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Resource allocation in OFDMA heterogeneous networks for maximizing weighted sum energy efficiency

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Abstract In this paper, a resource allocation algorithm for maximizing the weighted sum energy efficiency (EE) is investigated in orthogonal frequency division multiple access (OFDMA) heterogeneous networks (HetNets). We aim to balance the EE of macro cell and low power nodes by subchannel and power allocations. We formulate the problem as a nonlinear sum-of-ratios programming issue, and guarantee data rate requirements of users by using minimum rate constraints. Due to the nonconvexity of the problem, we develop a heuristic subchannel assignment algorithm, and then solve the power allocation problem by parameterized transformations and a first-order approximation based on an iterative algorithm. Numerical results illustrate the convergence and the effectiveness of the proposed algorithm.

Keywords resource allocation, energy efficiency, OFDMA, heterogeneous network, interference management

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1 Introduction

Recently, the information and communication technology (ICT) industry is responsible for about 3%–5% of the worldwide energy consumption, where the mobile communication system is one of the major contributors [1]. The energy-efficient design in wireless networks has therefore attracted increasing attention. On the other hand, heterogeneous network (HetNet) has been promoted as a key technology to provide higher throughput and wider coverage, which consists of various radio access nodes such as high power base stations (BSs) (e.g., macro or micro BSs) and low power BSs (e.g. pico, femto or relay stations) [2]. Also, HetNet can improve system energy efficiency (EE), since the users serviced by the low power BSs generally suffer lower path losses than macrocell users [3, 4]. However, the low power BSs embedded in the conventional macro-cellular networks may lead to cross-tier interferences, resulting in more complex interference management and resource allocation.

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The issue of radio resource allocation in HetNets has been investigated extensively, especially for orthogonal frequency division multiple access (OFDMA) multi-tier cellular network. Aiming to maximize the network capacity, some effective resource allocation and interference management schemes have been proposed. In [5], a fractional frequency reuse and power control method is proposed to coordinate the interference and maximize the long term log-scale throughput in HetNet. In [6], a resource allocation scheme for spectrum-sharing femtocell is investigated, where an interference temperature limit is introduced to protect the macro users from cross-tier interference. In [7], a joint of subchannel and power allocation is studied for maximizing the sum-rate of femto users with a total throughput guarantee of macro users. In [8], three distributed resource allocation approaches using different mathematical models for the heterogeneous multi-tier 5G network are presented. However, most of these works focus merely on achieving higher system throughput or spectral efficiency, ignoring the EE.

For energy-aware HetNet strategies, Ref. [9] minimizes the total energy consumption of the cellular system while satisfying users' data-rate requirements. Based on the high fluctuations in traffic demand over space and time in HetNets, Refs. [10–12] investigate the BS sleep strategies as well as the partial spectrum reuse scheme and the BS cooperation methods to reduce the energy cost. In [13], bit-per-Joule as a metric of EE is introduced to evaluate the system performance, where the authors consider a heterogeneous cloud radio access network architecture and then propose a soft fractional frequency reuse scheme. In [14], a game-theoretic power control algorithm for the multisource multirelay cooperative communication systems has been proposed, where the quality of service (QoS) constraint is formulated as an energy-efficient utility function. In [15], an interference migration strategy is proposed to deal with the interference non-uniform distribution phenomenon for maximizing EE. In [16–18], the energy-efficient user scheduling and power allocation are investigated in homogeneous and heterogeneous cellular networks. However, we note that different types of BSs have different roles to play in cellular networks, may be deployed in different scenarios and may have different EE requirements. The macro BSs are mainly used to provide larger coverage range while the low power BSs are deployed to achieve higher data rate and EE. Therefore, the weighted sum EE [19], may be a more appropriate EE criterion for the HetNets than the existing EE criteria used in [9, 13, 15–18].

In this paper, we propose a resource allocation strategy in OFDMA HetNets for maximizing the weighted sum EE. Such schemes can allow different BSs to have different EE weight factors, and can also balance the EE between macro BS and low power BSs. We formulate the optimization problem as a nonlinear sum-of-ratios programming issue [20], which is a mixed nonconvex problem and hard to be solved in general. Also, we consider the minimum data rate requirement of each user, making the problem more complex. To make the problem tractable, we first present a heuristic subchannel allocation algorithm to maximize the weighted sum-EE, and then solve the power allocation problem by some transformations and an iterative algorithm. Numerical results indicate that the convergence of proposed algorithm and the tradeoff between the macro BS and low power BSs.

The rest of this paper is organized as follows. In Section 2, we introduce the system model and formulate the optimization problem. Section 3 presents the solution of the optimization problem and provides an energy-efficient resource allocation algorithm. Section 4 describes the simulation and numerical results. Section 5 concludes the paper.

2 System model and problem formulation

Consider the downlink of a two-tier cellular network shown in Figure 1, where M pico BSs are overlaid on a macrocell with universal frequency reuse. Denote the set of all BSs as $\mathcal{M} = \{0, 1, \dots, M\}$, where the macro BS is indexed by 0. For the pico BSs, closed access is assumed. That is, only subscribing users can connect to the pico BSs. Denote the set of users associated with BS m as $\mathcal{K}_m$, and $\mathcal{K} = \mathcal{K}_0 \cup \cdots \cup \mathcal{K}_M$. The system bandwidth is divided into N OFDMA subchannels, and we denote the set of subchannels as $\mathcal{N}$.

The marcocell user equipments (MUEs) suffer cross-tier interference, while the picocell user equipments

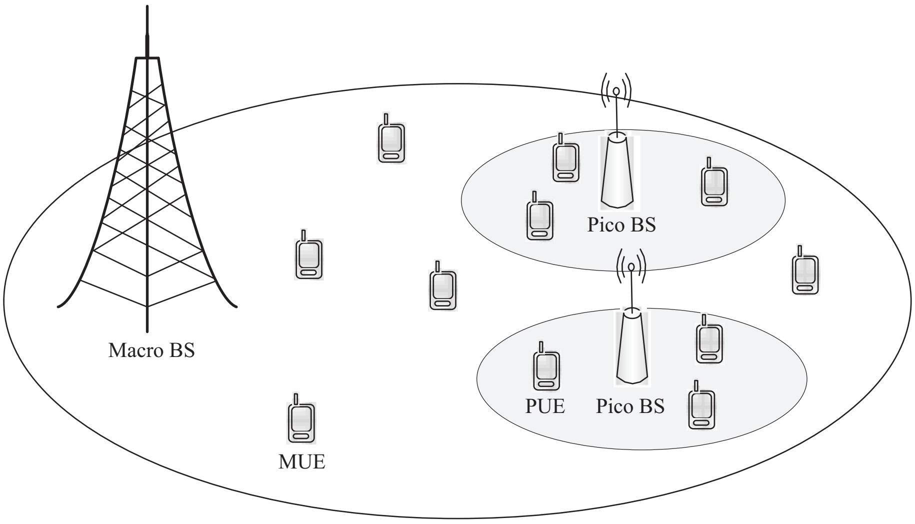

Figure 1 System model with $M = 2$.

(PUEs) suffer both cross-tier and inter-picocell interference, due to the cochannel deployment. The received signal to interference and noise ratio (SINR) at user $k \in \mathcal{K}_m$ on subchannel n is given by

$$\gamma_{m,k}(n) = \frac{p_m(n)|h_{m,(m,k)}(n)|^2}{\sum_{l=0,l\neq m}^{M} p_l(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2}, \quad k \in \mathcal{K}_m, \ \ m \in \mathcal{M}, \tag{1}$$

where $p_m(n)$ is the BS m transmit power on subchannel n, and $h_{l,(m,k)}(n)$ is the fading coefficient between BS l and user k in cell m on subchannel n. σ_z^2 represents the variance of additive white Gaussian noise (AWGN) at the receivers.

The overall data rate of cell m can be expressed as

$$R_m = \sum_{k\in\mathcal{K}_m}\sum_{n\in\mathcal{N}} s_{m,k}(n) B \log_2\left(1 + \gamma_{m,k}(n)\right), \quad m \in \mathcal{M}, \tag{2}$$

where B is the subchannel bandwidth, and $s_{m,k}(n)$ is the subchannel assignment index. If subchannel n is allocated to user k in cell m, $s_{m,k}(n) = 1$, and otherwise $s_{m,k}(n) = 0$.

For each macrocell or picocell, the power consumption of BS consists of a radio frequency (RF) transmit power and a fixed circuit power. The power consumption for BS m can be modeled as

$$P_m = \frac{1}{\xi_m}\sum_{n\in\mathcal{N}} p_m(n) + P_m^C, \quad m \in \mathcal{M}, \tag{3}$$

where ξ_m and P_m^C represent the power amplifier efficiency and the circuit power consumption at BS m, respectively. In general, the power consumption at macro BS P_0 is much larger than that of the pico BSs P_m, $\forall m \neq 0$, because the macro BS needs to provide a larger coverage range.

To balance the EE among the macorcell and the picocells, we define a weighted sum-EE as

$$U^{\mathrm{WEE}} = \sum_{m=0}^{M} \omega_m U_m^{\mathrm{EE}} = \sum_{m=0}^{M} \omega_m \frac{R_m}{P_m}. \tag{4}$$

In (4), ω_m denotes the EE weight factor for BS m, which means the level of importance and is determined by network operators, and U_m^{EE} is the EE of cell m.

Let $\mathcal{P} = \{p_m(n) \geqslant 0, \forall m, n\}$ and $\mathcal{S} = \{s_{m,k}(n) \in \{0,1\}, \forall m, n, k\}$ be the power and subchannel allocation policies, respectively. The BSs jointly determine the $\mathcal{P}$ and $\mathcal{S}$ to maximize the weighted sum-EE. The optimization problem can be formulated by

$$
\begin{aligned}
\max_{\mathcal{P},\mathcal{S}} \quad & \sum_{m=0}^{M} \omega_m U_m^{\mathrm{EE}}(\mathcal{P},\mathcal{S}) \\
\text{s.t.} \quad & C1: \sum_{n\in\mathcal{N}} s_{m,k}(n) B \log_2\left(1+\gamma_{m,k}(n)\right) \geqslant R_{m,k}^{\min}, \ \forall m\in\mathcal{M}, \ k\in\mathcal{K}_m, \\
& C2: \sum_{n\in\mathcal{N}} p_m(n) \leqslant P_m^{\max}, \ \forall m\in\mathcal{M}, \\
& C3: p_m(n) \geqslant 0, \ \forall m\in\mathcal{M}, \ n\in\mathcal{N}, \\
& C4: \sum_{k\in\mathcal{K}_m} s_{m,k}(n) = 1, \ \forall m\in\mathcal{M}, \ n\in\mathcal{N}, \\
& C5: s_{m,k}(n)\in\{0,1\}, \ \forall m\in\mathcal{M}, k\in\mathcal{K}_m, \ n\in\mathcal{N},
\end{aligned} \tag{5}
$$

where $P_m^{\max}$ is the maximum power constraint for BS m, and $R_{m,k}^{\min}$ is the minimum rate constraint for user k in cell m. $C4$ and $C5$ can guarantee that each subchannel is used by only one user in each cell to avoid intracell interference. Note that there still exist cross-tier interference and co-tier inter-picocell interference.

3 Solution to the optimization problem

In the problem (5), the objective function is a nonlinear sum-of-ratios function, which is nonconvex. Also, the constraints are nonconvex due to the existence of cochannel interference. And this problem is a mixed optimization problem incorporating both integer and continuous optimization variables. Such an optimization problem is computationally intractable in general, thus difficult to solve with traditional optimization methods.

To make the problem tractable, we apply a two-step approach. Note that the two-step algorithm will lead a suboptimal solution for the joint optimization problem (5). However, the convergence of the conventional iterative algorithms (e.g. [16]) can not be guaranteed due to the minimum rate constraint $C1$ in (5), and these algorithms also have a higher computational complexity due the iterations between the subproblems. Considered the realizability and low-complexity, we therefore adopt the following two-step method. We first present a heuristic subchannel allocation scheme to maximize weighted sum-EE, and then solve the problem by some effective transformations for the given subchannel allocation.

3.1 Subchannel allocation optimization problem

By simplifying the power allocation $\mathcal{P}$ as $\{p_m(n)=P_m^{\max}/N, \forall m,n\}$, we can derive an effective subchannel allocation scheme (i.e., finding $\mathcal{S}$). In this case, the constraints in $C2$ and $C3$ are satisfied. Now the optimization problem in (5) is reduced to

$$
\begin{aligned}
\max_{\mathcal{S}} \quad & \sum_{m=0}^{M} \frac{\omega_m}{P_m} \sum_{k\in\mathcal{K}_m} \sum_{n\in\mathcal{N}} s_{m,k}(n) R_{m,k}(n) \\
\text{s.t.} \quad & C1, C4, C5,
\end{aligned} \tag{6}
$$

where $R_{m,k}(n) = B\log_2\left(1+\gamma_{m,k}(n)\right)$. When the power allocation $\mathcal{P}$ is given, the problem in (6) is equivalent to a sum-rate maximization problem.

Therefore, each BS needs to maximize sum-rate of the cell by assigning subchannels to users, while satisfying the minimum rate requirement of each user.

To this end, a heuristic subchannel allocation algorithm is adopted. Let $\mathcal{Z}_m^{\mathcal{N}}$ be the set of remaining subchannels in cell m, which is initialized to $\mathcal{N}$. To satisfy the minimum rate requirements of users, we select the user whose rate is the farthest from its requirement, and allocate the subchannel with the highest SINR in $\mathcal{Z}_m^{\mathcal{N}}$ to the user. After the rate requirements of all users are satisfied, we then allocate subchannels from $\mathcal{Z}_m^{\mathcal{N}}$ based on maximizing sum-rate of each cell.

3.2 Power allocation optimization problem

For a given available $\mathcal{S}$ (which satisfies $C4$ and $C5$), the optimization problem (5) can be rewritten as

$$\max_{\mathcal{P}} \sum_{m=0}^{M} \omega_m U_m^{\text{EE}}(\mathcal{P}) \quad \text{s.t.} \;\; C1, C2, C3. \tag{7}$$

The problem (7) is a nonlinear sum-of-ratios optimization problem. Define the vectors $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_M)$ and $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_M)$. It is known from [20] that if $\mathcal{P}^*$ is the optimal solution to (7), then there exist $\boldsymbol{\theta}^*$ and $\boldsymbol{\phi}^*$ such that $\mathcal{P}^*$ is a solution to the following problem for $\boldsymbol{\theta} = \boldsymbol{\theta}^*$ and $\boldsymbol{\phi} = \boldsymbol{\phi}^*$,

$$\max_{\mathcal{P}} \sum_{m=0}^{M} \theta_m(\omega_m R_m - \phi_m P_m) \quad \text{s.t.} \; C1, C2, C3, \tag{8}$$

and $\mathcal{P}^*$ also satisfies the following equations for $\boldsymbol{\theta} = \boldsymbol{\theta}^*$ and $\boldsymbol{\phi} = \boldsymbol{\phi}^*$,

$$\begin{aligned} \theta_m &= \frac{1}{P_m}, \\ \omega_m R_m - \phi_m P_m &= 0, \;\; m = 0, 1, \ldots, M. \end{aligned} \tag{9}$$

Therefore, we can find the optimal $\mathcal{P}^*$ by solving the optimization problem (8). However, the problem (8) is still a nonconvex problem even for a given $(\boldsymbol{\theta}, \boldsymbol{\phi})$, due to the existence of cochannel interference.

We now stack all power variables into a power vector as $\boldsymbol{p} = (p_0(1), \ldots, p_0(N), \ldots, p_M(0), \ldots, p_M(N))$, and express the objective function in (8) as

$$F(\boldsymbol{p}, \boldsymbol{\theta}, \boldsymbol{\phi}) = \sum_{m=0}^{M} \theta_m(\omega_m R_m - \phi_m P_m) = \sum_{m=0}^{M} \theta_m \left(\omega_m \sum_{k \in \mathcal{K}_m} (g_{m,k} - f_{m,k}) - \phi_m P_m \right), \tag{10}$$

where

$$g_{m,k}(\boldsymbol{p}) = \sum_{n \in \mathcal{N}} s_{m,k}(n) B \log_2 \left(\sum_{l=0}^{M} p_l(n) |h_{l,(m,k)}(n)|^2 + \sigma_z^2 \right), \tag{11}$$

$$f_{m,k}(\boldsymbol{p}) = \sum_{n \in \mathcal{N}} s_{m,k}(n) B \log_2 \left(\sum_{l=0, l \neq m}^{M} p_l(n) |h_{l,(m,k)}(n)|^2 + \sigma_z^2 \right). \tag{12}$$

Then we use the following first-order Taylor expansion to linearize the objective function and the constraints [21]:

$$f_{m,k}(\boldsymbol{p}) \approx f_{m,k}(\bar{\boldsymbol{p}}) + \triangledown f_{m,k}^{\text{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}), \tag{13}$$

where $\triangledown f_{m,k}^{\text{T}}(\bar{\boldsymbol{p}})$ denotes the gradient at an arbitrary $\bar{\boldsymbol{p}}$. $\triangledown f_{m,k}^{\text{T}}(\boldsymbol{p})$ is a vector of length $N(M+1)$ with $(Nj+n)$-th entry as

$$\triangledown f_{m,k}^{\text{T}}(\boldsymbol{p})_{Nj+n} = \begin{cases} \frac{B s_{m,k}(n) |h_{j,(m,k)}(n)|^2}{\left(\sum_{l=0, l \neq m}^{M} p_l(n) |h_{l,(m,k)}(n)|^2 + \sigma_z^2\right) \ln 2}, & \forall j \neq m; \\ 0, & j = m. \end{cases} \tag{14}$$

Such that

$$\triangledown f_{m,k}^{\text{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) = \sum_{j=0, j \neq m}^{M} \sum_{n \in \mathcal{N}} \frac{B s_{m,k}(n) \left(p_j(n) - \bar{p}_j(n)\right) |h_{j,(m,k)}(n)|^2}{\left(\sum_{l=0, l \neq m}^{M} \bar{p}_l(n) |h_{l,(m,k)}(n)|^2 + \sigma_z^2\right) \ln 2}. \tag{15}$$

From (13), the optimization problem (8) can be rewritten as

$$\begin{aligned} \max_{\mathcal{P}} \quad & \sum_{m=0}^{M} \theta_m \left(\omega_m \sum_{k \in \mathcal{K}_m} \left(g_{m,k}(\boldsymbol{p}) - f_{m,k}(\bar{\boldsymbol{p}}) - \triangledown f_{m,k}^{\text{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) \right) - \phi_m P_m(\boldsymbol{p}) \right) \\ \text{s.t.} \quad & C1: \; g_{m,k}(\boldsymbol{p}) - f_{m,k}(\bar{\boldsymbol{p}}) - \triangledown f_{m,k}^{\text{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) \geqslant R_{m,k}^{\min}, \;\; \forall m \in \mathcal{M}, \;\; k \in \mathcal{K}_m, \\ & C2, C3. \end{aligned} \tag{16}$$

As proven in Appendix A, for a given $(\bar{\boldsymbol{p}}, \boldsymbol{\theta}, \boldsymbol{\phi})$, the problem (16) is a standard concave maximization problem with respect to $\mathcal{P}$. In the next subsection, we will develop an algorithm to find the optimal $(\boldsymbol{\theta}^*, \boldsymbol{\phi}^*)$ and determine $\bar{\boldsymbol{p}}$ for (16).

3.3 Iterative algorithm for finding auxiliary parameters

Define a vector $\chi(\boldsymbol{\theta}, \boldsymbol{\phi}) = [\chi_0(\boldsymbol{\theta}, \boldsymbol{\phi}), \chi_1(\boldsymbol{\theta}, \boldsymbol{\phi}), \ldots, \chi_{2M+1}(\boldsymbol{\theta}, \boldsymbol{\phi})]$, where

$$\begin{aligned} &\chi_m(\boldsymbol{\theta}, \boldsymbol{\phi}) = \theta_m P_m(\boldsymbol{p}) - 1, \\ &\chi_{M+1+m}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \omega_m \sum_{k \in \mathcal{K}_m} \left(g_{m,k}(\boldsymbol{p}) - f_{m,k}(\bar{\boldsymbol{p}}) - \nabla f_{m,k}^{\mathrm{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) \right) - \phi_m P_m(\boldsymbol{p}), \end{aligned} \tag{17}$$

for $m = 0, 1, \ldots, M$. It is easy to see that optimal $(\boldsymbol{\theta}^*, \boldsymbol{\phi}^*)$ can be achieved if and only if

$$\chi(\boldsymbol{\theta}, \boldsymbol{\phi}) = \boldsymbol{0}, \tag{18}$$

and the optimal $(\boldsymbol{\theta}^*, \boldsymbol{\phi}^*)$ is unique for (16). The proof is similar to [20]. The Eq. (18) can be solved by the Newton method as

$$\begin{aligned} \boldsymbol{\theta}^{(t+1)} &= \boldsymbol{\theta}^{(t)} - [\chi'(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)})]^{-1} \chi(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)}), \\ \boldsymbol{\phi}^{(t+1)} &= \boldsymbol{\phi}^{(t)} - [\chi'(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)})]^{-1} \chi(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)}), \end{aligned} \tag{19}$$

where t is the iteration index. From (19), we have

$$\theta_m^{(t+1)} = \left[\frac{1}{P_m(\mathbf{p})} \right]^{(t)}, \ m = 0, 1, \ldots, M, \tag{20}$$

$$\phi_m^{(t+1)} = \left[\frac{\omega_m \sum_{k \in \mathcal{K}_m} \left(g_{m,k}(\boldsymbol{p}) - f_{m,k}(\bar{\boldsymbol{p}}) - \nabla f_{m,k}^{\mathrm{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p} - \bar{\boldsymbol{p}}) \right)}{P_m(\boldsymbol{p})} \right]^{(t)}, \ m = 0, 1, \ldots, M. \tag{21}$$

The iterative algorithm for finding the optimal $(\boldsymbol{\theta}^*, \boldsymbol{\phi}^*)$ is summarized in Algorithm 1. For a given $(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)})$ in the t-th iteration, we solve the optimization problem (16) sequentially. $\bar{\boldsymbol{p}}$ is initialized to $\boldsymbol{p}^{(0,0)}$ and updated by $\boldsymbol{p}^{(i,t)}$ in an inner iteration (with index i). In this case, Eq. (13) shows that the inner iteration is a descent method. As proven in Appendix B, the Algorithm 1 can guarantee convergence.

The computational complexity of the proposed algorithm is acceptable. The problem (16) is a standard convex optimization and can be solved by the subgradient method, which generally has a polynomial convergence rate [22]. The complexity of the dominant calculations to solve the problem is $\mathcal{O}\left(2\delta_0(M+1)N\right)$, where δ_0 is the iterations of the subgradient method to converge. The outer loop is a sum-of-ratios optimization and needs δ_1 iterations to converge, which has a superlinear or quadratic convergence rate [20]. The inner loop is based on a descent method (with δ_2 iterations), which has a local linear convergence rate. Therefore, the total complexity of the algorithm is $\mathcal{O}\left(2\delta_0(M+1)N\delta_1\delta_2\right)$.

Algorithm 1 Iterative algorithm for finding auxiliary parameters

1: Initialize $t = 0$, maximum iterations $t_{\max}$; $\boldsymbol{p}^{(0,0)} = \left(p_0^{(0,0)}(1), \ldots, p_M^{(0,0)}(N)\right)$, where $p_m^{(0,0)}(n) = P_m^{\max}/N$;
$\boldsymbol{\theta}^{(0)} = (\theta_0^{(0)}, \theta_1^{(0)}, \ldots, \theta_M^{(0)})$ and $\boldsymbol{\phi}^{(0)} = (\phi_0^{(0)}, \phi_1^{(0)}, \ldots, \phi_M^{(0)})$, where $\theta_m^{(0)} = \frac{1}{P_m(\boldsymbol{p}_m^{(0)})}$ and $\phi_m^{(0)} = \frac{\omega_m R_m(\boldsymbol{p}_m^{(0)})}{P_m(\boldsymbol{p}_m^{(0)})}$;
2: **while** $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ convergence = **false**, and $t < t_{\max}$ **do**
3: Initialization $i = 0$, maximum iterations $i_{\max}$;
4: **while** $\boldsymbol{p}$ convergence = **false**, and $i < i_{\max}$ **do**
5: Compute the optimal $\boldsymbol{p}^*$ by solving (16) with $\bar{\boldsymbol{p}} = \boldsymbol{p}^{(i,t)}$ for a given $(\boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)})$;
6: $\boldsymbol{p}^{(i+1,t)} = \boldsymbol{p}^*$ and $i = i + 1$;
7: **end while**
8: Updating $\boldsymbol{\theta}^{(t+1)}$ and $\boldsymbol{\phi}^{(t+1)}$ according to (20) and (21);
9: $t = t + 1$;
10: **end while**

3.4 Solution to transformed power allocation subproblem

The problem (16) is a standard concave maximization problem, and can be solved by Lagrangian dual method [23]. We denote the non-negative dual variables associated with the maximum power constraints at BSs as $\boldsymbol{\lambda}$, and with the minimum rate constraints of users as $\boldsymbol{\alpha}$. The Lagrangian can be written as

$$\begin{aligned}\mathcal{L}^{(i+1,t)}(\mathcal{P},\boldsymbol{\alpha},\boldsymbol{\lambda}) =& \sum_{m=0}^{M}\sum_{k\in\mathcal{K}_m}\left(\omega_m\theta_m^{(t)}+\alpha_{m,k}\right)\left(g_{m,k}(\boldsymbol{p})-f_{m,k}(\boldsymbol{p}^{(i,t)})-\nabla f_{m,k}^{\mathrm{T}}(\boldsymbol{p}^{(i,t)})(\boldsymbol{p}-\boldsymbol{p}^{(i,t)})\right)\\ &-\sum_{m=0}^{M}\left(\lambda_m+\frac{\theta_m^{(t)}\phi_m^{(t)}}{\xi_m}\right)\sum_{n\in\mathcal{N}}p_m(n)+\sum_{m=0}^{M}\lambda_m P_m^{\max}-\sum_{m=0}^{M}\sum_{k\in\mathcal{K}_m}\alpha_{m,k}R_{m,k}^{\min}. \end{aligned} \tag{22}$$

The corresponding dual problem is

$$\min_{\boldsymbol{\lambda}\succeq 0,\boldsymbol{\alpha}\succeq 0}\max_{\mathcal{P}}\mathcal{L}^{(i+1,t)}(\mathcal{P},\boldsymbol{\alpha},\boldsymbol{\lambda}). \tag{23}$$

From the Karush-Kuhn-Tucker (KKT) conditions, we can find the optimal power allocation solution. The dual variables $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$ can be obtained by the subgradient method. The first-order derivative of (22) is given by

$$\begin{aligned}\frac{\partial\mathcal{L}^{(i+1,t)}(\mathcal{P},\boldsymbol{\alpha},\boldsymbol{\lambda})}{\partial p_m(n)} =& \sum_{c=0,c\neq m}^{M}\sum_{k\in\mathcal{K}_c}s_{c,k}(n)A_{c,k}\left(\frac{|h_{m,(c,k)}(n)|^2}{p_c(n)|h_{c,(c,k)(n)}|^2+\Xi_{c,k}(n)(\boldsymbol{p}_{-c})}-\frac{|h_{m,(c,k)}(n)|^2}{\Xi_{c,k}(n)\left(\boldsymbol{p}_{-c}^{(i,t)}\right)}\right)\\ &+\sum_{k\in\mathcal{K}_m}\frac{s_{m,k}(n)A_{m,k}|h_{m,(m,k)}(n)|^2}{p_m(n)|h_{m,(m,k)(n)}|^2+\Xi_{m,k}(n)(\boldsymbol{p}_{-m})}-\left(\lambda_m+\frac{\theta_m^{(t)}\phi_m^{(t)}}{\xi_m}\right)\\ \overset{(a)}{=}& \sum_{c=0,c\neq m}^{M}A_{c,k^*}\left(\frac{|h_{m,(c,k^*)}(n)|^2}{p_c(n)|h_{c,(c,k^*)(n)}|^2+\Xi_{c,k^*}(n)(\boldsymbol{p}_{-c})}-\frac{|h_{m,(c,k^*)}(n)|^2}{\Xi_{c,k^*}(n)\left(\boldsymbol{p}_{-c}^{(i,t)}\right)}\right)\\ &+\frac{A_{m,k^*}|h_{m,(m,k^*)}(n)|^2}{p_m(n)|h_{m,(m,k^*)(n)}|^2+\Xi_{m,k^*}(n)(\boldsymbol{p}_{-m})}-\left(\lambda_m+\frac{\theta_m^{(t)}\phi_m^{(t)}}{\xi_m}\right),\end{aligned} \tag{24}$$

where

$$A_{m,k}=B(\omega_m\theta_m^{(t)}+\alpha_{m,k})/\ln 2, \tag{25}$$

$$\Xi_{m,k}(n)(\boldsymbol{p}_{-m})=\sigma_z^2+\sum_{l=0,l\neq m}^{M}p_l(n)|h_{l,(m,k)}(n)|^2, \tag{26}$$

and $k^*=\arg_k\{s_{m,k}(n)=1\}$ is the optimal user at subchannel n in cell m, which is given in Subsection 3.1. The equality (a) holds, since $s_{m,k}(n)|_{k=k^*}=1$ and $s_{m,k}(n)|_{k\neq k^*}=0$. From the stationary condition $\frac{\partial\mathcal{L}^{(i+1,t)}(\mathcal{P},\boldsymbol{\alpha},\boldsymbol{\lambda})}{\partial p_m(n)}=0$, we can obtain the optimal power allocation as

$$p_m(n)=\left[\frac{B(\omega_m\theta_m^{(t)}+\alpha_{m,k^*})}{\left(\lambda_m+\frac{\theta_m^{(t)}\phi_m^{(t)}}{\xi_m}\right)\ln 2-\Psi_m(n)}-\frac{\Xi_{m,k^*}(n)(\boldsymbol{p}_{-m})}{|h_{m,(m,k^*)(n)}|^2}\right]^+, \tag{27}$$

where

$$\Psi_m(n)=\sum_{c=0,c\neq m}^{M}B(\omega_c\theta_c^{(t)}+\alpha_{c,k^*})\left(\frac{|h_{m,(c,k^*)}(n)|^2}{p_c(n)|h_{c,(c,k^*)(n)}|^2+\Xi_{c,k^*}(n)(\boldsymbol{p}_{-c})}-\frac{|h_{m,(c,k^*)}(n)|^2}{\Xi_{c,k^*}(n)\left(\boldsymbol{p}_{-c}^{(i,t)}\right)}\right). \tag{28}$$

The dual variables can be updated by

$$\lambda_m(u+1)=\left[\lambda_m(u)-\zeta_1(u)\left(P_m^{\max}-\sum_{n\in\mathcal{N}}p_m(n)\right)\right]^+, \tag{29}$$

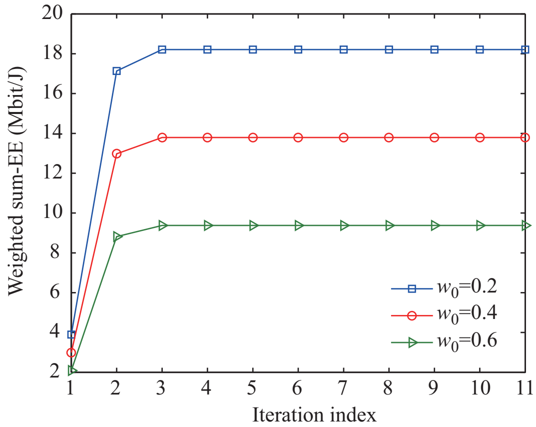

Figure 2 (Color online) Convergence of the Algorithm 1.

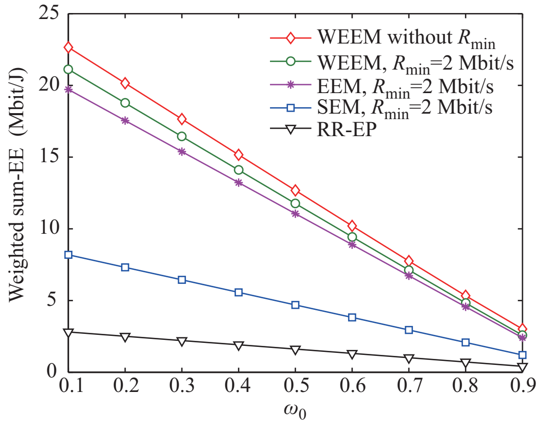

Figure 3 (Color online) Weighted sum-EE versus macrocell weight factor ω_0 for different algorithms.

$$\alpha_{m,k}(u+1) = \Bigg[\alpha_{m,k}(u) - \zeta_2(u)\Bigg(g_{m,k}(\boldsymbol{p}) - f_{m,k}(\boldsymbol{p}^{(i,t)}) - R_{m,k}^{\min} - \sum_{j=0,j\neq m}^{M}\sum_{n\in\mathcal{N}} \frac{Bs_{m,k}(n)\left(p_j(n) - p_j^{(i,t)}(n)\right)|h_{j,(m,k)}(n)|^2}{\left(\sum_{l=0,l\neq m}^{M} p_l^{(i,t)}(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2\right)\ln 2}\Bigg)\Bigg]^+, \tag{30}$$

where $[x]^+$ denotes $\max\{x,0\}$; u is the iteration index of the subgradient method; ζ_1 and ζ_2 are positive step sizes.

4 Simulation results

In this section, simulation results are given to evaluate the performance of the proposed resource allocation algorithms. In the simulation results, a macro BS is located at the center of the macrocell with 500-m radius. $M = 4$ pico BSs are placed with uniform intervals around a 300-m circle, and each picocell radius is 50 m. Assume the number of MUEs is $K_0 = 10$, and the number of PUEs in each picocell is $K_m = 5, \forall m \neq 0$. The users are uniformly distributed in the coverage area of their serving cells. The system bandwidth is 5 MHz, and the number of subchannels is $N = 16$. The power spectral density (PSD) of AWGN is -174 dBm. The pathloss models for macro BS and pico BSs are based on [2], and the small scale fading channel is modeled as the Rayleigh distribution. The maximum transmit power at macro BS is $P_0^{\max} = 46$ dBm, whereas at pico BS is $P_m^{\max} = 33$ dBm, $\forall m \neq 0$. The fixed circuit power consumption at macro BS is $P_0^C = 50$ W, whereas at pico BS is $P_m^C = 3$ W, $\forall m \neq 0$. The power amplifier efficiencies at the macro BS and pico BSs are $\xi_0 = 0.2$ and $\xi_m = 0.35, \forall m \neq 0$, respectively. We assume that the minimum rate requirements of MUEs and PUEs are $R_m^{\min} = R_{\min} = 2$ Mbit/s, $m = 0, 1, \ldots, M$.

In Figure 2, we study the convergence of weighted sum-EE for the proposed power allocation algorithm versus the number of iterations for different macrocell weight factors. We set the picocell weight factor $\omega_m = (1-\omega_0)/M, \forall m \neq 0$. It can be observed that the proposed scheme converges within 5 iterations. On the other hand, the higher weighted sum-EE will be achieved with smaller macrocell weight factor ω_0, since the picocells are more energy-efficient than the macrocell. Note that, the proposed power allocation algorithm has two-tier loops. Here, we only give the iterative results for the outer loop. The inner loop based on the descent method has an acceptable local linear convergence rate.

Figure 3 shows the weighted sum-EE versus macrocell weight factor. In this figure, we compare the proposed weighted sum-EE maximization (WEEM) algorithm with some other resource allocation algorithms, such as WEEM without considering users' minimum rate requirements, global EE maximization (EEM), spectral efficiency maximization (SEM), and round-robin scheduling with equal power allocation

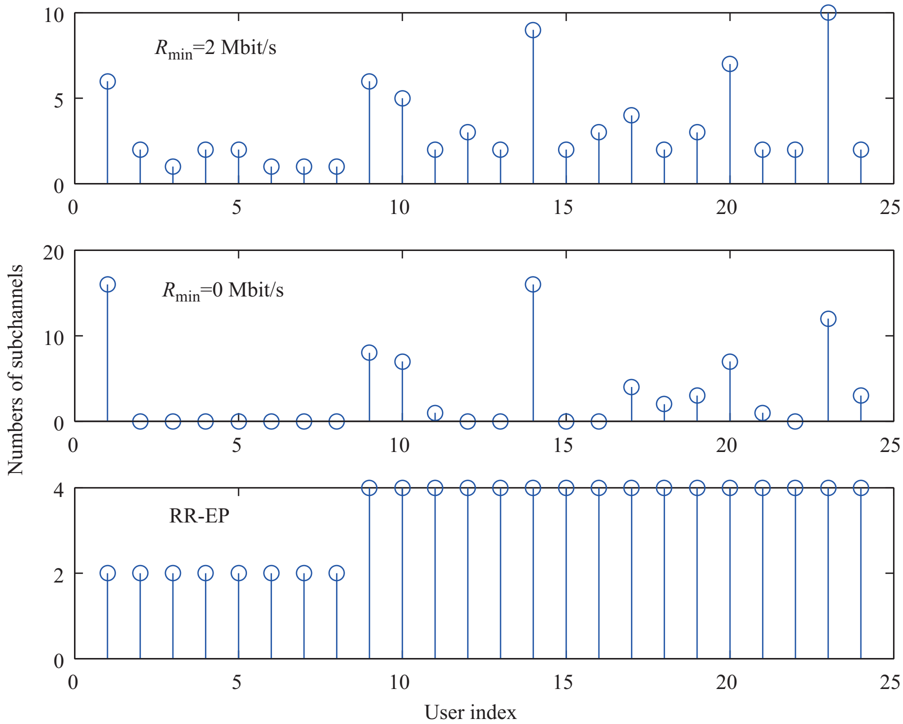

Figure 4 (Color online) Subchannel allocation among the users ($K_0 = 8$ and $K_m = 4, \forall m \neq 0$).

(RR-EP). The objective function in the EEM method is $\max_{\mathcal{S},\mathcal{P}}(\sum_{m=0}^{M} R_m / \sum_{m=0}^{M} P_m)$, while in SEM method is $\max_{\mathcal{S},\mathcal{P}} \sum_{m=0}^{M} R_m$. We observe that the proposed WEEM algorithm outperforms all these methods except the WEEM without rate constraint.

Figure 4 displays the subchannel allocation among the users for different algorithms. The results indicate that if we take no account of users' rate constraints (i.e., set $R_{\min} = 0$), a few users will almost take up all the system resources while the others have no resource. In this case, the users with unfavorable channel conditions will have no data rate.

Figure 5 illustrates weighted sum-EE, EE and sum rate versus the maximum transmit power at the macro BS $P_0^{\max}$. For studying the relationships between the maximum transmit power and the above performance indicators, we set $P_m^{\max} = P_0^{\max}/20, \forall m \neq 0$. As shown in the subplots, different performance indicators lead to different resource allocation strategies and system performance. On the other hand, the WEEM and EEM algorithms achieve a floor (corresponding to the optimal weighted sum-EE or EE) as $P_0^{\max}$ increases. And then no more power is consumed to further increase the rate, since now a little increase of rate needs much more power consumption, resulting in the weighted sum-EE or EE dropping rapidly.

Figure 6 demonstrates the EE tradeoff between the macrocell and the picocells. We provide the results of picocell EE versus the macrocell EE, by adjusting the macrocell weight factor ω_0. We also study the impacts of fixed circuit power consumption and noise power on the weighted sum-EE performance. Take $\omega_0 = 0.2$ as an example, the weighted sum-EEs U^{WEE} in Figures 6(a)–(c) are 20.1, 12.4 and 18.9 Mbit/J, respectively. It indicates that the weighted sum-EE decreases as both the circuit power and the noise power increase, which is similar to the conclusion in [16]. In addition, although the picocells have different EEs due to insufficient numbers of Monte-carlo simulation times, the results in these figures come to a same conclusion that the picocell EE U_m^{EE} decreases as the macrocell EE U_0^{EE} increases, and there exists a tradeoff between the U_0^{EE} and U_m^{EE}. The proposed algorithm can achieve the flexible EE tradeoff between macrocell and picocells, based on the system configurations and the channel conditions.

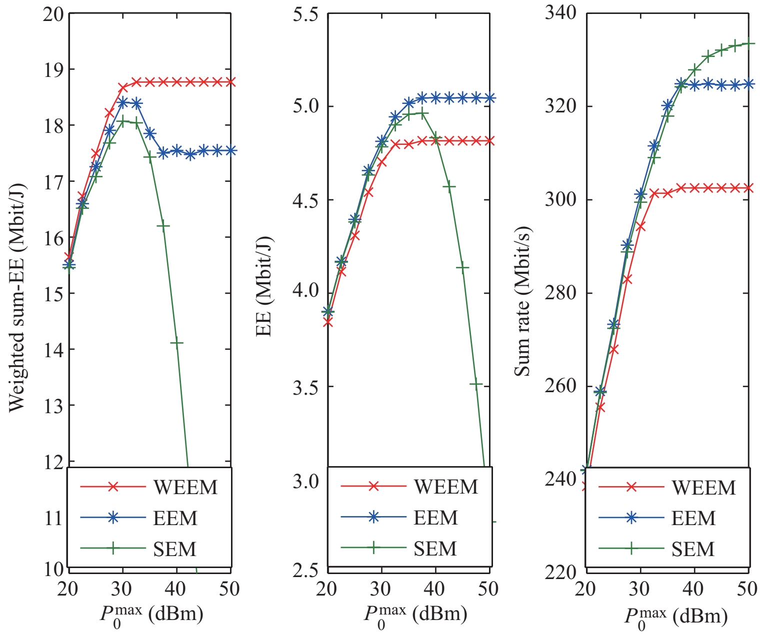

Figure 5 (Color online) Weighted sum-EE, EE and sum rate performance versus maximize transmit power of macrocell $P_0^{\max}$.

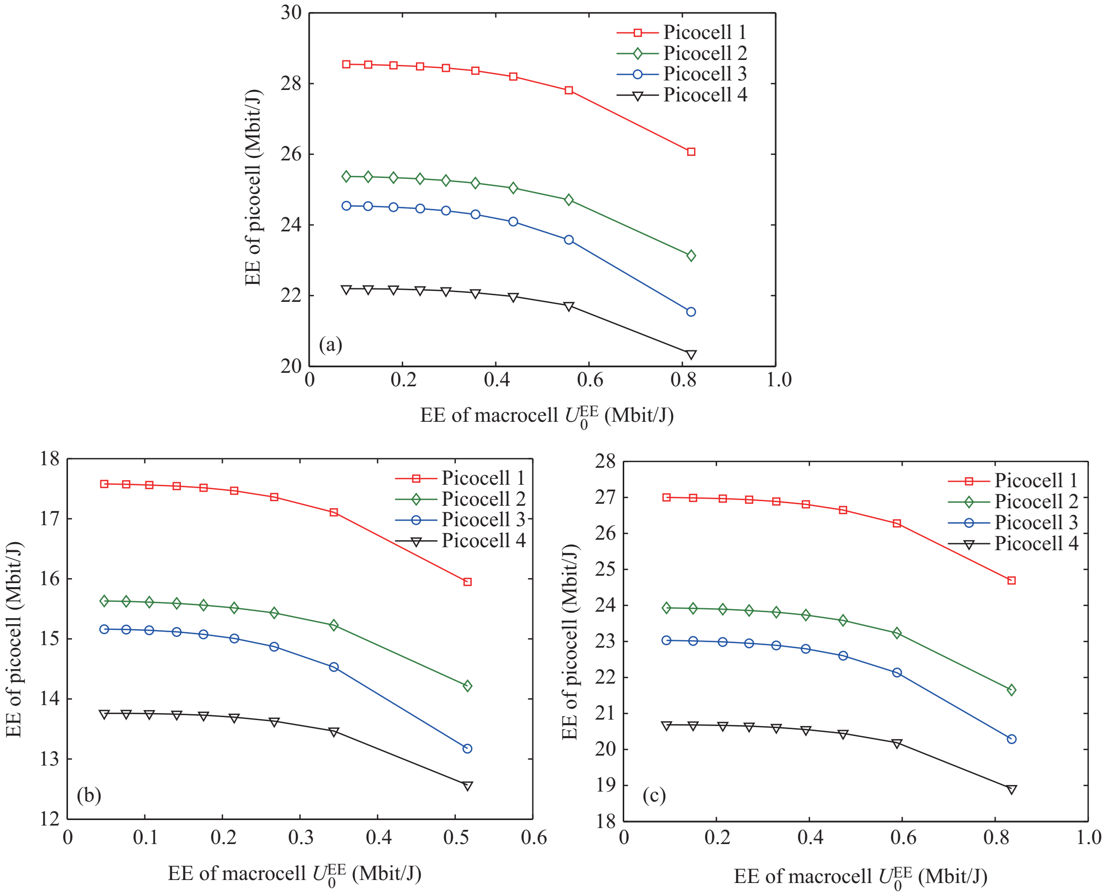

Figure 6 (Color online) Energy efficiency tradeoff between macrocell and picocells. (a) $P_0^C = 50$ W, $P_m^C = 3$ W, Noise PSD = −174 dBm; (b) $P_0^C = 80$ W, $P_m^C = 5$ W, Noise PSD = −174 dBm; (c) $P_0^C = 50$ W, $P_m^C = 3$ W, Noise PSD = −170 dBm.

5 Conclusion

We have studied the resource allocation issue in the downlink of an OFDMA two-tier network for maximizing weighted sum-EE of macrocell and picocells. We formulated the optimization problem as a nonlinear sum-of-ratios programming, where the data rate requirements of users were protected with minimum rate constraints. Due to the nonconvexity of the problem, we developed a heuristic subchannel assignment algorithm, and then solved the power allocation problem by parameterized transformations and a first-order approximation based on an iterative algorithm. Simulation results show that the proposed algorithm converges within a small number of iterations. Also, the proposed method can achieve a higher weighted sum-EE performance than the EE maximization and the SE maximization methods. An EE tradeoff exists between the macrocell and picocells for the proposed method.

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Conflict of interest The authors declare that they have no conflict of interest.

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Appendix A Proof of the concavity of the optimization problem

To prove the concavity of the optimization problem (16), we first define function $g_{m,n,k}(\boldsymbol{p}(n))$ as

$$g_{m,n,k}(\boldsymbol{p}(n)) = \log_2\left(\sum_{l=0}^{M} p_l(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2\right), \tag{A1}$$

and then derive the Hessian matrix of $g_{m,n,k}(\boldsymbol{p}(n))$ as

$$\begin{aligned}\nabla^2 g_{m,n,k}(\boldsymbol{p}(n)) =& \frac{-1}{\left(\sum_{l=0}^{M} p_l(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2\right)^2 \ln 2}\cdot \\ &\begin{bmatrix} |h_{0,(m,k)}(n)|^4 & |h_{0,(m,k)}(n)|^2|h_{1,(m,k)}(n)|^2 & \cdots & |h_{0,(m,k)}(n)|^2|h_{M,(m,k)}(n)|^2 \\ |h_{1,(m,k)}(n)|^2|h_{0,(m,k)}(n)|^2 & |h_{1,(m,k)}(n)|^4 & \cdots & |h_{1,(m,k)}(n)|^2|h_{M,(m,k)}(n)|^2 \\ \cdots & \cdots & \cdots & \cdots \\ |h_{M,(m,k)}(n)|^2|h_{0,(m,k)}(n)|^2 & |h_{M,(m,k)}(n)|^2|h_{1,(m,k)}(n)|^2 & \cdots & |h_{M,(m,k)}(n)|^4 \end{bmatrix} \\ =& -\frac{\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}}{\left(\sum_{l=0}^{M} p_l(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2\right)^2 \ln 2},\end{aligned} \tag{A2}$$

where $\boldsymbol{v} = \left(|h_{0,(m,k)}(n)|^2, |h_{1,(m,k)}(n)|^2, \ldots, |h_{M,(m,k)}(n)|^2\right)^{\mathrm{T}}$. For arbitrary $M \times 1$ dimensional vector $\boldsymbol{x}$, we have

$$\boldsymbol{x}^{\mathrm{T}}\nabla^2 g_{m,n,k}(\boldsymbol{p}(n))\,\boldsymbol{x} = -\frac{(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{v})^2}{\left(\sum_{l=0}^{M} p_l(n)|h_{l,(m,k)}(n)|^2 + \sigma_z^2\right)^2 \ln 2} \leqslant 0, \tag{A3}$$

which means the Hessian matrix is negative semidefinite and therefore $g_{m,n,k}(\boldsymbol{p}(n))$ is concave. We also know from (15) and (3) that both $-\nabla f_{m,k}^{\mathrm{T}}(\bar{\boldsymbol{p}})(\boldsymbol{p}-\bar{\boldsymbol{p}})$ and $-\phi_m P_m(\boldsymbol{p})$ are affine functions. The objective function in problem (16) is jointly concave in $\boldsymbol{p}$, since it is a non-negative linear combination of concave functions. Similarly, we can also prove that the constraints are convex. Thus the problem (16) is a standard concave optimization problem.

Appendix B Proof of the algorithm convergence

We now prove the convergence of the Algorithm 1. This algorithm is a two-loop iterative algorithm. As indicated in [20], the outer loop (i.e., sum-of-ratios algorithm) can guarantee convergence. For the inner loop, we have in i-th iteration:

$$\begin{aligned} F(\boldsymbol{p}^{(i,t)}, \boldsymbol{\theta}^{(t)}, \boldsymbol{\phi}^{(t)}) &= \sum_{m=0}^{M} \theta_m^{(t)}\left(\omega_m R_m(\boldsymbol{p}^{(i,t)}) - \phi_m^{(t)} P_m(\boldsymbol{p}^{(i,t)})\right) \\ &\overset{(a)}{\geqslant} \sum_{m=0}^{M} \theta_m^{(t)}\Bigg(\omega_m \sum_{k\in\mathcal{K}_m}\Big(g_{m,k}(\boldsymbol{p})^{(i,t)} - f_{m,k}(\boldsymbol{p}^{(i-1,t)}) \\ &\quad -\nabla f_{m,k}^{\mathrm{T}}(\boldsymbol{p}^{(i-1,t)})(\boldsymbol{p}^{(i,t)} - \boldsymbol{p}^{(i-1,t)})\Big) - \phi_m^{(t)} P_m(\boldsymbol{p}^{(i,t)})\Bigg) \\ &\overset{(b)}{\geqslant} \sum_{m=0}^{M} \theta_m^{(t)}\left(\omega_m R_m(\boldsymbol{p}^{(i-1,t)}) - \phi_m^{(t)} P_m(\boldsymbol{p}^{(i-1,t)})\right). \end{aligned} \tag{B1}$$

The inequality (a) holds, since $f_{m,k}$ is concave and then $f_{m,k}(\boldsymbol{p}^{(i,t)}) \leqslant f_{m,k}(\boldsymbol{p}^{(i-1,t)}) + \nabla f_{m,k}^{\mathrm{T}}(\boldsymbol{p}^{(i-1,t)})(\boldsymbol{p}^{(i,t)} - \boldsymbol{p}^{(i-1,t)})$. The inequality (b) also holds, since the left hand side of the inequality is the optimal solution for all feasible $\boldsymbol{p}$ obtained from the problem (16) in the i-th iteration. Therefore, $F(\boldsymbol{p}, \boldsymbol{\theta}, \boldsymbol{\phi})$ is either unchanged or improved after the i-th iteration, and the inner loop can also guarantee convergence.