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An optimization-based shared control framework with applications in multi-robot systems

Hao FANG, Chengsi SHANG & Jie CHEN*

School of Automation, Beijing Institute of Technology, Beijing 100081, *China*

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In recent years, researchers have been developing tools to allow human operators to work with multiple robots. To this end, results for autonomous systems can be helpful, e.g. controllability analysis [1], null-space approach [2], containment control [3]. On the other hand, results from teleoperation systems show that the shared control method is helpful in producing safe and efficient human-robot-interaction systems [4].

Among existing studies on shared control [5, 6], most approaches can be seen as variants under the policy blending framework (1),

$$u(t) = (1 - \lambda(t))u_{\mathrm{T}}(t) + \lambda(t)\mathcal{P}_{\mathrm{S}}(u_{\mathrm{H}}(t)), \qquad (1)$$

in which human input $u_{\mathrm{H}}(t)$ is first projected onto a safe/permissible space by $\mathcal{P}_{\mathrm{S}}$, then blended with an autonomous control signal $u_{\mathrm{T}}(t)$ by a weighting function $\lambda(t)$. However, this kind of methods does not explicitly address the control objectives of u_{H} and u_{T}, and a blend of actions as in (1) may be unsatisfactory for both sides. With such a potential defect, this method may not always be suitable for multi-agent systems, since control objectives for such systems, like formation adjustments and group separations, can be complex.

To solve this problem, the method in [7] is extended and an optimization based shared-control framework is proposed. In this framework, control objectives are expressed as cost functions and actions are computed by minimizing a blended cost. The framework is formulated using the model predictive control (MPC) method, which allows theoretical analysis on the stability of the closed-loop system. An experiment of one human operator controlling a four-robot system is also presented.

The shared control framework. In general, a shared control system tries to balance between a local task and a collaboration task. The local task considered here is to stabilize the system around the origin and the collaboration task is to comply with human control intentions.

For a system described by

$$x(k+1) = f(x(k), u(k)), \qquad (2)$$

the proposed controller framework consists of three components, a task model, a human model, and a task balancing algorithm, as shown in Figure 1. These components are formulated as follows.

First, the task model captures the local task for the autonomous controller. It consists of a pair of cost functions $l_{\mathrm{T}}(x, u)$ and $g_{\mathrm{T}}(x)$, a set of state and control constraint sets $\mathbb{X}$, $\mathbb{X}_f$ and $\mathbb{U}$, and a prediction window N, such that the local task is defined by the following optimization problem $\mathbb{P}_{\mathrm{T}}$:

$$\mathbb{P}_{\mathrm{T}}: \min_{\boldsymbol{z}\in\mathcal{Z}(x(k))} V_{\mathrm{T}}(\boldsymbol{z}) = \sum_{j=0}^{N-1} l_{\mathrm{T}}(z_j) + g_{\mathrm{T}}(x_N),$$

where $\mathbb{I}_n = \{0, \ldots, n\}$, $\boldsymbol{z} = \{z_j = (x_j, u_j)\}_{j\in\mathbb{I}_N}$ is a sequence of virtual control inputs and states, $l_{\mathrm{T}}(z_j)$ is a shorthand for $l_{\mathrm{T}}(x_j, u_j)$, and $\mathcal{Z}(\xi) = \{\boldsymbol{z}|x_0 = \xi; x_{j+1} = f(x_j, u_j), x_j \in \mathbb{X}, u_j \in \mathbb{U}, \forall j \in$

* Corresponding author (email: chenjie@bit.edu.cn)

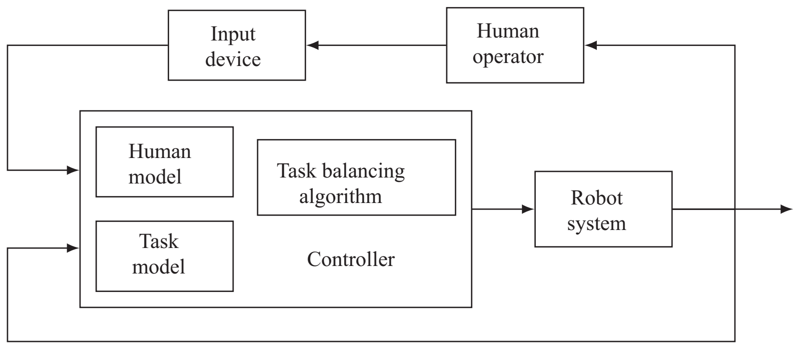

Figure 1 Structure of a shared control system.

$\mathbb{I}_N; x_N \in \mathbb{X}_f\}$ is the set of feasible solutions. The optimal cost $\min_{\boldsymbol{z}\in\mathcal{Z}(x)} V_{\mathrm{T}}(\boldsymbol{z})$ is denoted as $V_{\mathrm{T}}^*(x)$.

The MPC controller induced from $\mathbb{P}_{\mathrm{T}}$ is denoted as the nominal controller, and is assumed to be a stabilizing controller to the system in (2).

Second, the human model is designed to capture the collaboration task. It contains some algorithms to interpret human inputs such that at each time k, a reference signal $\boldsymbol{r}_k = \{r_{j|k}\}_{j\in\mathbb{I}_N}$ is generated for the shared control system. Furthermore, it contains a pair of collaboration costs $l_{\mathrm{H}}(x,u,r)$ and $g_{\mathrm{H}}(x,r)$ that penalizes deviation from such reference signals. The collaboration task is thus to minimize the following cost function:

$$V_{\mathrm{H}}(\boldsymbol{z}|\boldsymbol{r}) = \sum_{j=0}^{N-1} l_{\mathrm{H}}(z_j, r_{j|k}) + g_{\mathrm{H}}(x_N, r_{N|k}).$$

Third, the task balancing algorithm is an algorithm that at each time k, generates a weighting sequence $\boldsymbol{\lambda}_k = \{\lambda_{j|k}\}_{j\in\mathbb{I}_N}$ with $\lambda_{j|k} \in [0,1]$ to help balance the local task and the collaboration task.

Given these components, an MPC controller is constructed which, at each time k, solves the following optimization problem $\mathbb{P}$:

$$\mathbb{P}: \min_{\boldsymbol{z}\in\mathcal{Z}(x(k))} V(\boldsymbol{z}|\boldsymbol{r}_k, \boldsymbol{\lambda}_k) = \sum_{j=0}^{N-1} l_j(z_j) + g_N(x_N),$$

where the stage and terminal costs are defined as $l_j(z) = (1-\lambda_{j|k}) l_{\mathrm{T}}(z) + \lambda_{j|k} l_{\mathrm{H}}(z, r_{j|k})$ and $g_N(x) = (1-\lambda_{N|k}) g_{\mathrm{T}}(x) + \lambda_{j|k} g_{\mathrm{H}}(x, r_{N|k})$.

Let $\kappa_z(\boldsymbol{z})$ be a function that extracts u_0 from $\boldsymbol{z}$. The control law is then $u(k) = \kappa_z(\boldsymbol{z}_k^*)$ with $\boldsymbol{z}_k^* = \arg\min_{\boldsymbol{z}\in\mathcal{Z}(x(k))} V(\boldsymbol{z}|\boldsymbol{r}_k, \boldsymbol{\lambda}_k)$.

Given a stable nominal controller for the task model, it is desired that system designers can choose from a wide range of human models and task balancing algorithms without destabilizing the closed-loop system. In the following, a class of such models and algorithms is characterized.

Theoretical analysis. The stability of the shared control system is investigated under the following assumptions.

Assumption 1. $V_{\mathrm{T}}^*(x)$ is continuous, and there exists $\gamma, \alpha \in \mathcal{K}_\infty$ and an auxiliary controller κ_f, such that $\forall x \in \mathcal{X}_N$: $\gamma(\|x\|) \leqslant V_{\mathrm{T}}^*(x) \leqslant \alpha(\|x\|)$ and $\forall \boldsymbol{z} \in \mathcal{Z}(x)$, $\exists \boldsymbol{z}' \in \mathcal{Z}(f(x, \kappa_z(\boldsymbol{z}))$ satisfying that $V_{\mathrm{T}}(\boldsymbol{z}') - V_{\mathrm{T}}(\boldsymbol{z}) \leqslant -\gamma(\|x\|)$ [8].

Assumption 2. There exists a $\sigma \in \mathcal{K}$ such that $\forall x \in \mathcal{X}, u \in \mathcal{U}$: $0 \leqslant l_{\mathrm{H}}(x,u,r) \leqslant l_{\mathrm{T}}(x,u) + \sigma(\|r\|)$ and $0 \leqslant g_{\mathrm{H}}(x,r) \leqslant g_{\mathrm{T}}(x) + \sigma(\|r\|)$.

Assumption 3. The weighting function satisfies that $\exists \delta > 0$, $\exists \sigma_{\mathrm{M}} \in \mathcal{K}$ such that $\forall k$, $\|\boldsymbol{\lambda}_k\|_\infty \leqslant 1-\delta$ and $\sqrt{\mu(\|\boldsymbol{\lambda}_k\|_\infty)\alpha(\|x_k\|)} \leqslant \sigma_{\mathrm{M}}(\|\boldsymbol{r}_k\|_\infty)$, with $\mu(s) = s/(1-s)$ and α from Assumption 1.

We note that Assumption 1 is well-adopted in literature to design stable MPC systems; Assumption 2 in essence requires that l_{H}, g_{H} be in the same scale as l_{T} and g_{T} does; and Assumption 3 can be interpreted as that the local task should have more priority if the state error is large and the human reference signal is small.

The main result is as follows.

Theorem 1. If Assumptions 1 – 3 hold, the MPC controller that optimizes problem $\mathbb{P}$ renders the closed-loop system input-to-state stable (ISS).

Proof. At time k, denoting $\boldsymbol{z}^*$, $\boldsymbol{z}_{\mathrm{T}}^*$ respectively as the minimizer to $V(\cdot|\boldsymbol{r}_k)$, $V_{\mathrm{T}}(\cdot)$, let $\boldsymbol{z}'$ be the extended solution of $\boldsymbol{z}^*$ at time $k+1$ as in Assumption 1. Let $V_\Delta(\boldsymbol{z}|\boldsymbol{r}_k) = V(\boldsymbol{z}|\boldsymbol{r}_k) - V_{\mathrm{T}}(\boldsymbol{z})$, $\bar{\lambda} = \max_{j\in\mathbb{I}_N}\{\lambda_{j|k}\}$ and $\bar{r} = \max_{j\in\mathbb{I}_N}\{\|r_{j|k}\|\}$.

Along the state trajectory, from optimality and Assumption 1, we obtain that $V_{\mathrm{T}}^*(x_{k+1}) - V_{\mathrm{T}}^*(x_k) = V_{\mathrm{T}}^*(x_{k+1}) - V_{\mathrm{T}}(\boldsymbol{z}') + V_{\mathrm{T}}(\boldsymbol{z}') - V_{\mathrm{T}}(\boldsymbol{z}^*) + V_{\mathrm{T}}(\boldsymbol{z}^*) - V_{\mathrm{T}}(\boldsymbol{z}_{\mathrm{T}}^*) \leqslant -\gamma(\|x_k\|) + V_{\mathrm{T}}(\boldsymbol{z}^*) - V_{\mathrm{T}}(\boldsymbol{z}_{\mathrm{T}}^*)$.

Furthermore, from Assumption 2, we obtain that $V_{\mathrm{T}}(\boldsymbol{z}^*) - V_{\mathrm{T}}(\boldsymbol{z}_{\mathrm{T}}^*) = V(\boldsymbol{z}^*|\boldsymbol{r}_k) - V_{\mathrm{T}}(\boldsymbol{z}_{\mathrm{T}}^*) + V_{\mathrm{T}}(\boldsymbol{z}^*) - V(\boldsymbol{z}^*|\boldsymbol{r}_k) \leqslant V_\Delta(\boldsymbol{z}_{\mathrm{T}}^*|\boldsymbol{r}_k) - V_\Delta(\boldsymbol{z}^*|\boldsymbol{r}_k) \leqslant (N+1)\bar{\lambda}\sigma(\bar{r}) + \bar{\lambda} V_{\mathrm{T}}(\boldsymbol{z}_k^*)$.

Then, from Assumption 3, with $\sigma_r(s) = (\sigma_{\mathrm{M}} + (N+1)\sigma)(s)$ and $\sigma_\lambda(s) = \sqrt{\mu(s)} + \mu(s)$, we obtain that $V_{\mathrm{T}}^*(x_{k+1}) - V_{\mathrm{T}}^*(x_k) \leqslant -\gamma(\|x_k\|) + \sigma_\lambda(\bar{\lambda})\sigma_r(\bar{r})$.

V_{T}^* is thus a continuous ISS-Lyapunov function and the closed-loop system is ISS according to [9].

Experiment. An experiment was conducted where a human operator controlled the scaling and rotation of the formation of a four-robot system through a gamepad device. Details of the experiment are in the video attachments.

Conclusion. In this article, an optimization-based shared control framework was proposed. A class of collaboration cost functions was found, and they will not destabilize the shared control system. Experimental results demonstrated how this method can be used in facilitating interactions between human and multi-robot systems.

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