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Two performance enhanced control of flexible-link manipulator with system uncertainty and disturbances

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Abstract Precision control of flexible-link manipulator for space operation is challenging due to the dynamics coupling and system uncertainty. In this paper, to deal with system uncertainty and time-varying disturbance, two performance enhanced controller designs, named Composite Learning Control and Disturbance Observer Based Control are presented respectively. To overcome the nonminimum phase, using output redefinition, the dynamics is transformed to two subsystems: internal system and input-output system. For the internal dynamics, the PD (proportion differentiation) control is used with pole assignment. For the input-output subsystem, considering the unknown dynamics, the composite learning control is designed using neural modeling error while in case of disturbance, the disturbance observer based design is proposed. The stability analysis of the closed-loop system is presented via Lyapunov approach. Simulation of 2-degrees of freedom (DOF) flexible-link manipulator is conducted and the results show that the proposed methods can enhance the tracking performance.

Keywords flexible-link manipulator, composite learning control, disturbance observer, output redefinition, system uncertainty

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1 Introduction

Various advantages such as light weight, fast motion and low energy consumption, exist in flexible manipulators, which are widely used in many domains. For space operation, the manipulator could be served as maintenance and repair of the spacecraft with fault. As discussed in [1], space manipulator construction is preferred to be lightweight and long reach which increases the structural flexibility and causes vibrations. The control of flexible manipulators has received much attention among the research areas [2, 3].

In [4], the adaptive boundary control of a flexible manipulator is presented using time-scale decomposition. In [5], the control of flexible joint free-floating space manipulators is analyzed. In [1], the joint flexibilities are included in the dynamics. The nonlinear adaptive output feedback control is presented in [6] using boundary-layer control and quasi-steady-state control. In [7], the extended Kalman filter

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observer is used to estimate system states. In [8], the singular perturbation methodology is used to reduce the system order and the controller is with slow control and fast control. In [9], the impedance control of flexible mobile manipulator is analyzed using sliding mode law. In [10], a genetic algorithm -based hybrid fuzzy logic control strategy is designed for input tracking of the system. In [11], the output tracking controller for nonminimum phase systems of a causal reference trajectory is studied. In [12], the neural network (NN) based controller is proposed to control the tip position of flexible link manipulator. In [13], the NN based sliding mode control is applied to control the dynamics. In [14], NN is used for approximation to compensate the friction and experiment is conducted to show the performance.

From above-mentioned literature review, it is known that much progress has been achieved in topic of flexible manipulator control. However, with more specific on-orbit servicing application, high precision control has been put on agenda since the manipulator is promoted from cooperative target to non-cooperative target, which raises much challenge during the operation. As a result, the following two issues are required to be solved. In case of uncertainty, how can higher precision be achieved based on previous results? In case of disturbance, how to dynamically observe the time-varying information is also a key topic. In this paper, toward the mentioned problems, two performance enhanced control will be proposed for the flexible-link manipulator.

For system uncertainty, in [12] the NN based design is studied. It is noted that though NN is widely used for different applications such as hypersonic flight control [15, 16], robot control [17], marine vessel control [18], the issue how the NN is working as nonlinear function approximator, is not considered. Fortunately, in [19,20], the composite learning using modeling error is proposed which can greatly enhance the tracking precision. So in this paper, the first work is considering system uncertainty and the composite learning based control is designed for the dynamics. From literature review, there is not so much work on disturbance estimation based flexible-link manipulator control in case of disturbance. In this paper, we investigate the recent developed disturbance observer technique [21–23] to control the dynamics of flexible-link manipulator.

This paper is organized with the following structure. In Section 2, the dynamics of flexible-link manipulator is analyzed and the dynamics transformation is given. In Section 3, the PD control for internal dynamics is presented. For the control of nonlinear dynamics, composite learning control and disturbance observer based control two methods are designed in Section 4 and Section 5, respectively. The effectiveness of the proposed methods is verified via simulation in Section 6. Finally, Section 7 presents several comments and final remarks.

2 Dynamics model

The n DOF flexible-link manipulator's dynamics model [24] can be described as

$$M\begin{bmatrix}\ddot{\theta}\\ \ddot{\delta}\end{bmatrix}+\begin{bmatrix}S_1(\theta,\delta,\dot{\theta},\dot{\delta})\\ S_2(\theta,\delta,\dot{\theta},\dot{\delta})\end{bmatrix}+\begin{bmatrix}D_1 & 0\\ 0 & D_2\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \dot{\delta}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & K_2\end{bmatrix}\begin{bmatrix}\theta\\ \delta\end{bmatrix}=\begin{bmatrix}u\\ 0\end{bmatrix}+\begin{bmatrix}f_d\\ 0\end{bmatrix}, \tag{1}$$

where $M=\left[\begin{smallmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{smallmatrix}\right]$ is the positive definite symmetric inertia matrix, $S_1(\theta,\delta,\dot{\theta},\dot{\delta})$, $S_2(\theta,\delta,\dot{\theta},\dot{\delta})$ are Coriolis and centric fugal forces vectors, D_1 and D_2 are semi-definite damping matrices, K_2 is the stiffness matrix and u is the n-vector of joint torques.

The vectors $\theta\in\mathbb{R}^{n\times 1}$ and $\delta\in\mathbb{R}^{nm\times 1}$ are defined as

$$[\theta^{\mathrm{T}},\delta^{\mathrm{T}}]^{\mathrm{T}}=[\theta_1,\ldots,\theta_n,\delta_{1,1},\ldots,\delta_{1,m},\ldots,\delta_{n,1},\ldots,\delta_{n,m}]^{\mathrm{T}}, \tag{2}$$

where θ_i is the ith joint angle variable and $\delta_{i,j}$ is the ith link jth modal variable. In the range of operation, M is non-singular. So let $M^{-1}=\left[\begin{smallmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{smallmatrix}\right]$.

For simplicity, denote $S_1(\theta,\delta,\dot\theta,\dot\delta)$, $S_2(\theta,\delta,\dot\theta,\dot\delta)$ as S_1 and S_2. Then Eq. (1) can be written as

$$\begin{cases}\ddot\theta=-H_{11}(S_1+D_1\dot\theta)-H_{12}(S_2+D_2\dot\delta+K_2\delta)+H_{11}u+H_{11}f_d,\\ \ddot\delta=-H_{21}(S_1+D_1\dot\theta)-H_{22}(S_2+D_2\dot\delta+K_2\delta)+H_{21}u+H_{21}f_d.\end{cases}\tag{3}$$

To overcome the nonminimum phase, several approaches can be used. In [25–27], the tip position is output redefined with parameters α_i. In [28, 29], the principle of transmission zero assignment is used for output feedback control to achieve the nonminimum phase linear time-invariant system tracking via linearizing the model about an operation point. The redefinition of the output into slow and fast outputs in the context of integral manifold theory can be found in [30] where the singular perturbation parameter is employed for new variable definition.

In this paper, output redefinition is used:

$$y_i=\theta_i+\frac{\alpha_i}{l_i}\sum_{j=1}^{m}\phi_{i,j}\delta_{i,j},\tag{4}$$

where $\phi_{i,j}$ is the ith link jth modal function, l_i is the length of the ith link, $-1<\alpha_i<1$ and $\alpha_i=[\,\alpha_1,\alpha_2,\ldots,\alpha_n\,]^{\mathrm{T}}$ is respect to the output redefinition.

Rewrite (4) as

$$y=\theta+C\delta,\tag{5}$$

where $C=\begin{bmatrix}C_1 & & 0\\ & \ddots & \\ 0 & & C_n\end{bmatrix}\in\mathbb{R}^{n\times mn}$, $C_i=\frac{\alpha_i}{l_i}[\,\phi_{i,1}(l_i),\phi_{i,2}(l_i),\ldots,\phi_{i,m}(l_i)\,]$, $y=[y_1,\ldots,y_n]^{\mathrm{T}}$.

From (4), the following equation is obtained:

$$\ddot y=A(\alpha,\theta,\delta,\dot\theta,\dot\delta)+B(\alpha,\theta,\delta)u+d.\tag{6}$$

Denote $A(\alpha,\theta,\delta,\dot\theta,\dot\delta)$ and $B(\alpha,\theta,\delta)$ as A and B where

$$A=-(H_{11}+CH_{21})(S_1+D_1\dot\theta)-(H_{12}+CH_{22})(S_2+D_2\dot\delta+K_2\delta),\tag{7}$$

$$B=H_{11}+CH_{21},\tag{8}$$

$$d=(H_{11}+CH_{21})f_d.\tag{9}$$

Define $z=[\mu^{\mathrm{T}},\psi^{\mathrm{T}}]^{\mathrm{T}}$, where $\mu=[x_1^{\mathrm{T}},x_2^{\mathrm{T}}]^{\mathrm{T}}=[y^{\mathrm{T}},\dot y^{\mathrm{T}}]^{\mathrm{T}}$, $\psi=[\psi_1^{\mathrm{T}},\psi_2^{\mathrm{T}}]^{\mathrm{T}}=[\delta^{\mathrm{T}},\dot\delta^{\mathrm{T}}]^{\mathrm{T}}$. Then the dynamics is obtained as the input-output subsystem

$$\dot x_1=x_2,\quad \dot x_2=A+Bu_{\mathrm{ex}}+d,\tag{10}$$

and the internal dynamics

$$\dot\psi_1=\psi_2,\quad \dot\psi_2=E+Fu_{\mathrm{in}}+H_{21}f_d,\tag{11}$$

where $E=-H_{21}(S_1+D_1\dot\theta)-H_{22}(S_2+D_2\dot\delta+K_2\delta)$, $F=H_{21}$.

Remark 1. Using the methods from [31], the controller is with the following form

$$u=u_{\mathrm{ex}}+u_{\mathrm{in}},\tag{12}$$

where u_{ex} is the control input of input-output subsystem and u_{in} is the control input of internal dynamics subsystem. For more information, please refer to [31] while the design in this paper can be found in Sections 3 and 4.

3 Internal dynamics control

In order to stabilize the internal dynamics subsystem, the state feedback control is designed

$$u_{\mathrm{in}}=k_\delta\delta+k_{\dot\delta}\dot\delta,\tag{13}$$

where k_δ, $k_{\dot\delta}$ can be obtained via pole assignment method. Now the main focus is on the control of input-output subsystem.

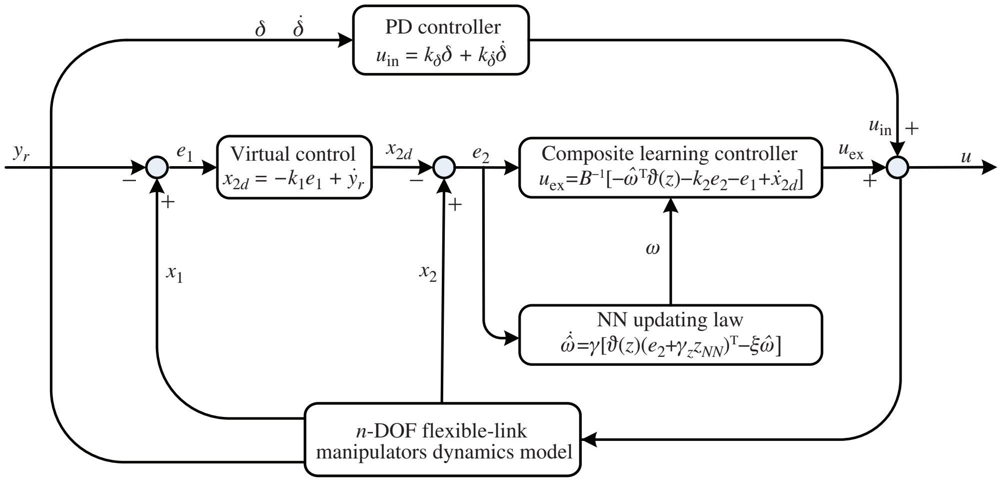

Figure 1 Composite learning control structure of flexible-link manipulator.

4 Composite learning control

4.1 Control goal

In this section, we consider the case there is no time varying disturbance which means $f_d = 0$. The control goal is to improve tracking accuracy using composite learning technique [19]. The control structure is shown in Figure 1. The main idea is to construct the neural modeling error and design the composite learning algorithm to improve the NN learning ability so that the tracking performance can be enhanced.

4.2 Controller design

Step 1: Define the tracking error

$$e_1 = x_1 - y_r. \tag{14}$$

Then the derivative of e_1 is obtained as

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_r. \tag{15}$$

Define the tracking error

$$e_2 = x_2 - x_{2d}, \tag{16}$$

and choose virtual control x_{2d} as

$$x_{2d} = -k_1 e_1 + \dot{y}_r, \tag{17}$$

where $k_1 \in \mathbb{R}^{n\times n}$ is the positive definite symmetric inertia matrix.

Furthermore, we know

$$\dot{x}_{2d} = -k_1 \dot{e}_1 + \ddot{y}_r = -k_1(\dot{x}_1 - \dot{y}_r) + \ddot{y}_r, \tag{18}$$

and

$$\dot{e}_1 = -k_1 e_1 + e_2. \tag{19}$$

Step 2: Using NN to approximate the unknown function A, we have

$$A = \omega^{\rm T}\vartheta(z) + \varepsilon, \tag{20}$$

where $\omega \in \mathbb{R}^{P\times n}$ is the optimal approximation weight vector, $\vartheta(\cdot) \in \mathbb{R}^{P\times 1}$ is a nonlinear vector function of the input, the components of $\vartheta(\cdot)$ are selected as Gaussian functions and ε is NN approximation error satisfying $\|\varepsilon\| \leqslant \|\varepsilon_m\|$.

Considering (10), we obtain

$$\dot{x}_2 = A + Bu_{\rm ex} = \omega^{\rm T}\vartheta + \varepsilon + Bu_{\rm ex}. \tag{21}$$

The control $u_{\rm ex}$ is designed as

$$u_{\rm ex} = B^{-1}\left[-\hat{\omega}^{\rm T}\vartheta - k_2 e_2 - e_1 + \dot{x}_{2d}\right], \tag{22}$$

where $\hat{\omega}$ is the estimation of ω, and $k_2 \in \mathbb{R}^{n\times n}$ is the positive definite symmetric inertia matrix.

From (10), the derivative of e_2 is obtained as follows:

$$\dot{e}_2 = A + Bu_{\rm ex} - \dot{x}_{2d} = \omega^{\rm T}\vartheta + \varepsilon + Bu_{\rm ex} - \dot{x}_{2d} = \tilde{\omega}^{\rm T}\vartheta + \varepsilon - k_2 e_2 - e_1, \tag{23}$$

where $\tilde{\omega} = \omega - \hat{\omega}$.

The prediction error is defined as

$$z_N = x_2 - \hat{x}_2, \tag{24}$$

where $\hat{x}_2$ is obtained from the serial-parallel estimation model

$$\dot{\hat{x}}_2 = \hat{\omega}^{\rm T}\vartheta + Bu_{\rm ex} + \beta z_N, \quad \hat{x}_2(0) = x_2(0), \tag{25}$$

where β is positive constant which is defined by user.

The derivative of modeling error can be written as

$$\dot{z}_N = \dot{x}_2 - \dot{\hat{x}}_2 = \tilde{\omega}^{\rm T}\vartheta + \varepsilon - \beta z_N. \tag{26}$$

Define $q = \tilde{\omega}^{\rm T}\vartheta$. Then we can obtain

$$\dot{z}_N^{\rm T} z_N = z_N^{\rm T}(q + \varepsilon) - \beta z_N^{\rm T} z_N. \tag{27}$$

Design NN updating law with z_N as below

$$\dot{\hat{\omega}} = \gamma\left[\vartheta(e_2 + \gamma_z z_N)^{\rm T} - \xi\hat{\omega}\right], \tag{28}$$

where γ, γ_z and ξ are positive design parameters.

4.3 Stability

Theorem 1. Consider system (10) with controller (22) and NN updating law (28). Then the tracking error and NN estimation error can be guaranteed to be bounded.

Proof. The Lyapunov function is chosen as

$$V = \frac{1}{2}\left(e_1^{\rm T} e_1 + e_2^{\rm T} e_2\right) + \frac{1}{2\gamma}{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega}\right) + \frac{1}{2}\gamma_z z_N^{\rm T} z_N. \tag{29}$$

Then the derivative of the Lyapunov function can be written as

$$\begin{aligned}\dot{V} &= e_1^{\rm T}\dot{e}_1 + e_2^{\rm T}\dot{e}_2 - \frac{1}{\gamma}{\rm tr}\left(\tilde{\omega}^{\rm T}\dot{\hat{\omega}}\right) + \gamma_z z_N^{\rm T}\dot{z}_N \\ &= -e_1^{\rm T} k_1 e_1 - e_2^{\rm T} k_2 e_2 + e_2^{\rm T}\varepsilon + \gamma_z z_N^{\rm T}\varepsilon - \gamma_z\beta z_N^{\rm T} z_N - \xi{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega} - \tilde{\omega}^{\rm T}\omega\right).\end{aligned} \tag{30}$$

Considering the following inequalities

$$e_2^{\rm T}\varepsilon \leqslant \frac{1}{2}e_2^{\rm T} e_2 + \frac{1}{2}\|\varepsilon\|^2, \quad z_N^{\rm T}\varepsilon \leqslant \frac{1}{2}z_N^{\rm T} z_N + \frac{1}{2}\|\varepsilon\|^2, \quad {\rm tr}\left(\tilde{\omega}^{\rm T}\omega\right) \leqslant \frac{1}{2}{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega}\right) + \frac{1}{2}{\rm tr}\left(\omega^{\rm T}\omega\right).$$

Then we have the following inequality

$$\begin{aligned}\dot{V} &\leqslant -e_1^{\rm T} k_1 e_1 - e_2^{\rm T} k_2 e_2 + \frac{1}{2}e_2^{\rm T} e_2 + \frac{1}{2}\|\varepsilon\|^2 + \frac{1}{2}\gamma_z z_N^{\rm T} z_N + \frac{1}{2}\gamma_z\|\varepsilon\|^2 \\ &\quad - \gamma_z\beta z_N^{\rm T} z_N + \frac{1}{2}\xi{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega}\right) + \frac{1}{2}\xi{\rm tr}\left(\omega^{\rm T}\omega\right) - \xi{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega}\right) \\ &\leqslant -e_1^{\rm T} k_1 e_1 - e_2^{\rm T} k_{20} e_2 - \beta_0\gamma_z z_N^{\rm T} z_N - \frac{1}{2}\xi{\rm tr}\left(\tilde{\omega}^{\rm T}\tilde{\omega}\right) + \chi,\end{aligned} \tag{31}$$

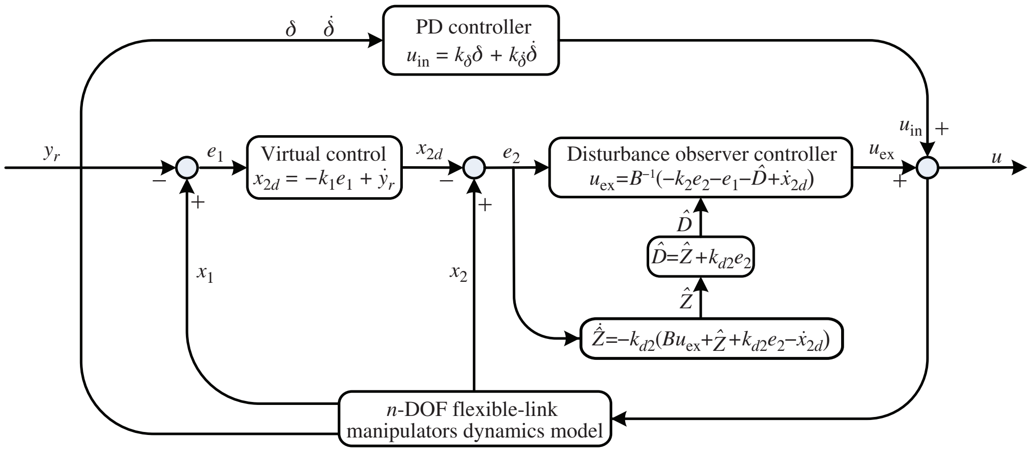

Figure 2 Disturbance observer based control structure of flexible-link manipulators.

where $k_{20} = k_2 - \frac{1}{2}$, $\beta_0 = \beta - \frac{1}{2}$, $\chi = \frac{1}{2}\|\varepsilon_m\|^2 + \frac{1}{2}\gamma_z\|\varepsilon_m\|^2 + \frac{1}{2}\xi\mathrm{tr}(\omega^{\mathrm{T}}\omega)$.

By selecting k_2, β to make k_{20} positive definite and $\beta_0 > 0$, then we have

$$\dot V \leqslant -\kappa V + \chi, \tag{32}$$

where

$$\kappa = \min\left[2\lambda_{\min}(k_1),\ 2\lambda_{\min}\left(k_2 - \frac{1}{2}\right),\ \xi,\ 2\left(\beta - \frac{1}{2}\right)\right]. \tag{33}$$

Furthermore, it can be obtained that

$$0 \leqslant V \leqslant \frac{\chi}{\kappa} + \left[V(0) - \frac{\chi}{\kappa}\right]\mathrm{e}^{-\kappa t}. \tag{34}$$

From (34), it is known that as $t \to \infty$, $V \to \frac{\chi}{\kappa}$. So all the signals included in the Lyapunov function (29) are bounded. This concludes the proof.

5 Disturbance observer based control

5.1 Control goal

In this section, time-varying disturbance exists in the dynamics. The control goal is to incorporate disturbance observer into the controller design so that the unknown effect could be efficiently compensated and better tracking performance is expected. The control structure is shown in Figure 2.

5.2 Controller design

Step 1: Similar to Step 1 in Subsection 4.2, with $e_1 = x_1 - y_r$, the virtual control x_{2d} is designed as

$$x_{2d} = -k_1e_1 + \dot y_r. \tag{35}$$

Furthermore, the derivative of x_{2d} can be calculated as

$$\dot x_{2d} = -k_1\dot e_1 + \ddot y_r = -k_1(x_2 - \dot y_r) + \ddot y_r. \tag{36}$$

Step 2: From (10), we have

$$\dot x_2 = A + Bu_{\rm ex} + d. \tag{37}$$

Define $D = A + d \in \mathbb{R}^{n\times 1}$ as compound disturbance and $e_2 = x_2 - x_{2d}$. Then the derivative of e_2 is written as

$$\dot e_2 = Bu_{\rm ex} + D - \dot x_{2d}. \tag{38}$$

The control signal $u_{\rm ex}$ is designed as

$$u_{\rm ex} = B^{-1}\big(-k_2 e_2 - e_1 - \hat{D} + \dot{x}_{2d}\big), \tag{39}$$

where $k_2 \in \mathbb{R}^{n\times n}$ is the positive definite symmetric inertia matrix, $\hat{D}$ is the estimation of D.
Step 3: Define $Z = D - k_{d2}e_2$ with k_{d2} as positive design parameter. The derivative of Z is obtained as

$$\dot{Z} = \dot{D} - k_{d2}\dot{e}_2 = \dot{D} - k_{d2}\left(Bu_{\rm ex} + D - \dot{x}_{2d}\right) = \dot{D} - k_{d2}\left(Bu_{\rm ex} + Z + k_{d2}e_2 - \dot{x}_{2d}\right). \tag{40}$$

Assumption 1. The compound disturbance D is bounded and its change rate $\dot{D}$ satisfies $\|\dot{D}\| \leqslant D_v$, where D_v is unknown positive constant.

Then the estimation of Z is proposed as

$$\dot{\hat{Z}} = -k_{d2}\big(Bu_{\rm ex} + \hat{Z} + k_{d2}e_2 - \dot{x}_{2d}\big). \tag{41}$$

The disturbance observer is designed as

$$\hat{D} = \hat{Z} + k_{d2}e_2. \tag{42}$$

Define $\tilde{Z} = Z - \hat{Z}$, $\tilde{D} = D - \hat{D}$. The following equality can be known

$$\tilde{Z} = \tilde{D}, \quad \dot{\tilde{Z}} = \dot{Z} - \dot{\hat{Z}} = \dot{D} - k_{d2}\tilde{Z}. \tag{43}$$

5.3 Stability

Theorem 2. Consider system (10) with controller (39) and disturbance observer design (42). Then the tracking error and disturbance estimation error can be guaranteed to be bounded.

Proof. The following Lyapunov function is considered

$$V = V_1 + V_2, \tag{44}$$

where

$$V_1 = \frac{1}{2}e_1^{\rm T}e_1, \tag{45}$$

$$V_2 = \frac{1}{2}e_2^{\rm T}e_2 + \frac{1}{2}\tilde{Z}^{\rm T}\tilde{Z}. \tag{46}$$

The derivative of V is calculated as

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2\\
&= e_1^{\rm T}\dot{e}_1 + e_2^{\rm T}\dot{e}_2 + \tilde{Z}^{\rm T}\dot{\tilde{Z}}\\
&= e_1^{\rm T}(e_2 - k_1e_1) + e_2^{\rm T}\big(-k_2e_2 - e_1 + \tilde{D}\big) + \tilde{Z}^{\rm T}\big(\dot{D} - k_{d2}\tilde{Z}\big)\\
&= -e_1^{\rm T}k_1e_1 - e_2^{\rm T}k_2e_2 + e_2^{\rm T}\tilde{D} - k_{d2}\tilde{Z}^{\rm T}\tilde{Z} + \tilde{Z}^{\rm T}\dot{D}\\
&\leqslant -e_1^{\rm T}k_1e_1 - e_2^{\rm T}k_2e_2 - k_{d2}\tilde{Z}^{\rm T}\tilde{Z} + \frac{1}{2}\left\|\tilde{Z}\right\|^2 + \frac{1}{2}\left\|\dot{D}\right\|^2\\
&= -e_1^{\rm T}k_1e_1 - e_2^{\rm T}\left(k_2 - \frac{1}{2}\right)e_2 - \left(k_{d2} - 1\right)\tilde{Z}^{\rm T}\tilde{Z} + \frac{1}{2}\left\|\dot{D}\right\|^2.
\end{aligned} \tag{47}$$

Define $L_1 = k_1$, $L_2 = k_2 - \frac{1}{2}$, $L_3 = k_{d2} - 1$, $\rho = \frac{1}{2}D_v^2$. When the selecting parameters satisfy the following conditions: $L_1 > 0$, $L_2 > 0$, $L_3 > 0$, with Assumption 1, Eq. (47) can be calculated as

$$\dot{V} \leqslant -\sigma V + \rho, \tag{48}$$

where $\sigma = 2\min\left[\lambda_{\min}(L_1), \lambda_{\min}(L_2), L_3\right]$. Then we have

$$0 \leqslant V \leqslant \frac{\rho}{\sigma} + \left[V(0) - \frac{\rho}{\sigma}\right]{\rm e}^{-\sigma t}. \tag{49}$$

From (49), it is known that as $t \to \infty$, $V \to \frac{\rho}{\sigma}$. So all the signals included in the Lyapunov function (44) are bounded. This concludes the proof.

Remark 2. Theoretical analysis indicates that as long as the disturbance is bounded and the derivative of the disturbance is bounded, the disturbance observer based method can be applied to guarantee the tracking performance shown in Theorem 2. In simulation, we employ sine disturbance as example to show the performance.

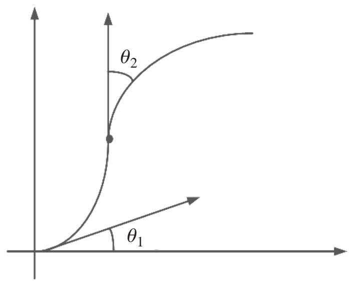

Figure 3 Structure of 2-DOF manipulators.

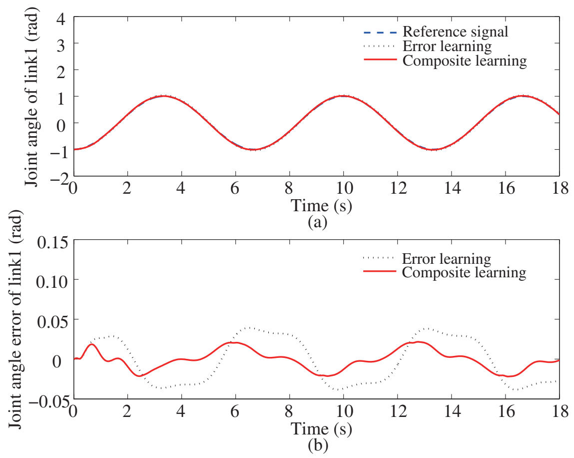

Figure 4 (Color online) Tracking response of link1.

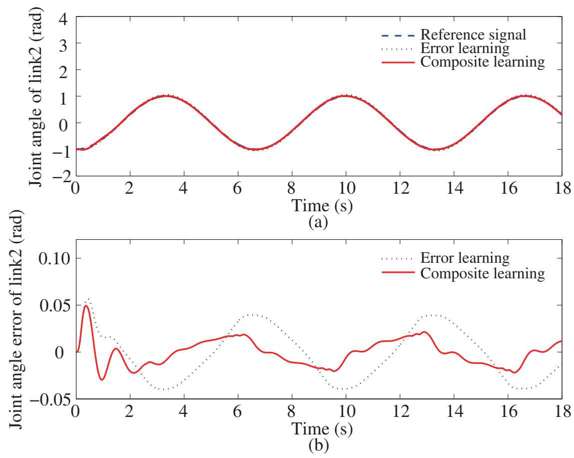

Figure 5 (Color online) Tracking response of link2.

6 Simulation

The structure of the 2-DOF flexible-link manipulator is shown in Figure 3. In order to verify the effectiveness of the control strategy, the simulation is performed in MATLAB software. The desired joint angle is set as $y_{r1} = -\cos(2\pi t)$, $y_{r2} = -\cos(2\pi t)$.

The length and the mass for each link are set as 0.5 m and 0.5 kg respectively while the constant flexural rigidity is set as 10 Nm^2. For more model information, please refer to [24]. The sampling period of the input-output subsystem $T_s = 0.01$ s, $\alpha = [0.9, 0.81]^{\mathrm{T}}$. For internal dynamics, we select $k_\delta = \begin{bmatrix} 0.1 & 0.1 & 0 & 0 \\ 0 & 0 & 10 & 10 \end{bmatrix}$, $k_{\dot{\delta}} = \begin{bmatrix} 0.6325 & 0.6325 & 0 & 0 \\ 0 & 0 & 6.3246 & 6.3246 \end{bmatrix}$.

Case 1. There is no disturbance where $f_d = 0$. To show the advantage of the control strategy, the comparison is conducted with the conventional NN learning [32, 33], where the NN updating law is whithout the modeling error. The learning method (28) in this paper is marked as "Composite Learning" while the conventional design is marked as "Error Learning". For the two-link manipulator, the matrix of $\hat{\omega}$ is with dimension 81×2. For each column, it is remarked as $\hat{\omega}_1 \in \mathbb{R}^{81\times 1}$ and $\hat{\omega}_2 \in \mathbb{R}^{81\times 1}$ where $\hat{\omega} = [\hat{\omega}_1\ \hat{\omega}_2]$.

The parameters of the controller are selected as $k_1 = \begin{bmatrix} 6 & 0 \\ 0 & 60 \end{bmatrix}$, $k_2 = \begin{bmatrix} 1 & 0 \\ 0 & 30 \end{bmatrix}$, $\beta = 10$, $\gamma = 0.5$, $\gamma_z = 10$, $\xi = 0.2$. The simulation results are shown in Figures 4–7.

Figures 4 and 5 show the tracking response of link1 and link2 and from the tracking error, the proposed method obtains better tracking performance with higher accuracy. From the NN weights updating depicted in Figure 7, the response of NN for the two methods is quite different since the composite learning based design is using additional NN modeling error. The control inputs are shown in Figure 6.

Case 2. In this case, the time-varying disturbance is considered and in simulation it is set as $f_d = 0.5 + 0.5\sin t$. The parameters of the controller are selected as $k_1 = \begin{bmatrix} 1 & 0 \\ 0 & 20 \end{bmatrix}$, $k_2 = \begin{bmatrix} 4 & 0 \\ 0 & 40 \end{bmatrix}$, $k_{d2} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$. For the disturbance estimation, we denote $\hat{D} = \left[\hat{D}_1, \hat{D}_2\right]$. The simulation results are shown in Figures 8–11.

The tracking performance of link1 and link2 is shown in Figures 8 and 9. It is obvious that with

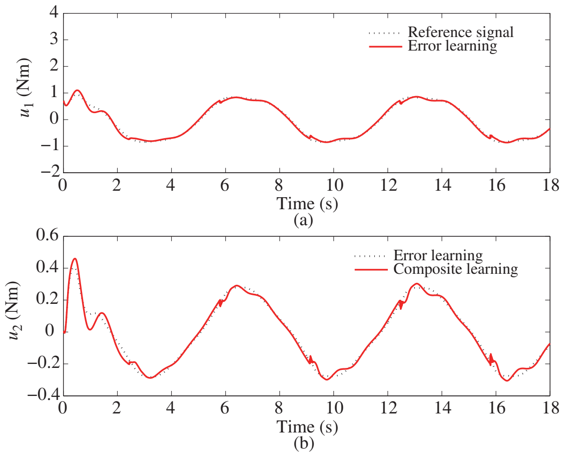

Figure 6 (Color online) Control input.

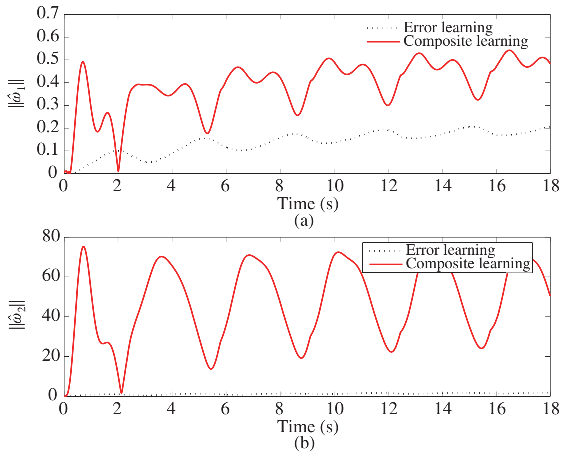

Figure 7 (Color online) NN updating law.

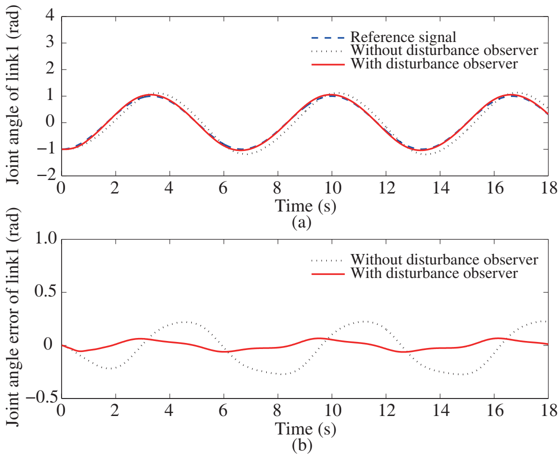

Figure 8 (Color online) Tracking response of link1.

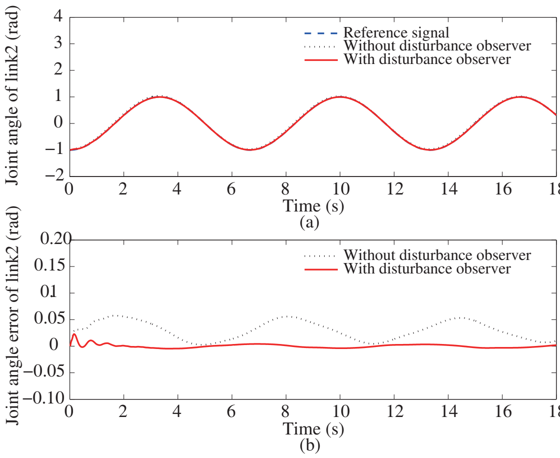

Figure 9 (Color online) Tracking response of link2.

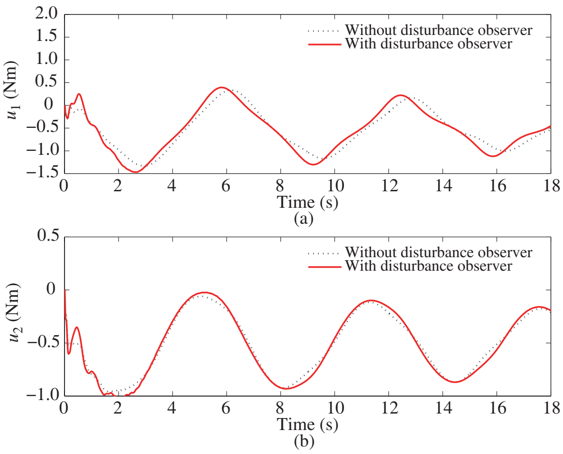

Figure 10 (Color online) Control input.

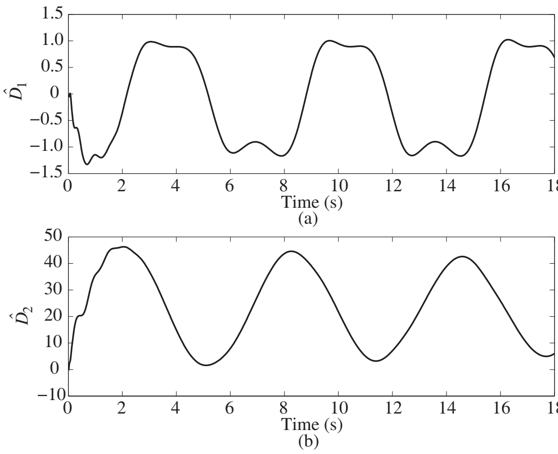

Figure 11 Disturbance estimation.

the disturbance observer, the performance in case of time-varying disturbance is greatly improved with smaller tracking error. The response of control inputs can be found in Figure 10 while the disturbance estimation is shown in Figure 11 where the information obtained from disturbance observer is with the same trend of the time-varying disturbance.

7 Conclusion and future work

In this paper, two performance enhanced controller designs are presented for flexible-link manipulator. Using output redefinition, the dynamics is transformed to two subsystems. For the internal dynamics, the PD control is used with pole assignment. For the input-output subsystem, considering the unknown dynamics, the composite learning control is designed using neural modeling error. In case of disturbance, the disturbance observer based design is proposed. The stability analysis of the closed-loop system is presented for the two approaches. Simulation is conducted and the results show that the methods can enhance the tracking performance. For future work, we will implement the approaches on ground system test in National Key Laboratory of Aerospace Flight Dynamics, Northwestern Polytechnical University while the improvement of NN learning and disturbance estimation will be the key step to enhance the tracking performance. The possible solution [34] can be adopted for more efficient learning in case of unknown system dynamics and time-varying disturbance.

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Conflict of interest The authors declare that they have no conflict of interest.

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