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Special Focus on Memristive Devices and Neuromorphic Computing

Morris-Lecar model of third-order barnacle muscle fiber is made of volatile memristors

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Dear editor,

Plenty of studies were introduced to understand the various properties of biological membranes. Hodgkin-Huxley (HH) axon model [1] is one among those studies which describes the initiation and propagation of an action potential in squid giant axon membrane. Recently, Ref. [2] claimed that according to the basic electrical circuit theory, the time-varying sodium and potassium conductance are actually time-invariant sodium and potassium generic memristors. Similarly, Morris and Lecar [3] developed a two-terminal electrical equivalent circuit model named Morris-Lecar (ML) model to reproduce the oscillations observed in calcium and potassium conductance in the giant barnacle muscle fiber.

This study asserts that the calcium and potassium ion-channel conductance in the original ML model are generic and volatile calcium and potassium ion-channel memristors which are verified via their respective frequency-dependent pinched hysteresis loops, power-off plot (POP) and dynamic route map (DRM).

Figure 1(a) shows the picture of Arthropod Barnacles which can be often seen on crabs, boats and rocks. Figure 1(b) depicts the two-terminal electrical equivalent circuit taken from [3]. Figure 1(c) shows the memristive third-order ML model. The ML model is defined by the following three differential state equations, in terms of three state variables V, M and N.

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} =& h_V(V,M,N;I)\\ =&[I - g_{\mathrm{L}}(V-E_{\mathrm{L}}) - g_{\mathrm{Ca}}M(V-E_{\mathrm{Ca}})\\ &- g_{\mathrm{K}}N(V-E_{\mathrm{K}})]/C_m, \end{aligned} \tag{1a}$$

$$\begin{aligned}\frac{\mathrm{d}M}{\mathrm{d}t} =& h_M(V,M;I)\\ =&\lambda_M(V)\,[M_\infty(V) - M] \triangleq h_M(M,V), \end{aligned} \tag{1b}$$

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}t} =& h_N(V,N;I)\\ =&\lambda_N(V)\,[N_\infty(V) - N] \triangleq h_N(N,V), \end{aligned} \tag{1c}$$

$$M_\infty(V) = \frac{1}{2}\left\{1+\tanh\left[\frac{V - V_1}{V_2}\right]\right\}, \tag{1d}$$

$$N_\infty(V) = \frac{1}{2}\left\{1+\tanh\left[\frac{V - V_3}{V_4}\right]\right\}, \tag{1e}$$

$$\lambda_M(V) = \bar{\lambda}_M \cosh\left[\frac{V - V_1}{2V_2}\right], \tag{1f}$$

$$\lambda_N(V) = \bar{\lambda}_N \cosh\left[\frac{V - V_3}{2V_4}\right]. \tag{1g}$$

The abbreviations of the parameters and the parameters values in (1) are given in Appendix A.

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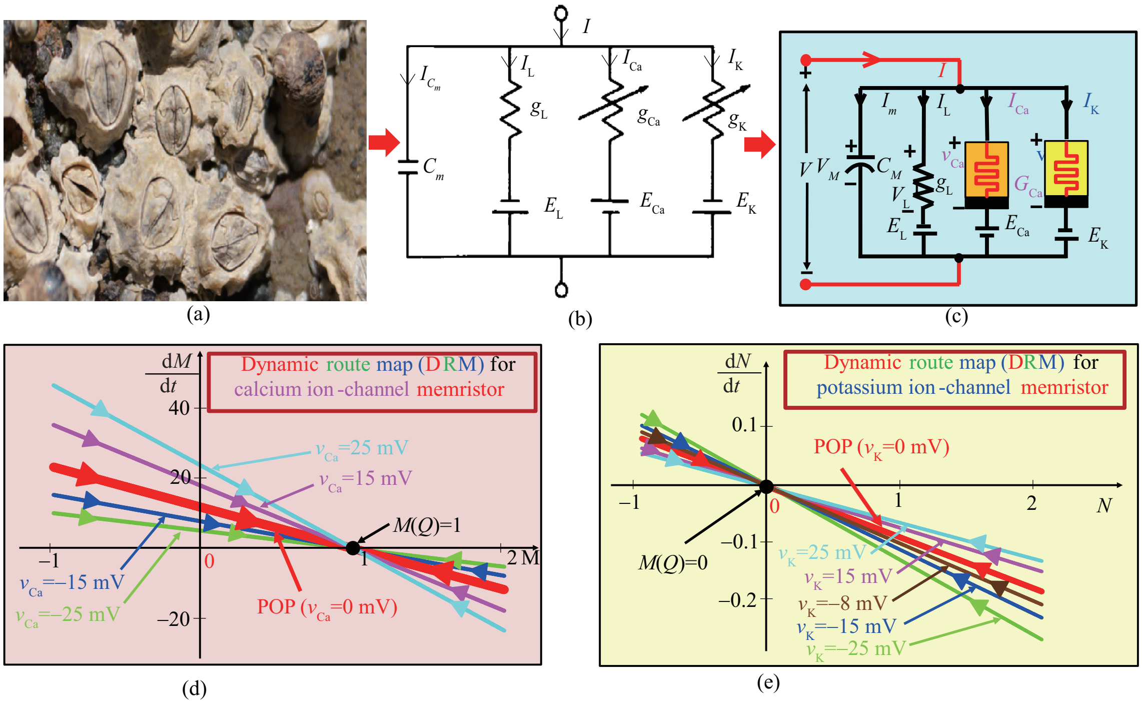

Figure 1 (Color online) (a) Arthropod barnacles (image taken from Wikipedia); (b) two-terminal electrical equivalent circuit of the original Morris-Lecar (ML) model taken from [3]; (c) memristive third-order Morris-Lecar (ML) circuit model; (d) DRM of calcium ion-channel memristor at $v_{\mathrm{Ca}} = -25, -15, 0, 15, 25$ mV, respectively; (e) DRM of potassium ion-channel memristor at $v_{\mathrm{K}} = -25, -15, -8, 0, 15, 25$ mV, respectively.

Since this model requires three differential equations, instead of the usual simplified version where only two state variables are chosen, it is called a third-order ML model.

Frequency-dependent pinched hysteresis loops of calcium and potassium ion-channel generic memristors. Rearrange the terms in 1(a) as

$$I = C_m \frac{\mathrm{d}V}{\mathrm{d}t} + g_{\mathrm{L}}(V - E_{\mathrm{L}}) + g_{\mathrm{Ca}}M(V - E_{\mathrm{Ca}}) + g_{\mathrm{K}}N(V - E_{\mathrm{K}}), \quad (2)$$

where the parameter values of g_L, g_{Ca}, g_{K}, E_L, E_{K}, E_{Ca}, respectively, are summarized in Table A1 in Appendix A. Let us rewrite the 3rd and the 4th term of (2) in terms of a voltage-controlled calcium and potassium ion-channel memristors [2], respectively.

$$i_{\mathrm{Ca}} = G_{\mathrm{Ca}}(M)v_{\mathrm{Ca}}, \quad (3a)$$

$$i_{\mathrm{K}} = G_{\mathrm{K}}(N)v_{\mathrm{K}}, \quad (3b)$$

where, $G_{\mathrm{Ca}}(M) = g_{\mathrm{Ca}}M$, $v_{\mathrm{Ca}} = V - E_{\mathrm{Ca}}$, $G_{\mathrm{K}}(N) = g_{\mathrm{K}}N$, $v_{\mathrm{K}} = V - E_{\mathrm{K}}$.

By applying a sinusoidal input to (3a) and (3b) at several frequencies, their frequency-dependent pinched hysteresis loops are obtained as shown in Appendix B (Figure B1(a) and (b)). Observe from Figure B1(a) and (b) in Appendix B that when $f > 40$ KHz (respectively, $f > 2$ KHz) for the calcium (respectively, potassium) ion-channel memristor, the pinched hysteresis loops of the calcium (respectively, potassium) ion-channel memristor tends to a straight line which confirms that both the calcium and potassium ion-channel memristors are generic memristors.

POP and DRM of calcium and potassium ion-channel generic memristors. The necessary and sufficient condition for a memristor to be volatile, or non-volatile, can be determined from POP [4].

The POP of the voltage-dependent calcium and potassium ion-channel generic memristor is obtained by plotting the power-off state equation $\mathrm{d}M/\mathrm{d}t|_{v_{\mathrm{Ca}}=0} = h_M(M, 0)$ and $\mathrm{d}N/\mathrm{d}t|_{v_{\mathrm{K}}=0} = h_N(N, 0)$ in the ($\mathrm{d}M/\mathrm{d}t$) vs. M plane, and in the ($\mathrm{d}N/\mathrm{d}t$) vs. N plane, respectively, for the calcium and potassium generic memristor as shown in red color in Figure 1(d) and (e), respectively.

The arrowheads pointing to the right on the POP in Figure 1(d) (respectively, Figure 1(e)) in the calcium (respectively, potassium) memristor indicates that $M(t)$ (respectively, $N(t)$) starting from any initial state $M(0) \neq M(Q)$ (respectively, $N(0) \neq N(Q)$) on the POP and above the M-axis (respectively, N-axis) must move towards east as long as $M(t)$ (respectively, $N(t)$) lies above the M-axis (respectively, N-axis). Conversely, the arrowheads pointing to the left on the POP in Figure 1(d) (respectively, Figure 1(e)) below the M-axis (respectively, N-axis) must move towards west as long as $M(t)$ (respectively, $N(t)$) lies below the M-axis (respectively, N-axis). The motion of the state variable M in Figure 1(d), and N in Figure 1(e), for constant applied DC voltage $v_{\rm Ca}$ in Figure 1(d) (respectively, $v_{\rm K}$ in Figure 1(e)), in the $({\rm d}M/{\rm d}t)$ vs. M plane (respectively, $({\rm d}N/{\rm d}t)$ vs. N plane) is called a dynamic route map (DRM). The DRM of the calcium and the potassium generic memristors is obtained by plotting $({\rm d}M/{\rm d}t)$ vs. M and $({\rm d}N/{\rm d}t)$ vs. N over the interval $-1 \leqslant M \leqslant 2$ and $-1 \leqslant N \leqslant 2$, respectively, for different values of $v_{\rm Ca} = -25, -15, 0, 15, 25$ mV and $v_{\rm K} = -25, -15, -8, 0, 15, 25$ mV, respectively, for the calcium and the potassium generic memristors as shown in Figure 1(d) and (e). Figure C1(a)–(d) in Appendix C shows the DRM plot of $({\rm d}M/{\rm d}t)$ vs. M and $({\rm d}N/{\rm d}t)$ vs. N over the interval $-100 \leqslant M \leqslant 200$ and $-100 \leqslant N \leqslant 200$, respectively, for different values of $v_{\rm Ca} = -25, -15, 0, 15, 25$ mV, and $v_{\rm K} = -25, -15, -8, 0, 15, 25$ mV, respectively. Observe from Figure 1(d) and (e) that the POP of the calcium and potassium generic memristors has only one stable equilibrium point located at $M(Q) = 1$, for -25 mV $\leqslant v_{\rm Ca} \leqslant 25$ mV and at $N(Q) = 0$, for -25 mV $\leqslant v_K \leqslant 25$ mV, respectively. Since, the POP of both the calcium and the potassium generic memristors has only one stable equilibrium point, which means that it forgets its past inputs when $M(t)$ reaches 1 (respectively, $N(t)$ reaches 0) in the ${\rm d}M/{\rm d}t|_{v_{\rm Ca}=0}$ vs. M plane, (respectively, ${\rm d}N/{\rm d}t|_{v_{\rm K}=0}$ vs. N plane), such memristors are called volatile memristors.

Conclusion. We confirmed that the time-varying calcium and potassium conductance in the original ML model are actually time-invariant generic memristors. We also presented a comprehensive analysis of the POP and DRM associated with the calcium and potassium generic memristors. Since the original ML model is made of calcium and potassium generic memristors and by analyzing the POP, we observe that there is only one stable equilibrium point for both the calcium and potassium generic memristors, which confirms that both the calcium and potassium generic memristors forget its past input signals when it reaches the stable equilibrium point, so we claim that the original ML model is made of volatile memristors.

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Supporting information Appendixes A–C. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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