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Special Focus on Advances in Disturbance/Uncertainty Estimation and Attenuation Techniques With Applications

Guaranteeing almost fault-free tracking performance from transient to steady-state: a disturbance observer approach

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Abstract In this paper, we propose an output-feedback fault-tolerant controller (FTC) for a class of uncertain multi-input single-output systems under float and lock-in-place actuator faults. Of particular interest is to recover a fault-free tracking performance of a (pre-defined) nominal closed-loop system, during the entire time period including the transients due to abrupt actuator faults. As a key component, a high-gain disturbance observer (DOB) is employed to rapidly compensate the lumped disturbance, a compressed expression of all the effect of actuator faults (as well as model uncertainty and disturbance) on the system. To implement this high-gain approach, a fixed control allocation (CA) law is presented in order to keep an extended system with a virtual scalar input to remain of minimum phase under any patterns of faults. It is shown via the singular perturbation theory that the proposed FTC, consisting of the high-gain DOB and the CA law, resolves the problem in an approximate but arbitrarily accurate sense. Simulations with the linearized lateral model of Boeing 747 are performed to verify the validity of the proposed FTC scheme.

Keywords disturbance observer, fault-tolerant control, robust control, performance recovery, actuator fault

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1 Introduction

In response to the growing need for assessing high-level objectives, modern control systems usually have complicated structures. Unfortunately, the increasing complexity often puts these systems into a dangerous situation resulting from abrupt faults and failures on the system components [1,2]. It is in this context that a tremendous research effort has been devoted to develop fault-tolerant control (FTC) schemes in the last two decades [3,4]. Two major approaches for the FTC designs have emerged in literature: passive and active FTC approaches. The first strategy is to employ a fixed controller during system operation without any detection and isolation of faults, while the other one is to reconfigure the controller structure via estimated information on the faults. Over the active FTC, the passive FTC has advantages that the design is simpler and faster response may be ensured, at the cost of requiring knowledge on the faults in the design stage. On the other hand, the active FTC schemes are able to handle even unanticipated faults outside an expected boundary by adjusting sudden changes of the system characteristics automatically, yet at the same time additional delays may take place because of the use of fault detection/isolation

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algorithms. (See [3, 4] and the references therein.)

While numerous researches in both directions have been performed in literature, the problem of guaranteeing a transient tracking performance at the moment of actuator faults still has not been fully dealt with yet. The management of the transient behavior may be of particular importance to critical control systems such as power grids [5] and aircrafts, whose transient malfunctions possibly result in considerable loss of efficiency and discomfort of mankind. To the best of the authors' knowledge, only a few studies have addressed the problem [6, 7]. In [6], an active FTC scheme was proposed by combining an adaptive sliding mode control with a backstepping control. While this approach preserves a satisfactory post-fault transient response as much as desired, it inherently leads to high computational complexity and requires the exact information on the high-frequency gain matrix of the plant, which restricts the class of model uncertainty dealt with. On the other hand, the authors of [7] introduced an adaptive-type FTC for the transient response control of spacecraft. Main drawbacks of this scheme are that full state information is explicitly used in the controller design and the transient response cannot be shaped arbitrarily.

In this paper, we present an output-feedback FTC scheme that recovers a fault-free nominal tracking performance during the entire time period, for a class of multi-input single-output (MISO) systems under actuator faults, parametric uncertainty, and external disturbances. Our central idea is to directly estimate a lumped signal representing all the effect of actuator faults, model uncertainty, and external disturbance at once and compensate it on-line, rather than try to detect the fault itself as in the traditional active FTC approaches (which possibly introduce an additional delay). To implement the idea, in this work a passive FTC is constructed by combining a high-gain disturbance observer (DOB) with a fixed control allocation (CA) law. Since its invention by the authors of [8], the DOB has been broadly used as a simple but powerful robust controller that attenuates the effect of model uncertainty and disturbance on the plant [9–13]. More recently, the authors of [14, 15] proposed a refined DOB structure based on the high-gain technique, which ensures the recovery of the nominal performance from transient to steady-state. When it comes to our problem, however, the DOB designs in the previous studies are not applicable directly, because (a) most of the studies dealt only with square systems, and (b) the high-gain DOB in [14, 15] requires the minimum phaseness of the plant, which may be lost when an actuator fault happens. This is why a fixed CA law comes into the picture in our FTC design [16]. In particular, provided that the MISO plant is of minimum phase in an input-wise sense, we propose a design guideline for the CA law with which the plant (with a virtual scalar input) remains of minimum phase robustly against any patterns of actuator faults and parametric uncertainties. Finally, we prove via the singular perturbation theory [17, 18] that the proposed DOB-based FTC, consisting of the high-gain DOB and the CA law, resolves the problem of interest. A few additional remarks on the proposed FTC are: (a) our result is semi-global so that any large (but bounded) parametric uncertainty and actuator faults can be handled; and (b) without utilizing knowledge on the generating model of the disturbance, the performance recovery of the DOB-based FTC is approximate; yet approximation error can be made small arbitrarily.

The remainder of this paper is organized as follows. First, Section 2 is devoted to formulate the problem of our interest. Then a DOB-based FTC is proposed in Section 3, while in Section 4 we prove that the proposed FTC recovers the fault-free nominal performance. Finally, to verify the validity of the proposed scheme, simulation results on the linearized lateral model of Boeing 747 are presented in Section 5.

Notation. For two sets $\mathcal{A} \subset \mathbb{R}^n$ and $\mathcal{B} \subset \mathbb{R}^n$ and a positive constant ϵ, we use $\mathcal{A} \overset{\epsilon}{\sqsubset} \mathcal{B}$ if $\mathcal{A} \subset \mathcal{B}$ and $\inf_{a\in\partial\mathcal{A}, b\in\partial\mathcal{B}}\{\|a-b\|\} > \epsilon$ where $\partial\mathcal{A}$ and $\partial\mathcal{B}$ indicate the boundary of $\mathcal{A}$ and $\mathcal{B}$, respectively, and $\mathcal{A} \sqsubset \mathcal{B}$ if there exists $\epsilon > 0$ such that $\mathcal{A} \overset{\epsilon}{\sqsubset} \mathcal{B}$. For two real numbers a and b, let $\mathcal{D}(a, b)$ be the closed disk in the complex plane whose diameter is the line segment connecting the points $-(1/a) + \mathrm{j}0$ and $-(1/b) + \mathrm{j}0$. For a positive integer m, the power set of $\{1, \dots, m\}$ excluding the empty set is denoted by $\mathcal{P}_m$. For a square and symmetric matrix P, $\underline{\lambda}(P)$ and $\overline{\lambda}(P)$ indicate the minimum and maximum eigenvalues of P, respectively.

2 Problem formulation

We consider an MISO linear plant

$$\dot{\mathsf{x}} = \mathsf{A}\mathsf{x} + \mathsf{B}(u + d), \quad y = \mathsf{C}\mathsf{x}, \tag{1}$$

where $\mathsf{x} \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}$ is the output, $d = (d_1, \ldots, d_m) \in \mathbb{R}^m$ is the disturbance, $u = (u_1, \ldots, u_m) \in \mathbb{R}^m$ is the actuator input, and $\mathsf{A} \in \mathbb{R}^{n\times n}$, $\mathsf{B} =: [\mathsf{B}_1, \ldots, \mathsf{B}_m] \in \mathbb{R}^{n\times m}$, and $\mathsf{C} \in \mathbb{R}^{1\times n}$ are unknown matrices satisfying that (A, B) is controllable and (C, A) is observable. The disturbance $d(t)$ is continuously differentiable, and $d(t)$ and its time derivative are bounded with known bounds. It is assumed that the initial condition $\mathsf{x}(0)$ belongs to a bounded set.

Throughout this paper, we pay our attention to a particular class of MISO systems (1) that are of minimum phase in an input-wise sense, stated as follows.

Assumption 1. Each single-input single-output (SISO) subsystem $\mathsf{C}(sI - \mathsf{A})^{-1}\mathsf{B}_i$, $i = 1, \ldots, m$, of (1) satisfies the following conditions:

(a) The system has the relative degree $\nu \geqslant 1$ uniformly on $i = 1, \ldots, m$; more precisely, $\mathsf{C}\mathsf{A}^j\mathsf{B}_i = 0$ for all $j = 0, \ldots, \nu - 2$, and $\mathsf{C}\mathsf{A}^{\nu-1}\mathsf{B}_i \neq 0$ with known sign;

(b) The numerator of its transfer function, denoted by $N_i(s)$, is included in the set of Hurwitz polynomials

$$\mathcal{N}_i := \left\{N_i(s) = N_{i,n-\nu}s^{n-\nu} + N_{i,n-\nu-1}s^{n-\nu-1} + \cdots + N_{i,0} : N_{i,j} \in [\underline{N}_{i,j}, \overline{N}_{i,j}]\right\}, \tag{2}$$

where $\underline{N}_{i,j}$ and $\overline{N}_{i,j}$ are known constants.

Without loss of generality, let $\mathsf{C}\mathsf{A}^{\nu-1}\mathsf{B}_i > 0$ for all $i = 1, \ldots, m$.

The first item of Assumption 1 admits a coordinate change $(x, \zeta) := [\mathsf{T}_x; \mathsf{T}_\zeta]\mathsf{x} \in \mathbb{R}^{\nu+(n-\nu)}$ for the state x such that $\mathsf{T}_x := [\mathsf{C}; \mathsf{C}\mathsf{A}; \ldots; \mathsf{C}\mathsf{A}^{\nu-1}] \in \mathbb{R}^{\nu\times n}$ and $[\mathsf{T}_x; \mathsf{T}_\zeta] \in \mathbb{R}^{n\times n}$ is nonsingular, by which the plant (1) of our interest is newly represented as the form

$$\dot{x} = A_\nu x + B_\nu \left(\Phi x + \Psi\zeta + G\left(u + d\right)\right), \quad y = C_\nu x, \tag{3a}$$

$$\dot{\zeta} = S\zeta + Mx + H\left(u + d\right). \tag{3b}$$

Here, for an integer $i \geqslant 1$, the matrices A_i, B_i, and C_i are defined as

$$A_i := \begin{bmatrix} 0_{i-1} & I_{i-1} \\ 0 & 0_{i-1}^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{i\times i}, \quad B_i := \begin{bmatrix} 0_{i-1} \\ 1 \end{bmatrix} \in \mathbb{R}^{i\times 1}, \quad C_i := \begin{bmatrix} 1 & 0_{i-1}^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{1\times i},$$

while the matrices $\Phi \in \mathbb{R}^{1\times\nu}$, $\Psi \in \mathbb{R}^{1\times(n-\nu)}$, $G := [G_1, \ldots, G_m] \in \mathbb{R}^{1\times m}$, $S \in \mathbb{R}^{(n-\nu)\times(n-\nu)}$, $M \in \mathbb{R}^{(n-\nu)\times\nu}$, and $H =: [H_1, \ldots, H_m] \in \mathbb{R}^{(n-\nu)\times m}$ are uncertain by definition. It is easy to see that $G_i = \mathsf{C}\mathsf{A}^{\nu-1}\mathsf{B}_i > 0$, and $0 < \underline{G}_i \leqslant G_i \leqslant \overline{G}_i$ with positive constants $\underline{G}_i$ and $\overline{G}_i$. Under the assumptions, one can also see that $x(0) \in \mathcal{X}^0$ and $\zeta(0) \in \mathcal{S}^0$ for some compact sets $\mathcal{X}^0 \subset \mathbb{R}^\nu$ and $\mathcal{S}^0 \subset \mathbb{R}^{n-\nu}$. Keeping the equivalence of (1) and (3) in mind, in what follows we mainly regard (3) as the plant to be controlled.

We suppose that in addition to the model uncertainty and the external disturbance, at most $m - 1$ actuator faults may take place during the operation. For its detailed description, let us define $T_i > 0$, $i = 1, \ldots, m$, as the time instant when the i-th actuator is under fault. To prevent the controllability of (3) from losing, it is natural to assume that there exists at least one $i \in \{1, \ldots, m\}$ satisfying that $T_i = \infty$. Then

$$\sigma(t) := \{i : \ t < T_i\} \in \mathcal{P}_m \tag{4}$$

stands for the set of indices, of which the actuators operate normally at the moment t. On the other hand, for given $\sigma \in \mathcal{P}_m$ the indicator matrix $\Lambda_\sigma \in \mathbb{R}^{m\times m}$ is defined as a diagonal matrix $\Lambda_\sigma = \mathrm{diag}(\Lambda_{\sigma,1}, \ldots, \Lambda_{\sigma,m})$ where $\Lambda_{\sigma,i} = 1$ if $i \in \sigma$ and $\Lambda_{\sigma,i} = 0$ otherwise. With these symbols, the following assumption on the actuator failure is made.

Assumption 2. The actuator input $u(t)$ of the plant (3) is of the form

$$u(t) = \Lambda_{\sigma(t)} u_{\rm con}(t) + \left(I - \Lambda_{\sigma(t)}\right) u^\star_{\rm flt}, \tag{5}$$

where $\sigma(t)$ is defined as (4), $u_{\rm con} =: \left(u_{{\rm con},1}, \ldots, u_{{\rm con},m}\right) \in \mathbb{R}^m$ is the input signal generated by a controller, and $u^\star_{\rm flt} =: \left(u^\star_{{\rm flt},1}, \ldots, u^\star_{{\rm flt},m}\right) \in \mathbb{R}^m$ is an unknown constant vector contained in an (arbitrarily large but) bounded set. Moreover, there exists a dwell time $\Delta > 0$ such that the failure moments T_i in (4) satisfy $\min_{i,j\in\{1,\ldots,m\}}(|T_i - T_j|) \geqslant \Delta$.

Remark 1. The class of actuator faults considered in Assumption 2 includes the lock-in-place (or stuck) fault (i.e., $u^\star_{{\rm flt},i} = u_i(T_i)$) and the floating fault (i.e., $u^\star_{{\rm flt},i} = u^\star_i$ for an unknown $u^\star_i$).

Roughly speaking, the main purpose of this paper is to force the considered plant (3) to behave as a fault-free nominal model, in view of the output trajectory. Since the worst scenario we may encounter (in the sense of actuator failure) is that only one healthy actuator is left, it would be reasonable to consider the following SISO system as a nominal model of (3):

$$\dot{x}_{\sf n} = A_\nu x_{\sf n} + B_\nu \left(\Phi_{\sf n} x_{\sf n} + \Psi_{\sf n} \zeta_{\sf n} + G_{\sf n} v_{\sf n}\right), \quad y_{\sf n} = C_\nu x_{\sf n}, \tag{6a}$$

$$\dot{\zeta}_{\sf n} = S_{\sf n} \zeta_{\sf n} + M_{\sf n} x_{\sf n} + H_{\sf n} v_{\sf n}, \tag{6b}$$

where $x_{\sf n} \in \mathbb{R}^\nu$ and $\zeta_{\sf n} \in \mathbb{R}^{n-\nu}$ are the nominal states, $v_{\sf n} \in \mathbb{R}$ is the nominal (scalar) input, and $y_{\sf n} \in \mathbb{R}$ is the nominal output. The matrices $\Phi_{\sf n}$, $\Psi_{\sf n}$, $G_{\sf n}$, $S_{\sf n}$, $M_{\sf n}$, and $H_{\sf n}$ are nominal components, and the initial conditions $x_{\sf n}(0)$ and $\zeta_{\sf n}(0)$ are located in $\mathcal{X}^0$ above and a compact subset $\mathcal{S}^0_{\sf n}$ of $\mathbb{R}^{(n-\nu)}$, respectively. (For a technical reason, let $\mathcal{S}^0_{\sf n}$ be larger than a bounded set $\{(H_{\sf n}/G_{\sf n}) B_\nu^{\rm T} x_{\sf n}(0) : x_{\sf n}(0) \in \mathcal{X}^0\}$.) It is supposed that without any uncertain factor, a (static or dynamic) output-feedback controller

$$\dot{c}_{\sf n} = E c_{\sf n} + F(r - y_{\sf n}), \quad v_{\sf n} = J c_{\sf n} + K(r - y_{\sf n}) \tag{7}$$

is constructed a priori for the nominal model (6), in which $c_{\sf n} \in \mathbb{R}^l$ is the controller state initiated in a compact set $\mathcal{C}^0_{\sf n}$, $r \in \mathbb{R}$ is the reference signal for $y_{\sf n}$ such that $r(t)$ is continuously differentiable, and $r(t)$ and $\dot{r}(t)$ are bounded, and E, F, J, and K are some matrices. We assume that the controller (7) is designed such that the nominal closed-loop system (6) and (7) is stable and the nominal tracking performance is satisfactory.

Now, we are ready to state the problem under consideration.

Problem of interest. Provided that Assumptions 1 and 2 hold and a threshold $\epsilon > 0$ is given, construct an output-feedback FTC

$$\dot{\psi} = f_{\rm con}(\psi, y, r, t), \quad u_{\rm con} = h_{\rm con}(\psi, y, r, t), \tag{8}$$

such that

(a) The plant state (x, ζ) of the closed-loop system (3) and (8) is bounded;

(b) Its output $y(t)$ satisfies

$$\|y(t) - y_{\sf n}(t)\| < \epsilon, \quad \text{for all } t \geqslant 0, \tag{9}$$

where $y_{\sf n}(t)$ is an output trajectory of the nominal closed-loop system (6) and (7) with $x(0) = x_{\sf n}(0) \in \mathcal{X}^0$ (or equivalently, $(y_{\sf n}(0), \ldots, y_{\sf n}^{(\nu-1)}(0)) = (y(0), \ldots, y^{(\nu-1)}(0)))$ and some $(\zeta_{\sf n}(0), c_{\sf n}(0)) \in \mathcal{S}^0_{\sf n} \times \mathcal{C}^0_{\sf n}$.

The second item in the problem statement, which is our primary concern, means that the FTC (8) recovers a pre-defined fault-free tracking performance of the nominal closed-loop system (6) and (7) in an approximate but arbitrarily accurate sense. More importantly, this recovery is desired to be achieved for transient (as well steady-state) periods including the failure moments, $t = T_i$.

3 Design of DOB-based FTC

In this section, we propose an output-feedback FTC (8) that solves the problem of interest, based on the DOB approach. It should be noted that the usual DOB designs in the literature are not directly applicable to our problem. This is because, the previous studies mostly took into account square systems (i.e., systems having the same number of inputs and outputs) with the number of inputs known, whereas the plant (3) and (5) considered here has an uncertain number of redundant inputs.

As a simple way to avoid this difficulty, we here adopt an auxiliary scalar input $v \in \mathbb{R}$ and allocate it into the control input $u_{\rm con}(t)$ in (5) as

$$u_{\rm con}(t) = \kappa v(t), \tag{10}$$

where $\kappa = [\kappa_1; \dots; \kappa_m] \in \mathbb{R}^{m\times 1}$ is a constant vector. The underlying rationale behind the fixed CA law (10) is that the plant (3) and (5) augmented with (10), computed by

$$\dot{x} = A_\nu x + B_\nu \left(\Phi x + \Psi\zeta + G\Lambda_\sigma \kappa v + G(1-\Lambda_\sigma)u^\star_{\rm flt} + Gd\right), \quad y = C_\nu x, \tag{11a}$$

$$\dot{\zeta} = S\zeta + Mx + H\Lambda_\sigma\kappa v + H(1-\Lambda_\sigma)u^\star_{\rm flt} + Hd, \tag{11b}$$

now can be viewed as an SISO system with respect to the auxiliary input v; more importantly, this property is invariant on the pattern of the actuator faults. Another advantage of (10) is that the design parameter κ explicitly appears in the new input matrix $[G\Lambda_\sigma\kappa; H\Lambda_\sigma\kappa]$ of the SISO system (11), which brings an opportunity to handle the system zeros of (11). This is of utter importance to us, since the DOB design basically follows the philosophy of high-gain technique [14, 15] where the controlled plant is necessarily of minimum phase. In Subsection 3.1, we will show that this requirement can always be obtained (even in the presence of model uncertainty and actuator faults) by selecting κ appropriately.

3.1 Design of static gain κ

We start by introducing a natural definition of the minimum phaseness for the system (11).

Definition 1. The system (11) with the switching signal $\sigma(t)$ in (4) is said to be of fault-invariant minimum phase if it is of minimum phase for any constant $\sigma \in \mathcal{P}_m$.

Lemma 1 then provides a simple necessary and sufficient condition for the fault-invariant minimum phaseness.

Lemma 1. All the systems (3) satisfying Assumption 1 are of fault-invariant minimum phase if and only if every polynomials in $\bigcup_{\sigma\in\mathcal{P}_m}\Gamma_\sigma$ where $\Gamma_\sigma := \{\sum_{i\in\sigma}\kappa_i N_i(s) : N_i(s) \in \mathcal{N}_i \text{ in (2)}\}$ are Hurwitz.

Proof. Lemma 1 can be easily proved by showing that for the transfer function of (3) with a constant set $\sigma \in \mathcal{P}_m$, its numerator is the same as $\sum_{i\in\sigma}\kappa_i N_i(s)$.

Motivated by the result of Lemma 1, the main goal of this subsection is to design the static gain κ in (10) that makes every elements of $\bigcup_{\sigma\in\mathcal{P}_m}\Gamma_\sigma$ in Lemma 1 Hurwitz. In the following, such κ will be selected in an iterative way. For this, let us consider the equality

$$\bigcup_{\sigma\in\mathcal{P}_{k+1}}\Gamma_\sigma = \Gamma_{\{k+1\}} \cup \left(\bigcup_{\sigma\in\mathcal{P}_k}\Gamma_\sigma\right) \cup \left(\bigcup_{\sigma\in\mathcal{P}_k}\Gamma_{\sigma\cup\{k+1\}}\right), \tag{12}$$

for $k = 0, \dots, m-1$, which presents a recursive decomposition of $\bigcup_{\sigma\in\mathcal{P}_{k+1}}\Gamma_\sigma$. We here note that the set $\bigcup_{\sigma\in\mathcal{P}_k}\Gamma_\sigma$ in the recursive relation (12) is well-defined only with partial components $\kappa_1, \dots, \kappa_k$ of κ. For simplicity, we assume without loss of generality that for any polynomial $N_i(s) \in \mathcal{N}_i$, its leading coefficient $N_{i,n-\nu}$ is the same as the i-th component $G_i > 0$ of the high-frequency gain matrix G of (3).

Theorem 1 shows the existence of a CA law guaranteeing the desired property.

Theorem 1. Suppose that Assumption 1 is satisfied. Then there exists κ such that the SISO system (11) is of fault-invariant minimum phase.

Proof. Theorem 1 is proved by the mathematical induction. First, it is obvious that once κ_1 is selected as any positive value, every polynomial in $\Gamma_{\{1\}}$ is Hurwitz by definition. Next, we claim that if $\kappa_i > 0$, $i = 1, \dots, k$ (with $k \leqslant m-1$), are chosen such that the polynomials in $\bigcup_{\sigma\in\mathcal{P}_k}\Gamma_\sigma$ are all Hurwitz, then there exists $\kappa_{k+1} > 0$ with which the polynomials in $\bigcup_{\sigma\in\mathcal{P}_{k+1}}\Gamma_\sigma$ are also Hurwitz. Under the hypothesis and item (b) of Assumption 1, the first two sets in the right hand-side of (12) naturally consist only of stable polynomials, regardless of the vaule of κ_{k+1}. On the other hand, for each $\sigma \in \mathcal{P}_k$ an element of $\Gamma_{\sigma\cup\{k+1\}}$ is of the form

$$\gamma_{\sigma\cup\{k+1\}}(s) := \kappa_{k+1}N_{k+1}(s) + \sum_{i\in\sigma}\kappa_i N_i(s). \tag{13}$$

Notice that because $\kappa_i N_{i,n-\nu} > 0$ for all $i = 1,\ldots,k$ and the latter polynomial $\sum_{i\in\sigma}\kappa_i N_i(s)$ in (13) is Hurwitz by definition, all the coefficients of $\gamma_{\sigma\cup\{k+1\}}(s)$ above must be positive as long as $\kappa_{k+1} > 0$. It is then obtained from the Kharitonov's theorem [19] that for given $\sigma \in \mathcal{P}_k$ and $\kappa_{k+1} > 0$, all the uncertain polynomials $\gamma_{\sigma\cup\{k+1\}}(s)$ in $\Gamma_{\sigma\cup\{k+1\}}$ are Hurwitz if and only if the following four extreme polynomials are Hurwitz:

$$\gamma^\star_{\sigma\cup\{k+1\},\mathsf{k}}(s) := \kappa_{k+1}N^\star_{k+1,\mathsf{k}}(s) + \sum\nolimits_{i\in\sigma}\kappa_i N^\star_{i,\mathsf{k}}(s), \quad \forall\ \mathsf{k} = \mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}, \tag{14}$$

where for $i = 1,\ldots,m$,

$$N^\star_{i,\mathsf{a}}(s) := \overline{N}_{i,n-\nu}s^{n-\nu} + \overline{N}_{i,n-\nu-1}s^{n-\nu-1} + \underline{N}_{i,n-\nu-2}s^{n-\nu-2} + \underline{N}_{i,n-\nu-3}s^{n-\nu-3} + \cdots, \tag{15a}$$
$$N^\star_{i,\mathsf{b}}(s) := \overline{N}_{i,n-\nu}s^{n-\nu} + \underline{N}_{i,n-\nu-1}s^{n-\nu-1} + \underline{N}_{i,n-\nu-2}s^{n-\nu-2} + \overline{N}_{i,n-\nu-3}s^{n-\nu-3} + \cdots, \tag{15b}$$
$$N^\star_{i,\mathsf{c}}(s) := \underline{N}_{i,n-\nu}s^{n-\nu} + \underline{N}_{i,n-\nu-1}s^{n-\nu-1} + \overline{N}_{i,n-\nu-2}s^{n-\nu-2} + \overline{N}_{i,n-\nu-3}s^{n-\nu-3} + \cdots, \tag{15c}$$
$$N^\star_{i,\mathsf{d}}(s) := \overline{N}_{i,n-\nu}s^{n-\nu} + \underline{N}_{i,n-\nu-1}s^{n-\nu-1} + \underline{N}_{i,n-\nu-2}s^{n-\nu-2} + \overline{N}_{i,n-\nu-3}s^{n-\nu-3} + \cdots. \tag{15d}$$

We emphasize that $N^\star_{i,\mathsf{k}}(s)$ in (15) are the very extreme polynomials of $N_i(s)$ of the set $\mathcal{N}_i$ in (2), all of which are thus Hurwitz by the Kharitonov's theorem and item (b) of Assumption 2. It then follows from the root locus technique that for each $\mathsf{k} \in \{\mathsf{a},\ldots,\mathsf{d}\}$, there exists sufficiently large $\underline{\kappa}_{\sigma\cup\{k+1\},\mathsf{k}} > 0$ such that $\gamma^\star_{\sigma\cup\{k+1\},\mathsf{k}}(s)$ in (14) is Hurwitz for all $\kappa_{k+1} > \underline{\kappa}_{\sigma\cup\{k+1\},\mathsf{k}} > 0$. At last, take κ_{k+1} sufficiently large to satisfy

$$\kappa_{k+1} > \max_{\sigma\in\mathcal{P}_k}\left\{\max_{\mathsf{k}=\{\mathsf{a},\mathsf{b},\mathsf{c},\mathsf{d}\}}\left\{\underline{\kappa}_{\sigma\cup\{k+1\},\mathsf{k}}\right\}\right\}, \tag{16}$$

so that all the uncertain polynomials of $\bigcup_{\sigma\in\mathcal{P}_k}\Gamma_{\sigma\cup\{k+1\}}$ (and thus those of $\bigcup_{\sigma\in\mathcal{P}_{k+1}}\Gamma_\sigma$ in the left hand-side of (12)) are Hurwitz. This concludes the proof of Theorem 1.

At last, we summarize the proof of the theorem as a design guideline for the gain κ of the CA law.

Design guideline for κ:

Step 0. Take $\kappa_1 > 0$ arbitrarily so that all the polynomials consisting of $\bigcup_{\sigma\in\mathcal{P}_1}\Gamma_\sigma$ $(= \Gamma_{\{1\}})$ are Hurwitz.

Step k $(k = 1,\ldots,m-1)$. For each $\sigma \in \mathcal{P}_{k+1}$ and each $\mathsf{k} \in \{\mathsf{a},\ldots,\mathsf{d}\}$, select $\underline{\kappa}_{\sigma\cup\{k+1\},\mathsf{k}} > 0$ such that with κ_i derived by the previous steps, the extreme polynomials $\gamma^\star_{\sigma\cup\{k+1\},\mathsf{k}}(s)$ in (14) are Hurwitz $\kappa_{k+1} > \underline{\kappa}_{\sigma\cup\{k+1\},\mathsf{k}} > 0$. Choose κ_{k+1} to satisfy (16).

Step m. With κ_i selected above, take $\kappa = [\kappa_1;\ldots;\kappa_m]$.

3.2 Representation to Byrnes-Isidori normal form

As a prerequisite for the DOB design, in this subsection we represent the augmented system (11) and the nominal model (6) into a Byrnes-Isidori normal form [20, Chapter 11].

For the former dynamics, we first introduce a variable

$$z_\sigma := \zeta - \frac{1}{G\Lambda_\sigma\kappa}H\Lambda_\sigma\kappa B_\nu^{\mathrm{T}}x = \zeta - \frac{1}{G\Lambda_\sigma\kappa}H\Lambda_\sigma\kappa x_\nu \in \mathbb{R}^{n-\nu}. \tag{17}$$

It is noted that with $\kappa_i > 0$,

$$0 < \min_{i\in\{1,\ldots,m\}}\{\underline{G}_i\kappa_i\} \leqslant G\Lambda_\sigma\kappa = \sum_{i\in\sigma}G_i\kappa_i \leqslant \sum_{i=1}^m\overline{G}_i\kappa_i \tag{18}$$

directly holds for all constant sets $\sigma \in \mathcal{P}_m$. This implies that (17) is well-defined for any $\sigma(t)$ in (4). Lemma 2 then says that for a time interval within which no additional fault occurs, the composite variable (x, z_σ) can serve as a new coordinate that represents (11) into a Byrnes-Isidori normal form.

Lemma 2. Let $\underline{T}$ and $\overline{T}$ be positive constants such that $\sigma(t)$ in (4) is fixed for $\underline{T} \leqslant t < \overline{T}$. Then during that period, the SISO system (11) is represented in the coordinate change (x, z_σ) with (17) as

$$\dot{x} = A_\nu x + B_\nu \left(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + \hat{G}_\sigma v + d_{\mathsf{x},\sigma}\right), \quad y = C_\nu x, \tag{19a}$$

$$\dot{z}_\sigma = \hat{S}_\sigma z_\sigma + \hat{M}_\sigma x + d_{\mathsf{z},\sigma}, \tag{19b}$$

where the matrices are defined as $\hat{\Phi}_\sigma := \Phi + (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)\Psi B_\nu^{\mathrm{T}}$, $\hat{\Psi}_\sigma := \Psi$, $\hat{G}_\sigma := G\Lambda_\sigma\kappa$, $\hat{S}_\sigma := S - (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)\Psi$, $\hat{M}_\sigma := M - (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)\Phi + (S - (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)\Psi)(H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)B_\nu^{\mathrm{T}}$, $\hat{H}_\sigma := H\Lambda_\sigma\kappa$, $d_{\mathsf{x},\sigma} := G(1-\Lambda_\sigma)u^\star_{\mathrm{flt}} + Gd$, and $d_{\mathsf{z},\sigma} := (H(1-\Lambda_\sigma) - (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)G(1-\Lambda_\sigma))u^\star_{\mathrm{flt}} + (H - (H\Lambda_\sigma\kappa/G\Lambda_\sigma\kappa)G)d$.

Proof. Lemma 2 is easily proved by differentiating $z_{\sigma(t)}(t) = z_\sigma(t)$ along with the (x,ζ)-dynamics (11), and we skip the details due to page limit.

Note that all the matrices and the external signals in the (x, z_σ)-dynamics are uncertain but bounded with known bounds, as those in the original dynamics (11) do. From this fact, one can find out some bounds for the uncertain parameters, uniformly on $\sigma \in \mathcal{P}_m$; in particular, let $\mathcal{D}_\mathsf{x} \subset \mathbb{R}$ and $\mathcal{D}_\mathsf{z} \subset \mathbb{R}$ be compact sets such that $d_{\mathsf{x},\sigma(t)}(t) \in \mathcal{D}_\mathsf{x}$, and $d_{\mathsf{z},\sigma(t)}(t) \in \mathcal{D}_\mathsf{z}$ for all $t \geqslant 0$ and all admissible $\sigma(t)$ in (4). On the other hand, we also observe that the set of the initial conditions $z_{\sigma(0)}(0) = z_{\{1,\ldots,m\}}(0)$ is bounded by $\{z_{\sigma(0)}(0) = \zeta(0) - (H\kappa/G\kappa)B_\nu^{\mathrm{T}}x(0) : \ \zeta(0) \in \mathcal{S}^0, \ x(0) \in \mathcal{X}^0\} \subset \mathcal{Z}^0$ for a σ-independent compact subset $\mathcal{Z}^0$ of $\mathbb{R}^{n-\nu}$.

A similar result can be derived for the nominal model (6). Indeed, in considering a coordinate $(x_\mathsf{n}, z_\mathsf{n})$ where

$$z_\mathsf{n} := \zeta_\mathsf{n} - \frac{1}{G_\mathsf{n}} H_\mathsf{n} B_\nu^{\mathrm{T}} x_\mathsf{n} = \zeta_\mathsf{n} - \frac{1}{G_\mathsf{n}} H_\mathsf{n} x_{\mathsf{n},\nu}, \tag{20}$$

one can readily express the nominal model (6) as the form

$$\dot{x}_\mathsf{n} = A_\nu x_\mathsf{n} + B_\nu \left(\hat{\Phi}_\mathsf{n} x_\mathsf{n} + \hat{\Psi}_\mathsf{n} z_\mathsf{n} + \hat{G}_\mathsf{n} v_\mathsf{n}\right), \quad y_\mathsf{n} = C_\nu x_\mathsf{n}, \tag{21a}$$

$$\dot{z}_\mathsf{n} = \hat{S}_\mathsf{n} z_\mathsf{n} + \hat{M}_\mathsf{n} x_\mathsf{n}, \tag{21b}$$

where $\hat{\Phi}_\mathsf{n} := \Phi_\mathsf{n} + (H_\mathsf{n}/G_\mathsf{n})\Psi_\mathsf{n} B_\nu^{\mathrm{T}}$, $\hat{\Psi}_\mathsf{n} := \Psi_\mathsf{n}$, $\hat{G}_\mathsf{n} := G_\mathsf{n}$, $\hat{S}_\mathsf{n} := S_\mathsf{n} - (H_\mathsf{n}/G_\mathsf{n})\Psi_\mathsf{n}$, and $\hat{M}_\mathsf{n} := M_\mathsf{n} - (H_\mathsf{n}/G_\mathsf{n})\Phi_\mathsf{n} + (S - (H_\mathsf{n}/G_\mathsf{n})\Psi_\mathsf{n})(H_\mathsf{n}/G_\mathsf{n})B_\nu^{\mathrm{T}}$. Similar to $\mathcal{Z}^0$, we denote the set of possible initial conditions $z_\mathsf{n}(0)$ for all $\zeta_\mathsf{n}(0) \in \mathcal{S}_\mathsf{n}^0$ and $x_\mathsf{n}(0) \in \mathcal{X}^0$ as $\mathcal{Z}_\mathsf{n}^0$.

We note in advance that the bounds for the plant (19) and the nominal model (21) will be utilized for the DOB design in Subsection 3.3.

3.3 Design of DOB-based controller

In this subsection, we complete the design of the FTC (8) by constructing a DOB-based controller for the augmented SISO system (11). As in other relevant studies, the DOB-based controller to be proposed here consists of two parts; baseline controller and DOB. Among them, the former part is designed by duplicating the existing structure (7) with y_n replaced by y as

$$\dot{c} = Ec + F(r - y), \quad v_{\mathrm{BL}} = Jc + K(r - y), \tag{22}$$

where $c \in \mathbb{R}^l$ is the state of (22) initiated in $\mathcal{C}_\mathsf{n}^0$. Hence, the remainder of this subsection is devoted to construct the DOB.

The first task for the DOB design is to choose a_i, $i = 0, \ldots, \nu - 1$, such that the transfer function

$$\frac{s^\nu + a_{\nu-1}s^{\nu-1} + \cdots + a_1 s + (\overline{G^\star}/\hat{G}_\mathsf{n})a_0}{s^\nu + a_{\nu-1}s^{\nu-1} + \cdots + a_1 s + (\underline{G^\star}/\hat{G}_\mathsf{n})a_0} \tag{23}$$

is strictly positive real [20, Chapter 6] where $\underline{G^\star} := \min\{\hat{G}_\mathsf{n}, \min_{i\in\{1,\ldots,m\}}\{\underline{G}_i\kappa_i\}\} > 0$ and $\overline{G^\star} := \max\{\hat{G}_\mathsf{n}, \sum_{i=1}^m \overline{G}_i\kappa_i\}$. For this, we recall that the transfer function in (23) is strictly positive real if and only if

(a) the denominator of (23) (i.e., $s^{\nu} + a_{\nu-1}s^{\nu-1} + \cdots + a_1 s + (\underline{G^\star}/\hat{G}_{\mathsf{n}})a_0$) is Hurwitz, and (b) the Nyquist plot of $a_0/(s^{\nu}+a_{\nu-1}s^{\nu-1}+\cdots+a_1 s)$ does not encounter the disk $\mathcal{D}(\underline{G^\star}/\hat{G}_{\mathsf{n}}, \overline{G^\star}/\hat{G}_{\mathsf{n}})$. Using this equivalence, a design guideline for a_i is presented as follows (for more detailed explanation, the readers are referred to [14]):

Design guideline for a_i.

Step 0. Choose a_i, $i = 1, \ldots, \nu - 1$, such that the polynomial $s^{\nu-1} + a_{\nu-1}s^{\nu-2} + \cdots + a_1$ is Hurwitz.

Step 1. Take $\overline{\mu}_1 > 0$ sufficiently small such that for all $\mu \in (0, \overline{\mu}_1)$, the polynomial $s^{\nu} + a_{\nu-1} + \cdots + a_1 s + \mu_1$ is Hurwitz. (The existence of such $\overline{\mu}_1$ is guaranteed by the root locus technique.)

Step 2. Select $\overline{\mu}_2 > 0$ satisfying that for all $\mu \in (0, \overline{\mu}_2)$, the Nyquist plot of $\mu/(s^{\nu}+a_{\nu-1}s^{\nu-1}+\cdots+a_1 s)$ does not encounter the disk $\mathcal{D}(\underline{G^\star}/\hat{G}_{\mathsf{n}}, \overline{G^\star}/\hat{G}_{\mathsf{n}})$.

Step 3. Take $a_0 < \min\{(\hat{G}_{\mathsf{n}}/\underline{G^\star})\overline{\mu}_1, \overline{\mu}_2\}$.

Next, based on the normal form expressions (19) and (21), we compute some compact sets in which the nominal state $(x_{\mathsf{n}}(t), z_{\mathsf{n}}(t), c_{\mathsf{n}}(t))$ and the partial actual state $z_{\sigma(t)}(t)$ are expected to remain during system operation. For the former variable, it is noted in advance that the nominal closed-loop system (7) and (21) will play a role as a (stable and time-invariant) reference model for the switched system (8) and (19), which experiences at most $m-1$ switches in the dynamics. To take into account the effect of switching dynamics, bounded sets for the nominal states $x(t)$, $z_{\mathsf{n}}(t)$, and $c_{\mathsf{n}}(t)$ are derived in a recursive way as follows (for initialization, let $\mathcal{X}^0_{\langle 0\rangle} := \mathcal{X}^0$, $\mathcal{Z}^0_{\mathsf{n},\langle 0\rangle} := \mathcal{Z}^0_{\mathsf{n}}$, and $\mathcal{C}^0_{\langle 0\rangle} := \mathcal{C}^0_{\mathsf{n}}$):

Design procedure for bounds of $(x_{\mathsf{n}}, z_{\mathsf{n}}, c_{\mathsf{n}})$.

Step j $(j = 0, \ldots, m-1)$**.** Select bounded sets $\mathcal{X}_{\langle j\rangle} \subset \mathbb{R}^{\nu}$, $\mathcal{Z}_{\mathsf{n},\langle j\rangle} \subset \mathbb{R}^{n-\nu}$, and $\mathcal{C}_{\mathsf{n},\langle j\rangle} \subset \mathbb{R}^{l}$ such that the solution $(x_{\mathsf{n}}(t), z_{\mathsf{n}}(t), c_{\mathsf{n}}(t))$ of (7) and (21) initiated in $\mathcal{X}^0_{\langle j\rangle} \times \mathcal{Z}^0_{\mathsf{n},\langle j\rangle} \times \mathcal{C}^0_{\mathsf{n},\langle j\rangle}$ belongs to $\mathcal{X}_{\langle j\rangle} \times \mathcal{Z}_{\mathsf{n},\langle j\rangle} \times \mathcal{C}_{\mathsf{n},\langle j\rangle}$ for all admissible reference signal r. Then take large compact sets $\overline{\mathcal{X}}_{\langle j\rangle}$, $\overline{\mathcal{Z}}_{\mathsf{n},\langle j\rangle}$, and $\overline{\mathcal{C}}_{\mathsf{n},\langle j\rangle}$ to satisfy

$$\mathcal{X}_{\langle j\rangle} \times \mathcal{Z}_{\mathsf{n},\langle j\rangle} \times \mathcal{C}_{\mathsf{n},\langle j\rangle} \overset{\epsilon/m}{\sqsubset} \overline{\mathcal{X}}_{\langle j\rangle} \times \overline{\mathcal{Z}}_{\mathsf{n},\langle j\rangle} \times \overline{\mathcal{C}}_{\mathsf{n},\langle j\rangle}, \tag{24}$$

where ϵ is given in problem of interest. Set $\mathcal{X}^0_{\langle j+1\rangle} := \overline{\mathcal{X}}_{\langle j\rangle}$, $\mathcal{Z}^0_{\mathsf{n},\langle j+1\rangle} := \overline{\mathcal{Z}}_{\mathsf{n},\langle j\rangle}$, and $\mathcal{C}^0_{\mathsf{n},\langle j+1\rangle} := \overline{\mathcal{C}}_{\mathsf{n},\langle j\rangle}$.

Step m**.** Take $\mathcal{X} := \mathcal{X}_{\langle m-1\rangle}$, $\overline{\mathcal{X}} := \overline{\mathcal{X}}_{\langle m-1\rangle}$, $\mathcal{Z}_{\mathsf{n}} := \mathcal{Z}_{\mathsf{n},\langle m-1\rangle}$, $\overline{\mathcal{Z}}_{\mathsf{n}} := \overline{\mathcal{Z}}_{\mathsf{n},\langle m-1\rangle}$, $\mathcal{C}_{\mathsf{n}} := \mathcal{C}_{\mathsf{n},\langle m-1\rangle}$, and $\overline{\mathcal{C}}_{\mathsf{n}} := \overline{\mathcal{C}}_{\mathsf{n},\langle m-1\rangle}$.

It will be seen in the next subsection that the nominal trajectory $(x_{\mathsf{n}}(t), z_{\mathsf{n}}(t), c_{\mathsf{n}}(t))$ of interest belongs to $\mathcal{X} \times \mathcal{Z}_{\mathsf{n}} \times \mathcal{C}_{\mathsf{n}}$, while a slightly larger set $\overline{\mathcal{X}} \times \overline{\mathcal{Z}}_{\mathsf{n}} \times \overline{\mathcal{C}}_{\mathsf{n}}$ will be used for the actual counterparts of the nominal state.

From now on, let us derive a bound for the partial state z_σ of the actual plant (19). The way of computing its bound is basically similar to that for the nominal state above, whereas the main difference comes from the fact that z_σ may jump at every failure moments. To take a look at the jumping behavior, we remark that the vector field of the (x, ζ)-dynamics (3) is piecewise continuous on t, by which the solution $(x(t), \zeta(t))$ is continuous on time [20, Chapter 3]. From the continuity of $\zeta(t)$, it follows that at each switching time $t = T \in \{T_1, \ldots, T_m\}$, $z_{\sigma(t)}(t)$ jumps from $\lim_{t\uparrow T} z_{\sigma(t)}(t)$ to

$$z_{\sigma(T)}(T) = \zeta(T) - \frac{H\Lambda_{\sigma(T)}\kappa}{G\Lambda_{\sigma(T)}\kappa} B_\nu^{\mathrm{T}} x(T) = \lim_{t\uparrow T} z_{\sigma(t)}(t) + \lim_{t\uparrow T} \frac{H\Lambda_{\sigma(t)}\kappa}{G\Lambda_{\sigma(t)}\kappa} B_\nu^{\mathrm{T}} x(t) - \frac{H\Lambda_{\sigma(T)}\kappa}{G\Lambda_{\sigma(T)}\kappa} B_\nu^{\mathrm{T}} x(T).$$

On the other hand, by the property of the Byrnes-Isidori normal form, the system (11) is of fault-invariant minimum phase if and only if the matrix $\hat{S}_\sigma$ in (19) is Hurwitz for all $\sigma \in \mathcal{P}_m$. Therefore with κ obtained by the proposed guideline and the external inputs x and $d_{\mathsf{x},\sigma}$ being bounded, the solution z_σ of (19b) must belong to a bounded region during the time period between two sequential failure moments. Summing up these findings, we compute a bound of z_σ as follows (let $\mathcal{Z}^0_{\langle 0\rangle} = \mathcal{Z}^0$ for initialization).

Design procedure for bound of z_σ.

Step j $(j = 0, \ldots, m-1)$**.** Take a compact set $\mathcal{Z}_{\langle j\rangle} \subset \mathbb{R}^{n-\nu}$ such that

$$\mathcal{Z}_{\langle j\rangle} \supset \Big\{ z_\sigma(t) \text{ of (19b) initiated in } \mathcal{Z}^0_{\langle j\rangle} : x \in \overline{\mathcal{X}},\ d_{\mathsf{z},\sigma} \in \mathcal{D}_{\mathsf{z}},\ \sigma \in \mathcal{P}_m \Big\}, \tag{25}$$

and choose a bounded set $\mathcal{Z}^0_{\langle j+1\rangle} \subset \mathbb{R}^{n-\nu}$ satisfying that

$$\mathcal{Z}^0_{\langle j+1\rangle} \supset \left\{ z + \frac{1}{\hat{G}_\sigma}\hat{H}_\sigma B_\nu^{\mathrm{T}} x - \frac{1}{\hat{G}_\mu}\hat{H}_\mu B_\nu^{\mathrm{T}} x : \ z \in \mathcal{Z}_{\langle j\rangle},\ x \in \overline{\mathcal{X}},\ \sigma,\mu \in \mathcal{P}_m \right\}.$$

Step m. Take $\mathcal{Z} := \mathcal{Z}_{\langle m-1\rangle}$.

To proceed, let us define a compact subset $\overline{\mathcal{W}} \sqsupset \mathcal{W}$ of $\mathbb{R}$ where

$$\begin{aligned}\mathcal{W} := \Bigg\{ &\frac{1}{\hat{G}_\sigma}\Big(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + \hat{G}_\sigma\left(Jc + K(r - C_\nu x)\right) + d_{\mathsf{x},\sigma} - \hat{\Phi}_{\mathsf{n}} x - \hat{\Psi}_{\mathsf{n}} z_{\mathsf{n}} - \hat{G}_{\mathsf{n}}\left(Jc + K(r - C_\nu x)\right)\Big) : \\ &x \in \overline{\mathcal{X}},\ z_\sigma \in \mathcal{Z},\ c \in \overline{\mathcal{C}}_{\mathsf{n}},\ d_{\mathsf{x},\sigma} \in \mathcal{D}_x,\ z_{\mathsf{n}} \in \overline{\mathcal{Z}}_{\mathsf{n}},\ \sigma \in \mathcal{P}_m \Bigg\}. \end{aligned} \tag{26}$$

Using the compact sets $\overline{\mathcal{W}}$ and $\overline{\mathcal{X}}$, we now design two saturation functions $\overline{s}_{\mathsf{w}} : \mathbb{R} \to \mathbb{R}$ and $\overline{s}_{\mathsf{x}} : \mathbb{R}^\nu \to \mathbb{R}^\nu$ that are continuously differentiable, bounded, and satisfy

$$\overline{s}_{\mathsf{w}}(w) = w \ \ \forall w \in \overline{\mathcal{W}},\ \ 0 \leqslant \frac{\mathrm{d}\overline{s}_{\mathsf{w}}}{\mathrm{d}w} \leqslant 1 \ \ \forall w \in \mathbb{R}, \ \ \text{and} \ \ \overline{s}_{\mathsf{x}}(q) = q \ \ \forall q \in \overline{\mathcal{X}},\ \ 0 \leqslant \frac{\mathrm{d}\overline{s}_{\mathsf{x}}}{\mathrm{d}q} \leqslant 1 \ \ \forall q \in \mathbb{R}^\nu. \tag{27}$$

Remark 2. Once the subset $\overline{\mathcal{W}} =: [\underline{w}, \overline{w}] \subset \mathbb{R}$ is determined, a simple candidate for $\overline{s}_{\mathsf{w}}(\cdot)$ is

$$\overline{s}_{\mathsf{w}}(w) = \begin{cases} \overline{p}(\overline{w}+\delta), & \text{if } \overline{w}+\delta \leqslant w, \\ \overline{p}(w), & \text{if } \overline{w} \leqslant w \leqslant \overline{w}+\delta, \\ w, & \text{if } \underline{w} \leqslant w \leqslant \overline{w}, \\ \underline{p}(w), & \text{if } \underline{w}-\delta \leqslant w \leqslant \underline{w}, \\ \underline{p}(\underline{w}-\delta), & \text{otherwise}, \end{cases}$$

where $\delta > 0$ can be any bounded value, and $\overline{p}(w)$ and $\underline{p}(w)$ are polynomials of w such that $\overline{p}'(\overline{w}) = 1$, $\overline{p}'(\overline{w}+\delta) = 0$, $\underline{p}'(\underline{w}) = 1$, and $\underline{p}'(\underline{w}-\delta) = 0$.

With the components presented above, we finally propose a DOB as

$$\dot{p} = \left(A_\nu - \underline{\Upsilon}^{-1}(\tau)\overline{\alpha} C_\nu\right) p + \frac{a_0}{\tau^\nu} B_\nu v, \tag{28a}$$

$$\dot{q} = \left(A_\nu - B_\nu \underline{\alpha}^{\mathrm{T}} \overline{\Upsilon}(\tau)^{-1}\right) q + \frac{a_0}{\tau^\nu} B_\nu y, \quad \dot{z}_{\mathsf{q}} = \hat{S}_{\mathsf{n}} z_{\mathsf{q}} + \hat{M}_{\mathsf{n}} \overline{s}_{\mathsf{x}}(q), \tag{28b}$$

$$v_{\mathrm{DOB}} = \overline{s}_{\mathsf{w}}\left(-C_\nu p + \frac{1}{\hat{G}_{\mathsf{n}}}\left(-\underline{\alpha}^{\mathrm{T}}\overline{\Upsilon}(\tau)^{-1} q + \frac{a_0}{\tau^\nu} y - \hat{\Phi}_{\mathsf{n}} q - \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}}\right)\right), \tag{28c}$$

where $\tau > 0$ is a sufficiently small design parameter that will be taken in Theorem 2, $\underline{\alpha} := [a_0; \ldots; a_{\nu-1}] \in \mathbb{R}^\nu$, $\overline{\alpha} := [a_{\nu-1}; \ldots; a_0] \in \mathbb{R}^\nu$, $\underline{\Upsilon}(\tau) := \mathrm{diag}(\tau, \tau^2, \ldots, \tau^\nu) \in \mathbb{R}^{\nu\times\nu}$, and $\overline{\Upsilon}(\tau) := \mathrm{diag}(\tau^\nu, \tau^{\nu-1}, \ldots, \tau) \in \mathbb{R}^{\nu\times\nu}$. The initial conditions $p(0)$ and $q(0)$ are taken arbitrarily to be contained in a compact set $\mathcal{F}^0$. On the other hand, for $z_{\mathsf{q}}(0)$ we take a restricted version $\underline{\mathcal{Z}}^0_{\mathsf{n}} \subset \mathcal{Z}^0_{\mathsf{n}}$ of $\mathcal{Z}^0_{\mathsf{n}}$ as a nonempty and bounded set such that $z_{\mathsf{q}}(0) - (H_{\mathsf{n}}/G_{\mathsf{n}})B_\nu^{\mathrm{T}} x_{\mathsf{n}}(0) \in \mathcal{S}^0_{\mathsf{n}}$ for all $z_{\mathsf{q}}(0) \in \underline{\mathcal{Z}}^0_{\mathsf{n}}$ and $x_{\mathsf{n}}(0) \in \mathcal{X}^0$. (Such nonempty $\underline{\mathcal{Z}}^0_{\mathsf{n}}$ always exists by the definition of $\mathcal{S}^0_{\mathsf{n}}$.) With the set, choose $z_{\mathsf{q}}(0) \in \underline{\mathcal{Z}}^0_{\mathsf{n}}$.

Remark 3. It can be readily seen that with $\overline{s}_{\mathsf{w}}$ and $\overline{s}_{\mathsf{x}}$ being inactive, the proposed DOB (28) becomes simplified into the conventional structure presented in literature, whose Q-filter has the form of a low-pass filter $Q(s) = a_0/((\tau s)^\nu + a_{\nu-1}(\tau s)^{\nu-1} + \cdots + a_1(\tau s) + a_0)$.

Summarizing the discussions so far, we construct the FTC (8) as the combination of the fixed CA law (10), the baseline controller (22), the DOB (28), and

$$v(t) = v_{\mathrm{BL}}(t) - v_{\mathrm{DOB}}(t). \tag{29}$$

The configuration of the overall system controlled by the proposed FTC is given in Figure 1.

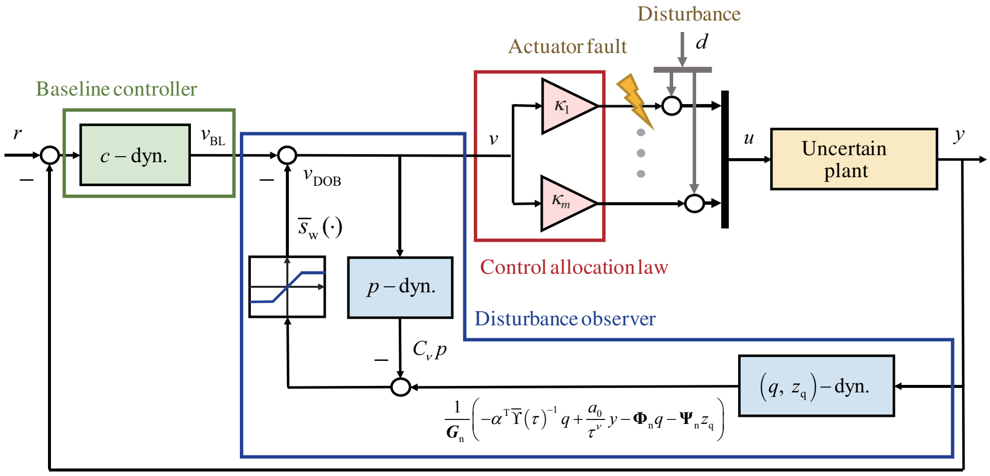

Figure 1 (Color online) Overall configuration of proposed DOB-based FTC consisting of input allocation law (10), baseline controller (22), and DOB (28).

4 Analysis of DOB-based FTC

This section aims to show that the proposed DOB-based FTC (10), (22), (28), and (29) with small τ solves the problem of interest, especially in the sense of Theorem 2.

Theorem 2. Suppose that Assumptions 1 and 2 hold. Then for given $\epsilon > 0$, there exists $\overline{\tau} > 0$ such that for all $\tau \in (0, \overline{\tau})$, the solution $(x(t), \zeta(t), z_{\mathsf{q}}(t), c(t), p(t), q(t))$ of the closed-loop system (3), (5), (10), (22), and (28), initiated in $\mathcal{X}^0 \times \mathcal{S}^0 \times \underline{\mathcal{Z}}_{\mathsf{n}}^0 \times \mathcal{C}^0 \times \mathcal{F}^0$, satisfies the following statements:

(a) $(x(t), \zeta(t)) \in \overline{\mathcal{X}} \times \overline{\mathcal{S}}_{\mathsf{n}}$ for all $t \geqslant 0$,

(b) $\|(x(t), z_{\mathsf{q}}(t), c(t)) - (x_{\mathsf{n}}^\star(t), z_{\mathsf{n}}^\star(t), c_{\mathsf{n}}^\star(t))\| < \epsilon, \quad \forall\, t \geqslant 0,$ (30)

where $(x_{\mathsf{n}}^\star(t), z_{\mathsf{n}}^\star(t), c_{\mathsf{n}}^\star(t))$ stands for the nominal state trajectory $(x_{\mathsf{n}}(t), z_{\mathsf{n}}(t), c_{\mathsf{n}}(t))$ of (6) and (7) initiated at $(x_{\mathsf{n}}(0), z_{\mathsf{n}}(0), c_{\mathsf{n}}(0)) = (x(0), z_{\mathsf{q}}(0), c(0)) \in \mathcal{X}^0 \times \mathcal{Z}_{\mathsf{n}}^0 \times \mathcal{C}_{\mathsf{n}}^0$.

We point out that in the coordinate $(x, \zeta_{\mathsf{q}}, c)$ with $\zeta_{\mathsf{q}} := z_{\mathsf{q}} + (H_{\mathsf{n}}/G_{\mathsf{n}})B_\nu^{\mathrm{T}} x_{\mathsf{n}}(0)$, the inequality (30) in Theorem 2 is rewritten by $\|(x(t), \zeta_{\mathsf{q}}(t), c(t)) - (x_{\mathsf{n}}^\star(t), \zeta_{\mathsf{n}}^\star(t), c_{\mathsf{n}}^\star(t))\| < \epsilon$ where $\zeta_{\mathsf{n}}^\star(t) := z_{\mathsf{n}}^\star(t) + (H_{\mathsf{n}}/G_{\mathsf{n}})B_\nu^{\mathrm{T}} x_{\mathsf{n}}^\star(t)$ initiated at $\zeta_{\mathsf{n}}^\star(0) = z_{\mathsf{n}}^\star(0) + (H_{\mathsf{n}}/G_{\mathsf{n}})B_\nu^{\mathrm{T}} x_{\mathsf{n}}^\star(0) = z_{\mathsf{q}}(0) + (H_{\mathsf{n}}/G_{\mathsf{n}})B_\nu^{\mathrm{T}} x(0) \in \mathcal{S}_{\mathsf{n}}^0$.

Theorem 2 will be proved in the following steps. First, with a coordinate transformation for p and q, we represent the overall system (together with the Byrnes-Isidori normal form (19) and (21)) into the standard singular perturbation form (Lemma 3). In particular, it will be seen that on the boundary layer in view of the singular perturbation theory, the (x, z_{q}, c)-dynamics, a part of the slow subsystem, behaves as the nominal closed-loop system (i.e., the $(x_{\mathsf{n}}, z_{\mathsf{n}}, c_{\mathsf{n}})$-dynamics (7) and (21)) for any patterns of actuator failure. Since the discontinuity on z_σ makes the singular perturbation theory inapplicable for the entire time period, we alternatively apply the Tickonov's theorem [17] to each subinterval of time between two sequential moments of failure (Lemma 4). By doing so, we will see that the actual state $(x(t), z_{\mathsf{q}}(t), c(t))$ could remain close to a nominal trajectory $(x_{\mathsf{n}}(t), z_{\mathsf{n}}(t), c_{\mathsf{n}}(t))$ at least for a while, even though the latter is not necessarily the same as $(x_{\mathsf{n}}^\star(t), z_{\mathsf{n}}^\star(t), c_{\mathsf{n}}^\star(t))$. Nonetheless, the difference between these nominal trajectories is negligible due to stability of the nominal closed-loop system, which concludes Theorem 2.

We begin the proof by representing the overall system into a singular perturbation form [18, 20].

Lemma 3. Let $\underline{T} > 0$ and $\overline{T} > 0$ be such that $\sigma(t)$ in (4) is constant for $\underline{T} \leqslant t < \overline{T}$. Then with the coordinate changes (17) and

$$\xi := \overline{\Upsilon}(\tau)^{-1}\left(\Pi(\tau)q - x\right) \quad \text{and} \quad \eta := \frac{1}{\tau}\underline{\Upsilon}(\tau)p + \frac{a_0}{\hat{G}_{\mathsf{n}}}\overline{\Upsilon}(\tau)^{-1}\left(\Pi(\tau)q - x\right) = \frac{1}{\tau}\underline{\Upsilon}(\tau)p + \frac{a_0}{\hat{G}_{\mathsf{n}}}\xi, \tag{31}$$

where

$$\Pi(\tau) := \frac{1}{a_0}\begin{bmatrix} a_0 & a_1\tau & \cdots & a_{\nu-1}\tau^{\nu-1} \\ 0 & a_0 & \ddots & \vdots \\ \vdots & & \ddots & a_1\tau \\ 0 & \cdots & & a_0 \end{bmatrix} \in \mathbb{R}^{\nu\times\nu}, \tag{32}$$

the closed-loop system (3), (5), (10), (22), (28), and (29) is transformed into a standard singular perturbation form for $\underline{T} \leqslant t < \overline{T}$, with respect to the perturbation parameter τ:

- Slow subsystems. The augmented plant (19), the baseline controller (22), and

$$\dot{z}_{\mathsf{q}} = \hat{S}_{\mathsf{n}} z_{\mathsf{q}} + \hat{M}_{\mathsf{n}} \bar{s}_{\mathsf{x}}\big(\Pi(\tau)^{-1}(x + \overline{\Upsilon}(\tau)\xi)\big). \tag{33}$$

- Fast subsystems.

$$\tau\dot{\xi} = (A_\nu - \overline{\alpha}C_\nu)\xi - B_\nu(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + \hat{G}_\sigma v + d_{\mathsf{x},\sigma}), \tag{34a}$$

$$\tau\dot{\eta} = (A_\nu - \overline{\alpha}C_\nu)\eta + a_0 B_\nu\left(\left(1 - \frac{\hat{G}_\sigma}{\hat{G}_{\mathsf{n}}}\right)v - \frac{1}{\hat{G}_{\mathsf{n}}}\left(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + d_{\mathsf{x},\sigma}\right)\right), \tag{34b}$$

where $v = Jc + K(r - C_\nu x) - \overline{s}_{\mathsf{w}}(-C_\nu\eta - (1/\hat{G}_{\mathsf{n}})(\hat{\Phi}_{\mathsf{n}}\Pi(\tau)^{-1}(x + \overline{\Upsilon}(\tau)\xi) + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}}))$.

Proof. Lemma 3 can be derived by straightforward computation.

To figure out the quasi-steady-state behavior of the singularly perturbed system (19), (22), (33), and (34), it is for now assumed that the slow variables σ, x, z_{q}, c, z_σ, $d_{\mathsf{x},\sigma}$, and $d_{\mathsf{z},\sigma}$ are frozen as $\sigma \in \mathcal{P}_m$, $(x, z_{\mathsf{q}}, c) \in \overline{\mathcal{X}} \times \mathcal{Z}_{\mathsf{n}} \times \mathcal{C}_{\mathsf{n}}$, $z_\sigma \in \mathcal{Z}$, $d_{\mathsf{x},\sigma} \in \mathcal{D}_{\mathsf{x}}$, and $d_{\mathsf{z},\sigma} \in \mathcal{D}_{\mathsf{z}}$. Under the hypothesis, we are interested in computing a (possibly σ-dependent) solution $\xi = \xi_\sigma^\star =: (\xi_{\sigma,1}^\star, \ldots, \xi_{\sigma,m}^\star)$ and $\eta = \eta_\sigma^\star =: (\eta_{\sigma,1}^\star, \ldots, \eta_{\sigma,m}^\star)$ of the following degenerating equation (which is obtained by putting $\tau = 0$ into the fast dynamics (34)):

$$\begin{aligned} 0 &= -a_{\nu-i}\xi_{\sigma,1}^\star + \xi_{\sigma,i+1}^\star, \quad \forall\, i = 1, \ldots, \nu-1, \\ 0 &= -a_0\xi_{\sigma,1}^\star - \left[\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + \hat{G}_\sigma\big(Jc + K(r - C_\nu x)\big) + d_{\mathsf{x},\sigma} - \hat{G}_\sigma \overline{s}_{\mathsf{w}}\left(-\eta_{\sigma,1}^\star - \frac{1}{\hat{G}_{\mathsf{n}}}(\hat{\Phi}_{\mathsf{n}} x + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}})\right)\right], \end{aligned} \tag{35a}$$

and

$$\begin{aligned} 0 =& - a_{\nu-i}\eta_{\sigma,1}^\star + \eta_{\sigma,i+1}^\star, \quad \forall\, i = 1, \ldots, \nu-1, \\ 0 =& - a_0\eta_{\sigma,1}^\star + a_0\Bigg[\left(1 - \frac{\hat{G}_\sigma}{\hat{G}_{\mathsf{n}}}\right)\big(Jc + K(r - C_\nu x)\big) - \left(1 - \frac{\hat{G}_\sigma}{\hat{G}_{\mathsf{n}}}\right)\overline{s}_{\mathsf{w}}\left(-\eta_{\sigma,1}^\star - \frac{1}{\hat{G}_{\mathsf{n}}}(\hat{\Phi}_{\mathsf{n}} x + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}})\right) \\ & - \frac{1}{\hat{G}_{\mathsf{n}}}(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + d_{\mathsf{x},\sigma})\Bigg], \end{aligned} \tag{35b}$$

(where $\Pi(0) = I$ and $\overline{\Upsilon}(0) = 0$ are used). It can be readily seen that the degenerating equation (35) admits a solution

$$\xi_{\sigma,1}^\star = -\frac{1}{a_0}\left(\hat{\Phi}_{\mathsf{n}} x + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}} + \hat{G}_{\mathsf{n}}\big(Jc + K(r - C_\nu x)\big)\right), \tag{36a}$$

$$\begin{aligned} \eta_{\sigma,1}^\star &= \left(\frac{\hat{G}_{\mathsf{n}}}{\hat{G}_\sigma} - 1\right)(Jc + K(r - C_\nu x)) + \left(\frac{\hat{G}_{\mathsf{n}}}{\hat{G}_\sigma} - 1\right)\frac{1}{\hat{G}_{\mathsf{n}}}(\hat{\Phi}_{\mathsf{n}} x + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}}) - \frac{1}{\hat{G}_\sigma}(\hat{\Phi}_\sigma x + \hat{\Psi}_\sigma z_\sigma + d_{\mathsf{x},\sigma}), \\ \xi_{\sigma,i}^\star &= 0 \ \text{ and } \ \eta_{\sigma,i}^\star = 0, \quad \forall i = 2, \ldots, \nu. \end{aligned} \tag{36b}$$

It should be emphasized that (36) is in fact the unique solution of (35), because the right hand-side of the last row of (35b) is a strictly decreasing function of $\eta_{\sigma,1}^\star$ by the property of $\overline{s}_{\mathsf{w}}$. In the computation of the solution, one may verify that with the slow variables frozen, the input of $\overline{s}_{\mathsf{w}}$,

$$w_\sigma := -\,\eta_{\sigma,1}^\star - \frac{1}{\hat{G}_{\mathsf{n}}}(\hat{\Phi}_{\mathsf{n}} x + \hat{\Psi}_{\mathsf{n}} z_{\mathsf{q}})$$

$$=\frac{1}{\hat{G}_\sigma}\left(\hat{\Phi}_\sigma x+\hat{\Psi}_\sigma z_\sigma+\hat{G}_\sigma\left(Jc+K(r-C_\nu x)\right)+d_{\mathsf{x},\sigma}-\hat{\Phi}_{\mathsf{n}}x-\hat{\Psi}_{\mathsf{n}}z_{\mathsf{q}}-\hat{G}_{\mathsf{n}}\left(Jc+K(r-C_\nu x)\right)\right) \quad (37)$$

belongs to $\mathcal{W}$ in (26) so that the saturation function is inactive.

It readily follows that on the boundary layer $(\xi,\eta)=(\xi^\star_\sigma,\eta^\star_\sigma)$ and with $\tau=0$, the singularly perturbed system (33) and (34) becomes reduced into the z_σ-dynamics (19b) and

$$\dot{x}=A_\nu x+B_\nu(\hat{\Phi}_{\mathsf{n}}x+\hat{\Psi}_{\mathsf{n}}z_{\mathsf{q}}+\hat{G}_{\mathsf{n}}(Jc+K(r-C_\nu x))), \quad (38a)$$

$$\dot{z}_{\mathsf{q}}=\hat{S}_{\mathsf{n}}z_{\mathsf{q}}+\hat{M}_{\mathsf{n}}x, \quad (38b)$$

$$\dot{c}=Ec+F(r-C_\nu x). \quad (38c)$$

It should be emphasize that (38a)–(38c) is decoupled from the remaining z_σ-dynamics, having exactly the same dynamics as the (σ-independent) stable nominal closed-loop system (7) and (21). Thus in the singular perturbation theoretic point of view, one may expect that the two trajectories $(x(t),z_{\mathsf{q}}(t),c(t))$ and $(x_{\mathsf{n}}(t),z_{\mathsf{n}}(t),c_{\mathsf{n}}(t))$ might be close to each other with small perturbation parameter τ. This is indeed the case for the subintervals of time between two sequential moments of failure, as in Lemma 4.

Lemma 4. Suppose that Assumptions 1 and 2 hold. Let $P=P^{\mathrm{T}}>0$ be the solution of the Lyapunov equation $P\Theta_{\mathsf{n}}+\Theta_{\mathsf{n}}^{\mathrm{T}}P=-I$ where Θ_{n} is the system matrix of the nominal closed-loop system (7) and (21). Then for given constant set $\sigma'\in\mathcal{P}_m$ and $\epsilon>0$, there exists $\overline{\tau}_{\sigma'}>0$ such that if

- $\sigma(t)=\sigma'$ for a time period $\underline{T}\leqslant t<\overline{T}$ satisfying $\overline{T}-\underline{T}>\Delta$;
- $(x(\underline{T}),z_{\mathsf{q}}(\underline{T}),c(\underline{T}))\in\mathcal{X}^0_{\langle j\rangle}\times\mathcal{Z}^0_{\mathsf{n},\langle j\rangle}\times\mathcal{C}^0_{\mathsf{n},\langle j\rangle}$, and $z_{\sigma'}(\underline{T})\in\mathcal{Z}^0_{\langle j\rangle}$ for some $j=0,\ldots,m-1$;
- $(p(\underline{T}),q(\underline{T}))\in\mathcal{F}$ where $\mathcal{F}\subset\mathbb{R}^{2\nu}$ is a bounded set independent of τ;

the state trajectory of the closed-loop system (3), (5), (10), (22), (28), and (29) satisfies the following statements for all $\tau\in(0,\overline{\tau}_{\sigma'})$:

(a) The partial state $(x(t),z_{\mathsf{q}}(t),c(t))$ belongs to $\overline{\mathcal{X}}_{\langle j\rangle}\times\overline{\mathcal{Z}}_{\mathsf{n},\langle j\rangle}\times\overline{\mathcal{C}}_{\mathsf{n},\langle j\rangle}$ for all $\underline{T}\leqslant t<\overline{T}$, and satisfies

$$\left\|\left(x(t),z_{\mathsf{q}}(t),c(t)\right)-\left(x_{\mathsf{n}}(t),z_{\mathsf{n}}(t),c_{\mathsf{n}}(t)\right)\right\|<\frac{\epsilon}{m}\sqrt{\frac{\underline{\lambda}(P)}{\overline{\lambda}(P)}}, \quad \forall\,\underline{T}\leqslant t<\overline{T}, \quad (39)$$

where $(x_{\mathsf{n}}(t),z_{\mathsf{n}}(t),c_{\mathsf{n}}(t))$ denotes the solution of the nominal closed-loop system (7) and (21) initiated at $(x_{\mathsf{n}}(\underline{T}),z_{\mathsf{n}}(\underline{T}),c_{\mathsf{n}}(\underline{T}))=(x(\underline{T}),z_{\mathsf{q}}(\underline{T}),c(\underline{T}))$;

(b) $z_{\sigma'}(t)$ remains in $\mathcal{Z}_{\langle j\rangle}$ for all $\underline{T}\leqslant t<\overline{T}$;

(c) There exists a τ-independent bounded set $\overline{\mathcal{F}}$ such that $(p(\overline{T}),q(\overline{T}))\in\overline{\mathcal{F}}$.

Proof. Because of page limit, we here briefly sketch the proof (especially for item (a)), while the detailed proof is similar to the studies in [14]. Roughly speaking, the lemma will be proved by applying the Tichonov's theorem [17] to the singularly perturbed form (19), (22), (33), and (34). It is clear that the reduced subsystem (38) is stable, because the internal dynamics (19) of the actual plant is stable by the selection of κ. We now investigate the stability of the fast subsystem (34), which is a requirement of the Tichonov's theorem [17]. To this end, by differentiating the error variables $\tilde{\xi}_{\sigma'}:=\xi-\xi_{\sigma'}$ and $\tilde{\eta}_{\sigma'}:=\eta-\eta_{\sigma'}$ with respect to a scaled time $\rho:=t/\tau$ (along with (34)) and by putting $\tau=0$ to the resulting equations (so that the slow variables are frozen in the time scale ρ), we obtain the boundary-layer system [20]

$$\frac{\mathrm{d}\tilde{\xi}_{\sigma'}}{\mathrm{d}\rho}=(A_\nu-\overline{\alpha}C_\nu)\tilde{\xi}_{\sigma'}+B_\nu\hat{G}_{\sigma'}\left(\overline{s}_{\mathsf{w}}\left(y_\eta+w_{\sigma'}\right)-w_{\sigma'}\right), \quad (40a)$$

$$\frac{\mathrm{d}\tilde{\eta}_{\sigma'}}{\mathrm{d}\rho}=(A_\nu-\overline{\alpha}C_\nu)\tilde{\eta}_{\sigma'}-a_0B_\nu u_\eta,\quad y_\eta:=-C_\nu\tilde{\eta}_{\sigma'},\quad u_\eta=-\left(\frac{\hat{G}_{\sigma'}}{\hat{G}_{\mathsf{n}}}-1\right)\left(\overline{s}_{\mathsf{w}}(y_\eta+w_{\sigma'})-w_{\sigma'}\right), \quad (40b)$$

where w_σ is defined in (37) and the origin is an equilibrium point of (40). We now claim that the origin of the boundary-layer system (40) is globally exponentially stable. Indeed, the transfer function of the linear subsystem (from u_η to y_η) in (40b) is given by $L(s):=a_0/(s^\nu+a_{\nu-1}s^{\nu-1}+\cdots+a_0)$, while the nonlinearity $(\hat{G}_{\sigma'}/\hat{G}_{\mathsf{n}}-1)(\overline{s}_{\mathsf{w}}(y_\eta+w_{\sigma'})-w_{\sigma'})$ in u_η belongs to the sector $[\underline{G^\star}/\hat{G}_{\mathsf{n}}-1,\overline{G^\star}/\hat{G}_{\mathsf{n}}-1]$.

Therefore, from the the circle criterion [20, Theorem 7.1] and the fact that

$$\frac{1+(\overline{G^\star}/\hat{G}_{\mathsf{n}}-1)L(s)}{1+(\underline{G^\star}/\hat{G}_{\mathsf{n}}-1)L(s)}=\frac{s^\nu+a_{\nu-1}s^{\nu-1}+\cdots+a_1s+(\overline{G^\star}/\hat{G}_{\mathsf{n}})a_0}{s^\nu+a_{\nu-1}s^{\nu-1}+\cdots+a_1s+(\underline{G^\star}/\hat{G}_{\mathsf{n}})a_0} \tag{41}$$

is strictly positive real, it is derived that the origin of the $\tilde{\eta}_{\sigma'}$-dynamics (40b) is globally exponentially stable. On the other hand, with the coefficients a_i of the strictly positive real transfer function (41) where $\underline{G^\star}/\hat{G}_{\mathsf{n}} \leqslant 1 \leqslant \overline{G^\star}/\hat{G}_{\mathsf{n}}$, it is clear that the polynomial $s^\nu+a_{\nu-1}s^{\nu-1}+\cdots+a_1s+a_0$ is Hurwitz (or equivalently, $A_\nu-\overline{\alpha}C_\nu$ is Hurwitz). This concludes the claim.

Here it should be pointed out that by definition, the initial value $(\xi(\underline{T}),\eta(\underline{T}))$ of the fast variables may diverges as τ goes to zero (this is the so-called peaking phenomenon [12]), by which the Tickonov's theorem cannot be directly applied to the time period $\underline{T}\leqslant t<\overline{T}$. To avoid this difficulty, with the help of the saturation functions $\overline{s}_{\mathsf{w}}$ and $\overline{s}_{\mathsf{x}}$ in (33), we take a small $0<\delta<\Delta/2$ such that the two trajectories $(x(t),z_{\mathsf{q}}(t),c(t))$ and $(x_{\mathsf{n}}(t),z_{\mathsf{n}}(t),c_{\mathsf{n}}(t))$ in item (a) remain close enough to each other for the transient period $\underline{T}\leqslant t\leqslant \underline{T}+\delta$, no matter how τ is selected. By analyzing the trajectory $(\xi(t),\eta(t))$ with $\tau<1$ for the transient period, it readily follows that $\|(\xi(\underline{T}+\delta),\eta(\underline{T}+\delta))-(\xi^\star_{\sigma'}(\underline{T}+\delta),\eta^\star_{\sigma'}(\underline{T}+\delta))\|\leqslant (k_1/\tau^\nu)\mathrm{e}^{-k_2(\delta/\tau)}$ with τ-independent constants k_1 and k_2. It is remarkable that the right hand-side of the inequality does not diverge but converges to zero as $\tau\to 0$. Thus, it is now possible to apply the Tickonov's theorem [17] to the truncated time period $\underline{T}+\delta\leqslant t<\overline{T}$, which brings item (a). Once item (a) is guaranteed, the proof of the remaining items is straightforward and therefore, it is omitted here.

To proceed, take $\overline{\tau}$ as $\overline{\tau}<\min_{\sigma'\in\mathcal{P}_m}\{\overline{\tau}_{\sigma'}\}$. In addition, denote the number of the actuator faults that occurs during system operation as $m_{\mathrm{flt}}\leqslant m-1$. Then the moments T_i of actuator faults, $i=1,\ldots,m$, can be rearranged in chronological order by $T_{\langle j\rangle}\in\{T_1,\ldots,T_m\}$, $j=1,\ldots,m$, to satisfy $T_{\langle 0\rangle}:=0< T_{\langle 1\rangle}<T_{\langle 2\rangle}<\cdots<T_{\langle m_{\mathrm{flt}}\rangle}<T_{\langle m_{\mathrm{flt}}+1\rangle}=\cdots=T_{\langle m-1\rangle}=T_{\langle m\rangle}=\infty$.

As the last step, we show that with $\tau<\overline{\tau}$, the distance between $(x(t),z_{\mathsf{q}}(t),c(t))$ and $\big(x^\star_{\mathsf{n}}(t),z^\star_{\mathsf{n}}(t),c^\star_{\mathsf{n}}(t)\big)$ is smaller than ϵ for each period $T_{\langle j\rangle}\leqslant t<T_{\langle j+1\rangle}$. For this, by repeating Lemma 4 iteratively during $T_{\langle j\rangle}\leqslant t<T_{\langle j+1\rangle}$, $j=0,\ldots,m_{\mathrm{flt}}$, we obtain

$$\big\|\big(x(t),z_{\mathsf{q}}(t),c(t)\big)-\big(x_{\mathsf{n},\langle j\rangle}(t),z_{\mathsf{n},\langle j\rangle}(t),c_{\mathsf{n},\langle j\rangle}(t)\big)\big\|<\frac{\epsilon}{m}\sqrt{\frac{\underline{\lambda}(P)}{\overline{\lambda}(P)}}\leqslant\frac{\epsilon}{m},\quad \forall\, T_{\langle j\rangle}\leqslant t<T_{\langle j+1\rangle}, \tag{42}$$

where $(x_{\mathsf{n},\langle j\rangle}(t),z_{\mathsf{n},\langle j\rangle}(t),c_{\mathsf{n},\langle j\rangle}(t))$ stands for the nominal trajectory $(x_{\mathsf{n}}(t),z_{\mathsf{n}}(t),c_{\mathsf{n}}(t))$ of (7) and (21) with the initial condition $(x_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),z_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),c_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}))=(x(T_{\langle j\rangle}),z_{\mathsf{q}}(T_{\langle j\rangle}),c(T_{\langle j\rangle}))$. (So the solution is defined for the truncated time period $T_{\langle j\rangle}\leqslant t<\infty$.) With a bundle of the nominal trajectories, we define error variables $\tilde{x}_{\mathsf{n},\langle j\rangle}:=x_{\mathsf{n},\langle j\rangle}-x_{\mathsf{n},\langle j-1\rangle}$, $\tilde{z}_{\mathsf{n},\langle j\rangle}:=z_{\mathsf{n},\langle j\rangle}-z_{\mathsf{n},\langle j-1\rangle}$, and $\tilde{c}_{\mathsf{n},\langle j\rangle}:=c_{\mathsf{n},\langle j\rangle}-c_{\mathsf{n},\langle j-1\rangle}$ for $j=1,\ldots,m_{\mathrm{flt}}$, whose dynamics is given by a stable and autonomous system

$$\big(\dot{\tilde{x}}_{\mathsf{n},\langle j\rangle},\dot{\tilde{z}}_{\mathsf{n},\langle j\rangle},\dot{\tilde{c}}_{\mathsf{n},\langle j\rangle}\big)=\Theta_{\mathsf{n}}\big(\tilde{x}_{\mathsf{n},\langle j\rangle},\tilde{z}_{\mathsf{n},\langle j\rangle},\tilde{c}_{\mathsf{n},\langle j\rangle}\big),\quad \forall\, T_{\langle j\rangle}\leqslant t<T_{\langle j+1\rangle}. \tag{43}$$

It is noted that since $(x_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),z_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),c_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}))=(x(T_{\langle j\rangle}),z_{\mathsf{q}}(T_{\langle j\rangle}),c(T_{\langle j\rangle}))$, the magnitude $\|(\tilde{x}_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),\tilde{z}_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),\tilde{c}_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}))\|$ of the initial condition of (43) is equal to

$$\big\|\big(x(T_{\langle j\rangle}),z_q(T_{\langle j\rangle}),c(T_{\langle j\rangle})\big)-\big(x_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),z_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle}),c_{\mathsf{n},\langle j\rangle}(T_{\langle j\rangle})\big)\big\|<\frac{\epsilon}{m}\sqrt{\frac{\underline{\lambda}(P)}{\overline{\lambda}(P)}}.$$

Now, we differentiate the Lyapunov function candidate $V_{\mathsf{n},\langle j\rangle}:=(\tilde{x}_{\mathsf{n},\langle j\rangle},\tilde{z}_{\mathsf{n},\langle j\rangle},\tilde{c}_{\mathsf{n},\langle j\rangle})^{\mathrm{T}}P(\tilde{x}_{\mathsf{n},\langle j\rangle},\tilde{z}_{\mathsf{n},\langle j\rangle},\tilde{c}_{\mathsf{n},\langle j\rangle})$ along with the j-th error dynamics (43), by which it is obtained that $\dot{V}_{\mathsf{n},\langle j\rangle}=-\|(\tilde{x}_{\mathsf{n},\langle j\rangle},\tilde{z}_{\mathsf{n},\langle j\rangle},\tilde{c}_{\mathsf{n},\langle j\rangle})\|^2 \leqslant-(1/\overline{\lambda}(P))V_{\mathsf{n},\langle j\rangle}$. Thus, the comparison lemma [20, Lemma 3.4] implies that

$$\big\|\big(\tilde{x}_{\mathsf{n},\langle j\rangle}(t),\tilde{z}_{\mathsf{n},\langle j\rangle}(t),\tilde{c}_{\mathsf{n},\langle j\rangle}(t)\big)\big\|\leqslant\sqrt{\frac{\overline{\lambda}(P)}{\underline{\lambda}(P)}}\mathrm{e}^{-t/(2\overline{\lambda}(P))}\big\|\big(\tilde{x}_{\mathsf{n},\langle j\rangle}(T_{\langle j-1\rangle}),\tilde{z}_{\mathsf{n},\langle j\rangle}(T_{\langle j-1\rangle}),\tilde{c}_{\mathsf{n},\langle j\rangle}(T_{\langle j-1\rangle})\big)\big\|<\frac{\epsilon}{m},$$

for $T_{\langle j\rangle}\leqslant t<T_{\langle j+1\rangle}$ and $j=1,\ldots,m_{\mathrm{flt}}$. It results from $(x_{\mathsf{n},\langle 0\rangle}(t),z_{\mathsf{n},\langle 0\rangle}(t),c_{\mathsf{n},\langle 0\rangle}(t))=(x^\star_{\mathsf{n}}(t),z^\star_{\mathsf{n}}(t),c^\star_{\mathsf{n}}(t))$, Young's inequality, and (42) that

$$\big\|\big(x(t),z_{\mathsf{q}}(t),c(t)\big)-\big(x^\star_{\mathsf{n}}(t),z^\star_{\mathsf{n}}(t),c^\star_{\mathsf{n}}(t)\big)\big\|$$

$$\leqslant \left\|\left(x(t), z_{\mathsf{q}}(t), c(t)\right) - \left(x_{\mathsf{n},\langle j\rangle}(t), z_{\mathsf{n},\langle j\rangle}(t), c_{\mathsf{n},\langle j\rangle}(t)\right)\right\| + \sum_{k=1}^{j} \left\|\left(\tilde{x}_{\mathsf{n},\langle k\rangle}(t), \tilde{z}_{\mathsf{n},\langle k\rangle}(t), \tilde{c}_{\mathsf{n},\langle k\rangle}(t)\right)\right\| < \epsilon,$$

for $T_{\langle j\rangle} \leqslant t < T_{\langle j+1\rangle}$ $(j = 0, \ldots, m_{\rm flt})$, which completes the proof of the theorem.

5 Simulation results: lateral control of Boeing 747

As an example, we take into account the 4-th order linearized lateral model of the Boeing 747 presented in [21, 22]. In particular, with unknown factors $W_{\mathsf{x},i} \in [0.7, 1.3]$ and $W_{\mathsf{u},i} \in [0.97, 1.03]$ that represents parametric uncertainty on the dimensional derivative of rolling moment, the system considered here is represented as (3) where $y = x \in \mathbb{R}$ is the yaw rate (rad/s), ζ_1 is the side-slip angle (rad), ζ_2 is the roll rate (rad/s), ζ_3 is the roll angle (rad), $u = (u_1, u_2, u_3) \in \mathbb{R}^3$ is the the control input (rad) that expresses the three rudder servos, and the matrices are defined as $\Phi = -0.115W_{\mathsf{x},1}$, $\Psi = [0.598W_{\mathsf{x},2}, -0.0318W_{\mathsf{x},3}, 0]$, $G = [0.4715W_{\mathsf{u},1}, 0.5W_{\mathsf{u},2}, 0.3W_{\mathsf{u},3}]$, and S, M, and H are known matrices with suitable dimensions. (Without loss of generality, we here multiply -1 into the original input matrix in [21] so that $G_i > 0$ holds.) We assume that $x(0) \in [-0.04, 0.04]$, $\zeta(0) \in [-0.004, 0.004] \times [-0.02, 0.02] \times [-0.015, 0.015]$, $\Delta \geqslant 10$ s, $\|d_i(t)\| \leqslant 0.005$, $\|\dot{d}_i(t)\| \leqslant 0.025$, and $\|u^\star_{\mathrm{flt},i}\| \leqslant 0.1$. The problem under consideration is to ensure the output $y(t)$ to track the reference signal $r(t) = 0.02\sin(0.2t)$ rad/s in the presence of both model uncertainty and actuator failures.

To address the problem, the proposed DOB-based FTC is constructed as follows. First, we take a nominal model (6) as the system with $S_{\mathsf{n}} = S$, $M_{\mathsf{n}} = M$, $N_{\mathsf{n}} = N_3$, $G_{\mathsf{n}} = G_3$, $\Phi_{\mathsf{n}} = \Phi$, and $\Psi_{\mathsf{n}} = \Psi$ where the uncertain parameters are set as $W_{\mathsf{x}} = W_{\mathsf{u}} = 0$. To achieve a satisfactory nominal tracking performance, the nominal controller (7) is designed as a proportional-integral (PI) controller with $E = 0$, $F = 1$, $J = K_{\rm int} := 17$, $K = K_{\rm prop} := 3.4$, and $c_{\mathsf{n}}(0) = 0$. Next, by the proposed design algorithm in Subsection 3.1, we set the gain κ of the input allocation law (10) as $\kappa = [1/3; 1/3; 1/3]$. After that, the DOB-based controller (22) and (28) is built up with $a_0 = 1$, $\tau = 0.04$, $c(0) = 0$, $p(0) = q(0) = 0$, $z_{\mathsf{q}}(0) = (0, 0, 0)$ and the saturation functions $\overline{s}_{\mathsf{w}}$ and $\overline{s}_{\mathsf{x}}$ obtained by $\overline{\mathcal{W}} = [-0.51, 0.51]$ and $\overline{\mathcal{X}} = [-0.041, 0.041]$.

For comparison, we perform the simulations with three types of controllers. The first two are the proposed DOB-based FTC and the PI controller with the input allocation law (10) (i.e., the proposed FTC without the DOB part). On the other hand, the last one is the adaptive FTC presented in [21], whose main purpose is to adjust the unknown parameters resulting from model uncertainty and actuator faults. In the following simulations, a measurement noise under uniform distribution enters the system whose maximum magnitude is 10^{-4} rad/s, while the input disturbances are set as $d_1(t) = 0.0035\sin(1.05t)$, $d_2(t) = 0.004\sin(2.1t)$, and $d_3(t) = 0.025\sin(4.5t)$. For the simulation, the uncertain parameters $W_{\mathsf{x},i}$ and $W_{\mathsf{u},i}$ are taken as 0.7 and 0.97, respectively.

Figures 2 and 3 show simulation results for the proposed FTC and the PI controller with different fault patterns. In particular, the simulation in Figure 2 is performed for the scenario when two lock-in-place actuator faults take place as: $u^\star_{\mathrm{flt},3} = u_3(T_3)$ for $T_3 := 50$ s and $u^\star_{\mathrm{flt},2} = u_2(T_2)$ for $T_2 := 100$ s. It is shown that unlike the PI control-based FTC, the proposed DOB-based FTC almost recovers the fault-free tracking performance for entire time period, in the presence of actuator faults as well as model uncertainty and external disturbance. A similar consequence can be found in Figure 3, in which the actuator faults have the floating form of $u^\star_{\mathrm{flt},3} = 0.06$ for $T_3 := 50$ s and of $u^\star_{\mathrm{flt},2} = 0.06$ for $T_2 := 100$ s, and additional output disturbance $d_{\mathsf{y}}(t) = 0.002\sin(4t)$ rad/s affects the system.

To compare the proposed FTC with the the adaptive FTC in [21], the simulation with the floating actuator faults is repeated in Figure 4. In the figure, one can observe that the adaptive FTC provides worse tracking performance than the proposed DOB-based FTC, in both transient and steady-state periods. This is mainly because the adaptive controller was constructed by taking the actuator faults into account only, and thus the persistent disturbance (of the sinusoidal form) may hinder the controller to adjust the uncertain parameters of the plant and the sudden actuator faults accurately. On the other hand, since the underlying principle of the proposed FTC is to compensate all the effect of the undesired

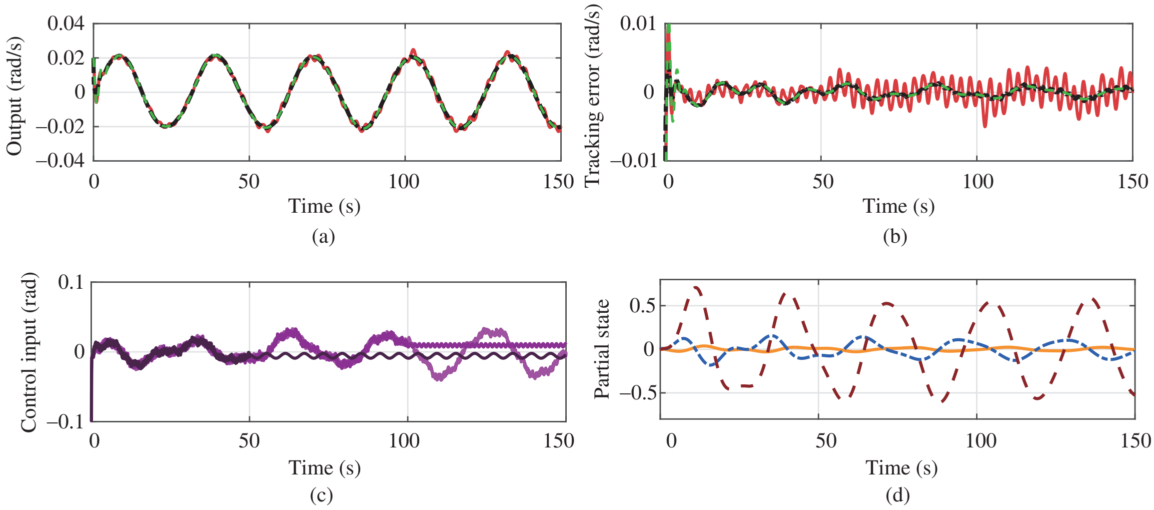

Figure 2 (Color online) Simulation results when two lock-in-place faults take place. (a) Output (rad/s): actual output $y(t)$ with (black dash-dotted) and without DOB (red solid), and nominal output $y_{\mathsf{n}}(t)$ (green dashed); (b) tracking error (rad/s): actual error $r(t)-y(t)$ with (black dash-dotted) and without DOB (red solid), and nominal error $r(t)-y_{\mathsf{n}}(t)$ (green dashed); (c) control input (rad) with the proposed FTC: $u_1(t)$ (darkest), $u_3(t)$ (intermediate), $u_2(t)$ (brightest); (d) partial state ζ with the proposed FTC: $\zeta_1(t)$ (rad/s) (yellow solid), $\zeta_2(t)$ (rad) (blue dash-dotted), $\zeta_3(t)$ (rad/s) (brown dashed).

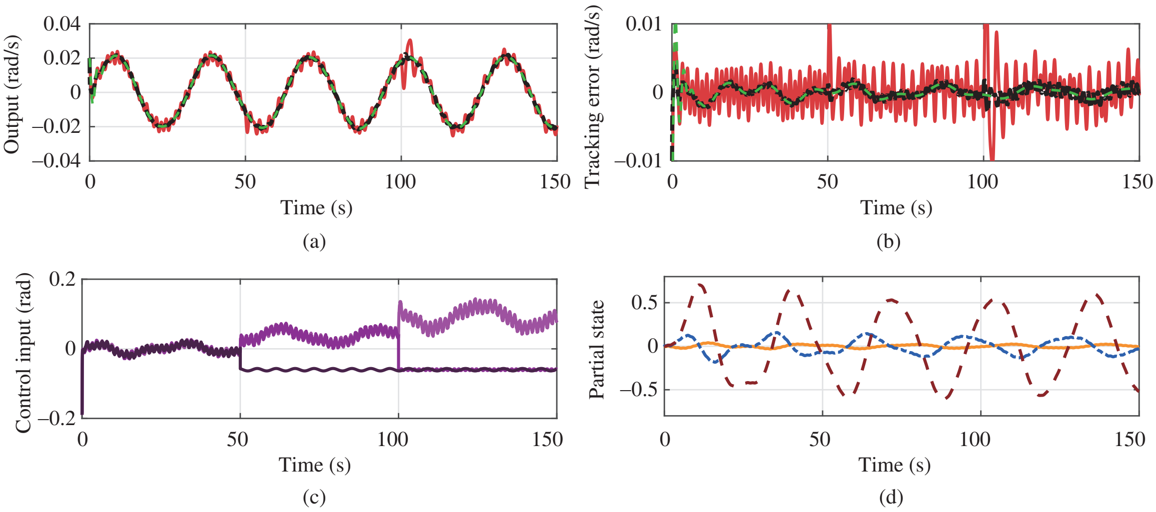

Figure 3 (Color online) Simulation results when two floating faults sequentially occur. (a) Output (rad/s): actual output $y(t)$ with (black dash-dotted) and without DOB (red solid), and nominal output $y_{\mathsf{n}}(t)$ (green dashed); (b) tracking error (rad/s): actual error $r(t)-y(t)$ with (black dash-dotted) and without DOB (red solid), and nominal error $r(t)-y_{\mathsf{n}}(t)$ (green dashed); (c) control input (rad): $u_1(t)$ (darkest), $u_3(t)$ (intermediate), $u_2(t)$ (brightest); (d) partial state ζ: $\zeta_1(t)$ (rad/s) (yellow solid), $\zeta_2(t)$ (rad) (blue dash-dotted), $\zeta_3(t)$ (rad/s) (brown dashed).

factors on the output $y(t)$ at once without explicit estimation of the fault, additional disturbance is not that problematic to the control performance, as seen in the simulation result.

6 Conclusion

In this paper we have proposed an output-feedback FTC for uncertain MISO plants with input redundancy, which are of minimum phase in an input-wise sense and that possibly suffers from actuator faults. In this paper we have employed a high-gain DOB-based controller as an FTC together with a CA law. By the proposed FTC, the actuator faults as well as model uncertainty and external disturbance are captured as a lumped form and then compensated quickly, by which fault-free tracking performance is

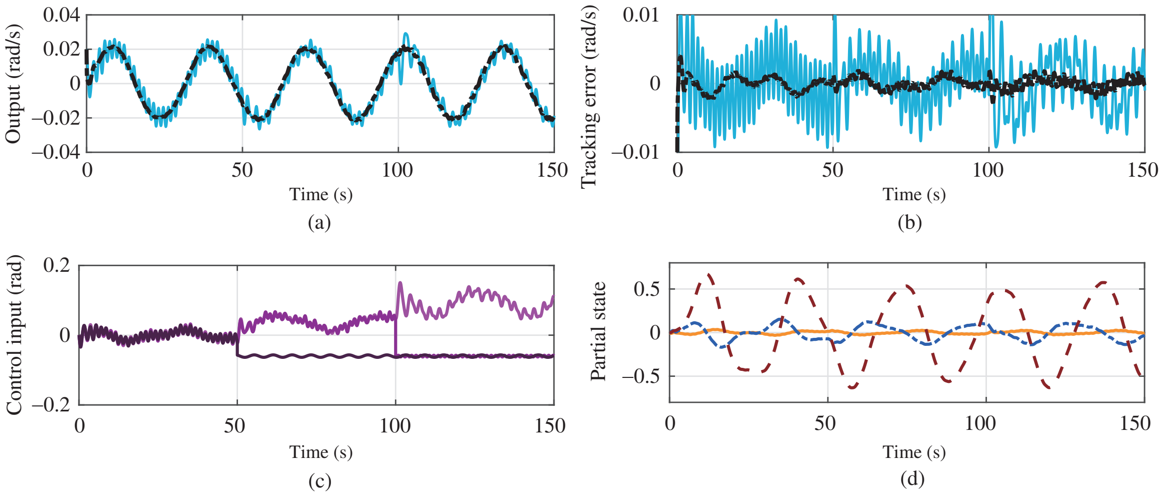

Figure 4 (Color online) Simulation results when two lock-in-place faults take place for comparison of the proposed FTC and the adaptive FTC in [21]. (a) Output (rad/s): actual output $y(t)$ with the proposed FTC (black dash-dotted) and the adaptive FTC in [21] (cyan solid); (b) tracking error (rad/s): actual error $r(t) - y(t)$ with the proposed FTC (black dash-dotted) and the adaptive FTC in [21] (cyan solid); (c) control input (rad) with the adaptive FTC in [21]: $u_1(t)$ (darkest), $u_3(t)$ (intermediate), $u_2(t)$ (brightest); (d) partial state ζ the adaptive FTC in [21]: $\zeta_1(t)$ (rad/s) (yellow solid), $\zeta_2(t)$ (rad) (blue dash-dotted), $\zeta_3(t)$ (rad/s) (brown dashed).

almost guaranteed during the entire operation. The performance of the proposed FTC has been proved from the perspective of the singular perturbation theory. Simulations for control of Boeing 747 have been presented to verify the validity of the proposed controller.

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