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A new stabilizing method for linear aperiodic sampled-data systems with time delay inputs and uncertainties

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Dear editor,

With the rapid development of communication, control, and computer technologies, networked control systems (NCSs) have been widely used in various fields owing to its advantages of sharing information resources, reducing system wiring, and increasing the flexibility and reliability of systems [1–3]. In NCSs, sampling-data systems commonly have aperiodic sampling characteristics owing to package dropouts, time triggering, network environment effects, among other issues [4]. Therefore, aperiodic sampled-data systems have been widely studied [5, 6]. An interesting idea on aperiodic sampled-data systems was proposed in [5], which proved that delays can impose a positive effect on the stability of some NCSs. The problem of robust stabilization of aperiodic uncertain sampled-data linear systems was addressed in [6], and synthesis conditions in forms of parameter-dependent linear matrix inequalities were proposed. However, in these studies, only the cases with time delays or uncertainties were considered [5–7], which cannot fully satisfy the actual requirements of controlled systems (e.g., aperiodic sampling, time delays, and uncertainties being present simultaneously in vehicle systems [8] and secure cooperative systems [9]). Therefore, the manner in which the aperiodic sampled-data systems with time delay inputs and uncertainties is efficiently stabilized is of practical significance, which motivates our current study.

The objective of this study is to introduce a new stabilizing method to handle time delay inputs and the uncertainties in a linear aperiodic sampled-data system. By constructing a new Lyapunov function, a less conservative stability criterion is presented in terms of matrix inequalities, which also provides the relationship among the parameters such as the upper and lower bounds of both sampling intervals and delays. Finally, two illustrative examples show the effectiveness and advantages of our proposed method.

Notations. The transposed blocks in a symmetric matrix are denoted by the symbol '$*$'. The superscript 'T' stands for matrix transposition. $\mathbb{N}$ is the set of non-negative integers. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with vector norm $\|\cdot\|$. $\mathbb{S}_+^{n\times n}$ represents the set of $n\times n$ positive definite matrices. $\mathbf{I}$ (or $\mathbf{I}_n$) means an identity matrix with appropriate dimensions (or n-dimension).

Problem formulation. Consider the following linear system with time delay inputs and uncertainties:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B}+\Delta\boldsymbol{B})\boldsymbol{u}(t-\tau_k), \quad (1)$$

where $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$ and $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ are the system state and control input, respectively; $\boldsymbol{A}$ and $\boldsymbol{B}$ are known real constant matrices with appro-

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priate dimensions; $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$ are time-varying parameter uncertainties, which are given by

$$\Delta\boldsymbol{A} = \boldsymbol{E}^*\boldsymbol{H}(t)\boldsymbol{F}_1^*,\ \Delta\boldsymbol{B} = \boldsymbol{E}^*\boldsymbol{H}(t)\boldsymbol{F}_2^*,$$

where $\boldsymbol{E}^*$, $\boldsymbol{F}_1^*$ and $\boldsymbol{F}_2^*$ are known real matrices with appropriate dimensions. For any time t, $\boldsymbol{H}(t)$ is an unknown matrix function satisfying $\boldsymbol{H}^{\mathrm{T}}(t)\boldsymbol{H}(t) \leqslant \boldsymbol{I}$. It is assumed that the system state is sampled aperiodically and the sampling instant sequence is denoted by set $\{s_k\}_{k\in\mathbb{N}}$, where $0 = s_0 < s_1 < \cdots < s_k < \cdots$ and $\lim_{k\to\infty} s_k = \infty$. Then, τ_k indicates the corresponding time delay of the input at sampling instant s_k and it is easy to obtain $t_k = s_k + \tau_k$. Note that t_k satisfies $0 < \tau_m \leqslant \tau_k \leqslant \tau_M$ for $k \in \mathbb{N}$, in which τ_m and τ_M are the lower and upper bounds of time delays, respectively. Suppose that the sampling intervals $T_k = s_{k+1} - s_k \in [T_m, T_M]$ for $k \in \mathbb{N}$, where T_m and T_M are the lower and upper bounds of aperiodic sampling intervals, respectively. For convenience, this study only focuses on the case that sampling intervals T_k are always larger than the corresponding delays τ_k, i.e., $T_m > \tau_M$.

To effectively handle the time delay inputs and analyze the stability of the system (1), a new extending impulsive-system approach with two modes is introduced. Then, the state-feedback control law of the system (1) is designed as

$$\boldsymbol{u}(t-\tau_k) = \begin{cases} \boldsymbol{K}\boldsymbol{x}(s_{k-1}), t \in (s_k, t_k], \\ \boldsymbol{K}\boldsymbol{x}(s_k), \quad t \in (t_k, s_{k+1}], \end{cases} \tag{2}$$

where $\boldsymbol{K}$ is the controller gain to be determined.

Thus, the system (1) is modeled as follows ($k \in \mathbb{N}$):

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{z}_1(t), \\ \qquad\qquad t \in (s_k, t_k], \\ \dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{z}_2(t), \\ \qquad\qquad t \in (t_k, s_{k+1}], \end{cases} \tag{3}$$

where $\boldsymbol{z}_1(t) = \boldsymbol{K}\boldsymbol{x}(s_{k-1})$ and $\boldsymbol{z}_2(t) = \boldsymbol{K}\boldsymbol{x}(s_k)$ for $t \in (s_k, s_{k+1}]$. At sampling instant s_k, we have $\boldsymbol{z}_2(s_k^+) = \boldsymbol{K}\boldsymbol{x}(s_k)$, $\boldsymbol{z}_1(s_k^+) = \boldsymbol{K}\boldsymbol{x}(s_{k-1}) = \boldsymbol{z}_1(s_k)$, and $\dot{\boldsymbol{z}}_1(s_k^+) = \dot{\boldsymbol{z}}_2(s_k^+) = 0$. Let $\boldsymbol{\xi}(t) = \mathrm{col}\{\boldsymbol{x}(t), \boldsymbol{z}_1(t), \boldsymbol{z}_2(t)\}$ belong to $\mathbb{R}^n$, $\boldsymbol{E}_1 = [\boldsymbol{I}_{n_x}\ \boldsymbol{0}\ \boldsymbol{0}]$, $\boldsymbol{E}_2 = [\boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{I}_{n_u}]$, $\boldsymbol{E}_3 = [\boldsymbol{0}\ \boldsymbol{I}_{n_u}\ \boldsymbol{0}]$, $\boldsymbol{E}_0 = \boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{E}^*$, and $\boldsymbol{F}_{\sigma(t)} = \boldsymbol{F}_1^*\boldsymbol{E}_1 + \boldsymbol{F}_2^*\boldsymbol{E}_{\sigma(t)+1}$. The system (3) is rewritten as follows:

$$\begin{cases} \dot{\boldsymbol{\xi}}(t) = (\boldsymbol{A}_{\sigma(t)} + \Delta\boldsymbol{A}_{\sigma(t)})\boldsymbol{\xi}(t), \\ \qquad\qquad t \notin \{s_k\}_{k\in\mathbb{N}} \cup \{t_k\}_{k\in\mathbb{N}}, \\ \boldsymbol{\xi}(t^+) = \boldsymbol{D}_{\sigma(t)}\xi(t),\ t \in \{s_k\}_{k\in\mathbb{N}} \cup \{t_k\}_{k\in\mathbb{N}}, \end{cases} \tag{4}$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{D}_1 = \begin{bmatrix} \boldsymbol{I}_{n_x} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{n_u} & \boldsymbol{0} \end{bmatrix}, \qquad \boldsymbol{D}_2 = \boldsymbol{I}_n,$$

and $\Delta\boldsymbol{A}_{\sigma(t)} = \boldsymbol{E}_0\boldsymbol{H}(t)\boldsymbol{F}_{\sigma(t)}$, in which $n = n_x + 2n_u$. Note that the switching law in (4) is described as when $t \in (s_k, t_k]$, $\sigma(t) = 1$ and when $t \in (t_k, s_{k+1}]$, $\sigma(t) = 2$. Furthermore, define $h_{1,k}$ and $h_{2,k}$ as the dwell times of impulsive switched system modes 1 and 2, respectively. From (3), it is known that $h_{1,k} = t_k - s_k = \tau_k \in [h_1^m, h_1^M] := [\tau_m, \tau_M]$ and $h_{2,k} = s_{k+1} - t_k = T_k - \tau_k \in [h_2^m, h_2^M] := [T_m - \tau_M, T_M - \tau_m]$, $k \in \mathbb{N}$, where h_i^m and h_i^M are the lower and upper bounds of $h_{i,k}$, $i = 1, 2$.

Main result. Based on the above discussions, we have the following result.

Theorem 1. Consider the system with time delay inputs and uncertainties (1). The globally exponentially stabilization problem of the system is solved by the control law (2), if there exist appropriate matrices $\boldsymbol{K} \in \mathbb{R}^{n_u\times n_x}$, $\boldsymbol{P}_i \in \mathbb{S}_+^{n\times n}$, and $\boldsymbol{S}_i \in \mathbb{S}_+^{n\times n}$, such that the following matrix inequalities hold for $h_i \in [h_i^m, h_i^M]$ with $i = 1, 2$:

$$\boldsymbol{\Omega}_i^1 = \begin{bmatrix} \boldsymbol{\Omega}_{11i}^1 & \boldsymbol{\Omega}_{12i}^1 \\ * & \boldsymbol{\Omega}_{22i}^1 \end{bmatrix} < 0, \tag{5}$$

$$\boldsymbol{\Omega}_i^2 = \begin{bmatrix} \boldsymbol{\Omega}_{11i}^2 & \boldsymbol{\Omega}_{12i}^2 \\ * & \boldsymbol{\Omega}_{22i}^2 \end{bmatrix} < 0, \tag{6}$$

where $\boldsymbol{\Omega}_{11i}^1 = \mathrm{Sym}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i) + \boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{F}_i + \boldsymbol{P}_i\boldsymbol{E}_0\boldsymbol{E}_0^{\mathrm{T}}\boldsymbol{P}_i - \boldsymbol{S}_i$, $\boldsymbol{\Omega}_{12i}^1 = \mathbf{S}_i\boldsymbol{D}_i$, $\boldsymbol{\Omega}_{22i}^1 = -\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{S}_i\boldsymbol{D}_i - \frac{1}{h_i}\boldsymbol{P}_{i-1}$, $\boldsymbol{\Omega}_{11i}^2 = 2(\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{F}_i + \boldsymbol{S}_i\boldsymbol{E}_0\boldsymbol{E}_0^{\mathrm{T}}\boldsymbol{S}_i) + \mathrm{Sym}(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{S}_i)$, $\boldsymbol{\Omega}_{12i}^2 = -\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{S}_i\boldsymbol{D}_i$, and $\boldsymbol{\Omega}_{22i}^2 = \boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{S}_i\boldsymbol{E}_0\boldsymbol{E}_0^{\mathrm{T}}\boldsymbol{S}_i\boldsymbol{D}_i$.

Proof. A Lyapunov function over the interval $(\alpha, \beta]$ is chosen as

$$V(t) = V_{\sigma(t)}^1(t) + (\beta - t)\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{S}_{\sigma(t)}\boldsymbol{e}(t) + \frac{\beta - t}{\beta - \alpha}V_{\sigma(\alpha)}^1(\alpha),$$

where α and β are two adjacent impulse instants of the system (4), $V_{\sigma(t)}^1(t) = \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{P}_{\sigma(t)}\boldsymbol{\xi}(t)$, and $\boldsymbol{e}(t) = \boldsymbol{\xi}(t) - \boldsymbol{\xi}(\alpha^+)$. If $\sigma(t) = i$ on $(\alpha, \beta]$, then $\sigma(\alpha) = i - 1$ for $i \in \mathbb{V}_N/\{1\}$ and $\sigma(\alpha) = 2$ for $i = 1$. As $\boldsymbol{P}_i > 0$ and $\boldsymbol{S}_i > 0$, $V(t)$ is positive. Let $\boldsymbol{\zeta}(t) = \mathrm{col}\{\boldsymbol{\xi}(t), \boldsymbol{\xi}(\alpha^-)\}$, $h_i = \beta - \alpha$ and $\varepsilon = t - \alpha \in (0, h_i]$. Then, the time derivative of $V(t)$ along the trajectory of (4) is given by

$$\dot{V}(t) = 2\dot{\boldsymbol{\xi}}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{\xi}(t) - \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{S}_i\boldsymbol{e}(t)$$

$$
\begin{aligned}
&+2(\beta-t)\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{S}_i\boldsymbol{e}(t)\\
&-\frac{1}{\beta-\alpha}\boldsymbol{\xi}^{\mathrm{T}}(\alpha)\boldsymbol{P}_{i-1}\boldsymbol{\xi}(\alpha)\\
\leqslant\ &\boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Omega}_i\boldsymbol{\zeta}(t).
\end{aligned}
$$

As $\boldsymbol{\Omega}_i=\boldsymbol{\Omega}_i^1+(h_i-\varepsilon)\boldsymbol{\Omega}_i^2$, $h_i-\varepsilon=b-t>0$, according to (5) and (6), we have $\boldsymbol{\Omega}_i<0$, in which $h_i\in[h_i^m,h_i^M]$. It follows that $\dot{V}(t)<0$. This completes the proof.

Illustrative examples. The active suspension system in vehicles can be formulated by (1) [8] with $\boldsymbol{x}(t)=[x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)]^{\mathrm{T}}$,

$$
\boldsymbol{A}=\begin{bmatrix}0&0&1&-1\\0&0&0&1\\-\frac{k_s}{m_s}&0&-\frac{c_s}{m_s}&\frac{c_s}{m_s}\\\frac{k_s}{m_u}&-\frac{k_t}{m_u}&\frac{c_s}{m_u}&-\frac{c_s+c_t}{m_s}\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}0\\0\\\frac{1}{m_s}\\-\frac{1}{m_u}\end{bmatrix},
$$

in which $m_s=973$ kg, $m_u=114$ kg, $k_t=10115$ N/m, $k_s=10111$ N/m, $c_s=1095$ Ns/m, $c_t=14.6$ Ns/m; $x_1(t),x_2(t),x_3(t),x_4(t)$ are the suspension deflection, tire deflection, sprung mass speed, and unsprung mass speed, respectively. The uncertain parameters caused by uneven road surfaces are expressed as $H(t)=\frac{\pi\tilde{H}V}{L}\sin(\frac{2\pi Vt}{L})$, $\boldsymbol{E}^*=[0.2\ 0.1\ 0.1\ 0.1]^{\mathrm{T}}$, $\boldsymbol{F}_1^*=[0.1\ 0.2\ 0.1\ 0.2]$, and $\boldsymbol{F}_2^*=0.2$, where $V=12.5$ m is the vehicle forward velocity; $\tilde{H}=0.006$ m and $L=5$ m are the height and the length of the bump, respectively. In this study, we choose $T_m=1.51$ s, $T_M=1.87$ s, $\tau_m=0.1$ s, and $\tau_M=0.5$ s. According to Theorem 1, one has $\boldsymbol{K}=-[5.3712\ \ 9.1587\ \ 5.3329\ \ 6.0106]$. The initial state of the system (1) is given by $\boldsymbol{x}(0)=[0.2\ 0\ 0.1\ \ -2]^{\mathrm{T}}$. Then, the system state trajectories are shown in Figure 1 and illustrate the effectiveness of the proposed result.

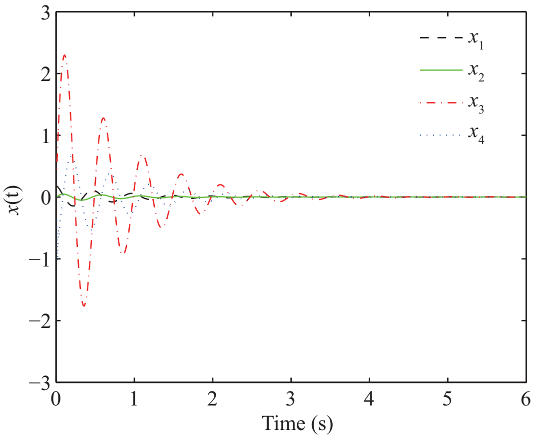

Figure 1 (Color online) State trajectories of the system (1).

An advantage of our proposed method is that h_i^m and h_i^M synthetically consider the upper and lower bounds of time delays and sampling intervals. Compared to traditional studies, which consider the aperiodic sampling case such as [6,7], our proposed method can relax the restrictions on the upper bound of aperiodic sampling intervals for its fixed lower bound. Next, another example described by the system (1) with $\boldsymbol{A}=[\begin{smallmatrix}0&1\\0&-0.1\end{smallmatrix}]$, $\boldsymbol{B}=[\begin{smallmatrix}0\\2\end{smallmatrix}]$ is given to show this advantage. Consider the uncertain parameters with $\boldsymbol{E}^*=[0.2\ 0.02]^{\mathrm{T}}$, $\boldsymbol{F}_1^*=[0\ 1]$, $\boldsymbol{F}_2^*=0$, $H(t)=0.2$. As there are no time delays in [6,7], we choose $\tau_k\equiv0$. Using Theorem 1, one has $\boldsymbol{K}=-[3.75\ 11.5]$. The initial state of the system (1) is given by $\boldsymbol{x}(0)=[5\ \ -5]^{\mathrm{T}}$. For the fixed lower bound $T_m=0.001$ s, refs. [6, 7] present that the maximum values of the upper bound T_M are 22.28 and 42.54, respectively. Our proposed method however can obtain $T_M=45.78$, which increases by 105.48% and 7.62% when compared with the results of [6,7], respectively. It can be seen that our method can provide less conservative results.

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