SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

November 2019, Vol. 62 212202:1–212202:15
https://doi.org/10.1007/s11432-018-9782-9

Asymptotic state estimation for linear systems with sensor and actuator faults

Chun-Hua XIE[1,2], Hui YANG[1,2*], Dianhui WANG[1,2,3] & Zhe LI[4]

[1]School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang 330013, China;
[2]Key Laboratory of Advanced Control & Optimization of Jiangxi Province, Nanchang 330013, China;
[3]Department of Computer Science and Information Technology, La Trobe University, Melbourne VIC 3086, Australia;
[4]College of Electrical and Information Engineering, Hunan University, Changsha 410082, China

Received 25 October 2018/Revised 19 December 2018/Accepted 31 January 2019/Published online 19 August 2019

Abstract This paper investigates the asymptotic state estimation problem for linear systems with sensor and actuator faults, where the faults are modeled via multiple modes. For the case of sensor faults, we first introduce a new notion of detectability, i.e., detectability of system against sensor faults. The notion helps to address the question of whether it is possible to asymptotically estimate the system state by using the control input and system output, irrespective of which mode the system is in and what values the fault signals are. A necessary and sufficient condition for the system to be detectable against sensor faults is given, and then two switched observers are proposed for asymptotic state estimation with the help of maximin strategy. For the system with ℓ fault modes, we provide the explicit form of the switched observer, which is based on a bank of $\frac{\ell(\ell+1)}{2}$ Luenberger-like or sliding-mode observers. Furthermore, extensions to the case of sensor and actuator faults are further studied. Finally, a simulation example of a reduced-order aircraft system is provided to show the effectiveness of the proposed approaches.

Keywords asymptotic state estimation, observer, linear systems, sensor faults, actuator faults

Citation Xie C-H, Yang H, Wang D H, et al. Asymptotic state estimation for linear systems with sensor and actuator faults. Sci China Inf Sci, 2019, 62(11): 212202, https://doi.org/10.1007/s11432-018-9782-9

1 Introduction

Observers were first proposed by Luenberger [1] as an effective method to estimate the state of a system using its available input and output measurements. Observer design problem has received considerable attention, owing to its applications in broad areas such as observer-based control, system monitoring, and fault detection and isolation (FDI) [2]. For the system only with sensor faults, the state estimation can be used directly to control the system, and the fault signals sometimes can be reconstructed from the system output and the state estimation. Motivated by the work of [1], many extensions have been developed, such as unknown input observers for linear systems [3], sliding-mode observers for linear systems [4], distributed observer schemes for sensor networks [5], observer design for linear singular systems [6], event-based estimators for linear systems [7], disturbance observer [8], and secure state estimators for cyber-physical systems [9].

In the practical applications of control systems, the sensor/actuator faults generally induce unknown fault signals entering into the system, result in poor state estimation performance or even cause the instability of the observer error system. This has resulted in an important research area on state estimation

* Corresponding author (email: yhshuo@263.net)

for the systems with sensor/actuator faults, as surveyed in [10–16] and the references therein. For this reason, we focus on the asymptotic estimator design problem for systems with sensor/actuator faults.

The main way to consider the observer design problem for the system with faults is to treat the fault signals as unknown inputs and solve the problem of state estimation in the existence of the unknown inputs. To name a few, in [10], the authors considered the observer design for linear systems subject to faults by combining the sliding-mode observer with the unknown input observer, and the result has been further extended to the case of linear systems with sensor faults [11]. Note that the traditional unknown input observer approach mainly focuses on the system where the unknown inputs enter into the system through state equation. In order to cope with the sensor faults, an output filter is introduced in [11] so that the fault signals appear in the state equation instead of the output equation of the augmented system. In [12], in the spirit of the unknown input observers, estimation schemes were designed for linear systems with actuator faults. The scheme is based on the idea of decoupling the state and output equations into fault-free subsystem, fault-dependent one, and sometimes also disturbance-dependent one. An unknown input observer is constructed for the first subsystem and is then used for state and fault estimation.

Nevertheless, a common limitation of these results [10–16] based on unknown input observer is that only the fault modeled by single mode can be addressed. The existing approaches and results cannot be applied to the case of the system with faults which modeled via multiple modes (In this paper, the fault distribution matrix belongs to a set but we do not know which one it is, and the corresponding fault signal is unknown). The main difficulty is: as the fault distribution matrix and the fault signal are uncertain, it turns out that the fault-dependent term is subject to a double uncertainty which makes most of the existing observer methods (e.g., [3, 4, 10–17]) non-applicable, where the distribution matrices of the unknown inputs/faults are required to be exactly known a prior. In many practical applications, to cover all the possible fault situations/scenarios, the fault is usually modeled through multiple modes (called multiple mode fault for brevity). Such modeling is common in FDI literature [18–20] and fault-tolerant control literature [21–26]. Unfortunately, to the best of our knowledge, there is no result available on the asymptotic observer design problem for systems with such faults. This motivates the present study.

This paper is concerned with the challenging problem of asymptotic state estimation for linear systems with multiple mode sensor/actuator faults. The main contributions are summarized as follows:

(1) For the system with multiple mode sensor faults, a necessary and sufficient condition is provided to answer the question that whether it is possible to asymptotically estimate the system state by using the control input and system output.

(2) Under the assumption that the system is detectable against sensor faults, two novel observers are proposed for asymptotically estimating the system state based on maximin strategy.

(3) For the system with simultaneous sensor and actuator faults, we show that an asymptotic observer can also be developed under certain conditions.

The paper is structured as follows. Section 2 provides preliminaries and background ideas including the notion of detectability of system against sensor faults. A necessary and sufficient condition for the system to be detectable against sensor faults is subsequently given in Section 2. The observer design and its analytic results for the system with sensor faults are presented in Sections 3 and 4. Extensions to the case of sensor and actuator faults are further studied in Section 5. Section 6 gives an example to illustrate the results. Finally, Section 7 concludes the paper.

Notations. For a matrix A, A^{-1} and $\lambda_{\min}(A)$ denote its inverse and minimum eigenvalue, respectively. $0_{m\times n}$ and I_n denote the zero matrix with $m\times n$ dimensions and the identity matrix with $n\times n$ dimensions, respectively, and their subscripts will be omitted for simplicity without confusion. The notion $A > 0$ means that A is a symmetric positive definite matrix. Let $\mathrm{diag}\{p_1, \ldots, p_n\}$ denote the diagonal matrix with $p_1, \ldots, p_n$ on its main diagonal. For a constant matrix F, $\mathrm{img}(F)$ denotes the image space (or column space) of F, and $F^{\perp}$ denotes a basis matrix of the orthogonal complement space of $\mathrm{img}(F)$ if the orthogonal complement space is not equal to $\{0\}$, otherwise $F^{\perp} = 0$. Finally, the cardinality of a finite set $\mathcal{J}$ is denoted by $\mathrm{card}(\mathcal{J})$.

2 Problem statement and preliminaries

The background ideas and preliminaries are introduced in this section.

2.1 System model and fault model

Let us first consider the system only with sensor faults

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t) + Ff_s(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the system state, control input and system output, respectively. A, B, C and D are known constant matrices. The pair $(F, f_s(t))$ belongs to a finite set of fault modes given by

$$\mathcal{M} \triangleq \{ (F_1, f_{s1}(t)), \ldots, (F_i, f_{si}(t)), \ldots, (F_\ell, f_{s\ell}(t)) \}, \tag{2}$$

where $(F_i, f_{si}(t))$ corresponds to the i-th fault mode, and the corresponding fault distribution matrix and unknown fault signal are F_i and $f_{si}(t)$, respectively. $f_{si}(t)$ is assumed to be piecewise continuous. The positive integer $\ell \geqslant 1$ represents the number of the total possible fault modes. Without loss of generality, assume $F_1, \ldots, F_\ell$ to be of full column rank. Note that, Eq. (1) with zero fault input can be used to represent the system operating in fault-free case. Moreover, it is also worth remarking that the system (1) will be reduced to the one in [13] when $D = 0$ and $\ell = 1$.

For the input $u(t)$, the initial state $x(0)$, the fault distribution matrix F_i and the fault signal $f_{si}(t)$, the solution of (1) is denoted as $x(t) = x(t, x(0), u(t))$. The corresponding measured output is denoted as $y(t) = y(t, x(0), u(t), f_{si}(t))$.

Remark 1. The considered fault model (2) is common in FDI literature [18,19] but has also been found in fault-tolerant control literature [22]. Furthermore, an example is given to support the reasonability of the multiple mode sensor faults; i.e., the following sensor faults [20] can be transformed into the form in (1) and (2):

$$y^F(t) = \mathrm{diag}\{\rho_{i1}, \ldots, \rho_{ip}\} y(t) + \mathrm{diag}\{\jmath_{i1}, \ldots, \jmath_{ip}\} [\bar{y}_{i1}(t), \ldots, \bar{y}_{ip}(t)]^{\mathrm{T}} = y(t) + F_i f_{si}(t), \tag{3}$$

with

$$F_i = [\mathrm{diag}\{\rho_{i1} - 1, \ldots, \rho_{ip} - 1\}, \mathrm{diag}\{\jmath_{i1}, \ldots, \jmath_{ip}\}], \quad f_{si}(t) = \left[y(t)^{\mathrm{T}}, \bar{y}_{i1}(t), \ldots, \bar{y}_{ip}(t)\right]^{\mathrm{T}}, \tag{4}$$

where $i = 1, \ldots, \ell$ represents the i-th fault mode, $y(t) \in \mathbb{R}^p$ denotes the input signal of the sensors, $y^F(t) \in \mathbb{R}^p$ denotes the output signal of the sensors, and $\bar{y}_{ij}(t)$ represents the stuck value or bias value of the j-th sensor in the i-th mode. When $\rho_{ij} = 1$ and $\jmath_{ij} = 0$, there is no fault for the j-th sensor in the i-th mode. When $\rho_{ij} = 0$ and $\jmath_{ij} = 0$, the j-th sensor is outage in the i-th mode. When $\rho_{ij} = 0$ and $\jmath_{ij} = 1$, the j-th sensor is stuck at the value $\bar{y}_{ij}(t)$ in the i-th mode. When $0 < \rho_{ij} < 1$ and $\jmath_{ij} = 0$, the type of sensor faults is loss of effectiveness in the j-th sensor under the i-th mode. When $\rho_{ij} = 1$ and $\jmath_{ij} = 1$, the j-th sensor suffers from bias fault in the i-th mode and $\bar{y}_{ij}(t)$ represents the bias value.

Another intuitive example is that, if no more than 1 sensor is subject to bias fault, the sensor's output can be transformed into the following form:

$$y^F(t) = y(t) + Ff_s(t), \tag{5}$$

where $F \in \{\mathrm{diag}(1, 0, \ldots, 0), \mathrm{diag}(0, 1, 0, \ldots, 0), \ldots, \mathrm{diag}(0, \ldots, 0, 1)\}$ and $f_s(t)$ represents the bias fault.

Remark 2. Motivated by unknown input observers, some methods have been proposed for state or fault reconstruction problems [10–17]. However, a common limitation of these results is that only single mode fault (i.e., $\ell = 1$) can be addressed. This limits the applicability of these results. Compared with them, a more general case is investigated in this paper that ℓ can be any positive integer.

Remark 3. As the fault distribution matrix F and the fault signal $f_s(t)$ are uncertain, it turns out that the fault-dependent term $Ff_s(t)$ is subject to a double uncertainty. This double uncertainty makes most of the existing observer methods (e.g., [3, 4, 10–17]) non-applicable for the system (1). As a result, the problem under consideration is quite different and more challenging than those in [3, 4, 10–17], where the distribution matrices of the unknown input or fault are known constant matrices. When considering the linear system (1) with multiple mode faults, the problem to design an observer for asymptotically estimating the system state is still open. To address it, we first design a bank of observers for the system under consideration, and then a switched observer is proposed to recover the system state with the help of the maximin strategy [9].

Remark 4. Typically, the disturbances are always inevitable in many practical applications. Compared with the sensor/actuator fault signals, the disturbances are usually bounded and small in size and thus the effects of the disturbances on the state estimation are smaller. For this reason, the disturbances are not taken into account in the system under consideration, and we focus on the state estimation problem for the system with faults instead of disturbances, which allows us to find a simple solution to the problem from which we glean significant insights. It is also worth mentioning that the proposed asymptotic switched observer to recover the system state is based on a bank of Luenberger-like (or sliding-mode) observers, and thereby the robustness of the switched observer with respect to disturbances can be enhanced by properly designing the Luenberger-like (or sliding-mode) observers, separately.

Design objective. Construct an observer such that the system state can be estimated asymptotically.

2.2 Detectability of system against sensor faults

This subsection is devoted to deriving conditions under which the system state can be asymptotically estimated from the control input signal $u(t)$ and output signal $y(t)$, regardless of which mode the system is in and what values the fault signals are. This motivates the following definition of "detectability of system against sensor faults".

Definition 1. The system (1) is detectable against the multiple mode sensor faults (2) if for every initial states $x_1(0) \in \mathbb{R}^n$, $x_2(0) \in \mathbb{R}^n$, control input $u(t) \in \mathbb{R}^m$ and faults $f_{si}(t)$, $f_{sj}(t)$ with $i, j \in \{1, \dots, \ell\}$, we have

$$\begin{aligned} & y\big(t, x_1(0), u(t), f_{si}(t)\big) = y\big(t, x_2(0), u(t), f_{sj}(t)\big), \\ & \forall t \geqslant 0 \Rightarrow x\big(t, x_1(0), u(t)\big) \to x\big(t, x_2(0), u(t)\big) \text{ as } t \to \infty. \end{aligned} \tag{6}$$

In essence, this definition means that, when a system is detectable under multiple mode sensor faults, the deviation between the possible system states $x(t, x_1(0), u(t))$ and $x(t, x_2(0), u(t))$ that are compatible with the input signal $u(t)$ and the measured output $y(t)$ will converge to zero, regardless of which mode the system is in and what values the fault signals are.

Remark 5. The only difference between the detectability of system against sensor faults in Definition 1 and the strong detectability in [27, Definition 1.2] is that the fault distribution matrix in the latter is required to be unique (the one in this paper belongs to a finite set). One can see that Definition 1 is a general version of strong detectability and will be reduced to the latter when $\ell = 1$.

Now, an important theorem is given to provide the conditions for the system to be detectable.

Theorem 1. The following statements are equivalent:

(i) The system (1) is detectable against the multiple mode sensor faults (2);

(ii) The pair $(A, (F_{ij}^{\perp})^{\mathrm{T}} C)$ is detectable for all $i, j \in \{1, \dots, \ell\}$, where $F_{ij} \triangleq [F_i, F_j]$.

Proof. Note that, in view of the usual definition of detectability [28], the condition (ii) can be equivalently re-stated as the following condition:

(ii)$'$ For every $i, j \in \{1, \dots, \ell\}$ and initial condition $x(0) \in \mathbb{R}^n$, we have $(F_{ij}^{\perp})^{\mathrm{T}} C \mathrm{e}^{At} x(0) = 0$, $\forall t \geqslant 0 \Rightarrow \mathrm{e}^{At} x(0) \to 0$ as $t \to \infty$.

Hence, to prove this theorem, we just need to show the equivalence of the conditions (i) and (ii)$'$. Now we are ready to prove it.

(i) $\Rightarrow$ (ii)$'$: Suppose for the sake of contradiction that (ii)$'$ does not hold and (i) holds. Then, there exist $i, j \in \{1, \dots, \ell\}$ and an initial condition $x(0)$ such that

$$(F_{ij}^{\perp})^{\mathrm{T}} C \mathrm{e}^{At} x(0) = 0, \quad \forall t \geqslant 0 \text{ and } \lim_{t\to\infty} \mathrm{e}^{At} x(0) \neq 0. \tag{7}$$

Let

$$\begin{aligned} f_{si}(t) &= F_i^{\mathrm{T}} B_{F_{ij}} \left((B_{F_{ij}})^{\mathrm{T}} F_{ij} F_{ij}^{\mathrm{T}} B_{F_{ij}}\right)^{-1} (B_{F_{ij}})^{\mathrm{T}} C \left(x(t, x(0), u(t)) - \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau \right), \\ f_{sj}(t) &= -F_j^{\mathrm{T}} B_{F_{ij}} \left((B_{F_{ij}})^{\mathrm{T}} F_{ij} F_{ij}^{\mathrm{T}} B_{F_{ij}}\right)^{-1} (B_{F_{ij}})^{\mathrm{T}} C \left(x(t, x(0), u(t)) - \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau \right), \end{aligned} \tag{8}$$

where $x(t, x(0), u(t)) = \mathrm{e}^{At} x(0) + \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau$, and $B_{F_{ij}}$ is any basis matrix of $\mathrm{img}(F_{ij})$. It can be verified that $(B_{F_{ij}})^{\mathrm{T}} F_{ij}$ is of full row rank.

With such choices of $f_{si}(t)$ and $f_{sj}(t)$, one can check that

$$\begin{aligned} &\left[F_{ij}^{\perp}, B_{F_{ij}}\right]^{\mathrm{T}} \left(C x(t, x(0), u(t)) + D u(t) - F_i f_{si}(t) + F_j f_{sj}(t)\right) \\ &= \begin{bmatrix} (F_{ij}^{\perp})^{\mathrm{T}} \\ (B_{F_{ij}})^{\mathrm{T}} \end{bmatrix} \left(D u(t) + C \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau \right) \end{aligned} \tag{9}$$

holds for all $t \geqslant 0$, where we have used the facts that $(F_{ij}^{\perp})^{\mathrm{T}} F_{ij} = 0$ and $(F_{ij}^{\perp})^{\mathrm{T}} C \mathrm{e}^{At} x(0) = 0$, $\forall t \geqslant 0$. From the definitions of $F_{ij}^{\perp}$ and $B_{F_{ij}}$, it can be seen that either $[F_{ij}^{\perp}, B_{F_{ij}}]$ or $B_{F_{ij}}$ is invertible. Thus, Eq. (9) is equivalent to

$$\mathrm{e}^{At} \times 0 + C \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau + D u(t) + F_i f_{si}(t) = C x\big(t, x(0), u(t)\big) + D u(t) + F_j f_{sj}(t), \ \forall t \geqslant 0. \tag{10}$$

We can view the left-hand side of (10) as the output $y(t, 0, u(t), f_{si}(t))$ and the right-hand side of (10) as the output $y(t, x(0), u(t), f_{sj}(t))$. If the system (1) is detectable against the multiple mode sensor faults (2), it can be obtained from Definition 1 that

$$\lim_{t\to\infty} \left(\mathrm{e}^{At} \times 0 + \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau \right) = \lim_{t\to\infty} \left(\mathrm{e}^{At} x(0) + \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau \right), \tag{11}$$

which means that $\lim_{t\to\infty} \mathrm{e}^{At} x(0) = 0$ and it contradicts (7).

(ii)$'$ $\Rightarrow$ (i): Suppose by contradiction that (i) does not hold and (ii)$'$ holds. Thus, there exist two initial conditions $x_1(0)$, $x_2(0)$, a control input $u(t)$ and some fault signals $f_{si}(t)$, $f_{sj}(t)$ with $i, j \in \{1, \dots, \ell\}$ such that

$$y\big(t, x_1(0), u(t), f_{si}(t)\big) = y\big(t, x_2(0), u(t), f_{sj}(t)\big), \quad \forall t \geqslant 0, \tag{12}$$

$$\lim_{t\to\infty} x\big(t, x_1(0), u(t)\big) \neq \lim_{t\to\infty} x\big(t, x_2(0), u(t)\big). \tag{13}$$

It can be seen from (12) that

$$\begin{aligned} &C \mathrm{e}^{At} x_1(0) + C \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau + D u(t) + F_i f_{si}(t) \\ &= C \mathrm{e}^{At} x_2(0) + C \int_0^t \mathrm{e}^{A(t-\tau)} B u(\tau) \mathrm{d}\tau + D u(t) + F_j f_{sj}(t) \end{aligned} \tag{14}$$

holds for all $t \geqslant 0$, which implies

$$C \mathrm{e}^{At} \big(x_1(0) - x_2(0)\big) + F_{ij} \left[f_i^{\mathrm{T}}(t), -f_j^{\mathrm{T}}(t)\right]^{\mathrm{T}} = 0, \quad \forall t \geqslant 0. \tag{15}$$

Premultiply (15) by $(F_{ij}^{\perp})^{\mathrm{T}}$ and then we have

$$(F_{ij}^{\perp})^{\mathrm{T}} C \mathrm{e}^{At} \big(x_1(0) - x_2(0)\big) = 0, \quad \forall t \geqslant 0. \tag{16}$$

If (ii)$'$ holds, it can be obtained from (16) that $\lim_{t\to\infty} \mathrm{e}^{At}(x_1(0) - x_2(0)) = 0$, which means that

$$\lim_{t\to\infty}\left(\mathrm{e}^{At}x_1(0) + \int_0^t \mathrm{e}^{A(t-\tau)}Bu(\tau)\mathrm{d}\tau\right) = \lim_{t\to\infty}\left(\mathrm{e}^{At}x_2(0) + \int_0^t \mathrm{e}^{A(t-\tau)}Bu(\tau)\mathrm{d}\tau\right). \tag{17}$$

The left-hand side of (17) is $\lim_{t\to\infty} x(t, x_1(0), u(t))$ and the right-hand side of (17) is $\lim_{t\to\infty} x(t, x_2(0), u(t))$. Thus, (17) is equivalent to $\lim_{t\to\infty} x(t, x_1(0), u(t)) = \lim_{t\to\infty} x(t, x_2(0), u(t))$, which contradicts (13).

3 Observer design for the system with sensor faults

In the previous section, we have quantified the necessary and sufficient conditions for the detectability of system against sensor faults. Nevertheless, we did not discuss how to estimate system state from the input signal and output signal. In this section, we focus on constructing a switched observer based on a bank of $\frac{\ell(\ell+1)}{2}$ Luenberger-like observers to asymptotically estimate the system state.

3.1 Observer design for the case that $\ell = 1$

According to Theorem 1, the system (1) is detectable for the case that $\ell = 1$ if and only if the pair $\big(A, (F_1^{\perp})^{\mathrm{T}}C\big)$ is detectable. Thus, the following Luenberger-like observer can be established to estimate the system state for the case that $\ell = 1$.

$$\begin{aligned} \dot{\hat{x}}_1(t) &= A\hat{x}_1(t) + Bu(t) + L_1(F_1^{\perp})^{\mathrm{T}}\big(\hat{y}_1(t) - y(t)\big),\\ \hat{y}_1(t) &= C\hat{x}_1(t) + Du(t), \end{aligned} \tag{18}$$

where the observer matrix L_1 is selected a priori such that $A + L_1(F_1^{\perp})^{\mathrm{T}}C$ is Hurwitz. It is pretty easy to prove that, if the system (1) is detectable for the case that $\ell = 1$, there always exists a matrix L_1 such that $A + L_1(F_1^{\perp})^{\mathrm{T}}C$ is Hurwitz and then the Luenberger-like observer (18) is a exponential observer for the system (1). Clearly, it is easy to design an observer for the case that $\ell = 1$. Hence, we only consider the case $\ell \geqslant 2$ in the remainder part of this section, which is difficult to solve.

Remark 6. In [13], the authors proposed sliding-mode observers for the system with sensor faults, where single fault mode is considered, i.e., $\ell = 1$. Compared with [13], there are several advantages of the observer (18). First and foremost, the design condition is more relaxed than that in [13], where the observability of the pair (A, C) is needed. Furthermore, the Luenberger-like observer (18) is easier to implement and simpler than the sliding-mode one.

3.2 Switched observer design for the case that $\ell \geqslant 2$

Note that, if the system (1) is detectable against the multiple mode sensor faults (2), then the pair $(A, (F_{\mathcal{J}}^{\perp})^{\mathrm{T}}C)$ is detectable (please see Theorem 1 for details) and there always exists a matrix $L_{\mathcal{J}}$ such that $A + L_{\mathcal{J}}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}}C$ is Hurwitz, where $\mathcal{J} \subseteq \{1, \ldots, \ell\}$, $1 \leqslant \mathrm{card}(\mathcal{J}) \leqslant 2$, and $F_{\mathcal{J}}$ is defined as follows:

$$F_{\mathcal{J}} = \begin{cases} F_{ii}, \text{ if } \mathcal{J} = \{i\},\\ F_{ij}, \text{ if } \mathcal{J} = \{i, j\} \text{ and } i < j,\\ F_{ji}, \text{ if } \mathcal{J} = \{i, j\} \text{ and } j < i. \end{cases} \tag{19}$$

Motivated by this point and following the same framework as the Luenberger-like observer, we construct an observer for the set $\mathcal{J}$, which satisfies $\mathcal{J} \subseteq \{1, \ldots, \ell\}$ and $1 \leqslant \mathrm{card}(\mathcal{J}) \leqslant 2$, as follows:

$$\begin{aligned} \dot{\hat{x}}_{\mathcal{J}}(t) &= A\hat{x}_{\mathcal{J}}(t) + Bu(t) + L_{\mathcal{J}}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}}\big(\hat{y}_{\mathcal{J}}(t) - y(t)\big),\\ \hat{y}_{\mathcal{J}}(t) &= C\hat{x}_{\mathcal{J}}(t) + Du(t), \end{aligned} \tag{20}$$

where the observer matrix $L_{\mathcal{J}}$ is selected a priori such that $A + L_{\mathcal{J}}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}}C$ is Hurwitz.

Theorem 2. Consider the system (1) with $\ell \geqslant 2$, and the observer (20) with $\mathcal{J} \subseteq \{1, \ldots, \ell\}$ and $1 \leqslant \mathrm{card}(\mathcal{J}) \leqslant 2$. Assume that the system (1) is detectable against the multiple mode sensor faults (2). If the fault mode that the system operates in belongs to the set $\mathcal{J}$, then there always exists an observer matrix $L_{\mathcal{J}}$ such that the observer estimation error converges to zero, i.e., $\hat{x}_{\mathcal{J}}(t) \rightarrow x(t)$ as $t \rightarrow \infty$.

Proof. Without loss of generality, one can assume that the system operates in the i-th mode, i.e., $(F, f_s(t)) = (F_i, f_{si}(t))$, but does not know the value of i. From the assumption of the theorem, one can conclude that $F = F_i$, $\{i\} \subseteq \mathcal{J} \subseteq \{1, \ldots, \ell\}$, and thus $(F_{\mathcal{J}}^{\perp})^{\mathrm{T}} F = (F_{\mathcal{J}}^{\perp})^{\mathrm{T}} F_i = 0$. Therefore, we have

$$(F_{\mathcal{J}}^{\perp})^{\mathrm{T}} \big(\hat{y}_{\mathcal{J}}(t) - y(t)\big) = (F_{\mathcal{J}}^{\perp})^{\mathrm{T}} \big(C\hat{x}_{\mathcal{J}}(t) - Cx(t) - Ff_s(t)\big) = (F_{\mathcal{J}}^{\perp})^{\mathrm{T}} C \big(\hat{x}_{\mathcal{J}}(t) - x(t)\big). \tag{21}$$

Combining (1), (20) and (21), one can obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(\hat{x}_{\mathcal{J}}(t) - x(t)\big) = \big(A + L_{\mathcal{J}}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}} C\big)\big(\hat{x}_{\mathcal{J}}(t) - x(t)\big). \tag{22}$$

As the system (1) is assumed to be detectable against sensor faults, it can be seen from Theorem 1 that the pair $(A, (F_{\mathcal{J}}^{\perp})^{\mathrm{T}} C)$ is detectable. Hence, an observer matrix $L_{\mathcal{J}}$ can be chosen such that $A + L_{\mathcal{J}}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}} C$ is Hurwitz, and further we have $\lim_{t\rightarrow\infty} \big(\hat{x}_{\mathcal{J}}(t) - x(t)\big) = 0$. Thus, the proof is completed.

According to Theorem 2, the state can be estimated asymptotically according to the observer (20) if the fault mode that the system operates in belongs to the set $\mathcal{J}$. Unfortunately, the aforementioned condition cannot be confirmed a priori. This is to say, the state estimation cannot be got based on only one $\hat{x}_{\mathcal{J}}(t)$. To overcome the difficulty, a switched observer is proposed with the help of the maximin strategy.

For each subset $\mathcal{N} \subseteq \{1, \ldots, \ell\}$ satisfying $\mathrm{card}(\mathcal{N}) = 1$, let $\pi_{\mathcal{N}}(t)$ denote the largest deviation between the estimation $\hat{x}_{\mathcal{N}}(t)$ and any estimation $\hat{x}_{\mathcal{O}}(t)$, where $\mathcal{O}$ is a set which satisfies $\mathcal{N} \subseteq \mathcal{O} \subseteq \{1, \ldots, \ell\}$ and $\mathrm{card}(\mathcal{O}) = 2$:

$$\pi_{\mathcal{N}}(t) = \max_{\{1,\ldots,\ell\}\supseteq\mathcal{O}\supseteq\mathcal{N},\, \mathrm{card}(\mathcal{O})=2} \big\|\big(\hat{x}_{\mathcal{N}}(t) - \hat{x}_{\mathcal{O}}(t)\big)\big\|. \tag{23}$$

If $F = F_{\mathcal{N}}$, one has $\hat{x}_{\mathcal{N}}(t) \rightarrow 0$ and $\hat{x}_{\mathcal{O}}(t) \rightarrow 0$ as $t \rightarrow \infty$. That is, $\hat{x}_{\mathcal{N}}(t)$ and all $\hat{x}_{\mathcal{O}}(t)$ provide the effective estimations for the system state $x(t)$. In other words, $\pi_{\mathcal{N}}(t)$ is small, which motivates the following scheme for obtaining state estimation:

$$\hat{x}(t) = \hat{x}_{\vartheta(t)}(t),\ \vartheta(t) \in \operatorname*{arg\,min}_{\mathcal{N}\subseteq\{1,\ldots,\ell\},\, \mathrm{card}(\mathcal{N})=1} \pi_{\mathcal{N}}(t). \tag{24}$$

One can see that the switched observer (24) is based on a bank of $\frac{\ell(\ell+1)}{2}$ Luenberger-like observers (20). In the sequel, we give the following result on the performance of the proposed switched observer.

Theorem 3. Consider the system (1) with $\ell \geqslant 2$, which is detectable against the sensor faults (2). Then, the switched observer (24) can be implemented to asymptotically estimate the system state.

Proof. Without loss of generality, one can assume that the system (1) operates in the i-th mode, i.e., $(F, f_s(t)) = (F_i, f_{si}(t))$, but does not know the value of i. Let $\mathcal{I} = \{i\}$. Clearly, $\mathrm{card}(\mathcal{I}) = 1$.

According to Theorem 2, it follows readily that

$$\begin{aligned} \lim_{t\rightarrow\infty} \pi_{\mathcal{I}}(t) &= \lim_{t\rightarrow\infty} \left(\max_{\{1,\ldots,\ell\}\supseteq\mathcal{O}\supseteq\mathcal{I},\, \mathrm{card}(\mathcal{O})=2} \|\hat{x}_{\mathcal{I}}(t) - \hat{x}_{\mathcal{O}}(t)\| \right) \\ &= \max_{\{1,\ldots,\ell\}\supseteq\mathcal{O}\supseteq\mathcal{I},\, \mathrm{card}(\mathcal{O})=2} \left\| \lim_{t\rightarrow\infty} \big(\hat{x}_{\mathcal{I}}(t) - \hat{x}_{\mathcal{O}}(t)\big) \right\| = 0. \end{aligned} \tag{25}$$

Recall from (24), and then one has $\pi_{\vartheta(t)}(t) \leqslant \pi_{\mathcal{I}}(t)$. Observer that for the set $\vartheta(t)$, there is at least one set $\mathcal{P}(t)$ satisfying $\mathcal{I} \subseteq \mathcal{P}(t)$, $\{1, \ldots, \ell\} \supseteq \mathcal{P}(t) \supseteq \vartheta(t)$ and $\mathrm{card}(\mathcal{P}(t)) = 2$. More specifically, $\mathcal{P}(t)$ can be chosen as $\mathcal{P}(t) = \vartheta(t) \cup \mathcal{I} \cup \mathcal{Z}(t)$, where $\mathcal{Z}(t)$ is any set such that $\mathcal{Z}(t) \subseteq \{1, \ldots, \ell\} \setminus (\vartheta(t) \cup \mathcal{I})$ and $\mathrm{card}(\mathcal{Z}(t)) = 2 - \mathrm{card}(\vartheta(t) \cup \mathcal{I})$. Furthermore, with such choice of $\mathcal{P}(t)$, one has

$$\big\|\hat{x}_{\vartheta(t)}(t) - x(t)\big\| \leqslant \big\|\hat{x}_{\vartheta(t)}(t) - \hat{x}_{\mathcal{P}(t)}(t)\big\| + \big\|\hat{x}_{\mathcal{P}(t)}(t) - x(t)\big\|. \tag{26}$$

Note that

$$\pi_{\vartheta(t)}(t) = \max_{\{1,\ldots,\ell\}\supseteq\mathcal{O}\supseteq\vartheta(t),\,\mathrm{card}(\mathcal{O})=2} \left\|\hat{x}_{\vartheta(t)}(t) - \hat{x}_{\mathcal{O}}(t)\right\|. \tag{27}$$

Thus, from (27) and the facts that $\{1,\ldots,\ell\} \supseteq \mathcal{P}(t) \supseteq \vartheta(t)$ and $\mathrm{card}(\mathcal{P}(t)) = 2$, we have

$$\left\|\hat{x}_{\vartheta(t)}(t) - \hat{x}_{\mathcal{P}(t)}(t)\right\| \leqslant \pi_{\vartheta(t)}(t) \leqslant \pi_{\mathcal{I}}(t). \tag{28}$$

Furthermore, from the definition of $\mathcal{P}(t)$, one can get that $\mathcal{I} \subseteq \mathcal{P}(t) \subseteq \{1,\ldots,\ell\}$ and $\mathrm{card}(\mathcal{P}(t)) = 2$. It can be further seen from Theorem 2 that

$$\begin{aligned} \lim_{t\to\infty} \left\|\hat{x}_{\mathcal{P}(t)}(t) - x(t)\right\| &\leqslant \lim_{t\to\infty} \left(\max_{\mathcal{I}\subseteq\mathcal{Q}\subseteq\{1,\ldots,\ell\},\,\mathrm{card}(\mathcal{Q})=2} \|\hat{x}_{\mathcal{Q}}(t) - x(t)\| \right) \\ &= \max_{\mathcal{I}\subseteq\mathcal{Q}\subseteq\{1,\ldots,\ell\},\,\mathrm{card}(\mathcal{Q})=2} \left\| \lim_{t\to\infty} (\hat{x}_{\mathcal{Q}}(t) - x(t)) \right\| = 0. \end{aligned} \tag{29}$$

Combining (25), (26), (28) and (29), one has $\lim_{t\to\infty} \|\hat{x}_{\vartheta(t)}(t) - x(t)\| = 0$, which completes the proof.

4 Another observer approach for system only with sensor faults

The observer design problem for the system with sensor faults has been well solved in Section 3. However, the observer (24) cannot be extended to the case of the system with actuator faults. In this section, another observer approach is proposed, which is extended to the case of the system with simultaneous actuator and sensor faults in the next section.

Consider a new state $x_f(t) \in \mathbb{R}^p$, which is a filtered version of $y(t)$, satisfying

$$\dot{x}_f(t) = A_f x_f(t) + y(t), \tag{30}$$

where $A_f \in \mathbb{R}^{p\times p}$ is a Hurwitz matrix. Obviously, Eqs. (1) and (30) can be rewritten as the following augmented system of order $n + p$.

$$\begin{aligned} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}_f(t) \end{bmatrix}}_{\dot{x}_a(t)} &= \underbrace{\begin{bmatrix} A & 0 \\ C & A_f \end{bmatrix}}_{A^a} \underbrace{\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}}_{x_a(t)} + \underbrace{\begin{bmatrix} B \\ D \end{bmatrix}}_{B^a} u(t) + \underbrace{\begin{bmatrix} 0 \\ F \end{bmatrix}}_{F^a} f_s(t), \\ \underbrace{\begin{bmatrix} (F^{\perp})^{\mathrm{T}} y(t) \\ x_f(t) \end{bmatrix}}_{y_a(t)} &= \underbrace{\begin{bmatrix} (F^{\perp})^{\mathrm{T}} C & 0 \\ 0 & I \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}}_{x_a(t)} + \underbrace{\begin{bmatrix} (F^{\perp})^{\mathrm{T}} D \\ 0 \end{bmatrix}}_{D^a} u(t). \end{aligned} \tag{31}$$

One should note that, $y_a(t)$ is unavailable because we only know $F \in \{F_1,\ldots,F_\ell\}$, but we do not know which one it is. For simplification purpose, let us define

$$F_{ij}^a = \begin{bmatrix} 0 \\ F_{ij} \end{bmatrix}, \quad C_{ij}^a = \begin{bmatrix} (F_{ij}^{\perp})^{\mathrm{T}} C & 0 \\ 0 & I \end{bmatrix}, \quad D_{ij}^a = \begin{bmatrix} (F_{ij}^{\perp})^{\mathrm{T}} D \\ 0 \end{bmatrix}. \tag{32}$$

Now, we establish the following lemma, which is helpful to prove our main results.

Lemma 1. The following statements are equivalent:

(a) The system (1) is detectable against the multiple mode sensor faults (2);

(b) The invariant zeros of $(A^a, F_{ij}^a, C_{ij}^a)$ lie in the left half plane for all $i, j \in \{1,\ldots,\ell\}$; i.e., for all s with non-negative real part,

$$\mathrm{rank} \begin{bmatrix} sI_{n+p} - A^a & -F_{ij}^a \\ C_{ij}^a & 0 \end{bmatrix} = (n+p) + \mathrm{rank}(F_{ij}^a), \quad \forall i, j \in \{1,\ldots,\ell\}. \tag{33}$$

Proof. It can be obtained from Theorem 1 that the condition (a) is equivalent to the following condition:

(a)′ The pair $(A,(F_{ij}^{\perp})^{\mathrm{T}}C)$ is detectable for all $i,j\in\{1,\ldots,\ell\}$.

Hence, to prove this theorem, we just need to prove the equivalence between (a)′ and (b). By performing elementary column and row operations, one can obtain that

$$\begin{aligned}
\operatorname{rank}\begin{bmatrix} sI_{n+p}-A^a & -F_{ij}^a \\ C_{ij}^a & 0\end{bmatrix} &= \operatorname{rank}\begin{bmatrix} sI_n-A & 0 & 0 \\ -C & sI_p-A_f & -F_{ij} \\ (F_{ij}^{\perp})^{\mathrm{T}}C & 0 & 0 \\ 0 & I_p & 0\end{bmatrix} \overset{\text{(e1)}}{=} \operatorname{rank}\begin{bmatrix} sI_n-A & 0 & 0 \\ -C & 0 & -F_{ij} \\ 0 & I_p & 0\end{bmatrix}\\
&= \operatorname{rank}\begin{bmatrix} sI_n-A & 0 \\ C & F_{ij}\end{bmatrix}+p \overset{\text{(e2)}}{=} \operatorname{rank}\begin{bmatrix} sI_n-A & 0 \\ (F_{ij}^{\perp})^{\mathrm{T}}C & 0 \\ (B_{F_{ij}})^{\mathrm{T}}C & (B_{F_{ij}})^{\mathrm{T}}F_{ij}\end{bmatrix}+p\\
&\overset{\text{(e3)}}{=} \operatorname{rank}\begin{bmatrix} sI_n-A & 0 \\ (F_{ij}^{\perp})^{\mathrm{T}}C & 0 \\ 0 & (B_{F_{ij}})^{\mathrm{T}}F_{ij}\end{bmatrix}+p = \operatorname{rank}\begin{bmatrix} sI_n-A \\ (F_{ij}^{\perp})^{\mathrm{T}}C\end{bmatrix}+p+\operatorname{rank}(F_{ij}),
\end{aligned}\tag{34}$$

where (e1), (e2) and (e3) are deduced from the facts that $(F_{ij}^{\perp})^{\mathrm{T}}F_{ij}=0$, either $[F_{ij}^{\perp},B_{F_{ij}}]$ or $B_{F_{ij}}$ is invertible, and $(B_{F_{ij}})^{\mathrm{T}}F_{ij}$ is of full row rank. One can see from (34) that (b) is equivalent to that

$$\operatorname{rank}\begin{bmatrix} sI_n-A \\ (F_{ij}^{\perp})^{\mathrm{T}}C\end{bmatrix}=n,\quad \forall i,j\in\{1,\ldots,\ell\}\tag{35}$$

holds for all s with non-negative real part, which is equivalent to that the pair $(A,(F_{ij}^{\perp})^{\mathrm{T}}C)$ is detectable for all $i,j\in\{1,\ldots,\ell\}$. Hence, the proof is completed.

According to Lemma 1, if the system (1) is detectable against the faults (2), then the invariant zeros of (A^a,F_{ij}^a,C_{ij}^a) are stable for all $i,j\in\{1,\ldots,\ell\}$. Note that, $\operatorname{rank}(C_{ij}^aF_{ij}^a)=\operatorname{rank}(F_{ij}^a)$ is always satisfied. From [29], $\operatorname{rank}(C_{ij}^aF_{ij}^a)=\operatorname{rank}(F_{ij}^a)$ and the invariant zeros of the system model given by the triple (A^a,F_{ij}^a,C_{ij}^a) are stable if and only if there exist $P_{ij}^a>0$, $Q_{ij}^a>0$, L_{ij}^a and R_{ij}^a such that

$$He\big(P_{ij}^a(A^a+L_{ij}^aC_{ij}^a)\big)=-Q_{ij}^a \text{ and } R_{ij}^aC_{ij}^a=(F_{ij}^a)^{\mathrm{T}}P_{ij}^a.\tag{36}$$

Let $C_{\mathcal{J}}^a$, $D_{\mathcal{J}}^a$, $F_{\mathcal{J}}^a$, $P_{\mathcal{J}}^a$, $Q_{\mathcal{J}}^a$, $L_{\mathcal{J}}^a$ and $R_{\mathcal{J}}^a$ be constructed in a same way as $F_{\mathcal{J}}$. Thus, Eq. (36) can be rewritten as

$$He\big(P_{\mathcal{J}}^a(A^a+L_{\mathcal{J}}^aC_{\mathcal{J}}^a)\big)=-Q_{\mathcal{J}}^a \text{ and } R_{\mathcal{J}}^aC_{\mathcal{J}}^a=(F_{\mathcal{J}}^a)^{\mathrm{T}}P_{\mathcal{J}}^a.\tag{37}$$

Inspired by (37), a sliding-mode observer is proposed for the set $\mathcal{J}\subseteq\{1,\ldots,\ell\}$ and $1\leqslant\operatorname{card}(\mathcal{J})\leqslant 2$ as follows:

$$\begin{aligned}
\dot{\hat{x}}_{a\mathcal{J}}(t) &= A^a\hat{x}_{a\mathcal{J}}(t)+B^au(t)+L_{\mathcal{J}}^a\big(\hat{y}_{a\mathcal{J}}(t)-y_{a\mathcal{J}}(t)\big)\\
&\quad+\hat{\rho}_{\mathcal{J}}(t)(P_{\mathcal{J}}^a)^{-1}(C_{\mathcal{J}}^a)^{\mathrm{T}}(R_{\mathcal{J}}^a)^{\mathrm{T}}\Gamma\left(R_{\mathcal{J}}^a\big(y_{a\mathcal{J}}(t)-\hat{y}_{a\mathcal{J}}(t)\big)\right),\\
\hat{y}_{a\mathcal{J}}(t) &= C_{\mathcal{J}}^a\hat{x}_{a\mathcal{J}}(t)+D_{\mathcal{J}}^au(t),\\
y_{a\mathcal{J}}(t) &= \begin{bmatrix}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}}y(t)\\ x_f(t)\end{bmatrix}=C_{\mathcal{J}}^ax_a(t)+D_{\mathcal{J}}^au(t)+\begin{bmatrix}(F_{\mathcal{J}}^{\perp})^{\mathrm{T}}F\\ 0\end{bmatrix}f_s(t),
\end{aligned}\tag{38}$$

where

$$\Gamma\left(R_{\mathcal{J}}^a\big(y_{a\mathcal{J}}(t)-\hat{y}_{a\mathcal{J}}(t)\big)\right)=\begin{cases}\dfrac{R_{\mathcal{J}}^a\big(y_{a\mathcal{J}}(t)-\hat{y}_{a\mathcal{J}}(t)\big)}{\big\|R_{\mathcal{J}}^a\big(y_{a\mathcal{J}}(t)-\hat{y}_{a\mathcal{J}}(t)\big)\big\|}, & \text{if } \big\|R_{\mathcal{J}}^a\big(y_{a\mathcal{J}}(t)-\hat{y}_{a\mathcal{J}}(t)\big)\big\|\neq 0,\\ 0, & \text{otherwise.}\end{cases}\tag{39}$$

Solutions of (38) need to be understood in a Filippov sense [30]. In (38), $\hat{\rho}_{\mathcal{J}}(t)$ is the estimation of the unknown constant $\rho \triangleq \sup_{t\geqslant 0}\|f_s(t)\|$, and it is tuned as

$$\dot{\hat{\rho}}_{\mathcal{J}}(t) = -r_{\mathcal{J}}\sigma_{\mathcal{J}}(t)\hat{\rho}_{\mathcal{J}}(t) + 2r_{\mathcal{J}}\left\|R^a_{\mathcal{J}}\big(y_{a\mathcal{J}}(t) - \hat{y}_{a\mathcal{J}}(t)\big)\right\|, \tag{40}$$

where $r_{\mathcal{J}}$ denotes any positive constant and the function $\sigma_{\mathcal{J}}(t)$ represents the positive uniform continuous function satisfying

$$\lim_{t\to\infty}\int_0^t \sigma_{\mathcal{J}}(\tau)\mathrm{d}\tau \leqslant \bar{\sigma}_{\mathcal{J}} < \infty \tag{41}$$

with any given positive constant $\bar{\sigma}_{\mathcal{J}}$. Define $\hat{\rho}_{\mathcal{J}}(0)$ to be positive, and thus $\hat{\rho}_{\mathcal{J}}(t)$ is always positive. One can check that $\sigma_{\mathcal{J}}(t)$ can be chosen as $\sigma_{\mathcal{J}}(t) = \theta_{\mathcal{J}_1}\mathrm{e}^{-\theta_{\mathcal{J}_2}t}$ with $\theta_{\mathcal{J}_1}/\theta_{\mathcal{J}_2} \leqslant \bar{\sigma}_{\mathcal{J}}$ and $\theta_{\mathcal{J}_1}, \theta_{\mathcal{J}_2} > 0$.

Remark 7. Compared with the sliding-mode observer proposed in [11], the upper bound of the faults is not required to be known beforehand in (38). Furthermore, a drawback of the aforementioned result is that only single fault mode can be addressed, while there is not such a constraint in our approach.

Let $\tilde{x}_{a\mathcal{J}}(t) = \hat{x}_{a\mathcal{J}}(t) - x_a(t)$ and $\tilde{\rho}_{\mathcal{J}}(t) = \hat{\rho}_{\mathcal{J}}(t) - \rho$, and then one can get the following error system:

$$\begin{aligned}\dot{\tilde{x}}_{a\mathcal{J}}(t) = {} & \big(A^a + L^a_{\mathcal{J}}C^a_{\mathcal{J}}\big)\tilde{x}_{a\mathcal{J}}(t) - F^a f_s(t) - L^a_{\mathcal{J}}\big[F^{\mathrm{T}}F^{\perp}_{\mathcal{J}}, 0\big]^{\mathrm{T}} f_s(t) - \hat{\rho}_{\mathcal{J}}(t)(P^a_{\mathcal{J}})^{-1}(C^a_{\mathcal{J}})^{\mathrm{T}}(R^a_{\mathcal{J}})^{\mathrm{T}} \\ & \times \Gamma\left(R^a_{\mathcal{J}}\left(C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t) - \big[F^{\mathrm{T}}F^{\perp}_{\mathcal{J}}, 0\big]^{\mathrm{T}} f_s(t)\right)\right),\end{aligned} \tag{42}$$

$$\dot{\tilde{\rho}}_{\mathcal{J}}(t) = -r_{\mathcal{J}}\sigma_{\mathcal{J}}(t)\tilde{\rho}_{\mathcal{J}}(t) - r_{\mathcal{J}}\sigma_{\mathcal{J}}(t)\rho + 2r_{\mathcal{J}}\left\|R^a_{\mathcal{J}}\left(C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t) - \big[F^{\mathrm{T}}F^{\perp}_{\mathcal{J}}, 0\big]^{\mathrm{T}} f_s(t)\right)\right\|. \tag{43}$$

Similar to Section 3, we only consider the case that $\ell \geqslant 2$ in the sequel.

Theorem 4. Consider the original system (1) and the augmented system (31) with $\ell \geqslant 2$, and the observer (38)–(41) with $\mathcal{J} \subseteq \{1, \ldots, \ell\}$ and $1 \leqslant \mathrm{card}(\mathcal{J}) \leqslant 2$. Assume that the original system is detectable against the multiple mode sensor faults (2). If the fault mode that the system operates in belongs to the set $\mathcal{J}$, then there always exist observer parameters $L^a_{\mathcal{J}}$, $P^a_{\mathcal{J}}$ and $R^a_{\mathcal{J}}$ such that all signals in the resulting closed-loop system are uniformly ultimately bounded and $\hat{x}_{a\mathcal{J}}(t) \to x_a(t)$ as $t \to \infty$.

Proof. From Lemma 1 and the analysis below it, one can see that there always exist $P^a_{\mathcal{J}} > 0$, $Q^a_{\mathcal{J}} > 0$, $L^a_{\mathcal{J}}$ and $R^a_{\mathcal{J}}$ such that

$$He\big(P^a_{\mathcal{J}}(A^a + L^a_{\mathcal{J}}C^a_{\mathcal{J}})\big) = -Q^a_{\mathcal{J}} \text{ and } R^a_{\mathcal{J}}C^a_{\mathcal{J}} = (F^a_{\mathcal{J}})^{\mathrm{T}}P^a_{\mathcal{J}}, \tag{44}$$

if and only if the system (1) is detectable against the sensor faults (2). With such choices of $P^a_{\mathcal{J}}$, $Q^a_{\mathcal{J}}$, $L^a_{\mathcal{J}}$, and $R^a_{\mathcal{J}}$, we further consider the following Lyapunov function for the error system (42) and (43):

$$V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(t), \tilde{\rho}_{\mathcal{J}}(t)) = \tilde{x}^{\mathrm{T}}_{a\mathcal{J}}(t)P^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t) + \frac{1}{2r_{\mathcal{J}}}\tilde{\rho}^{\mathrm{T}}_{\mathcal{J}}(t)\tilde{\rho}_{\mathcal{J}}(t). \tag{45}$$

Without loss of generality, one can assume that the system operates in the i-th mode, i.e., $(F, f_s(t)) = (F_i, f_{si}(t))$, but does not know the value of i. From the assumption of the theorem, one has $F = F_i$ and $\{i\} \subseteq \mathcal{J} \subseteq \{1, \ldots, \ell\}$. One can check that $(F^{\perp}_{\mathcal{J}})^{\mathrm{T}}F = (F^{\perp}_{\mathcal{J}})^{\mathrm{T}}F_i = 0$, and (42) and (43) can be rewritten as follows:

$$\dot{\tilde{x}}_{a\mathcal{J}}(t) = \big(A^a + L^a_{\mathcal{J}}C^a_{\mathcal{J}}\big)\tilde{x}_{a\mathcal{J}}(t) - F^a f_s(t) - \hat{\rho}_{\mathcal{J}}(t)(P^a_{\mathcal{J}})^{-1}(C^a_{\mathcal{J}})^{\mathrm{T}}(R^a_{\mathcal{J}})^{\mathrm{T}}\Gamma\left(R^a_{\mathcal{J}}C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t)\right), \tag{46}$$

$$\dot{\tilde{\rho}}_{\mathcal{J}}(t) = -r_{\mathcal{J}}\sigma_{\mathcal{J}}(t)\tilde{\rho}_{\mathcal{J}}(t) - r_{\mathcal{J}}\sigma_{\mathcal{J}}(t)\rho + 2r_{\mathcal{J}}\left\|R^a_{\mathcal{J}}C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t)\right\|. \tag{47}$$

Taking the derivatives of $V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(t), \tilde{\rho}_{\mathcal{J}}(t))$ along the trajectory of (46) and (47), together with (44), $\rho = \sup_{t\geqslant 0}\|f_s(t)\|$ and $-\tilde{\rho}^2_{\mathcal{J}}(t) - \rho\tilde{\rho}_{\mathcal{J}}(t) \leqslant \rho^2/4$ yields

$$\begin{aligned}\dot{V}_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(t), \tilde{\rho}_{\mathcal{J}}(t)) \leqslant {} & \tilde{x}^{\mathrm{T}}_{a\mathcal{J}}(t)He\big(P^a_{\mathcal{J}}A^a + P^a_{\mathcal{J}}L^a_{\mathcal{J}}C^a_{\mathcal{J}}\big)\tilde{x}_{a\mathcal{J}}(t) - 2\tilde{x}^{\mathrm{T}}_{a\mathcal{J}}(t)P^a_{\mathcal{J}}F^a f_s(t) - 2\hat{\rho}_{\mathcal{J}}(t)\left\|R^a_{\mathcal{J}}C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t)\right\| \\ & + \frac{\rho^2}{4}\sigma_{\mathcal{J}}(t) + 2\tilde{\rho}_{\mathcal{J}}(t)\left\|R^a_{\mathcal{J}}C^a_{\mathcal{J}}\tilde{x}_{a\mathcal{J}}(t)\right\|\end{aligned}$$

$$\leqslant \tilde{x}_{a\mathcal{J}}^{\rm T}(t)He\big(P_{\mathcal{J}}^aA^a+P_{\mathcal{J}}^aL_{\mathcal{J}}^aC_{\mathcal{J}}^a\big)\tilde{x}_{a\mathcal{J}}(t)+2\rho\left\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\right\|-2\hat{\rho}_{\mathcal{J}}(t)\left\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\right\| + \frac{\rho^2}{4}\sigma_{\mathcal{J}}(t)+2\tilde{\rho}_{\mathcal{J}}(t)\left\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\right\|$$
$$= -\tilde{x}_{a\mathcal{J}}^{\rm T}(t)Q_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)+\frac{\rho^2}{4}\sigma_{\mathcal{J}}(t), \tag{48}$$

where the second inequality is derived from the facts that (a) $-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF^af_s(t)=-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF_{\mathcal{J}}^af_s(t) \leqslant 2\rho\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\|$ if $\{i\}=\mathcal{J}$; (b) $-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF^af_s(t)=-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF_{\mathcal{J}}^a[f_s^{\rm T}(t),0]^{\rm T}\leqslant 2\rho\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\|$ if $\{i\}\subsetneqq\mathcal{J}, i=\min_{\zeta\in\mathcal{J}}\zeta$; and (c) $-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF^af_s(t)=-2\tilde{x}_{a\mathcal{J}}^{\rm T}(t)P_{\mathcal{J}}^aF_{\mathcal{J}}^a[0,f_s^{\rm T}(t)]^{\rm T}\leqslant 2\rho\|R_{\mathcal{J}}^aC_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(t)\|$ if $\{i\}\subsetneqq\mathcal{J}, i=\max_{\zeta\in\mathcal{J}}\zeta$.

Integrating (48) over $[0,t]$ yields

$$0\leqslant\int_0^t\tilde{x}_{a\mathcal{J}}^{\rm T}(\tau)Q_{\mathcal{J}}^a\tilde{x}_{a\mathcal{J}}(\tau){\rm d}\tau\leqslant V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(0),\tilde{\rho}_{\mathcal{J}}(0))-V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(t),\tilde{\rho}_{\mathcal{J}}(t))+\frac{\rho^2}{4}\int_0^t\sigma_{\mathcal{J}}(\tau){\rm d}\tau$$
$$\leqslant V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(0),\tilde{\rho}_{\mathcal{J}}(0))+\frac{\rho^2}{4}\int_0^t\sigma_{\mathcal{J}}(\tau){\rm d}\tau, \tag{49}$$

which implies that

$$V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(t),\tilde{\rho}_{\mathcal{J}}(t))\leqslant V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(0),\tilde{\rho}_{\mathcal{J}}(0))+\frac{\rho^2}{4}\bar{\sigma}_{\mathcal{J}}, \tag{50}$$

and

$$\lambda_{\min}(Q_{\mathcal{J}}^a)\int_0^\infty\left\|\tilde{x}_{a\mathcal{J}}(\tau)\right\|^2{\rm d}\tau\leqslant V_{\mathcal{J}}(\tilde{x}_{a\mathcal{J}}(0),\tilde{\rho}_{\mathcal{J}}(0))+\frac{\rho^2}{4}\bar{\sigma}_{\mathcal{J}}. \tag{51}$$

From (50), the solutions $(\tilde{x}_{a\mathcal{J}}(t),\tilde{\rho}_{\mathcal{J}}(t))$ are uniformly bounded, and then we have that $\tilde{x}_{a\mathcal{J}}(t)$ is uniformly continuous. According to (51) and Barbalat Lemma [31], one can obtain $\lim_{t\to\infty}\|\tilde{x}_{a\mathcal{J}}(t)\|=0$. Thus, the proof is completed.

Analogous to Subsection 3.2, for each subset $\mathcal{N}\subseteq\{1,\ldots,\ell\}$ satisfying ${\rm card}(\mathcal{N})=1$, let $\pi_{a\mathcal{N}}(t)$ denote the largest deviation between $\hat{x}_{a\mathcal{N}}(t)$ and $\hat{x}_{a\mathcal{O}}(t)$, where $\mathcal{O}$ is a set which satisfies $\mathcal{N}\subseteq\mathcal{O}\subseteq\{1,\ldots,\ell\}$ and ${\rm card}(\mathcal{O})=2$:

$$\pi_{a\mathcal{N}}(t)=\max_{\{1,\ldots,\ell\}\supseteq\mathcal{O}\supseteq\mathcal{N},\,{\rm card}(\mathcal{O})=2}\left\|\big(\hat{x}_{a\mathcal{N}}(t)-\hat{x}_{a\mathcal{O}}(t)\big)\right\|. \tag{52}$$

If $F=F_{\mathcal{N}}$, one can see from Theorem 4 that $\hat{x}_{a\mathcal{N}}(t)\to 0$ and $\hat{x}_{a\mathcal{O}}(t)\to 0$ as $t\to\infty$. That is, $\hat{x}_{a\mathcal{N}}(t)$ and all $\hat{x}_{a\mathcal{O}}(t)$ provide the effective estimations for $x_a(t)$, which motivates the following scheme for obtaining the state estimation:

$$\hat{x}_a(t)=\hat{x}_{a\vartheta(t)}(t),\quad \vartheta(t)\in\mathop{\arg\min}_{\mathcal{N}\subseteq\{1,\ldots,\ell\},\,{\rm card}(\mathcal{N})=1}\pi_{a\mathcal{N}}(t). \tag{53}$$

One can see that the switched observer (53) is based on a bank of $\frac{\ell(\ell+1)}{2}$ sliding-mode observers (38).

Next, we will give the following result on the performance of the above switched observer.

Theorem 5. Consider the original system (1) and the augmented system (31) with $\ell\geqslant 2$. Assume that the system (1) is detectable against the multiple mode sensor faults (2). Then, the switched observer (53) can be implemented to asymptotically estimate the augmented system state $x_a(t)$.

Proof. Analogous to the proof of Theorem 3.

5 Extensions for the system with simultaneous sensor and actuator faults

In this section, the approach in Section 4 is extended to the system subject to both the actuator faults and sensor faults, which is described as follows:

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)+Ef_a(t),\\ y(t)&=Cx(t)+Du(t)+Ff_s(t),\end{aligned} \tag{54}$$

where $(E, F, f_a(t), f_s(t)) \in \{(E_1, F_1, f_{a1}(t), f_{s1}(t)), \ldots, (E_\ell, F_\ell, f_{s\ell}(t), f_{a\ell}(t))\}$, and $(E_i, F_i, f_{ai}(t), f_{si}(t))$ corresponds to the i-th fault mode for $i = 1, \ldots, \ell$. The positive integer $\ell \geqslant 1$ denotes the number of the total possible fault modes. For the i-th fault mode, $E_i \in \mathbb{R}^{n\times v_i}$, $F_i \in \mathbb{R}^{p\times q_i}$ are the fault distribution matrices, $f_{ai}(t) \in \mathbb{R}^{v_i}$ denotes the unknown actuator fault signal, and $f_{si}(t) \in \mathbb{R}^{q_i}$ represents the unknown sensor fault signal. $f_{ai}(t)$ and $f_{si}(t)$ are assumed to be piecewise continuous and bounded.

Analogous to Section 4, consider a new state $x_f(t) \in \mathbb{R}^p$, which satisfies

$$\dot{x}_f(t) = A_f x_f(t) + y(t), \tag{55}$$

where $A_f \in \mathbb{R}^{p\times p}$ is a Hurwitz matrix. Eqs. (54) and (55) can be rewritten as the following augmented system:

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}_f(t) \end{bmatrix}}_{\dot{x}_a(t)} = \underbrace{\begin{bmatrix} A & 0 \\ C & A_f \end{bmatrix}}_{A^a} \underbrace{\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}}_{x_a(t)} + \underbrace{\begin{bmatrix} B \\ D \end{bmatrix}}_{B^a} u(t) + \underbrace{\begin{bmatrix} E & 0 \\ 0 & F \end{bmatrix}}_{F^{\rm new}} \underbrace{\begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}}_{f^{\rm new}(t)},$$

$$\underbrace{\begin{bmatrix} (F^\perp)^{\rm T} y(t) \\ x_f(t) \end{bmatrix}}_{y_a(t)} = \underbrace{\begin{bmatrix} (F^\perp)^{\rm T} C & 0 \\ 0 & I \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}}_{x_a(t)} + \underbrace{\begin{bmatrix} (F^\perp)^{\rm T} D \\ 0 \end{bmatrix}}_{D^a} u(t). \tag{56}$$

Eq. (56) is in the form of (31). As described in Section 4, a switched observer can be designed by replacing F^a with $F^{\rm new}$ for the system (56). Because the switched observer follows the same scheme as the one in Section 4, we just point out the existence conditions for the switched observer with respect to the system (56):

(i) The invariant zeros of $(A, E_{ij}, (F_{ij}^\perp)^{\rm T} C)$ lie in the left half plane for all $i, j \in \{1, \ldots, \ell\}$, where $E_{ij} \triangleq [E_i, E_j]$;

(ii) $\mathrm{rank}((F_{ij}^\perp)^{\rm T} C E_{ij}) = \mathrm{rank}(E_{ij})$ is satisfied for all $i, j \in \{1, \ldots, \ell\}$.

Remark 8. If the fault signals are large with rapid change, the conventional observer may results in that the estimation error evolves away from the origin. Therefore, the conventional robust observer cannot guarantee a desirable estimation performance against severe adversarial attacks. Nevertheless, the proposed approach can be used to asymptotically estimate the system state, regardless of the magnitude and the change of the attack signals.

6 Simulation studies

In this section, a simulation example of a reduced-order aircraft system is given to show the effectiveness of the proposed approaches.

Example 1. The effectiveness of the switched observer will be verified in a linearized reduced-order aircraft system borrowed from [32]. The system has the form as (54), where $x(t) = [\beta(t), p(t), \gamma(t)]^{\rm T}$ and $u(t) = [\delta_{\rm DT}, \delta_{\rm AI}, \delta_{\rm RU}, \delta_{\rm RTV}, \delta_{\rm YTV}]$. The physical meanings of $\beta(t)$, $p(t)$, $\gamma(t)$, $\delta_{\rm DT}$, $\delta_{\rm AI}$, $\delta_{\rm RU}$, $\delta_{\rm RTV}$ and $\delta_{\rm YTV}$ can be found in [32].

In this example, the attack angle, Mach number, and altitude are selected as 29.73°, 0.2, and 10000ft, respectively. Then we have

$$A = \begin{bmatrix} -0.0590 & 0.4960 & -0.8680 \\ -5.5130 & -0.9390 & 0.6650 \\ 0.0680 & 0.0260 & -0.1040 \end{bmatrix},\ C = I_3,\ D = 0_{2\times 5},\ B = \begin{bmatrix} 0.0060 & 0.0060 & 0.0040 & 0 & 0.0900 \\ 1.8790 & 1.3280 & 0.0290 & 0.6750 & 0.2170 \\ -0.1090 & -0.0960 & -0.0840 & 0.0070 & -2.9740 \end{bmatrix}.$$

Here, the following three possible fault modes are considered:

- Fault mode 1. $E_1 = [0, 1, 0]^{\rm T}$, $F_1 = [1, 1, 0]^{\rm T}$, $f_{a1} = 3 + 3\sin(0.1t)$ and $f_{s1} = 3 + 3\sin(0.2t)$;

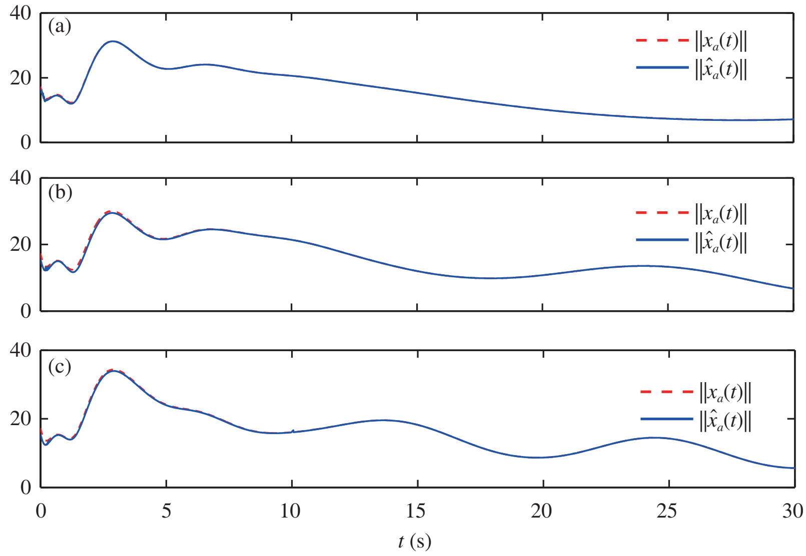

Figure 1 (Color online) $\|x_a(t)\|$ and its estimation $\|\hat{x}_a(t)\|$ for (a) $(E,F)=(E_1,F_1)$, (b) $(E,F)=(E_2,F_2)$, and (c) $(E,F)=(E_3,F_3)$, respectively.

- Fault mode 2. $E_2=[0,1,0]^{\mathrm{T}}$, $F_2=[0,1,1]^{\mathrm{T}}$, $f_{a2}=3-3\cos(0.3t)$ and $f_{s2}=3-3\cos(0.4t)$;
- Fault mode 3. $E_3=[0,1,0]^{\mathrm{T}}$, $F_3=[1,1,1]^{\mathrm{T}}$, $f_{a3}=3+3\sin(0.5t)$ and $f_{s3}=3+3\sin(0.6t)$.

Through some validations, the conditions (i) and (ii) below (56) are satisfied. Thus, a switched observer can be reconstructed. By applying the theory developed in this paper, the observer parameters $R^a_{\mathcal{J}}$, $P^a_{\mathcal{J}}$, $L^a_{\mathcal{J}}$ and $F^{\perp}_{\mathcal{J}}$ for all $\mathcal{J}\subseteq\{1,2,3\}$ satisfying $1\leqslant\mathrm{card}(\mathcal{J})\leqslant 2$ can be obtained, e.g.,

$$P^a_{\{2,3\}}=\begin{bmatrix} 65.2262 & -0.0002 & -44.3730 & 0.0001 & -27.3839 & 27.3838\\ -0.0002 & 1.1653 & -1.1655 & 0.0831 & 0.5905 & -0.6430\\ -44.3730 & -1.1655 & 391.0014 & -0.0830 & 14.9032 & -14.8507\\ 0.0001 & 0.0831 & -0.0830 & 2.7041 & -0.9879 & -0.0342\\ -27.3839 & 0.5905 & 14.9032 & -0.9879 & 34.7703 & -33.3997\\ 27.3838 & -0.6430 & -14.8507 & -0.0342 & -33.3997 & 34.4436 \end{bmatrix},\quad F^{\perp}_{\{2,3\}}=\begin{bmatrix}-0.0000\\ -0.7071\\ 0.7071\end{bmatrix},$$

$$R^a_{\{2,3\}}=\begin{bmatrix} -1.6482 & 0.0831 & 0.5905 & -0.6430\\ -1.6482 & 0.0831 & 0.5905 & -0.6430\\ 0.0742 & -1.0221 & 1.3707 & 1.0440\\ -0.0432 & 1.6820 & 0.3828 & 1.0098 \end{bmatrix},\quad L^a_{\{2,3\}}=\begin{bmatrix} 0.0057 & -0.0550 & -0.0939 & 0.1013\\ 6.1122 & 0.3967 & -1.3044 & 0.7337\\ 0.0221 & -0.0026 & -0.0137 & 0.0118\\ -0.2216 & -2.3886 & -1.0965 & -1.1555\\ 0.0286 & -1.1627 & -2.8674 & -2.8260\\ 0.1164 & -1.0802 & -2.7946 & -2.9127 \end{bmatrix}. \quad (57)$$

For simulation purpose, let $x_a(0)=[x^{\mathrm{T}}(t),x_f^{\mathrm{T}}(t)]^{\mathrm{T}}=[10,10,10,0,0,0]^{\mathrm{T}}$, $\hat{x}_{a\mathcal{J}}(0)=[9,9,9,0,0,0]^{\mathrm{T}}$, $\hat{\rho}_{\mathcal{J}}(0)=3$, $r_{\mathcal{J}}=50$, $\sigma_{\mathcal{J}}=\mathrm{e}^{-t}$ and $u(t)=0_{5\times1}$. The system states and their estimations are shown in Figure 1 with $(E,F)=\{(E_1,F_1),(E_2,F_2),(E_3,F_3)\}$, respectively. Also, the observer estimation errors are plotted in Figure 2. It can be seen from Figures 1 and 2 that the system state can be estimated asymptotically.

7 Conclusion

In this paper, the asymptotic state estimation problem for linear systems with multiple mode faults has been studied. For the case of sensor faults, we introduced a new notion, i.e., detectability of system

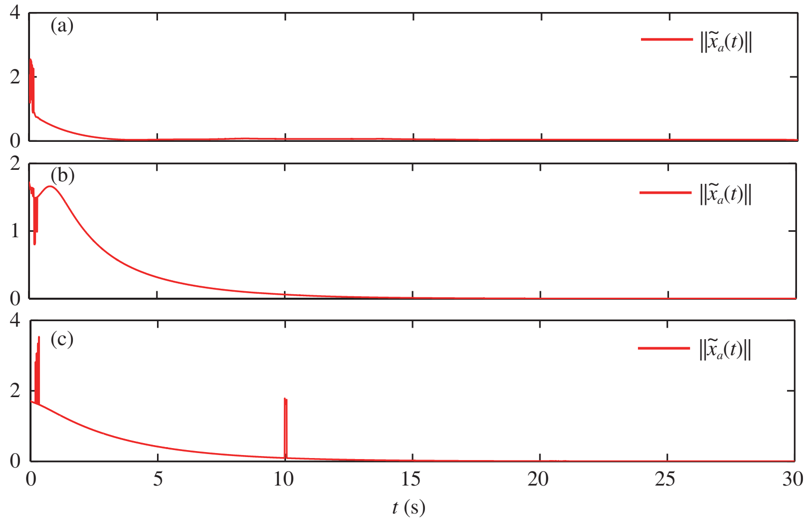

Figure 2 (Color online) $\|\tilde{x}_a(t)\|$ for (a) $(E,F)=(E_1,F_1)$, (b) $(E,F)=(E_2,F_2)$, and (c) $(E,F)=(E_3,F_3)$, respectively.

against sensor faults. A necessary and sufficient condition for the system to be detectable against sensor faults has been given. Two switched observers have been developed for state estimation with the help of maximin strategy. Extensions to the case of sensor and actuator faults have been investigated.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 61733005, 61673172, 61663013, 61803155, 51565012) and Science and Technology Research Project of Jiangxi Education Department (Grant No. 170376).

References

1 Luenberger D. Observers for multivariable systems. IEEE Trans Autom Control, 1966, 11: 190–197
2 Zhou D H, Qin L G, He X, et al. Distributed sensor fault diagnosis for a formation system with unknown constant time delays. Sci China Inf Sci, 2018, 61: 112205
3 Kudva P, Viswanadham N, Ramakrishna A. Observers for linear systems with unknown inputs. IEEE Trans Autom Control, 1980, 25: 113–115
4 Kalsi K, Lian J M, Hui S, et al. Sliding-mode observers for systems with unknown inputs: a high-gain approach. Automatica, 2010, 46: 347–353
5 de Souza C E, Coutinho D, Kinnaert M. Mean square state estimation for sensor networks. Automatica, 2016, 72: 108–114
6 Zheng G, Bejarano F J. Observer design for linear singular time-delay systems. Automatica, 2017, 80: 1–9
7 Shi D W, Chen T W, Darouach M. Event-based state estimation of linear dynamic systems with unknown exogenous inputs. Automatica, 2016, 69: 275–288
8 Park G, Shim H. Guaranteeing almost fault-free tracking performance from transient to steady-state: a disturbance observer approach. Sci China Inf Sci, 2018, 61: 070224
9 Chong M S, Wakaiki M, Hespanha J P. Observability of linear systems under adversarial attacks. In: Processings of American Control Conference, 2015. 2439–2444
10 Edwards C, Spurgeon S K, Patton R J. Sliding mode observers for fault detection and isolation. Automatica, 2000, 36: 541–553
11 Edwards C, Tan C P. Sensor fault tolerant control using sliding mode observers. Control Eng Pract, 2006, 14: 897–908
12 Park T G. Estimation strategies for fault isolation of linear systems with disturbances. IET Control Theory Appl, 2010, 4: 2781–2792
13 Alwi H, Edwards C, Tan C P. Sliding mode estimation schemes for incipient sensor faults. Automatica, 2009, 45: 1679–1685
14 Rafaralahy H, Richard E, Boutayeb M, et al. Sensor diagnosis and state estimation for a class of skew symmetric time-varying systems. Automatica, 2012, 48: 2284–2289
15 Lan J, Patton R J. A new strategy for integration of fault estimation within fault-tolerant control. Automatica, 2016, 69: 48–59
16 Wang X, Tan C P, Zhou D. A novel sliding mode observer for state and fault estimation in systems not satisfying matching and minimum phase conditions. Automatica, 2017, 79: 290–295
17 Gao Z, Ding S X. Actuator fault robust estimation and fault-tolerant control for a class of nonlinear descriptor systems. Automatica, 2007, 43: 912–920
18 Yang G H, Wang H. Fault detection for a class of uncertain state-feedback control systems. IEEE Trans Control Syst

Technol, 2010, 18: 201–212

19 Shen Q K, Jiang B, Shi P. Adaptive fault diagnosis for T-S fuzzy systems with sensor faults and system performance analysis. IEEE Trans Fuzzy Syst, 2014, 22: 274–285

20 Li X J, Yang G H. Fault detection in finite frequency domain for Takagi-Sugeno fuzzy systems with sensor faults. IEEE Trans Cybern, 2014, 44: 1446–1458

21 Yang G H, Ye D. Reliable H_∞ control of linear systems with adaptive mechanism. IEEE Trans Autom Control, 2010, 55: 242–247

22 Jiang B, Yang H, Shi P. Switching fault tolerant control design via global dissipativity. Int J Syst Sci, 2010, 41: 1003–1012

23 Tong S C, Huo B Y, Li Y M. Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures. IEEE Trans Fuzzy Syst, 2014, 22: 1–15

24 Liu M, Ho D W C, Shi P. Adaptive fault-tolerant compensation control for Markovian jump systems with mismatched external disturbance. Automatica, 2015, 58: 5–14

25 Li Y X, Yang G H. Adaptive asymptotic tracking control of uncertain nonlinear systems with input quantization and actuator faults. Automatica, 2016, 72: 177–185

26 Deng C, Yang G H. Distributed adaptive fault-tolerant control approach to cooperative output regulation for linear multi-agent systems. Automatica, 2019, 103: 62–68

27 Hautus M L J. Strong detectability and observers. Linear Algebra Appl, 1983, 50: 353–368

28 Zhang W, Zhang H, Chen B S. Generalized Lyapunov equation approach to state-dependent stochastic stabilization/detectability criterion. IEEE Trans Autom Control, 2008, 53: 1630–1642

29 Corless M, Tu J. State and input estimation for a class of uncertain systems. Automatica, 1998, 34: 757–764

30 Filippov A F. Differential Equations with Discontinuous Right-Hand Sides. Dordrecht: KluwerAcademic Publishers, 1988

31 Slotine J J E, Li W. Applied Nonlinear Control. Englewood Cliffs: Prentice-Hall, 1991

32 Siwakosit W, Hess R A. Multi-input/multi-output reconfigurable flight control design. J Guid Control Dyn, 2001, 24: 1079–1088