

Anti-disturbance control of hypersonic flight vehicles with input saturation using disturbance observer

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Abstract This paper proposes an anti-disturbance control scheme for the near space vehicle (NSV) based on terminal sliding mode (TSM) technique and disturbance observer method. To tackle the system uncertainty and the time-varying unknown external disturbance of the NSV, a disturbance observer based on TSM technique is designed which can render the disturbance estimate error convergent in finite time. Furthermore, an auxiliary design system is introduced to analyze the input saturation effect. Based on the developed disturbance observer and the auxiliary design system, an anti-disturbance attitude control scheme is developed for the NSV using the TSM technique to speed up the convergence of all signals in closed-loop system. For the closed-loop system, the stability is rigorously proved by using the Lyapunov method and we guarantee the finite time convergence of all closed-loop system signals in the presence of the integrated affection of the system uncertainty, the input saturation, and the unknown external disturbance. Simulation study results are given to show the effectiveness of the developed TSM anti-disturbance attitude control scheme using the disturbance observer and the auxiliary system for the NSV.

Keywords near space vehicles, anti-disturbance control, sliding mode control, input saturation, disturbance observer

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1 Introduction

Nowadays, the hypersonic flight vehicle (HFV) has attracted much attention due to its wide application in both civilian and military areas [1]. Indeed, there are a number of robust flight control schemes developed for the HFV [2,3]. In [4], a direct neural discrete control scheme was proposed for the HFV. Fuzzy tracking control scheme was studied for the HFV via Takagi-Sugeno (T-S) model in [5]. In [6], an adaptive neural control scheme was proposed based on high gain observer for HFVs. The online support vector regression compensated nonlinear generalized predictive control was developed for the HFV in [7]. The near space vehicle (NSV) as one of the most important HFVs has received increasing interest in recent years due to working in the near space and performing various tasks. As a kind of HFV, the NSV processes a wide range of flight envelopes and the flight environment is sharply changeable with time variation. Consequently, it is necessary to develop efficient flight control schemes to enhance the control robustness. As is well known, the NSV has important features, such as high control channel coupling and

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strong nonlinearity, which will further increase the design difficulty of the robust flight control scheme by considering the unknown external disturbance. Thus, the robust flight control design of the NSV is a significant challenge in the control area.

With the NSV being a multiple-input multiple-output (MIMO) nonlinear system subject to external disturbances and large uncertainties, it is natural to develop efficient robust flight control schemes for the NSV [8–13]. For an NSV, an adaptive flight control scheme was developed using functional link neural network in [14]. In [15], a robust control scheme was studied for the NSV with input backlash-like hysteresis. Using T-S fuzzy technique, a fault-tolerant flight control scheme was developed for the NSV in [16]. In [17], a fault tolerant-control scheme was studied for the NSV based on backstepping method. By reviewing these research results, it is clear that the robust flight control design needs to be further investigated for the NSV subjected to unknown external disturbance and input saturation.

To efficiently handle the unknown disturbance and fully use the dynamic information of disturbance, the disturbance observer can be introduced to design the flight control scheme of the NSV. In the past several decades, the disturbance-observer based control (DOBC) schemes have been extensively studied [18–22]. An adaptive tracking control scheme was developed using fuzzy logic system for a MIMO nonlinear systems with disturbance observer in [23]. In [24], an adaptive control scheme was proposed using neural network and disturbance observer for the nonaffine nonlinear system. For the bank-to-turn missile, a robust autopilot was designed based on disturbance observer in [25]. In [26], a disturbance suppression control scheme was proposed for nonlinear systems using DOBC method. During the past years, some disturbance observer-based robust flight control schemes have been developed for the NSV to improve the disturbance rejection ability. Robust optimal predictive control was developed for the NSV based on functional link network disturbance observer in [27]. In [28], a robust disturbance observer based-attitude control was proposed for the NSV with external disturbance. Sliding mode disturbance observer (SMDO) has been extensively studied as a class of nonlinear disturbance observers. However, the convergence speed of the disturbance estimate error needs to be further improved to meet the control requirement of the NSV. In this paper, recalling the terminal sliding mode disturbance observer (TSMDO) developed for the single-input and single-output system [29], the disturbance observer with finite time convergence property will be developed for the NSV.

On the other hand, there exists the input saturation due to only limited forces and moments provided by the equipped actuators of the NSV [30–35]. To ensure satisfactory control performance of the NSV, the input saturation needs to be considered. In [36], an adaptive tracking control was studied using neural network for a class of stochastic nonlinear systems subject to unknown input saturation. Tracking control scheme was studied for nonlinear systems with bounded controls and control rates in [37]. In [38], a guaranteed transient performance-based control was proposed for the NSV with input saturation. Although some control schemes have been designed to control an NSV with input saturation, the faster control speed is desired for the NSV to complete the expected control objective. In this setting, the terminal sliding mode (TSM) technique can be employed to develop the flight control scheme of the NSV. Up to now, the TSM control schemes have been studied for various aircrafts. In [39], the six-degree-of-freedom robust and adaptive TSM control scheme was studied for the formation flying of spacecrafts. Second-order TSM control scheme was proposed for the cruising flight of a hypersonic vehicle based on the SMDO in [40]. In [41], a second-order nonsingular TSM control was developed for a reusable launch vehicle. To speed the control convergence, the TSM control strategy should be further investigated for the NSV in the presence of input saturation, system uncertainty, and unknown disturbance.

According to the above discussion, the disturbance observer-based TSM control to follow the desired attitude command signals of the NSV subject to the unknown disturbance and the input saturation. The layout of the paper is as follows. The problem description of the TSM attitude control for the NSV is given in Section 2. Section 3 describes the tracking control design for a class of MIMO nonlinear systems using the TSM technique and the disturbance observer. The TSM attitude control schemes are presented for the NSV using the developed disturbance observer-based TSM control scheme in Section 4. Simulation study results are provided to show the effectiveness of the proposed SMDO-based attitude control approach in Section 5. Some conclusion remarks are addressed in Section 6.

2 Problem description

To develop the anti-disturbance attitude control scheme using the TSM technique for the NSV, the attitude motion model is achieved according to the six-degrees-of-freedom and twelve-state kinematic equations, which is expressed as [7]

$$\dot{\Omega} = F_s(\Omega) + G_s(\Omega)\omega + D_s(\Omega, d_s), \quad y_s = \Omega, \quad (1)$$

$$\dot{\omega} = F_f(\omega) + G_f(\omega)M_c + D_f(\omega, d_f), \quad y_f = \omega, \quad (2)$$

where $\Omega = [\alpha, \beta, \mu]^T$ is a attitude angle vector consisting of angle of attack, sideslip angle, and flight-path roll angle, respectively; $\omega = [P, Q, R]^T$ is the fast-loop state vector including three body-axis angular velocities; and $M_c = [l_{ctrl}, m_{ctrl}, n_{ctrl}]^T$ is a control moment vector of the NSV. The expressions of F_s , G_s , F_f , and G_f are omitted here and can be found in [7]. $D_s(\Omega, d_s) = \Delta F_s(\Omega) + d_s(t)$, $D_f(\omega, d_f) = \Delta F_f(\omega) + d_f(t)$, where $\Delta F_s(\Omega)$ and $\Delta F_f(\omega)$ denote the system uncertainties, and $d_s(t)$ and $d_f(t)$ are the unknown external disturbances. In this paper, the attitude angle vector Ω and the body-axis angular velocity vector ω are assumed to be measurable.

In general, there exist the constraints of rated angular velocities in flight process of the NSV, which lead to the bounded angular velocities. On the other hand, the bounded moments are generated by the bounded surface deflection angles. Thus, the angular velocity vector ω and the control moment vector M_c satisfy the nonsymmetric saturation constraints. To easily describe these constraints, the angular velocity vector ω and the control moment vector M_c are uniformly replaced by a vector $\ell = [\ell_1, \ell_2, \ell_3]^T$. Define $\ell_c = \text{sat}(\ell) = [\text{sat}(\ell_1), \text{sat}(\ell_2), \text{sat}(\ell_3)]^T$ and the saturation operation $\text{sat}(\ell_i)$ is defined as

$$\text{sat}(\ell_i) = \begin{cases} \ell_{ri \max}, & \text{if } \ell_i > \ell_{ri \max}, \\ \ell_i, & \text{if } \ell_{li \max} \leq \ell_i \leq \ell_{ri \max}, \\ \ell_{li \max}, & \text{if } \ell_i < \ell_{li \max}, \end{cases} \quad (3)$$

where $\ell = [\ell_1, \ell_2, \ell_3]^T$ is the input of the saturation operation. $\ell_{ri \max} > 0$ and $\ell_{li \max} < 0$, $i = 1, 2, 3$ are the known saturation levels of the angular velocity and the control moment.

To design the TSM control for the uncertain attitude motion dynamics (1) and (2) of the NSV, we give the following lemma and assumptions:

Lemma 1 ([29,42]). For a continuous and positive function $V(t)$, if the following condition is held:

$$\dot{V}(t) + \alpha V(t) + \lambda V^\gamma(t) \leq 0, \quad \forall t > t_0. \quad (4)$$

Then, $V(t)$ converges in finite time t_s and t_s is decided by

$$t_s \leq t_0 + \frac{1}{\alpha(1+\gamma)} \ln \frac{\alpha V^{1-\gamma}(t_0) + \lambda}{\lambda}, \quad (5)$$

where $\alpha > 0$, $\lambda > 0$, and $0 < \gamma < 1$.

Assumption 1 ([29,42]). For the attitude motion dynamics (1) and (2) of the NSV, matrices G_s and G_f are invertible.

Assumption 2. For the disturbances D_s and D_f of the NSV, there exist positive constants λ_{si} and λ_{fi} such that $\lambda_{si} > |D_{si}|$ and $\lambda_{fi} > |D_{fi}|$, $i = 1, 2, 3$.

In this paper, we design the TSM attitude control scheme to follow a desired attitude signal of the NSV with input saturation and external disturbance. Furthermore, all closed-loop system signals are convergent in finite time under the developed TSM attitude control.

3 Anti-disturbance control of MIMO uncertain nonlinear systems

According to (1) and (2), the attitude motion dynamics of the NSV can be described by an uncertain MIMO nonlinear system. To facilitate the design of the TSM attitude control scheme, we first consider

the TSM control of a general MIMO nonlinear system, which is written as

$$\dot{x} = F(x) + G(x)u + D(x, t), \quad y = x, \tag{6}$$

where $x \in \mathbb{R}^{n \times 1}$ is the system measurable state, $y \in \mathbb{R}^{n \times 1}$ is the system output, and $u \in \mathbb{R}^{n \times 1}$ is the control input vector. $F(x) \in \mathbb{R}^{n \times 1}$ and $G(x) \in \mathbb{R}^{n \times n}$ are the known vector and matrix of system function and control gain, respectively. $D(x, t) = \Delta F(x) + d(t)$ is a compound disturbance which includes the system uncertainty $\Delta F(x) \in \mathbb{R}^{n \times 1}$ and the disturbance $d(t) \in \mathbb{R}^{n \times 1}$.

Usually, there may exist the input saturation for the uncertain MIMO nonlinear system (6). We assume that the control input $u = [u_1, u_2, \dots, u_n]^T$ satisfies nonsymmetric saturation. That is, the control signal $u(t) = \text{sat}(v) = [\text{sat}(v_1), \text{sat}(v_2), \dots, \text{sat}(v_n)]^T$ is constrained by

$$\text{sat}(v_i) = \begin{cases} v_{ri \max}, & \text{if } v_i > v_{ri \max}, \\ v_i, & \text{if } v_{li \max} \leq v_i \leq v_{ri \max}, \\ v_{li \max}, & \text{if } v_i < v_{li \max}, \end{cases} \tag{7}$$

where $v = [v_1, v_2, \dots, v_n]^T$ is the input of the saturation operation. $v_{ri \max} > 0$ and $v_{li \max} < 0$, $i = 1, 2, \dots, n$ are the known saturation levels of the control input.

Define $\Delta u = u - v$. Suppose that the difference Δu between the desired control input v and the actual control input $\text{sat}(u)$ satisfies the following assumption:

Assumption 3. For the Δu , it is assumed to satisfy the following condition:

$$\|\Delta u\| \leq \theta, \tag{8}$$

where θ is a known constant.

For a given system output y_d , the developed TSM control scheme should render all closed-loop signals convergent in finite time. For the studied MIMO nonlinear system (6), the matrix $G(x)$ is assumed to be invertible in the anti-disturbance control design.

Remark 1. From the view of a practical control system, the difference Δu between the desired control input u and the actual control input $\text{sat}(u)$ cannot be large. The reason is that the system controllability should be satisfied when control input saturation occurs. For example, the provided control moment M_c is bounded due to the bounded surface deflection angle for the NSV. However, the NSV is controllable under the bounded surface deflection angles. Thus, we can conclude that the difference Δu is bounded. To satisfy Assumption 3, the parameter θ can be large.

3.1 Design of disturbance observer with finite time convergence property

To design a TSMDO for the disturbance D of the uncertain MIMO nonlinear system (6), an auxiliary variable is defined as [29]

$$\sigma = z - x, \tag{9}$$

where z is given by

$$\dot{z} = -K\sigma - \Lambda \text{sign}(\sigma) - \Gamma \sigma^{p_0/q_0} - |F(x)|\text{sign}(\sigma) + G(x)u, \tag{10}$$

where q_0 and p_0 are odd positive integers with $p_0 < q_0$. $K = \text{diag}\{k_i\}_{n \times n}$, $\Lambda = \text{diag}\{\lambda_i\}_{n \times n}$, $\Gamma = \text{diag}\{\tau_i\}_{n \times n}$, $k_i > 0$, $\lambda_i > 0$ and $\tau_i > 0$ are design parameters and $\lambda_i > |D_i|$, $\sigma^{p_0/q_0} = [\sigma_1^{p_0/q_0}, \sigma_2^{p_0/q_0}, \dots, \sigma_n^{p_0/q_0}]^T$, $\text{sign}(\sigma) = [\text{sign}(\sigma_1), \text{sign}(\sigma_2), \dots, \text{sign}(\sigma_n)]^T$, and $|F(x)| = \text{diag}\{|F_i(x)|\}_{n \times n}$.

The disturbance estimate \hat{D} of the TSMDO is decided by

$$\hat{D} = -K\sigma - \Lambda \text{sign}(\sigma) - \Gamma \sigma^{p_0/q_0} - |F(x)|\text{sign}(\sigma) - F(x). \tag{11}$$

Differentiating (9), and considering (6) and (10) yields

$$\dot{\sigma} = \dot{z} - \dot{x} = -K\sigma - \Lambda \text{sign}(\sigma) - \Gamma \sigma^{p_0/q_0} - |F(x)|\text{sign}(\sigma) - F(x) - D. \tag{12}$$

The above design of the TSMDO can be included in the following theorem.

Theorem 1. For an MIMO nonlinear system described by (6), the TSMDO is designed in accordance with (9)–(11). Then, the finite time convergence of the estimate error can be guaranteed for the developed TSMDO.

Proof. Consider the following Lyapunov function candidate

$$V_o = \frac{1}{2}\sigma^T\sigma. \tag{13}$$

Considering (12), we have

$$\begin{aligned} \dot{V}_o &= \sum_{i=1}^n \sigma_i \dot{\sigma}_i = \sum_{i=1}^n [\sigma_i(-k_i\sigma_i - \lambda_i \text{sign}(\sigma_i) - \tau_i \sigma_i^{p_0/q_0} - |F_i(x)|\text{sign}(\sigma_i) - F_i(x) - D_i)] \\ &\leq -\sum_{i=1}^n k_i^2 \sigma_i^2 - \sum_{i=1}^n \lambda_i |\sigma_i| - \sum_{i=1}^n \tau_i \sigma_i^{(p_0+q_0)/q_0} - \sum_{i=1}^n |F_i(x)| |\sigma_i| + \sum_{i=1}^n |F_i(x)| |\sigma_i| + \sum_{i=1}^n |\sigma_i| |D_i| \\ &\leq -\sum_{i=1}^n k_i^2 \sigma_i^2 - \sum_{i=1}^n \tau_i \sigma_i^{(p_0+q_0)/q_0} \\ &\leq -2kV_o - 2^{(p_0+q_0)/2q_0} \tau V_o^{(p_0+q_0)/2q_0}, \end{aligned} \tag{14}$$

where $k = \min(k_i)$ and $\tau = \min(\tau_i)$.

Define $\tilde{D} = D - \hat{D}$. Considering (6) and (9)–(11), we obtain

$$\begin{aligned} \tilde{D} &= D - \hat{D} = D + K\sigma + \Lambda \text{sign}(\sigma) + \Gamma \sigma^{p_0/q_0} + |F(x)|\text{sign}(\sigma) + F(x) \\ &= \dot{x} - F(x) - G(x)u + K\sigma + \Lambda \text{sign}(\sigma) + \Gamma \sigma^{p_0/q_0} + |F(x)|\text{sign}(\sigma) + F(x) \\ &= \dot{x} + K\sigma + \Lambda \text{sign}(\sigma) + \Gamma \sigma^{p_0/q_0} + |F(x)|\text{sign}(\sigma) - G(x)u \\ &= \dot{x} - \dot{z} = -\dot{\sigma}. \end{aligned} \tag{15}$$

Based on Lemma 1 and (14), we can guarantee the auxiliary variable σ converge to zero in finite time. Since the auxiliary variable σ converges to zero in finite time, we know that the derivative of σ can converge to zero in finite time. Thus, the finite time convergence of the disturbance estimate error \tilde{D} of the designed TSMDO can also be guaranteed according to (15). This concludes the proof.

3.2 Design of tracking control based on TSM technique

In this subsection, the TSM control scheme will be studied for the uncertain MIMO system (6). To handle the input saturation of the uncertain MIMO nonlinear system (6), the following auxiliary system is constructed to compensate for the effect of the input saturation with the same order as the MIMO nonlinear system [43]:

$$\dot{\xi} = -M\xi - W\xi^{p_1/q_1} - \theta g \text{sign}(\xi) + G(x)\Delta u, \tag{16}$$

where q_1 and p_1 are odd positive integers with $p_1 < q_1$. $\xi \in \mathbb{R}^n$ is the state of the auxiliary system. $M = \text{diag}\{m_i\}_{n \times n}$, $W = \text{diag}\{w_i\}_{n \times n}$, $m_i > 0$ and $w_i > 0$ are design parameters. $\text{sign}(\xi) = [\text{sign}(\xi_1), \text{sign}(\xi_2), \dots, \text{sign}(\xi_n)]^T$, $g = \text{diag}\{\|g_i(x)\|\}_{n \times n}$, $\xi^{p_1/q_1} = [\xi_1^{p_1/q_1}, \xi_2^{p_1/q_1}, \dots, \xi_n^{p_1/q_1}]^T$, $g_i(x)$ is the i th row of the control gain matrix $G(x)$.

Choose the Lyapunov function candidate as

$$V_a = \frac{1}{2}\xi^T\xi. \tag{17}$$

Invoking (16), we have

$$\begin{aligned} \dot{V}_a &= -\xi^T M \xi - \sum_{i=1}^n w_i \xi_i^{(p_1+q_1)/q_1} - \theta \xi^T g \text{sign}(\xi) + \xi^T G(x)\Delta u \\ &\leq -\sum_{i=1}^n m_i^2 \xi_i^2 - \sum_{i=1}^n w_i \xi_i^{(p_1+q_1)/q_1} - \sum_{i=1}^n \theta \|g_i(x)\| |\xi_i| + \xi^T G(x)\Delta u \end{aligned}$$

$$\leq -2mV_a - 2^{(p_1+q_1)/2q_1} wV_a^{(p_1+q_1)/2q_1}, \tag{18}$$

where $m = \min(m_i)$ and $w = \min(w_i)$.

To design the TSM control scheme for the studied MIMO nonlinear system (6), we define the sliding surface as

$$s = z - y_d - \xi. \tag{19}$$

In accordance with (6), (9), and $\Delta u = u - v$, the derivative of s is

$$\begin{aligned} \dot{s} &= \dot{x} + \dot{\sigma} - \dot{y}_d - \dot{\xi} \\ &= F(x) + G(x)u + D(x, t) - \dot{y}_d + M\xi + W\xi^{p_1/q_1} + \theta g \text{sign}(\xi) - G(x)\Delta u + \dot{\sigma} \\ &= F(x) + G(x)(v + \Delta u) + D(x, t) - \dot{y}_d + M\xi + W\xi^{p_1/q_1} + \theta g \text{sign}(\xi) - G(x)\Delta u + \dot{\sigma} \\ &= F(x) + G(x)v + D(x, t) - \dot{y}_d + M\xi + W\xi^{p_1/q_1} + \theta g \text{sign}(\xi) + \dot{\sigma}. \end{aligned} \tag{20}$$

Using the designed TSMDO and the auxiliary system, the TSM tracking control scheme is proposed as

$$v = -G^{-1}(x)(F(x) - \dot{y}_d + \hat{D} + M\xi + W\xi^{p_1/q_1} + \theta g \text{sign}(\xi) + Bs + Ls^{p_2/q_2}), \tag{21}$$

where q_2 and p_2 are odd positive integers with $p_2 < q_2$. $B = \text{diag}\{b_i\}_{n \times n}$, $L = \text{diag}\{l_i\}_{n \times n}$, $b_i > 0$ and $l_i > 0$ are design parameters and $s^{p_2/q_2} = [s_1^{p_2/q_2}, s_2^{p_2/q_2}, \dots, s_n^{p_2/q_2}]^T$.

The above development of the TSM control using the TSMDO for the MIMO nonlinear system (6) is summarized as follows:

Theorem 2. Considering an uncertain MIMO nonlinear system (6) subject to the disturbance and the input saturation (7), the TSMDO is designed as (9)–(11). Using the developed disturbance observer-based TSM tracking control scheme (21), the finite time convergence of all closed-loop signals are guaranteed.

Proof. Considering (20) and (21), we have

$$\dot{s} = -Bs - Ls^{p_2/q_2} + D - \hat{D} + \dot{\sigma} = -Bs - Ls^{p_2/q_2} + \tilde{D} + \dot{\sigma}. \tag{22}$$

Invoking (15), we obtain

$$\dot{s} = -Bs - Ls^{p_2/q_2}. \tag{23}$$

Consider the Lyapunov function candidate

$$V_c = \frac{1}{2} s^T s. \tag{24}$$

Invoking (23) and (24), the time derivative of V is

$$\dot{V}_c = -\sum_{i=1}^n b_i s_i^2 - \sum_{i=1}^n l_i s_i^{(p_2+q_2)/q_2} \leq -2bV_c - l2^{(p_2+q_2)/2q_2} V_c^{(p_2+q_2)/2q_2}, \tag{25}$$

where $b = \min(b_i)$ and $l = \min(l_i)$.

According to Lemma 1 and (25), we obtain $s \rightarrow 0$ in finite time, which means $z - y_d \rightarrow 0$ in finite time by considering $\xi \rightarrow 0$ in finite time. From the definition of z , we know $\sigma + x - y_d \rightarrow 0$. Since $\sigma \rightarrow 0$ in finite time, we can obtain $x \rightarrow y_d$ in finite time. Namely, the tracking error is convergent in finite time under the designed TSM control scheme.

For the whole system, including the disturbance observer and the auxiliary system, the Lyapunov function candidate is chosen as

$$V = V_o + V_a + V_c. \tag{26}$$

Considering (14), (18), and (25), we have

$$\dot{V} = \dot{V}_o + \dot{V}_a + \dot{V}_c \leq -2\delta_1 V - \delta_2 2^{(p+q)/2q} V^{(p+q)/2q}, \tag{27}$$

where

$$\delta_1 = \min(k, m, r), \quad \delta_2 = \min(\tau_i, w_i, l_i), \quad \frac{p+q}{2q} = \min\left(\frac{p_0+q_0}{2q_0}, \frac{p_1+q_1}{2q_1}, \frac{p_2+q_2}{2q_2}\right).$$

From (27) and Lemma 1, we know that the finite time convergence of all closed-loop system signals can be guaranteed. This concludes the proof.

Remark 2. To estimate the unknown disturbance in finite time, the TSMDO is designed for an uncertain MIMO nonlinear system in this paper. From (11), we can see that the disturbance estimate involves the sign function. To obtain the smooth disturbance estimate, an extra low-pass filter can be introduced in the designed disturbance observer as the same as the developed SMDO in [44]. On the other hand, the estimated output of TSMDO is employed to design TSM tracking control (21). Thus, the developed TSM tracking control based on TSMDO has a good disturbance reject ability for the uncertain MIMO nonlinear system (6).

4 Anti-disturbance attitude control design

In this section, the attitude tracking control schemes will be developed for the NSV using the designed TSM tracking control approach in Section 3.

4.1 Terminal sliding mode control design for attitude angle loop

According to (9) and (10), for the attitude angle dynamic (1) of the NSV, we have

$$\sigma_s = z_s - \Omega, \quad (28)$$

where z_s is designed as

$$\dot{z}_s = -K_s \sigma_s - \Lambda_s \text{sign}(\sigma_s) - \Gamma_s \sigma_s^{p_{s0}/q_{s0}} - |F_s(\Omega)| \text{sign}(\sigma_s) + G_s(\Omega) \omega, \quad (29)$$

where q_{s0} and p_{s0} are odd positive integers with $p_{s0} < q_{s0}$. $K_s = \text{diag}\{k_{si}\}_{3 \times 3}$, $\Lambda_s = \text{diag}\{\lambda_{si}\}_{3 \times 3}$, $\Gamma_s = \text{diag}\{\tau_{si}\}_{3 \times 3}$, $k_{si} > 0$, $\lambda_{si} > 0$ and $\tau_{si} > 0$, $i = 1, 2, 3$ are design parameters. Here, $\lambda_{si} > |D_{si}|$, $\sigma_s^{p_{s0}/q_{s0}} = [\sigma_{s1}^{p_{s0}/q_{s0}}, \sigma_{s2}^{p_{s0}/q_{s0}}, \sigma_{s3}^{p_{s0}/q_{s0}}]^T$, $\text{sign}(\sigma_s) = [\text{sign}(\sigma_{s1}), \text{sign}(\sigma_{s2}), \text{sign}(\sigma_{s3})]^T$, and $|F_s(\Omega)| = \text{diag}\{|F_{s1}(\Omega)|, |F_{s2}(\Omega)|, |F_{s3}(\Omega)|\}$.

In accordance with (11), the TSMDO of the attitude angle loop is proposed as

$$\hat{D}_s = -K_s \sigma_s - \Lambda_s \text{sign}(\sigma_s) - \Gamma_s \sigma_s^{p_{s0}/q_{s0}} - |F_s(\Omega)| \text{sign}(\sigma_s) - F_s(\Omega). \quad (30)$$

To handle the constraint of rated angular velocity in flight process, the following auxiliary system is constructed according to (16):

$$\dot{\xi}_s = -M_s \xi_s - W_s \xi_s^{p_{s1}/q_{s1}} - \theta_s g_s \text{sign}(\xi_s) + G_s(\Omega) \Delta \omega, \quad (31)$$

where q_{s1} and p_{s1} are odd positive integers with $p_{s1} < q_{s1}$. $\xi_s \in \mathbb{R}^3$ is the state of the auxiliary system. $M_s = \text{diag}\{m_{si}\}_{3 \times 3}$, $W_s = \text{diag}\{w_{si}\}_{3 \times 3}$, $m_{si} > 0$ and $w_{si} > 0$ are design parameters. $\text{sign}(\xi_s) = [\text{sign}(\xi_{s1}), \text{sign}(\xi_{s2}), \text{sign}(\xi_{s3})]^T$, $g_s = \text{diag}\{\|g_{si}(x)\|\}_{3 \times 3}$, $\xi_s^{p_{s1}/q_{s1}} = [\xi_{s1}^{p_{s1}/q_{s1}}, \xi_{s2}^{p_{s1}/q_{s1}}, \xi_{s3}^{p_{s1}/q_{s1}}]^T$, g_{si} is the i th row of the control gain matrix $G_s(\Omega)$, $\Delta \omega = \omega - v_\omega$, $\|\Delta \omega\| \leq \theta_s$, and v_ω is a designed command input.

To design TSM controller of the attitude angle dynamic (1), the sliding surface is designed as

$$s_s = z_s - \Omega_d - \xi_s, \quad (32)$$

where Ω_d is the desired signal of the attitude angle Ω .

Invoking (21), the TSM control law of the attitude angle dynamic (1) is proposed as

$$v_\omega = -G_s^{-1}(\Omega)(F_s(\Omega) - \dot{\Omega}_d + \hat{D}_s + M_s \xi_s + W_s \xi_s^{p_{s1}/q_{s1}} + \theta_s g_s \text{sign}(\xi_s) + B_s s_s + L_s s_s^{p_{s2}/q_{s2}}), \quad (33)$$

where q_{s2} and p_{s2} are odd positive integers with $p_{s2} < q_{s2}$. $B_s = \text{diag}\{b_{si}\}_{3 \times 3}$, $L_s = \text{diag}\{l_{si}\}_{3 \times 3}$, $b_{si} > 0$ and $l_{si} > 0$, $i = 1, 2, 3$ are design parameters, and $s_s^{p_{s2}/q_{s2}} = [s_{s1}^{p_{s2}/q_{s2}}, s_{s2}^{p_{s2}/q_{s2}}, s_{s3}^{p_{s2}/q_{s2}}]^T$.

4.2 Terminal sliding mode control design for attitude angular velocity loop

For the attitude angular velocity dynamic (2) of the NSV, according to (9) and (10), we have

$$\sigma_f = z_f - \omega, \tag{34}$$

where z_f is designed as

$$\dot{z}_f = -K_f \sigma_f - \Lambda_f \text{sign}(\sigma_f) - \Gamma_f \sigma_f^{p_{f0}/q_{f0}} - |F_f(\omega)| \text{sign}(\sigma_f) + G_f(\omega) M_c, \tag{35}$$

where q_{f0} and p_{f0} are odd positive integers with $p_{f0} < q_{f0}$. $K_f = \text{diag}\{k_{fi}\}_{3 \times 3}$, $\Lambda_f = \text{diag}\{\lambda_{fi}\}_{3 \times 3}$, $\Gamma_f = \text{diag}\{\tau_{fi}\}_{3 \times 3}$, $k_{fi} > 0$, $\lambda_{fi} > 0$ and $\tau_{fi} > 0, i = 1, 2, 3$ are design parameters and $\lambda_{fi} > |D_{fi}|$, $\sigma_f^{p_{f0}/q_{f0}} = [\sigma_{f1}^{p_{f0}/q_{f0}}, \sigma_{f2}^{p_{f0}/q_{f0}}, \sigma_{f3}^{p_{f0}/q_{f0}}]^T$, $\text{sign}(\sigma_f) = [\text{sign}(\sigma_{f1}), \text{sign}(\sigma_{f2}), \text{sign}(\sigma_{f3})]^T$, and $|F_f(\omega)| = \text{diag}\{|F_{f1}(\omega)|, |F_{f2}(\omega)|, |F_{f3}(\omega)|\}$.

In accordance with (11), the TSMDO of the attitude angle velocity loop is proposed as

$$\hat{D}_f = -K_f \sigma_f - \Lambda_f \text{sign}(\sigma_f) - \Gamma_f \sigma_f^{p_{f0}/q_{f0}} - |F_f(\omega)| \text{sign}(\sigma_f) - F_f(\omega). \tag{36}$$

To handle the constraint of the control moment, the following auxiliary system is constructed according to (16):

$$\dot{\xi}_f = -M_f \xi_f - W_f \xi_f^{p_{f1}/q_{f1}} - \theta_f g_f \text{sign}(\xi_f) + G_f(\omega) \Delta M_c, \tag{37}$$

where q_{f1} and p_{f1} are odd positive integers with $p_{f1} < q_{f1}$. $\xi_f \in \mathbb{R}^3$ is the state of the auxiliary system. $M_f = \text{diag}\{m_{fi}\}_{3 \times 3}$, $W_f = \text{diag}\{w_{fi}\}_{3 \times 3}$, $m_{fi} > 0$ and $w_{fi} > 0$ are design parameters. $\text{sign}(\xi_f) = [\text{sign}(\xi_{f1}), \text{sign}(\xi_{f2}), \text{sign}(\xi_{f3})]^T$, $g_f = \text{diag}\{\|g_{fi}(x)\|\}_{3 \times 3}$, $\xi_f^{p_{f1}/q_{f1}} = [\xi_{f1}^{p_{f1}/q_{f1}}, \xi_{f2}^{p_{f1}/q_{f1}}, \xi_{f3}^{p_{f1}/q_{f1}}]^T$, g_{fi} is the i th row of the control gain matrix $G_f(\omega)$, $\Delta M_c = M_c - v_M$, $\|\Delta M_c\| \leq \theta_f$, and v_M is a designed command input.

To design TSM controller of the attitude angle dynamic (1), the sliding surface is given by

$$s_f = z_f - \omega_d - \xi_f, \tag{38}$$

where ω_d is the desired signal of the attitude angle ω .

Invoking (21), the TSM control law of the attitude angle dynamic (2) is proposed as

$$v_M = -G_f^{-1}(\Omega)(F_f(\Omega) - \dot{\Omega}_d + \hat{D}_f + M_f \xi_f + W_f \xi_f^{p_{f1}/q_{f1}} + \theta_f g_f \text{sign}(\xi_f) + B_f s_f + L_f s_f^{p_{f2}/q_{f2}}), \tag{39}$$

where q_{f2} and p_{f2} are odd positive integers with $p_{f2} < q_{f2}$. $B_f = \text{diag}\{b_{fi}\}_{3 \times 3}$, $L_f = \text{diag}\{l_{fi}\}_{3 \times 3}$, $b_{fi} > 0$ and $l_{fi} > 0, i = 1, 2, 3$ are design parameters, and $s_f^{p_{f2}/q_{f2}} = [s_{f1}^{p_{f2}/q_{f2}}, s_{f2}^{p_{f2}/q_{f2}}, s_{f3}^{p_{f2}/q_{f2}}]^T$.

Remark 3. From (30), (31), (36), and (37), we note that the disturbance upper boundaries λ_{si} and λ_{fi} and the upper boundaries θ_s and θ_f of the differences between the desired control inputs and the actual control inputs are used to design the TSM disturbance observers and the auxiliary systems, respectively. It is clear that these parameters will affect the disturbance estimate performance and the auxiliary system states, which should be properly chosen to satisfy the design requirement.

5 Numerical simulation

To illustrate the effectiveness of the proposed TSM attitude control scheme of the NSV, the simulation results are presented in this section. The TSM attitude control schemes are designed in accordance with (28)–(39).

In this simulation, we suppose that the flight velocity is $V = 2500$ m/s and the flight altitude is $h = 30$ km for the NSV. The initial values of the attitude angles and attitude angular velocities are assumed as $\alpha_0 = 2^\circ$, $\beta_0 = 1^\circ$, $\mu_0 = 0^\circ$, and $p_0 = q_0 = r_0 = 0^\circ/\text{s}$ for the NSV. The saturation levels are chosen as $v_{ri \max} = 1.4 \times 10^6$ Nm and $v_{li \max} = -1.5 \times 10^6$ Nm, $i = 1, 2, 3$. At the same time, the system uncertainties $\Delta F_s(\Omega)$ and $\Delta F_f(\omega)$ are assumed as the 10% uncertainties in aerodynamic coefficients and

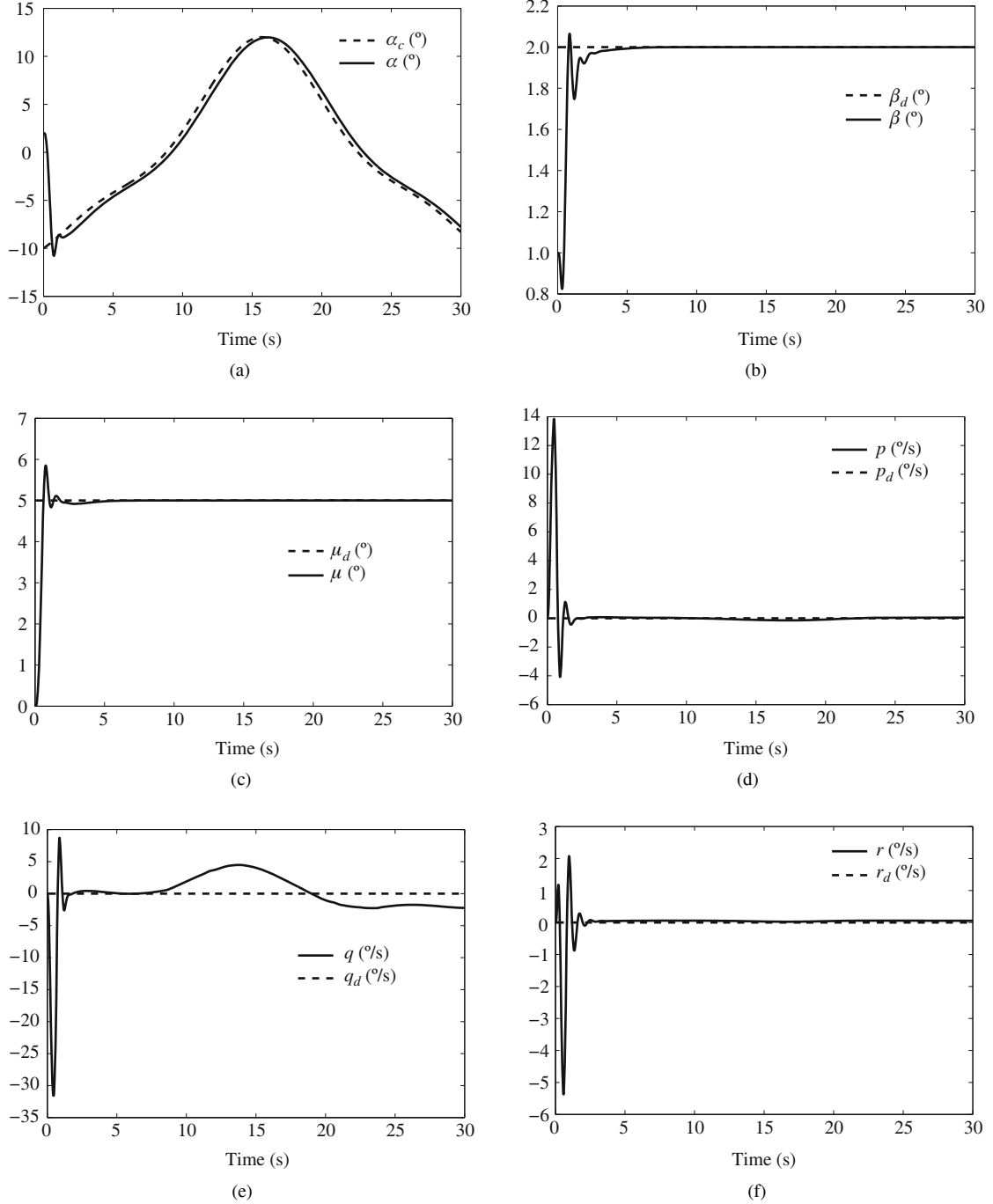


Figure 1 Control result of (a) angle of attack, (b) sideslip angle, (c) flight-path roll angle, (d) roll angular velocity, (e) pitch angular velocity, and (f) yaw angular velocity.

aerodynamic moment coefficients, respectively. The external disturbances $d_s(t)$ and $d_f(t)$ of the NSV are chosen as [7]

$$d_{s1}(t) = \sin(0.5t) + 0.25, \quad d_{s2}(t) = 2(\sin(0.2t) + 0.2), \quad d_{s3}(t) = 1.5(\sin(0.5t) + 0.1), \quad (40)$$

and

$$\begin{aligned} d_{f1}(t) &= 150000(\sin(2t) + 0.1) \text{ Nm}, \\ d_{f2}(t) &= 200000(\sin(2t) + 0.2) \text{ Nm}, \end{aligned} \quad (41)$$

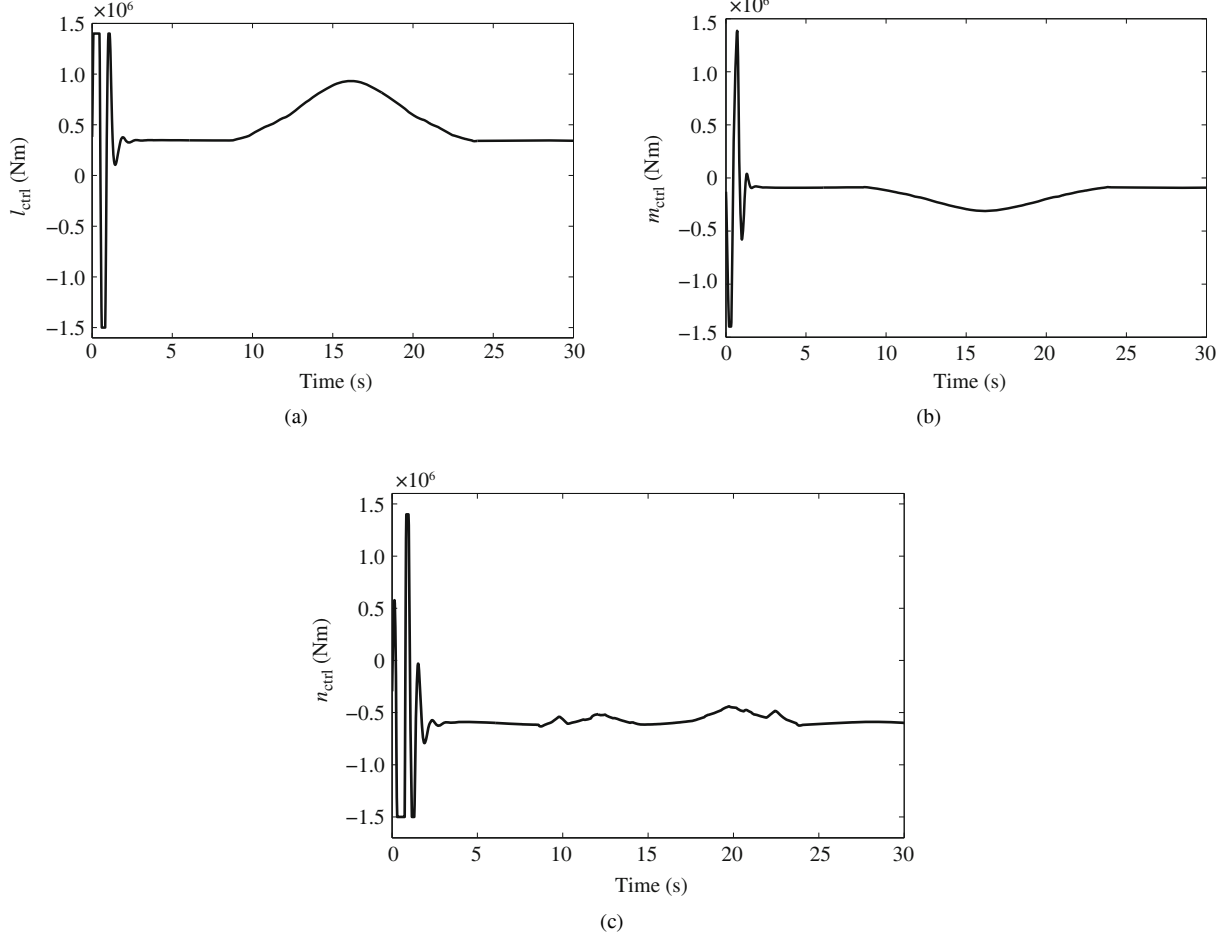


Figure 2 (a) Roll, (b) pitching, and (c) yaw control moment.

$$d_{f3}(t) = 200000(\sin(2t) + 0.2)) \text{ Nm.}$$

To design the TSM attitude controllers, all design parameters are chosen as $q_{s0} = q_{s1} = q_{s2} = q_{f0} = q_{f1} = q_{f2} = 9$, $p_{s0} = p_{s1} = p_{s2} = p_{f0} = p_{f1} = p_{f2} = 5$, $K_s = \text{diag}\{200\}_{3 \times 3}$, $\Lambda_s = \text{diag}\{50\}_{3 \times 3}$, $\Gamma_s = \text{diag}\{0.5\}_{3 \times 3}$, $M_s = \text{diag}\{150\}_{3 \times 3}$, $W_s = \text{diag}\{0.5\}_{3 \times 3}$, $\theta_s = 50$, $B_s = \text{diag}\{50\}_{3 \times 3}$, $L_s = \text{diag}\{0.5\}_{3 \times 3}$, $K_f = \text{diag}\{200\}_{3 \times 3}$, $\Lambda_f = \text{diag}\{300000\}_{3 \times 3}$, $\Gamma_f = \text{diag}\{0.5\}_{3 \times 3}$, $M_f = \text{diag}\{150\}_{3 \times 3}$, $W_f = \text{diag}\{0.5\}_{3 \times 3}$, $\theta_f = 10000$, $B_f = \text{diag}\{100\}_{3 \times 3}$, and $L_f = \text{diag}\{0.5\}_{3 \times 3}$.

The desired attitude signals are taken as $\alpha_d = 2 \sin(0.5t) - 10 \cos(0.2t)$, $\beta_d = 2$, $\mu_d = 5$, and $p_d = q_d = r_d = 0$. Under the developed TSM attitude control schemes, the control results of the attitude angles and the attitude angular velocities are presented in Figure 1. From this figure, we observe that the satisfactory tracking performance is obtained for the desired attitude signals of the NSV with time-varying unknown disturbance and input saturation. At the same time, the corresponding control moments are shown in Figure 2, which are bounded and convergent.

Based on the above simulation results, we conclude that the developed TSM attitude control scheme is valid for the NSV in the presence of unknown disturbance and input saturation.

6 Conclusion

In this article, the TSM attitude control scheme has been studied for the NSV subject to unknown disturbance and input saturation. The TSMDO has been proposed to guarantee the finite time convergence of the disturbance estimate error. At the same time, an auxiliary design system has been designed to

compensate for the input saturation effect. Using the developed disturbance observer and the auxiliary system, the TSM attitude control scheme has been developed to improve the anti-disturbance ability and to speed up the convergence of all closed-loop system signals. Numerical simulation results have been presented to show the effectiveness of the developed TSM attitude control scheme under the integrated affection of the unknown compound disturbance and the input saturation. In future, the developed TSM attitude control scheme based on the TSMDO can be extended to other nonlinear systems.

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