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From frequency domain to time domain: performance analysis on cyclic prefixed multi-user single-carrier transmission

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Abstract This paper analyzes the performance of cyclic prefixed single-carrier transmission under multi-user interference (MUI). For multi-user receivers using single carrier frequency domain adaptive antenna array, we derive the signal to interference plus noise ratio by analyzing the time domain estimated signal after the transformation from frequency domain to time domain. By assuming the MUI follows the Gaussian distribution, we can derive the analytical expression for the bit error rate (BER) performance. The analytical result is verified by the numerical results generated by Monte Carlo method. It is concluded from the analytical results that if the number of receive antennas goes to infinity as assumed in massive multiple input multiple output (MIMO) system, the BER performance will be determined by two ratios: signal to noise ratio as well as the N_r to U ratio, where N_r is the number of receive antennas and U is the number of users.

Keywords single-carrier transmission, multi-user interference, frequency domain processing, MIMO, SC-FDAAA

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1 Introduction

In high speed data communication, a long data sequence is usually divided into blocks and then delivered sequentially. When the data rate is large, for example, 1 Gb/s, wireless channel will exhibit strong frequency selectivity due to the multi-path delay, therefore incurring inter-block interference (IBI) and inter-symbol interference (ISI) at the receiver side [1]. To address the problems, multi-carrier transmission methods such as orthogonal frequency division multiplexing (OFDM) [2–5] transmit different symbols in orthogonal subcarriers so that the broadband transmission can be converted to parallel narrowband transmission. However, OFDM modulation causes a high peak-to-average power ratio (PAPR), and

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therefore requires a wide dynamic range of digital-to-analog converter and analog-to-digital converter devices at the transceiver. Also, OFDM modulation is sensitive to the inter-carrier interference caused by the carrier frequency offset (CFO) between the transmitter and receiver [6]. Recently, more attention has been attracted to single-carrier (SC) transmission, which is proved to be free of the high PAPR and the CFO sensitivity issues associated with OFDM [7].

To avoid IBI, a guard interval (GI) with the length longer than the channel dispersion length can be inserted to the beginning of each block [7]. GI can be filled by either a copy of the block tail or a group of redundant zeros. The former is known as cyclic prefix (CP), while the latter is known as Zero-padding (ZP). A comparison between CP and ZP inserted SC block transmission has been made in [8] and [9] and it is shown that almost the same block error rate is achieved by both schemes when the block size is large. Here, we only consider CP inserted SC transmission in this study and refer to it as SC transmission hereafter for brevity. At the receiver side, equalization is carried out after CP removal to suppress the ISI. Time domain equalization (TDE) is a conventional way to combat ISI when the data rate is moderate so that the multi-path delayed signal replicas can be coherently resolved at the receiver. The optimal TDE receiver is based on transversal equalization as well as feedback equalization [10, 11] and relies on maximum likelihood (ML) detection [12]. Consequently, the computational complexity to realize the optimal TDE is exponential to the block size and becomes prohibitively high. By contrast, frequency domain equalization (FDE) [13, 14] uses one-tap equalization on each frequency, its computational complexity increases linearly with the block size and thus is much lower than TDE. Also, when multiple antennas are available at the receiver side, multiple access interference can also be suppressed by frequency domain weight control.

Single-carrier transmission with frequency domain processing has recently received increasing attention. In our previous study [15, 16], single-carrier frequency domain adaptive antenna array (SC-FDAAA) algorithm has been proposed for cellular system to suppress co-channel interference as well as multi-user interference (MUI). It has been shown that SC-FDAAA receiver can accommodate any number of users if it is no more than the number of receive antennas, and its performance is not sensitive to the angle of arrival spread of the received waveforms. Moreover, single-carrier frequency division multiple access (SC-FDMA [17]), also known as discrete Fourier transform spread OFDM (DFTS-OFDM) [18, 19], has been adopted as uplink (link from mobile station to base station) multi-user access solution [20].

In the previous study [7], the performance of SC with ZP is compared with OFDM in terms of PAPR and sensitivity to CFO. In [21] and [22], the average bit error rate (BER) performance of SC-FDE is compared with OFDM to show that SC-FDE can achieve comparable performance as OFDM. However, the comparison is made through simulation instead of theoretical analysis. The authors in [23] derived effective end-to-end signal-to-noise ratio for SC receivers using QR decomposition (QRD)-based block detection. However, QRD-based block detection requires high computational complexity and is less practical. A more practical solution is the one-tap FDE receiver due to less complexity. The BER performance analysis was carried out in [24] for direct sequence code division multiple access receiver using FDE. However, to the best of our knowledge, no performance analysis for uncoded SC multi-user transmission is available in literature. In this paper, we try to fill this gap. The analysis starts from SC-FDAAA receiver with multiple antennas. To obtain the BER performance, signal to interference plus noise ratio (SINR) of the time domain signal is derived by considering the frequency domain weight control and the transform from frequency domain to time domain. The analysis is then verified by Monte Carlo simulation results. The analytical results under a single-path fading assumption demonstrate that when the number of receive antennas, N_r, increases to infinity, the BER performance will be determined by two ratios: the signal to noise ratio (SNR) as well as the N_r to U ratio with U being the number of users. In another word, the number of users can also be increased as long as it is kept proportional to the number of receive antennas.

The rest of the paper is organized as follows. The performance of SC-FDAAA receiver will be analyzed in Section 2. Simulations are then carried out to verify the analysis and the results are presented in Section 3. Given the single-path assumption, a discussion on the analytical result is presented in Section 4. Finally, conclusions are given in Section 5.

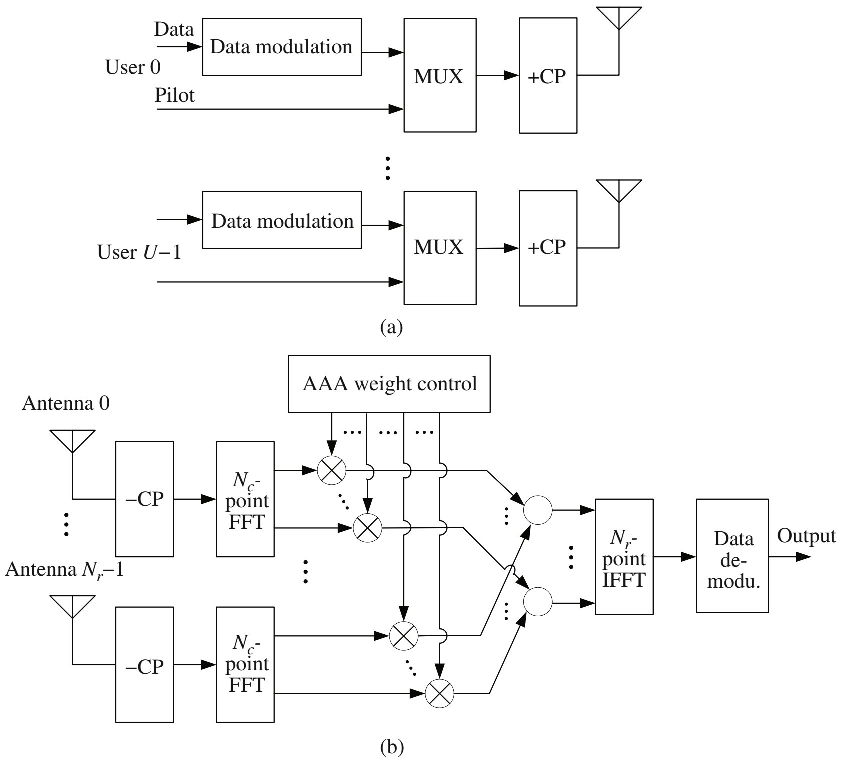

Figure 1 Multi-user SC transmission with multi-antenna FDAAA receiver. (a) Transmitter structure; (b) receiver structure.

2 Analysis on SC-FDAAA receiver and its generalization

Multi-user SC transmission with multiple single-antenna transmitters and a multi-antenna FDAAA receiver is shown in Figure 1. The receiver is equipped with N_r receive antennas. The number of users is U and each user has one transmit antenna. In this paper, we consider $U \leqslant N_r$, and the channel between the transmitter and the receiver follows block fading, that is, the channel remains unchanged during the transmission period of a block but varies over blocks. This paper hereafter considers the symbol-spaced discrete time representation of signal. Assuming an L-path channel, the impulse response of the channel can be expressed as

$$h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \tag{1}$$

where h_l and τ_l are the path gain and the time delay of the lth path, respectively, and h_l follows the complex Gaussian distribution and satisfies $\sum_{l=0}^{L-1} E\{|h_l|^2\} = 1$, where $E\{\cdot\}$ represents the expectation operation. Let a symbol duration be the time unit, and assume the time delay $\tau_l = l$. Suppose that CP inserted block transmission is used to avoid IBI, as mentioned in Section 1, and that the CP is longer than the maximum path delay of the channel. In the following, we omit the insertion and removal of CP for the purpose of simplicity.

The received signal block $\{r_m(n) : n = 0, \ldots, N_c - 1\}$ of N_c symbols with $L \leqslant N_c$ at the mth antenna is given by

$$r_m(n) = \sqrt{P_0} \sum_{l=0}^{L-1} h_{0,m}(l) s_0(n-l) + \sum_{u=1}^{U-1} \sqrt{P_u} \sum_{l=0}^{L-1} h_{u,m}(l) s_u(n-l) + z_m(n), \tag{2}$$

where $s_u(n)$ and P_u are the transmit signal and the transmit signal power of user u $(u = 0, \ldots, U-1)$ respectively, $h_{u,m}(l)$ is the impulse response of the lth path of the channel between user u and the mth receive antenna. Note that without loss of generality, here we assume that different users may use different transmit powers. Let the transmit signal from the $u = 0$th user be the desired signal, and the transmit signals from the other users be the interfering signals. $z_m(n)$ is the additive white Gaussian noise (AWGN). The first term in the right-hand side of (2) is the desired user's signal, the second term is the MUI signal, and the last term is the noise signal. According to (2), we have the frequency domain representation of the received signal on the kth frequency as

$$R_m\left(k\right) = \sqrt{P_0} H_{0,m}\left(k\right) S_0\left(k\right) + \sum_{u=1}^{U-1} \sqrt{P_u} H_{u,m}\left(k\right) S_u\left(k\right) + Z_m\left(k\right), \tag{3}$$

where

$$\begin{cases} S_u(k) = \sum_{n=0}^{N_c-1} s_u(n) \exp\left(-\mathrm{j}2\pi k \frac{n}{N_c}\right), \\ H_{u,m}\left(k\right) = \sum_{n=0}^{N_c-1} h_{u,m}\left(n\right) \exp\left(-\mathrm{j}2\pi k \frac{n}{N_c}\right), \\ Z_m(k) = \sum_{n=0}^{N_c-1} z_m(n) \exp\left(-\mathrm{j}2\pi k \frac{n}{N_c}\right). \end{cases} \tag{4}$$

Note that according to the Parseval's theorem [25], the noise component $Z_m(k)$ has the same statistical property as its counterpart in time domain. The frequency domain received signal vector on the kth frequency $\boldsymbol{R}\left(k\right)$ is then expressed as

$$\boldsymbol{R}\left(k\right) = \sqrt{P_0} \boldsymbol{H}_0\left(k\right) S_0\left(k\right) + \sum_{u=1}^{U-1} \sqrt{P_u} \boldsymbol{H}_u\left(k\right) S_u\left(k\right) + \boldsymbol{Z}\left(k\right), \tag{5}$$

where $\boldsymbol{H}_u\left(k\right) = [H_{u,0}(k), H_{u,1}(k), \ldots, H_{u,N_r-1}(k)]^{\mathrm{T}}$ and $\boldsymbol{Z}(k) = [Z_0(k), Z_1(k), \ldots, Z_{N_r-1}(k)]^{\mathrm{T}}$ with the superscript "T" representing transpose operation.

In FDAAA receiver [15], AAA weight control is applied on each frequency. The AAA weight control on the kth frequency is given [26, 27] by

$$\hat{R}_{\text{FDAAA}}\left(k\right) = \boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}\left(k\right) \boldsymbol{R}\left(k\right), \tag{6}$$

where $\boldsymbol{W}_{\text{FDAAA}}\left(k\right) = \left[W_{\text{FDAAA},0}\left(k\right), \ldots, W_{\text{FDAAA},N_r-1}\left(k\right)\right]^{\mathrm{T}}$ is the SC-FDAAA weight control vector. Following the minimum mean square error (MMSE) criterion, $\boldsymbol{W}_{\text{FDAAA}}\left(k\right)$ is calculated by

$$\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}\left(k\right) = \boldsymbol{C}_{sr}\left(k\right) \boldsymbol{C}_{rr}^{-1}\left(k\right), \tag{7}$$

where

$$\boldsymbol{C}_{sr}\left(k\right) = E\left\{\sqrt{P_0} S_0\left(k\right) \boldsymbol{R}^{\mathrm{H}}\left(k\right)\right\} = P_0 E\left\{S_0\left(k\right) S_0^*\left(k\right)\right\} \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) = N_c P_0 \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) \tag{8}$$

with $E\{\cdot\}$ denoting the expectation operation, and

$$\begin{aligned} \boldsymbol{C}_{rr}\left(k\right) &= E\left\{\boldsymbol{R}\left(k\right) \boldsymbol{R}^{\mathrm{H}}\left(k\right)\right\} \\ &= P_0 E\left\{S_0\left(k\right) S_0^*\left(k\right)\right\} \boldsymbol{H}_0\left(k\right) \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) + \sum_{u=1}^{U-1} P_u E\left\{S_u\left(k\right) S_u^*\left(k\right)\right\} \boldsymbol{H}_u\left(k\right) \boldsymbol{H}_u^{\mathrm{H}}\left(k\right) \\ &\quad + E\left\{\boldsymbol{Z}_u\left(k\right) \boldsymbol{Z}_u^{\mathrm{H}}\left(k\right)\right\} \\ &= N_c P_0 \boldsymbol{H}_0\left(k\right) \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) + N_c \sum_{u=1}^{U-1} P_u \boldsymbol{H}_u\left(k\right) \boldsymbol{H}_u^{\mathrm{H}}\left(k\right) + N_c N_0 \boldsymbol{I}_{N_r}. \end{aligned} \tag{9}$$

Substituting (8) and (9) into (7) gives

$$\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}\left(k\right) = P_0 \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) \left(P_0 \boldsymbol{H}_0\left(k\right) \boldsymbol{H}_0^{\mathrm{H}}\left(k\right) + \sum_{u=1}^{U-1} P_u \boldsymbol{H}_u\left(k\right) \boldsymbol{H}_u^{\mathrm{H}}\left(k\right) + N_0 \boldsymbol{I}_{N_r}\right)^{-1}. \tag{10}$$

Then, by substituting (10) into (6), we have the weighted receive signal on the kth frequency as

$$\hat{R}_{\text{FDAAA}}\left(k\right) = \sqrt{P_0} \boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}\left(k\right) \boldsymbol{H}_0\left(k\right) S_0\left(k\right)$$

$$+\sum_{u=1}^{U-1}\sqrt{P_u}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_u(k)\,S_u(k)+\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{Z}(k)\,, \tag{11}$$

and the time domain signal estimate is obtained after an N_c-point inverse fast Fourier transform (IFFT), which is given as

$$\begin{aligned}\hat{r}_{\text{FDAAA}}(n) &= \frac{1}{N_c}\sum_{k=0}^{N_c-1}\hat{R}_{\text{FDAAA}}(k)\exp\left(-\mathrm{j}2\pi k\frac{n}{N_c}\right),\\ &= \frac{1}{N_c}\sum_{k=0}^{N_c-1}\Big[\sqrt{P_0}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_0(k)\,S_0(k)+\sum_{u=1}^{U-1}\sqrt{P_u}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_u(k)\,S_u(k)\\ &\quad+\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{Z}(k)\Big]\times\exp\left(\mathrm{j}2\pi k\frac{n}{N_c}\right)\\ &= \hat{s}_0(n)+I(n)+\hat{z}(n)\,,\end{aligned} \tag{12}$$

where the desired signal component is given by

$$\hat{s}_0(n)=\sqrt{P_0}s_0(n)\sum_{k=0}^{N_c-1}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_0(k), \tag{13}$$

the ISI in combination with the MUI component is given by

$$\begin{aligned}I(n)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\Bigg[&\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_0(k)\sum_{i=0,i\neq n}^{N_c-1}\sqrt{P_0}s_0(i)\exp\left(-\mathrm{j}2\pi k\frac{i}{N_c}\right)\\ &+\sum_{u=1}^{U-1}\sqrt{P_u}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{H}_u(k)\,S_u(k)\times\exp\left(\mathrm{j}2\pi k\frac{n}{N_c}\right)\Bigg],\end{aligned} \tag{14}$$

and the noise component is

$$\hat{z}(n)=\frac{1}{N_c}\sum_{k=0}^{N_c-1}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,\boldsymbol{Z}(k)\exp\left(\mathrm{j}2\pi k\frac{n}{N_c}\right). \tag{15}$$

We start the analysis on SC-FDAAA receiver from deriving the power of the noise component as

$$\begin{aligned}&E\left\{\hat{z}(n)\,\hat{z}^*(n)\right\}\\ &\quad=\frac{1}{N_c^2}\sum_{k=0}^{N_c-1}\sum_{k'=0}^{N_c-1}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{T}}(k)\,E\left\{\boldsymbol{Z}(k)\,\boldsymbol{Z}^{\mathrm{H}}(k')\right\}\boldsymbol{W}_{\text{FDAAA}}^{\mathrm{H}}(k')\exp\left(\mathrm{j}2\pi k\frac{n}{N_c}\right)\exp\left(-\mathrm{j}2\pi k'\frac{n}{N_c}\right).\end{aligned} \tag{16}$$

Since $\boldsymbol{Z}(k)=[Z_0(k),Z_1(k),\ldots,Z_{N_r-1}(k)]^{\mathrm{T}}=\sum_{n=0}^{N_c-1}[z_0(n),z_1(n),\ldots,z_{N_r-1}(n)]\exp(-\mathrm{j}2\pi k\frac{n}{N_c})$, we have

$$E\left\{\boldsymbol{Z}(k)\,\boldsymbol{Z}^{\mathrm{H}}(k')\right\}=N_cN_0\boldsymbol{I}_{N_r}\delta(k-k')\,, \tag{17}$$

and therefore $E\left\{\hat{z}(n)\,\hat{z}^*(n)\right\}$ conditioned on the instantaneous channel realization is obtained by

$$E\left\{\hat{z}(n)\,\hat{z}^*(n)\right\}=\frac{N_0}{N_c}\sum_{k=0}^{N_c-1}W_{\text{FDAAA}}^{\mathrm{T}}(k)\left(W_{\text{FDAAA}}^{\mathrm{T}}(k)\right)^{\mathrm{H}}. \tag{18}$$

On the other hand, the power of the conditional desired signal component in (13) is given by

$$E\left\{\hat{s}_0(n)\,\hat{s}_0^*(n)\right\}=\frac{P_0}{N_c^2}\left(\sum_{k=0}^{N_c-1}W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_0(k)\right)\left(\sum_{k=0}^{N_c-1}W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_0(k)\right)^{\mathrm{H}}. \tag{19}$$

Let $\hat{a}(n) \equiv \hat{s}_0(n) + I(n)$. The power of $\hat{a}(n)$ is obtained by

$$\begin{aligned} E\left\{\hat{a}(n)\hat{a}^*(n)\right\} &= \frac{P_0}{N_c}\sum_{k=0}^{N_c-1} W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_0(k)\left(W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_0(k)\right)^{\mathrm{H}} \\ &\quad + \sum_{u=1}^{U-1}\frac{P_u}{N_c}\sum_{k=0}^{N_c-1} W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_u(k)\left(W_{\text{FDAAA}}^{\mathrm{T}}(k)\boldsymbol{H}_u(k)\right)^{\mathrm{H}}. \end{aligned} \tag{20}$$

Therefore, the SINR for the desired user 0 at the output of SC-FDAAA receiver conditioned on an instantaneous channel realization of $\{\boldsymbol{H}_0, \ldots, \boldsymbol{H}_{U-1}\}$ can be obtained from (18)–(20) as

$$\Gamma_{\text{FDAAA}} = \frac{E\{\hat{s}_0(n)\hat{s}_0^*(n)\}}{E\{\hat{I}(n)\hat{I}^*(n)\} + E\{\hat{z}(n)\hat{z}^*(n)\}} \tag{21}$$

$$= \frac{E\{\hat{s}_0(n)\hat{s}_0^*(n)\}}{E\{\hat{a}(n)\hat{a}^*(n)\} - E\{\hat{s}_0(n)\hat{s}_0^*(n)\} + E\{\hat{z}(n)\hat{z}^*(n)\}}. \tag{22}$$

Using the derived conditional SINR and the assumption that the interference plus noise component follows the Gaussian distribution, the average BER performance of an SC-FDAAA receiver using QPSK modulation can be obtained by averaging the conditional BER of Γ_{FDAAA} over the distribution of $\{\boldsymbol{H}_0, \ldots, \boldsymbol{H}_{U-1}\}$ [1]:

$$P_b^{\text{FDAAA}} = E\left\{\frac{1}{\sqrt{\pi}}\int_{0.5\Gamma_{\text{FDAAA}}}^{\infty}\exp\left(-t^2/2\right)\mathrm{d}t\right\}. \tag{23}$$

From the above analysis on SC-FDAAA receiver, it can be concluded that the interference component(s) in the output signal after IFFT cannot be evaluated straightforwardly due to the fact that SC transmission spreads each symbol in the whole bandwidth and each frequency carries the components of all the symbols. Therefore, it is difficult, if not impossible, to directly give the interference power for each symbol. However, the interference power can be calculated by a more sophisticated way when we are evaluating the SINR, that is, the interference power is obtained by

$$\hat{P}_I = \hat{P}_{S+I} - \hat{P}_S, \tag{24}$$

where $\hat{P}_{S+I}$ and $\hat{P}_S$ are the evaluated power of desired signal plus interference and the evaluated power of the desired signal after IFFT, respectively. The SINR of the desired signal is then obtained by

$$\hat{\Gamma} = \frac{\hat{P}_S}{\hat{P}_{S+I} - \hat{P}_S + \hat{P}_Z}, \tag{25}$$

where $\hat{P}_Z$ is the evaluated power of the noise component after IFFT.

It should be noted that the analysis for the SC-FDAAA receiver is also applicable to other types of receivers for the SC transmission. For example, the performance of SC-FDMA receiver can also be analyzed within the same framework. Also, the multiple antennas could be used to provide diversity gain or multiplexing gain, and an optimal tradeoff could be obtained, which can maximize the system sum rate. It is believed that the optimal tradeoff could be obtained by performance analysis, and this remains as interesting work for the interested readers.

3 Numerical results

In this section, the analysis on SC-FDAAA receiver will be testified by numerical results generated by Monte Carlo simulations. The parameters used to generate the results are shown in Table 1. Four-antenna receiver and eight-antenna receiver are used and a slow transmit power control (TPC) is assumed to mitigate the path loss effect so that each user has the same average receive SNR. When the number of users vary from one (single user case) to four (the maximal number of users that can be accommodated by

Table 1 Parameters used for simulations

Number of antennas		4, 8
Modulation		qpsk
Transmit power control		slow
Target receive SNR		−10 dB–10 dB
Number of users		1−4
Channel	Number of paths L	16
	Power delay profile	Uniform
	Estimation	Ideal
FFT		$N_c = 256$

a four-antenna SC-FDAAA receiver), the simulation results and theoretical results calculated following (22) for average BER performance of user 0 are marked by “simu” and “theo”, respectively. Figure 2 shows the result of four-antenna receiver and Figure 3 shows the result of eight-antenna receiver. A good match between the simulation results and the theoretical results is observed, which verifies our proposed analysis and shows the effectiveness of the Gaussian assumption as well.

4 Discussion

Due to the frequency selectivity of the channel response, it is difficult to interpret the SINR expression in (22) in its current format. To get some insight, a special case of single-path channel is assumed here. Using this assumption, the frequency domain channel response will be constant for all the frequencies and the SINR in (22) can be rewritten as

$$\gamma' = \frac{P_0\left(\sum_{m=0}^{N_r-1}\frac{P_0|H_{0,m}|^2}{\sum_{u=0}^{U-1}P_u|H_{u,m}|^2+N_0}\right)^2}{\sum_{u=1}^{U-1}P_u\left(\sum_{m=0}^{N_r-1}\frac{\sqrt{P_0P_u}H_{0,m}^*H_{u,m}}{\sum_{u=0}^{U-1}P_u|H_{u,m}|^2+N_0}\right)^2+N_0\sum_{m=0}^{N_r-1}\frac{P_0|H_{0,m}|^2}{\sum_{u=0}^{U-1}P_u|H_{u,m}|^2+N_0}}, \tag{26}$$

where $H_{u,m}$ is the frequency response for the channel between the uth user and the mth antenna.

It is very interesting to investigate the case when the number of receive antennas increases to infinity, as the assumption used in massive MIMO system [28]. Meanwhile, the number of users that can be simultaneously detected by SC-FDAAA can also be increased infinitely as long as the N_r to U ratio is kept. Given the fixed ratio N_r/U, when $N_r \to \infty$, (26) can be approximated by

$$\lim_{N_r\to\infty}\gamma' \approx \frac{P_0}{N_0}\frac{N_rP_0}{UP_0+N_0}, \tag{27}$$

where it is assumed that $P_0 = P_u$. Then, this result can be further simplified as

$$\lim_{N_r\to\infty}\gamma' \approx \frac{P_0}{N_0}\frac{N_r}{U} \tag{28}$$

if $\frac{P_0}{N_0} \gg \frac{1}{U}$. Given fixed target SNRs P_0/N_0, the BER performance versus the N_r/U ratio is shown in Figure 4.

5 Conclusion

In this paper, a general performance analysis for multi-user SC transmission has been proposed. The SC-FDAAA receiver, which was proposed in our previous study, has been analyzed. The SINR of the time domain signal estimate after the frequency domain array processing as well as the transform from frequency domain to time domain has been derived and the BER expression based on the Gaussian

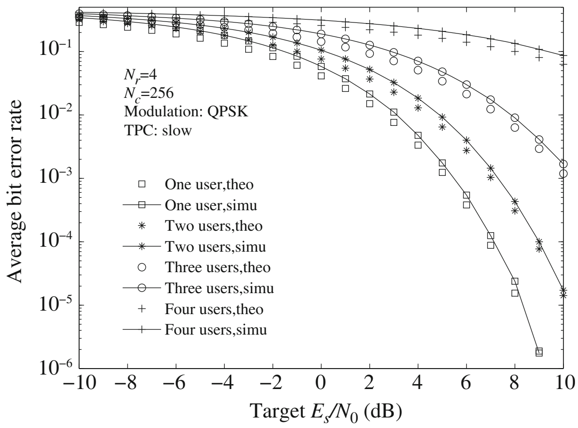

Figure 2 Average BER performance of user 0, four-antenna SC-FDAAA receiver.

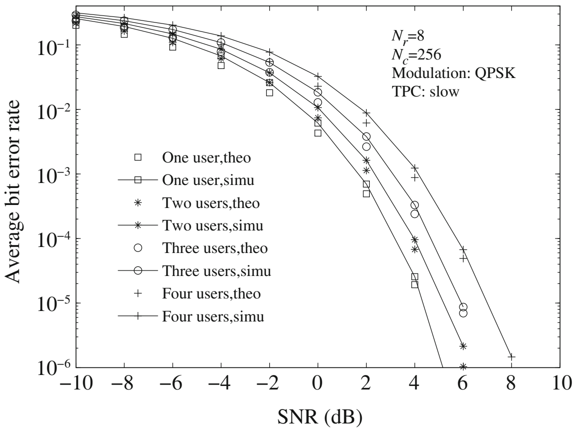

Figure 3 Average BER performance of user 0, eight-antenna SC-FDAAA receiver.

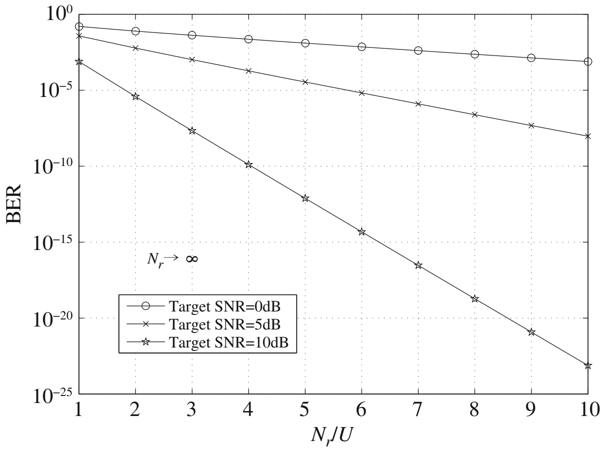

Figure 4 Analytical BER performance under the assumption of single-path channel when $N_r \to \infty$.

assumption of interference plus noise has been given. Simulations have been carried out to testify the proposed analysis and a good match between the theoretical results and the simulation ones has been shown by the numerical results. Under the assumption of single-path channel, when the number of antennas increases to infinity, it is found that the BER of SC-FDAAA receiver is determined by two factors: SNR and the N_r to U ratio. That is to say, the number of users can be increased infinitely as long as the N_r to U ratio is kept.

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Conflict of interest The authors declare that they have no conflict of interest.

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