j)\mathrm{d}\tau\Big\}, \tag{19}$$

where $C_1(t)$ is defined as the sum of C_t to $\dot{q}$ and G_t to $\dot{q}$.

Through the division of divisional points and similar items, we can obtain the following relationships:

$$
\begin{aligned}
\Delta V^j \leqslant \frac{1}{\beta(j+1)}\Big\{ &- \exp(-\rho\tau)\delta y^{j\mathrm{T}} D\delta y^j(t) - \rho\int_0^t \exp(-\rho\tau)\delta y^{j\mathrm{T}} D\delta y^j\mathrm{d}\tau \\
&- \Lambda\exp(-\rho\tau)\delta e^{j\mathrm{T}} l_p\delta e^j - \rho\Lambda\int_0^t \exp(-\rho\tau)\delta e^{j\mathrm{T}} l_p\delta e^j\mathrm{d}\tau \\
&- \int_0^t \exp(-\rho\tau)\omega\mathrm{d}\tau\Big\}.
\end{aligned}
\tag{20}
$$

Further, using the Cauchy-Schwarz inequality, $\Delta V^j \leqslant 0$ is obtained, that is, $V^{j+1} \leqslant V^j$. So when $j \to \infty$, we can obtain $\lim_{x\to\infty} e^j(t) \to 0, \dot{e}^j(t) \to 0, t \in t_f$.

5 Simulations and experiments

5.1 Iterative learning simulations

In order to verify the effectiveness of the proposed iterative learning-based trajectory tracking control method, the simulations are conducted to track the fish body wave of each joint of a bionic robotic fish. The simulation steps are as follows:

(1) Set the number of iterations, and generate the desired speed trajectory and the initial control target;

(2) By using the initial position, determine the initial output of the system at the initial control target, and the corresponding initial state at the initial speed;

(3) Input control signals to the system, and simultaneously sample and record the output and system error;

(4) Calculate the new input using the last error and iterative information according to the corresponding learning law;

(5) Check the new error, stop the iterative operation when the tracking error is less than a preset value or when the maximum number of iterations is reached; otherwise return to step (3).

In this subsection, the simulations of the three-joint bionic robotic fish are presented. The expected trajectories of the three joints are set as $q_1 = \sin(2t)$, $q_2 = \cos(2t)$, $q_3 = \sin(3t)$, respectively. The parameters of the bionic robotic fish are $m_1 = 10$, $m_2 = 8$, $m_3 = 7$, $l_1 = 0.5$, $l_2 = 0.5$, $l_3 = 0.5$, and the results are shown in Figure 2.

As the number of iterations increases, the tracking trajectory gradually converges to the desired trajectory. Thus, the joint motions of the bionic robotic fish could eventually track the fish body wave. The tracking error gradually approaches to zero, which verifies the effectiveness of the proposed scheme for bionic robotic fish tracking based on the fish body wave by using the iterative learning control.

5.2 Swimming experiment

The joint iteration data generated after the above iterative learning simulations are input into the controller in the form of an array, as shown in Table 1.

Figure 3 shows the experimental video snapshots using the post-learning data to drive a bionic robotic fish. The experimental results reveal that both the swing amplitude and the speed increase owing to the iterative learning of the fish body wave. This conclusion is consistent with the expected result of the fish body wave evolution with time.
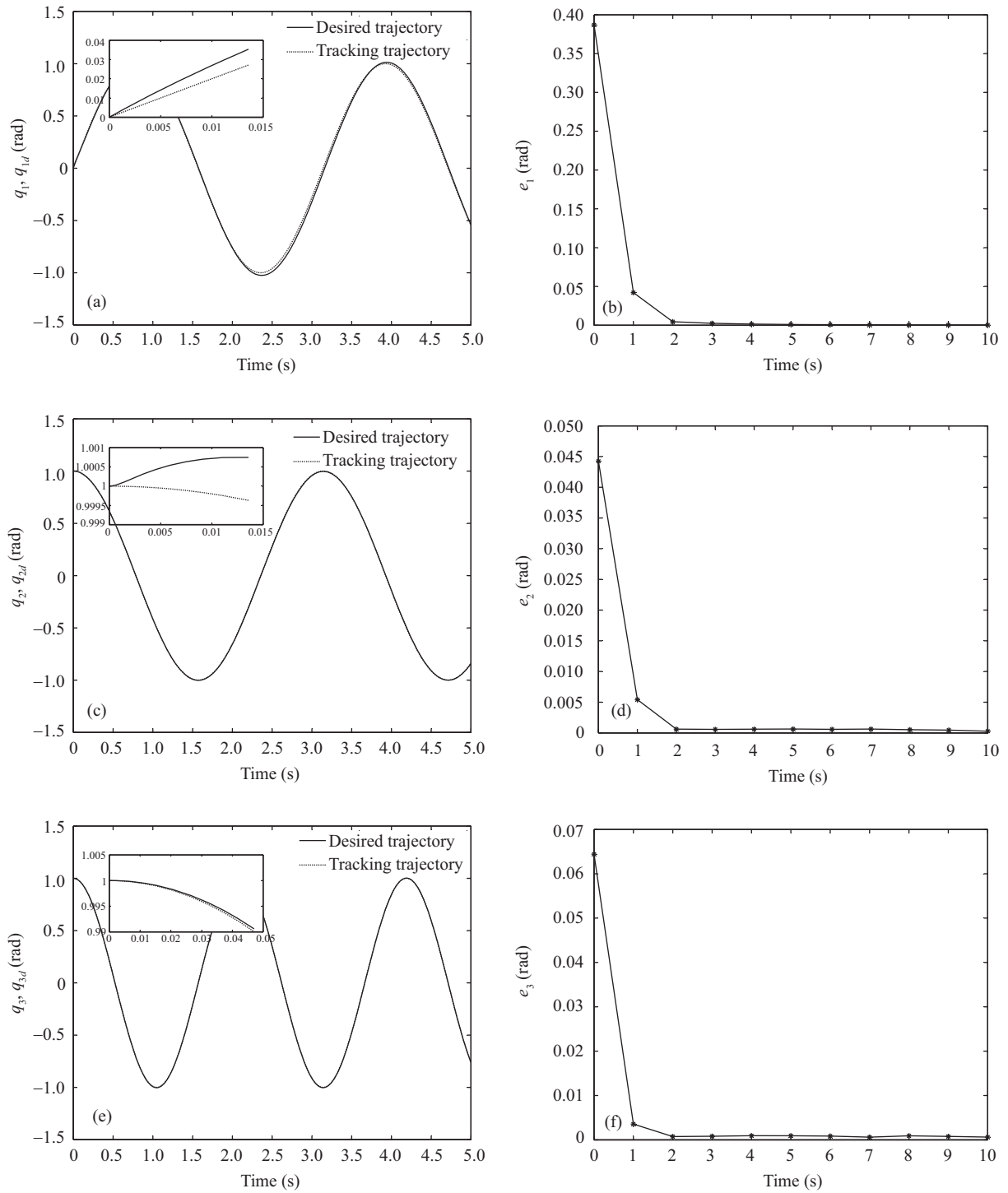
Figure 2 The tracking trajectories of the 10th iteration (a, c, e) and their corresponding convergent tracking errors (b, d, f) of joints 1, 2, and 3, respectively.

6 Conclusion

The learning control for a bionic robotic fish can enhance the adaptability of the bionic robotic fish to the aquatic environment. In this paper, a dynamic model of the three-joint bionic robotic fish was established. The iterative learning controller was designed. The convergence of the iterative learning controller was analyzed. Both the simulations and experiments of learning control were performed. The results showed

Table 1   Joint angle values (rad)

q_1	q_2	q_3
0.018599	0.982761	0.668262
0.019931	0.982589	0.671295
0.021075	0.982439	0.674106
0.022140	0.982299	0.676608
0.023320	0.982147	0.679044
0.024707	0.981975	0.681737
0.025980	0.981793	0.685615
0.027107	0.981623	0.687857
0.028215	0.981471	0.690092
0.029449	0.981322	0.693007
0.030827	0.981151	0.696253
0.032189	0.980964	0.699150
0.033389	0.980785	0.701525
0.034488	0.980627	0.703754
0.035652	0.980471	0.706347
0.036927	0.980300	0.709420
0.038324	0.980108	0.712490
0.039721	0.979917	0.715080
0.040961	0.979745	0.717330
0.042073	0.979586	0.719713
0.043216	0.979418	0.722542
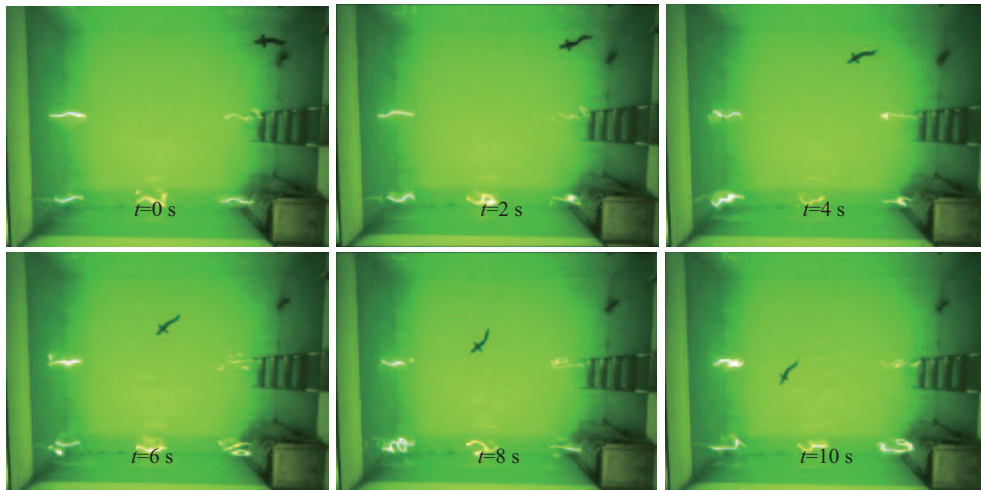
Figure 3   (Color online) Screenshots of bionic robotic fish swimming.

that the iterative learning method proposed for bionic robotic fish control is feasible. The proposed method can be used to learn fish behavior in an unfamiliar environment. In the near future, we expect to achieve autonomous swimming control in dynamic aquatic environments with some disturbances that will substantially promote large-scale applications of the bionic robotic fish.

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