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Joint range ambiguity resolving and multiple maneuvering targets tracking in clutter via MMPHDF-DA

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Abstract It is difficult to track multiple maneuvering targets of which the number is unknown and time-varying, especially when there is range ambiguity. The random finite sets (RFS) based probability hypothesis density filter (PHDF) is an effective solution to the problem of multiple targets tracking. However, when tracking multiple targets via the range ambiguous radar, the problem of range ambiguity has to be solved. In this paper, a multiple model PHDF and data association (MMPHDF-DA) based method is proposed to address multiple maneuvering targets tracking with range ambiguous radar in clutter. Firstly, by introducing the turn rate of target and the discrete pulse interval number (PIN) as components of target state vector, and modeling the incremental variable of the PIN as a three-state Markov chain, the problem of multiple maneuvering targets tracking with range ambiguity is converted into a hybrid state filtering problem. Then, by implementing a novel "track-estimate" oriented association with the filtering results of the hybrid filter, target tracks are provided at each time step. Simulation results demonstrate that the MMPHDF-DA can estimate target state as well as the PIN simultaneously, and succeeds in multiple maneuvering target tracking with range ambiguity in clutter. Simulation results also demonstrate that the MMPHDF-DA can overcome the limitation of the Chinese Remainder Theorem for range ambiguity resolving.

Keywords probability hypothesis density, particle filter, range ambiguity, maneuvering target tracking, data association

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1 Introduction

In recently years, the random finite set (RFS) [1] has proven to be an effective method for multiple targets tracking. In the RFS framework, the state set and measurement set are modeled as RFSs so that the problem of dynamically multiple targets tracking in clutter is cast in the Bayesian framework. However, the RFS based optimal multiple targets Bayesian filter requires dealing with a combinatorial sum of high dimensional integrals, and thus is vary computationally expensive and intractable in general.

To alleviate the intractability and reduce the complexity, the probability hypothesis density filter (PHDF) [2–12] was devised as a suboptimal solution to the Bayesian multiple targets filter. The PHDF

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is an effective treatment for multiple targets tracking in clutter, which estimates targets number and target states simultaneously. To give a recursive solution of the PHDF, particle filter approximation of PHDF (Particle-PHDF) [3–5] and Gaussian mixture (GM) implementation of PHDF (GM-PHDF) [6–9] were devised by Zajic, Vo and Ma et al. The RFS based approaches have been successfully used for many real-world problems [10–13].

However, to provide precise velocities of targets, the airborne pulse Doppler (PD) radar usually adopts the middle or high pulse repetition frequency (M/HPRF) working mode, which results in problem that the range measurements of target are ambiguous [14] and makes the multiple maneuvering targets tracking problem much more complex. As a result, the multiple maneuvering targets tracking with PD radar involves estimating a time varying number target states based on a time varying number of ambiguous measurements. This requires that the algorithm should be capable of resolving range ambiguity and estimating the target number and target states simultaneously, and is an intractable problem to be solved.

At present, there have been lots of data processing based researches on single target tracking with range ambiguity and some effective methods have been proposed, such as Chinese Remainder Theorem [15,16], permutation and combination method [17], multiple hypothesis (MH) [18], hybrid filter [19], and so on. However, when tracking multiple targets in clutters with range ambiguity, the matching of range gates of different PRFs before range ambiguity resolving is required. The existing literatures for multiple targets tracking with range ambiguity mainly focus on target tracking with the precondition that the matching has been done correctly, and not involve how to deal with the outliers due to wrong matching [18]. The essence of this kind of methods is converting the problem of multiple targets tracking into multiple single-target tracking. To make correct matching of range gates, correct associations between targets and observations are required, which is also the difficulty of multiple targets tracking.

The distinct merit of the PHDF is that it can avoid the association between targets and measurements, and hence the matching of target range gates before target tracking. Hence, the PHDF is a feasible solution to multiple targets tracking with range ambiguity in theory. Under this consideration, the authors proposed a PHDF based range ambiguity resolving and multiple targets tracking method in [20]. This method uses all of the possible measurements for states update. Although of high efficiency, it is very computationally intensive and cannot adapt the situation of maneuvering target tracking. This paper proposes a novel algorithm based on the multiple model PHDF and data association (MMPHDF-DA), which enables joint range ambiguity solving and multiple maneuvering targets tracking. In the proposed algorithm, the ambiguous measurements are utilized for states update directly, which reduces the computation load dramatically. Furthermore, by introducing the turn rate as components of target state and setting up corresponding dynamic equations, the proposed method performs well in maneuvering target tracking. It should be noticed that the multiple model setup in this paper is not the traditional multiple target moving model [4], but the incremental variable of the PIN transitional model, which is also an innovation of this paper and plays a very important part in range ambiguity resolving. The effectiveness of the proposed algorithm is verified by simulation results.

2 Background and problem formulation

In this section, a short review of RFS and the PHDF for multiple targets filtering are presented.

2.1 Multiple targets filtering

Define a random set stochastic process $\boldsymbol{X} = \{\boldsymbol{X}_k | k \in \mathbb{N}\}$ and a random set stochastic process $\boldsymbol{Z} = \{\boldsymbol{Z}_k | k \in \mathbb{N}^+\}$. The multiple target states are modeled by RFS $\boldsymbol{X}_k = \{\boldsymbol{x}_k^1, \ldots, \boldsymbol{x}_k^{M_k}\} \subset \boldsymbol{E}_s$, where $\boldsymbol{E}_s$ is the state space, M_k is the targets number, and $\boldsymbol{x}_k^i$ is the ith state of target. The multiple target measurements are given by RFS $\boldsymbol{Z}_k = \{\boldsymbol{z}_k^1, \ldots, \boldsymbol{z}_k^{N_k}\} \subset \boldsymbol{E}_o$, where $\boldsymbol{E}_o$ is the measurement space, N_k is the measurements number, and $\boldsymbol{z}_k^j$ is the jth measurement of target.

The problem of multiple targets filtering is how to estimate the states $\hat{\boldsymbol{X}}_k = \{\hat{\boldsymbol{x}}_k^1, \ldots, \hat{\boldsymbol{x}}_k^{\hat{N}_k}\}$ based on the observations $\boldsymbol{Z}_{1:k} = \{\boldsymbol{Z}_1, \ldots, \boldsymbol{Z}_k\}$, where $\hat{N}_k$ is the estimated targets number, and $\hat{\boldsymbol{x}}_k^i$ is the ith estimated target state.

2.2 The PHDF

Let $D_{k-1|k-1}(\boldsymbol{x}) = D_{k-1|k-1}(\boldsymbol{x}|\boldsymbol{Z}_{1:k-1})$ denote the PHD at time $k-1$, and $D_{k-1|k}(\boldsymbol{x}) = D_{k-1|k}(\boldsymbol{x}|\boldsymbol{Z}_{1:k})$ denote the predicted PHD at time k, given the single target state vector $\boldsymbol{x}$ and observations $\boldsymbol{Z}_{1:k}$, the prediction equation and update equation are respectively given by [2]

$$D_{k|k-1}(\boldsymbol{x}) = \gamma_k(\boldsymbol{x}) + \int \Phi_{k|k-1}(\boldsymbol{x}|\boldsymbol{x}_{k-1}) D_{k-1|k-1}(\boldsymbol{x}_{k-1}) \mathrm{d}\boldsymbol{x}_{k-1}, \tag{1}$$

and

$$D_{k|k}(\boldsymbol{x}) = \left[v_k(\boldsymbol{x}) + \sum_{\boldsymbol{z} \in \boldsymbol{Z}_k} \frac{\psi_{k,\boldsymbol{z}}(\boldsymbol{x})}{\kappa_k(\boldsymbol{z}) + \langle D_{k|k-1}, \psi_{k,\boldsymbol{z}} \rangle} \right] D_{k|k-1}(\boldsymbol{x}), \tag{2}$$

with $v_k(\boldsymbol{x}) = 1 - P_{D,k}(\boldsymbol{x})$, $\psi_{k,\boldsymbol{z}}(\boldsymbol{x}) = P_{D,k}(\boldsymbol{x}) g_k(\boldsymbol{z}|\boldsymbol{x})$, $\Phi_{k|k-1}(\boldsymbol{x}|\boldsymbol{x}_{k-1}) = \mathrm{e}_{k|k-1}(\boldsymbol{x}_{k-1}) f_{k|k-1}(\boldsymbol{x}|\boldsymbol{x}_{k-1}) + b_{k|k-1}(\boldsymbol{x}|\boldsymbol{x}_{k-1})$, and $\kappa_k(\boldsymbol{z}) = \lambda_k c_k(\boldsymbol{z})$, where $P_{D,k}(\boldsymbol{x}_k)$ and $\mathrm{e}_{k|k-1}(\boldsymbol{x}_{k-1})$ respectively denote the target detection and target survival probability, $\gamma_k(\cdot)$ and $b_{k|k-1}(\cdot|\boldsymbol{x}_{k-1})$ respectively denote the PHD of new birth target $\boldsymbol{\Gamma}_k$ and spawned target $B_{k|k-1}(\boldsymbol{x}_{k-1})$, $f_{k|k-1}(\cdot|\boldsymbol{x}_{k-1})$ and $g_k(\cdot|\boldsymbol{x}_k)$ respectively denote the single target transition function and single target likelihood, while λ_k and $c_k(\cdot)$ respectively represent the false alarms number per scan and clutter points probability distribution, and $\langle \cdot, \cdot \rangle$ is the inner product defined as

$$\langle \varphi, \phi \rangle = \int \varphi(\boldsymbol{x}) \phi(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}. \tag{3}$$

In general, neither of the integrals of (1) and (2) has closed solution, which can be implemented by the particle filter based numerical methods [3–5].

2.3 Range ambiguity

Assume that $R_{\max}$ is the maximum range of interest, F_r is the PRF used by the radar. The maximum unambiguous range $R_{u,\max}$ corresponding to F_r is given by

$$R_{u,\max} = \frac{C}{2F_r}, \tag{4}$$

where C is the speed of light. Let (r_k, θ_k, d_k) denote the true position of target, where r_k, θ_k and d_k respectively denote the range, bearing and Doppler of target. Then the range measurement generated by radar will appear to be $r_{k,\mathrm{amb}} = \mathrm{mod}(r_k, R_{u,\max})$, where the function $\mathrm{mod}(x, y)$ denotes the remainder of x/y. Consequently, the true range of target must be one of the values given by the set

$$\{r_k^i | r_k^i = (i-1) \times R_{u,\max} + r_{k,\mathrm{amb}}; i = 1, 2, \ldots, N_{u,\max}\}, \tag{5}$$

where

$$N_{u,\max} = \mathrm{Floor}\left(\frac{R_{\max}}{R_{u,\max}} \right) \tag{6}$$

denotes the maximum unambiguous number corresponding to PRF F_r and the function $\mathrm{Floor}(x)$ means to get the nearest integer equal to or less than x. The value $i \in \{1, 2, \ldots, N_{u,\max}\}$ is defined as the pulse interval number (PIN) corresponding to PRF F_r such that r_k^i reflects the true range of target at time k. Eq. (5) demonstrates that the true range of target must be one of ranges represented by (5); however, it is impossible to get the true range of target directly.

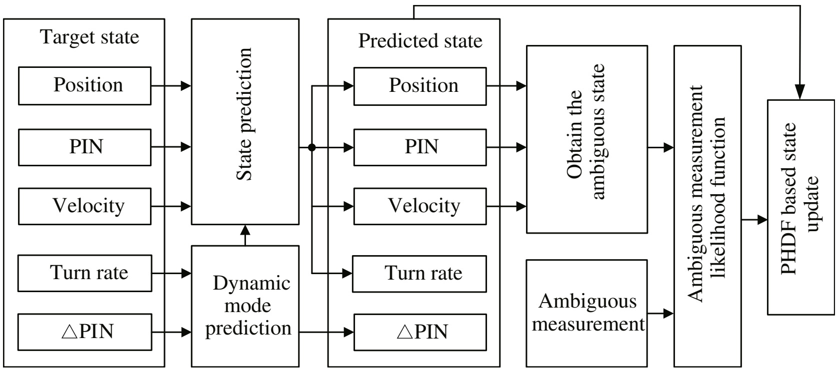

Figure 1 The hybrid estimation system.

3 System setup

Assume that the range measurements generated by the radar are ambiguous. According to the hybrid filtering system, Figure 1 shows the general block diagram of the proposed method.

The dynamic model and measurement model are described as follows.

3.1 Dynamic model

Let $\boldsymbol{x}_{s,k} = [x_k\ \dot{x}_k\ y_k\ \dot{y}_k\ \omega_k]^{\mathrm{T}}$ be the state of target at time k, where (x_k, y_k), $(\dot{x}_k, \dot{y}_k)$ and ω_k respectively denote the position, velocity and turn rate of target, and $[\cdot]^{\mathrm{T}}$ represents the transpose of a matrix $[\cdot]$. The state propagation from time k to $k+1$ is given by

$$\boldsymbol{x}_{s,k+1} = \boldsymbol{F}_{s,k}\boldsymbol{x}_{s,k} + \boldsymbol{G}_{s,k}\boldsymbol{V}_{s,k}, \tag{7}$$

with

$$\boldsymbol{F}_{s,k} = \begin{bmatrix} 1 & \frac{\sin\omega_k T}{\omega_k} & 0 & -\frac{1-\cos\omega_k T}{\omega_k} & 0 \\ 0 & \cos\omega_k T & 0 & -\sin\omega_k T & 0 \\ 0 & \frac{1-\cos\omega_k T}{\omega_k} & 1 & \frac{\sin\omega_k T}{\omega_k} & 0 \\ 0 & \sin\omega_k T & 0 & \cos\omega_k T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

and

$$\boldsymbol{G}_{s,k} = \begin{bmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

respectively denoting the transition matrix and the distribution matrix of process noise, where T is the sampling interval, and $\boldsymbol{V}_{s,k}$ is process noise with zero mean and covariance

$$\boldsymbol{Q}_{s,k} = \begin{bmatrix} a_{x,\max}/3 & 0 & 0 \\ 0 & a_{y,\max}/3 & 0 \\ 0 & 0 & \sigma_\omega^2 \end{bmatrix}, \tag{10}$$

where $a_{x,\max}$ and $a_{y,\max}$ are the maximum accelerations in x direction and y direction, respectively, and σ_ω is the stand deviation for turn rate.

Let $\Delta\mathrm{PIN}_k = \mathrm{PIN}_{k+1} - \mathrm{PIN}_k$ denote the incremental variable of PIN_k, where PIN_k is the PIN corresponding to the PRF at time k. Assuming that the increase or decrease of target range can not exceed the maximum unambiguous range in one sampling interval. Let $m_k = 1, 2, 3$ respectively denote that PIN_k moves backward to the previous one pulse interval, and target maintains the current pulse interval and target moves forward to the next one pulse interval. Then state model of PIN_k can be reasonably simplified into

$$\mathrm{PIN}_{k+1} = \begin{cases} \mathrm{PIN}_k - 1, & m_k = 1, \\ \mathrm{PIN}_k, & m_k = 2, \\ \mathrm{PIN}_k + 1, & m_k = 3, \end{cases} \tag{11}$$

and $\Delta\mathrm{PIN}_k$ can be obtained as follows:

$$\Delta\mathrm{PIN}_k = \begin{cases} -1, & m_k = 1, \\ 0, & m_k = 2, \\ 1, & m_k = 3. \end{cases} \tag{12}$$

Take the PIN as an element of the state vector, i.e.,

$$\boldsymbol{x}_k = [\boldsymbol{x}_{s,k}\ \mathrm{PIN}_k]^{\mathrm{T}}. \tag{13}$$

The extended dynamic equation is given by

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_k \boldsymbol{x}_k + \boldsymbol{B}\Delta\mathrm{PIN}_k + \boldsymbol{V}_k, \tag{14}$$

with

$$\boldsymbol{F}_k = \begin{bmatrix} \boldsymbol{F}_{s,k} & \boldsymbol{O}_{1\times 5}^{\mathrm{T}} \\ \boldsymbol{O}_{1\times 5} & 1 \end{bmatrix}, \tag{15}$$

and

$$\boldsymbol{V}_k = \begin{bmatrix} \boldsymbol{G}_{s,k} V_{s,k} \\ u_k \end{bmatrix}, \tag{16}$$

where $\boldsymbol{O}_{1\times 5}$ is a 1×5 zero vector, $\boldsymbol{B} = [0\ 0\ 0\ 0\ 0\ 1]^{\mathrm{T}}$ and $u_k = 0$.

Since m_k takes one discrete value out of $\boldsymbol{S} = \{1,\ 2,\ 3\}$ at a random time step, it can be modeled by a three-state Markov chain [21] with

$$\pi_{\mathrm{pq}} \triangleq P(m_k = q | m_{k-1} = p),\ p, q \in \boldsymbol{S}, \tag{17}$$

denoting the model transitional probability that m_k switches from p at time $k-1$ to q at time k. In summary the transitional probability matrix is given by

$$\prod\nolimits_m = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}, \tag{18}$$

which is assumed to be known and the initial model probabilities denoted by $\varphi_1 = P\{m_0 = 1\}$, $\varphi_2 = P\{m_0 = 2\}$ and $\varphi_3 = P\{m_0 = 3\}$ satisfying $\varphi_1 + \varphi_2 + \varphi_3 = 1$ are also assumed to be known.

3.2 Measurement model

Let $\boldsymbol{z}_{k,\mathrm{amb}} = [r_{k,\mathrm{amb}}\ \theta_k\ d_k]^{\mathrm{T}}$ be the ambiguous measurement generated by the radar at time k, where $r_{k,\mathrm{amb}}$, θ_k and d_k respectively denote the ambiguous range, bearing and Doppler measurements of target. Then the measurement equation is given by

$$\boldsymbol{z}_{k,\mathrm{amb}} = \begin{bmatrix} \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2} - \mathrm{PIN}_k R_{u,\max} \\ \arctan\left(\dfrac{y_k - y_s}{x_k - x_s}\right) \\ \dfrac{x_k \dot{x}_k + y_k \dot{y}_k}{\sqrt{(x_k - x_s)^2 + (y_k - y_s)^2}} \end{bmatrix} + \boldsymbol{W}_k, \tag{19}$$

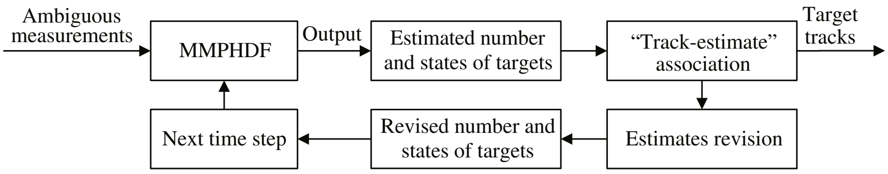

Figure 2 The proposed method.

where (x_s, y_s) is the location of the radar, $R_{u,\max}$ is the maximum unambiguous range, and $\boldsymbol{W}_k$ is independent, zero-mean while Gaussian measurement noise with variance

$$\boldsymbol{R}_k = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_d^2 \end{bmatrix}, \tag{20}$$

with σ_r, σ_θ and σ_d respectively denoting the stand deviation for range, bearing, and Doppler measurements.

4 Particle implementation

In this section, the recursive Bayesian solution of the hybrid systems described in the previous section will be implemented with the MMPHDF-DA, which can directly approximate the state and measurement equations required by (14) and (19). As presented in Figure 2, the implementation of proposed method mainly includes three parts: MMPHDF with range ambiguity, "track-estimate" DA with the outputs of MMPHDF and estimates revision with the updated target tracks.

As is well known, the estimated states are more precise than the measurements, and thus, the association with the outputs of the PHDF rather than the initial measurements should give better performance. Thus, we consider a "track-estimate" based DA method here. The basic procedures of the MMPHDF-DA are as follows:

Let $\{\boldsymbol{x}_k^p, m_k^p, w_k^p\}_{p=1}^{L_k}$ and $\Re_k = \{\tau_{k_i,k}^q | q = 1, \ldots, \mathrm{Tr}_k\}$ respectively denote the particle set and target track set at time k, where $\boldsymbol{x}_k^p$, m_k^p, $w_k^p (p = 1, \ldots, L_k)$ and L_k respectively denote the particle state, PIN changing model, particle weight and number of particles, $\tau_{k_q,k}^q$ denote the qth track, k_q is the initialized time step, and Tr_k is the target tracks number. Given the target tracks set $\Re_{k-1} = \{\tau_{k_q,k-1}^q | q = 1, \ldots, \mathrm{Tr}_{k-1}\}$ and the particle set $\{\boldsymbol{y}_{k-1}^p, w_{k-1}^p\}_{p=1}^{L_{k-1}}$, the main procedure of the proposed MMPHDF-DA method are presented as follows:

Step 1. Initialization at $k = 0$.

Let L_0 be the particles number representing one target and $\hat{N}_0$ be the expected initial target number, $\forall p \in \{1, \ldots, L_0\}$ sample $\boldsymbol{x}_{0|0}^p$ from $D_{0|0}$ and set $k = 1$. The weight associated with each particle is $\hat{N}_0 / L_0$.

Step 2. Prediction.

This step contains two procedures.

$\forall p \in \{1, \ldots, L_{k-1}\}$, sample the PIN incremental model variable $m_{k|k-1}^p$ according the $m_{k-1|k-1}^p$ and the model transitional matrix $\prod_m$;

$\forall p \in \{1, \ldots, L_{k-1}\}$, sample $\boldsymbol{x}_{k|k-1}^p$ from the proposal density $q_k(\cdot | \boldsymbol{x}_{k-1}^p, m_{k|k-1}^p, \boldsymbol{Z}_{k,\mathrm{amb}})$, and evaluate the predicted weight

$$w_{k|k-1}^p = \frac{\Phi_{k|k-1}(\boldsymbol{x}_{k|k-1}^p | x_{k-1}^p)}{q_k(\boldsymbol{x}_{k|k-1}^p | \boldsymbol{x}_{k-1}^p, \boldsymbol{Z}_{k,\mathrm{amb}})}; \tag{21}$$

$\forall p \in \{L_{k-1} + 1, \ldots, L_{k-1} + J_k\}$, sample the model variable $m_{k|k-1}^p$ according the initial model probabilities φ_1, φ_2 and φ_3;

$\forall p \in \{L_{k-1} + 1, \ldots, L_{k-1} + J_k\}$, sample $\boldsymbol{x}_{k|k-1}^p$ from the proposal density $p_k(\cdot | m_{k|k-1}^p \boldsymbol{Z}_{k,\mathrm{amb}})$, and assign a weight

$$w_{k|k-1}^p = \frac{1}{J_k} \frac{\gamma_k(\boldsymbol{x}_{k|k-1}^p)}{p_k(\boldsymbol{x}_{k|k-1}^p | \boldsymbol{Z}_{k,\mathrm{amb}})} \tag{22}$$

to each new born particle.

Step 3. Update.

After the new ambiguous measurement set $\boldsymbol{Z}_{k,\mathrm{amb}}$ is available, the associated weights are updated with the single target likelihood function.

$\forall \boldsymbol{z} \in \boldsymbol{Z}_{k,\mathrm{amb}}$, compute

$$C_k(\boldsymbol{z}) = \sum_{p=1}^{L_{k-1}+J_k} \psi_{k,\boldsymbol{z}}(\boldsymbol{x}^p_{k|k-1}) w^p_{k|k-1}; \tag{23}$$

$\forall p \in \{1, \ldots, L_{k-1} + J_k\}$, update particle weights [3]

$$w^p_{k|k} = \left[1 - P_D(\boldsymbol{x}^p_{k|k-1}) + \sum_{\boldsymbol{z}\in \boldsymbol{Z}_{k,\mathrm{amb}}} \frac{\psi_{k,\boldsymbol{z}}(\boldsymbol{x}^p_{k|k-1})}{\kappa_k(\boldsymbol{z}) + C_k(\boldsymbol{z})}\right] w^p_{k|k-1}. \tag{24}$$

Step 4. Resampling [21].

Compute the mass of particles

$$\hat{N}_{k|k} = \sum_{p=1}^{L_{k-1}+J_k} w^p_{k|k}, \tag{25}$$

and resample particle set $\{\boldsymbol{x}^p_{k|k-1}, m^p_{k|k-1}, w^p_{k|k} \big/ \hat{N}_{k|k}\}_{p=1}^{L_{k-1}+J_k}$ to get $\{\boldsymbol{x}^p_k, m^p_k, w^p_k \big/ \hat{N}_{k|k}\}_{p=1}^{L_k}$, where L_k is particles number used by the filter at time k. Then, the estimated targets number is approximated by $\hat{N}_k = \mathrm{round}(\hat{N}_{k|k})$, where the function round(x) means to get the integer nearest to x.

Step 5. Estimation.

If $\hat{N}_k \neq 0$, analyze the resampled particle set $\{\boldsymbol{x}^p_k\}_{p=1}^{L_k}$ with the k-mean algorithm, and obtain $\hat{N}_k$ clusters $\{\boldsymbol{x'}^p_k\}_{p=1}^{L_{k,n}} (n = 1, \ldots, \hat{N}_k)$, where $\boldsymbol{x'}^p_k \in \{\boldsymbol{x}^p_k\}_{p=1}^{L_k}$, and $L_{k,n}$ is the particles number of the nth cluster satisfying $\sum_{n=1}^{\hat{N}_k} L_{k,n} = L_k$. The estimated target states $\hat{\boldsymbol{X}}_k = \{\hat{\boldsymbol{x}}_{k,n}\}_{n=1}^{\hat{N}_k}$ and covariance $\hat{\boldsymbol{Q}}_k = \{\hat{\boldsymbol{Q}}_{k,n}\}_{n=1}^{\hat{N}_k}$ are given by

$$\hat{\boldsymbol{x}}_{k,n} = \frac{1}{L_{k,n}} \sum_{p=1}^{L_{k,n}} \boldsymbol{x'}^p_k,\ n = 1, \ldots, \hat{N}_k, \tag{26}$$

$$\hat{\boldsymbol{Q}}_{k,n} = \frac{1}{L_{k,n}} \sum_{p=1}^{L_{k,n}} (\boldsymbol{x'}^p_k - \hat{\boldsymbol{x}}_{k,n})^{\mathrm{T}} (\boldsymbol{x'}^p_k - \hat{\boldsymbol{x}}_{k,n}),\ n = 1, \ldots, \hat{N}_k. \tag{27}$$

Step 6. "Track-estimate" association.

$\forall m \in \{1, 2, \ldots, \mathrm{Tr}_{k-1}\}$ and $\forall n \in \{1, 2, \ldots, \hat{N}_k\}$, define the statistical distance between the mth track and the nth estimate

$$d^2_{\mathrm{mn}}(\gamma) = \left[\hat{\boldsymbol{y}}_{k,n} - \hat{\boldsymbol{y}}_{k|k-1,m}\right]^{\mathrm{T}} \boldsymbol{S}^{-1}_{k,m} \left[\hat{\boldsymbol{y}}_{k,n} - \hat{\boldsymbol{y}}_{k|k-1,m}\right], \tag{28}$$

where $\hat{\boldsymbol{y}}_{k,n} - \hat{\boldsymbol{y}}_{k|k-1,m}$ is the innovation, $\hat{\boldsymbol{y}}_{k|k-1,m}$ and $\boldsymbol{S}_{k,m}$ are the predicted state and filtering covariance of the mth track, and the square root $g = \sqrt{\gamma}$ is the "σ data" of gate [22].

The basic ideas of the "track-estimate" association method are as follows:

(1) If $\hat{N}_k = 0$ and $\mathrm{Tr}_{k-1} = 0$, jump to step 7;

(2) If $\hat{N}_k > 0$ and $\mathrm{Tr}_{k-1} = 0$, $\forall \hat{\boldsymbol{x}}_{k,n} \in \hat{\boldsymbol{X}}_k$, initialize a track with the 3/4 logic method [22];

(3) If $\hat{N}_k = 0$ and $\mathrm{Tr}_{k-1} > 0$, $\forall \tau^m_{k_r,k-1} \in \Re_{k-1}$, terminate $\tau^m_{k_r,k-1}$ if $\tau^m_{k_r,k-1}$ is a temporary track, otherwise, obtain the temporary track $\tau^m_{k_r,k}$ by updating $\tau^m_{k_r,k-1}$ with the predicted state $\hat{\boldsymbol{x}}_{k|k-1,m}$;

(4) If $\hat{N}_k > 0$ and $\mathrm{Tr}_{k-1} > 0$, $\forall m \in \{1, 2, \ldots, \mathrm{Tr}_{k-1}\}$ and $\forall n \in \{1, 2, \ldots, \hat{N}_k\}$, compute the statistical distance $d^2_{\mathrm{mn}}(\gamma)$ between track $\tau^m_{k_r,k-1}$ and estimate $\hat{\boldsymbol{x}}_{k,n}$ according to (28), and then

• $\forall \tau^m_{k_r,k-1} \in \Re_{k-1}$, if $\exists \hat{\boldsymbol{x}}_{k,n} \in \hat{\boldsymbol{X}}_k$, s.t. $d^2_{\mathrm{mn}}(\gamma) \leqslant \gamma$, obtain the track $\tau^m_{k_r,k}$ by updating $\tau^m_{k_r,k-1}$ with state $\hat{\boldsymbol{x}}$, and save $\tau^m_{k_r,k}$ into $\Re_k$, where the state $\hat{\boldsymbol{x}}$ satisfies

$$\hat{\boldsymbol{x}} = \arg \min_{\hat{\boldsymbol{x}}_{k,n} \in \hat{\boldsymbol{X}}_k} d^2_{\mathrm{mn}}(\gamma); \tag{29}$$

• $\forall\tau^m_{k_r,k-1}\in\Re_{k-1}$, if there is no $\hat{\boldsymbol{x}}_{k,n}\in\hat{\boldsymbol{X}}_k$, s.t. $d^2_{\mathrm{mn}}(\gamma)\leqslant\gamma$, terminate $\tau^m_{k_r,k-1}$ if $\tau^m_{k_r,k-1}$ is a temporary track, otherwise, obtain the temporary track $\tau^m_{k_r,k}$ by updating $\tau^m_{k_r,k-1}$ with the predicted state $\hat{\boldsymbol{x}}_{k|k-1,m}$;

• $\forall\hat{\boldsymbol{x}}_{k,n}\in\hat{\boldsymbol{X}}_k$, if $\hat{\boldsymbol{x}}_{k,n}$ is not used for the update of any track, $\hat{\boldsymbol{x}}_{k,n}$ is used as track head for track initialization.

Step 7. Estimates revision

If there are tracks updated by the estimates in step 6, the estimated number and estimated states of targets are revised by the updated tracks. Let $k=k+1$, and jump to step 2 until the radar stops working.

5 Simulations

To investigate the effectiveness of the MMPHDF-DA proposed for multiple maneuvering targets tracking with range ambiguity, a multiple maneuvering targets tracking scenario is simulated.

5.1 Scenario

For simplicity, consider a two-dimensional tracking scenario. Assume that the targets are observed in clutters over the surveillance region S. The targets move according to dynamic equation proposed in subsection 3.1. The initial state of newborn target follows a Gaussian distribution, of which the mean $\boldsymbol{x}_0$ and covariance $\boldsymbol{Q}_b$ are given by

$$\boldsymbol{x}_0=\begin{bmatrix}30\ \mathrm{km}\\0\ \mathrm{km/s}\\30\ \mathrm{km}\\0\ \mathrm{km/s}\\0\ \mathrm{rad/s}\end{bmatrix},\ \boldsymbol{Q}_b=\mathrm{Diag}\left(\begin{bmatrix}1\ \mathrm{km}^2\\0.5\ (\mathrm{km/s})^2\\1\ \mathrm{km}^2\\0.5\ (\mathrm{km/s})^2\\(\pi/6)^2(\mathrm{rad/s})^2\end{bmatrix}\right). \tag{30}$$

For simplicity, we assume there are no spawning targets.

Assume that the radar can provide the target ambiguous range, bearing and Doppler measurements. The MMPHDF uses the single target transition function and initial state density as importance sampling proposal densities. The initial model probabilities $\varphi_1=\varphi_2=\varphi_3=1/3$ and model transition matrix are given by

$$\prod_m=\begin{bmatrix}0.80&0.20&0\\0.10&0.80&0.10\\0&0.20&0.80\end{bmatrix}, \tag{31}$$

The choice of the model transition matrix implies that the increase or decrease of radial range of target during one sampling interval cannot exceed the unambiguous range. The remainding main parameters are shown in Table 1.

Figure 3 shows the real positions of a simulated scenario with 3 tracks. Figure 4 displays the x positions and y positions of the tracks given in Figure 3 respectively.

5.2 Effectiveness verification

To verify the effectiveness of MMPHDF-DA, comparisons with the PHDF methods proposed in [20] are made without considering clutters.

Figures 5 and 6 respectively show the filtering results given by the PHDF and MMPHDF-DA, where the crosses "×" are true positions of targets, the small circles "o" are estimates of target given by the PHDF, and the lines with circles "-o-" are estimates of target tracks given by the MMPHDF-DA. Figure 7 gives the comparisons between the true targets number and estimated targets number given by the two methods at each time step. Figure 8 shows the Wasserstein distance [23] (multiple targets miss distance)

Table 1   Parameters for simulation

Item	Value
Location of radar (km, km)	(0, −10)
Total simulation step (s)	50
Maximum range of interest (km)	120
Pulse repetition interval (kHz)	31.25
Sampling interval (s)	1
Average target birth rate per scan	0.2
Target survival probability	0.95
Probability of detection	0.95
Stand deviation for turn rate (rad/s)	0.35
Stand deviation for range measurement (km)	0.2
Stand deviation for bearing measurement (rad)	0.0087
Stand deviation for Doppler measurement (km/s)	0.04
Probability of gate in association	0.95
Particles number for representing a target	3000
Particles number for searching new targets	4000

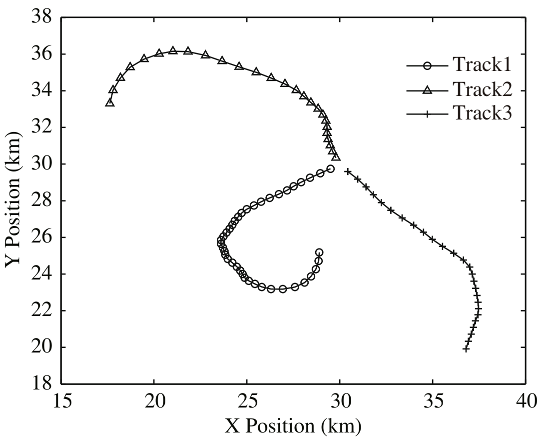

Figure 3   Three true tracks generated by the simulation.

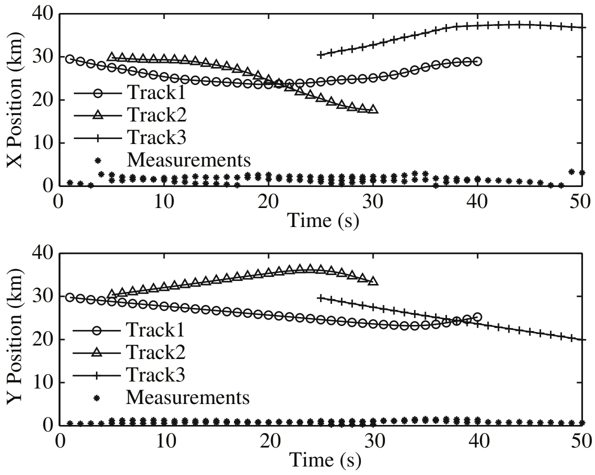

Figure 4   Target tracks given in Figure 3 and ambiguous measurements generated by the radar.

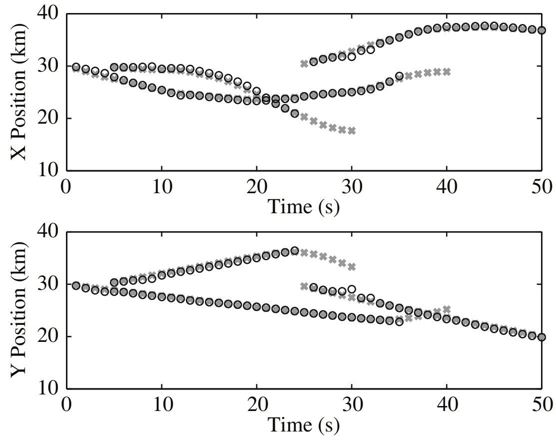

Figure 5   Target states estimates given by the PHDF.

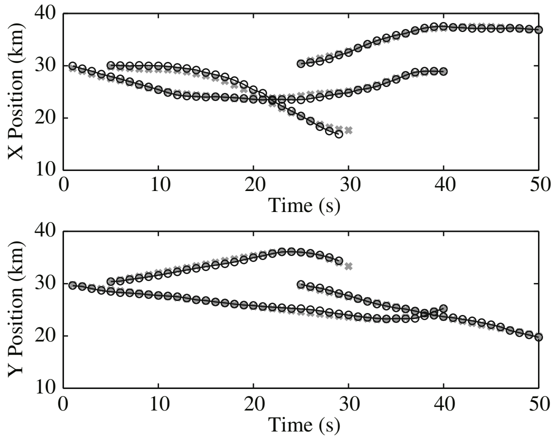

Figure 6   Target tracks estimates given by the MMPHDF-DA.

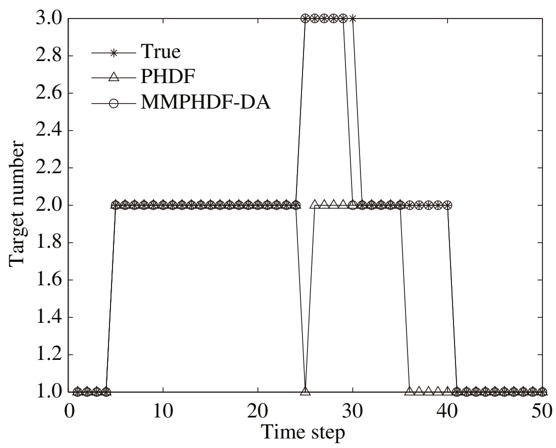

Figure 7 Target numbers estimates against time step.

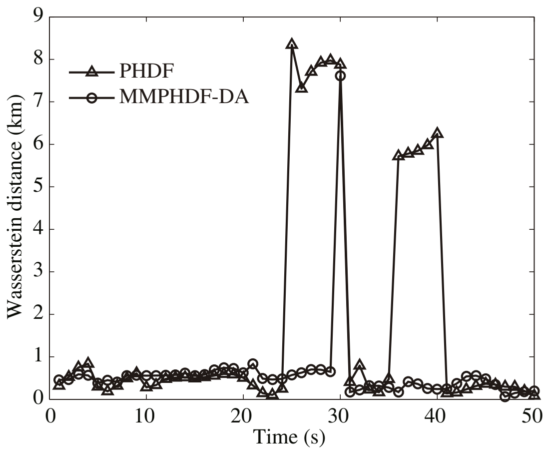

Figure 8 Multiple targets miss distance for the estimates.

Table 2 Simulation results for different maximum ranges of interest

	PHDF		MMPHDF-DA	
$R_{\max}$ (km)	Average errors (km)	Average time (s)	Average errors (km)	Average time (s)
60	1.8339	6.5813	0.6044	1.8876
80	1.9335	8.9967	0.5145	2.0388
100	1.9483	10.9621	0.6962	2.0621

for the estimates of tracks given in Figure 3. It should be noted that the miss distance presented in Figure 8 grow suddenly at some time steps, which can be explained by the fact that the estimated numbers of targets are unequal to the true numbers of targets at those time steps.

Simulation results are demonstrating that:

(1) The MMPHDF-DA performs well in range ambiguity resolving and multiple maneuvering targets tracking, while the performance of PHDF degrade rapidly when the target is maneuvering;

(2) only one PRF is adopted by the radar in the simulation, which means that the MMPHDF-DA overcomes the limitation of range ambiguity resolving with the Chinese Remainder Theorem that three or more than three PRFs are required and the numbers of range cells corresponding to PRFs must be coprime.

5.3 Simulation with different parameters

To analyze the quality of the proposed algorithm, simulations with different maximum range of interest $R_{\max}$, probability of detection P_D and average clutter per scan λ_k are made. The simulations are carried out on a computer with quad 2.66 GHz Intel Core$^{\mathrm{TM}}$ 2 processors and 3 GB RAM.

Setting the average clutter per scan $\lambda_k = 0$ and the probability of detection $P_D = 0.95$, Table 2 shows the average multiple targets miss distance (average error) and average running time per step of the PHDF and MMPHDF-DA when the maximum range of interest varies from 60 km to 100 km.

Table 2 shows that:

(1) The average miss distance of the MMPHDF-DA is much less than that of the PHDF despite the maximum range of interest.

(2) The average running time of the PHDF almost grows linearly with the maximum range of interest. This is because the PHDF uses all of the possible measurements for states update, while the number of possible measurements is related to the maximum range of interest and the maximum unambiguous range.

(3) The MMPHDF-DA uses the ambiguous measurements directly for states update; thus the average running time is hardly affected by the maximum range of interest.

Table 3 Simulation results for different probabilities of detection

P_D	PHDF		MMPHDF-DA	
	Average errors (km)	Average time (s)	Average errors (km)	Average time (s)
0.65	2.0728	10.3725	1.2167	4.4231
0.80	1.9025	7.7420	1.0888	3.1533
0.95	1.8339	6.5813	0.6044	1.8876

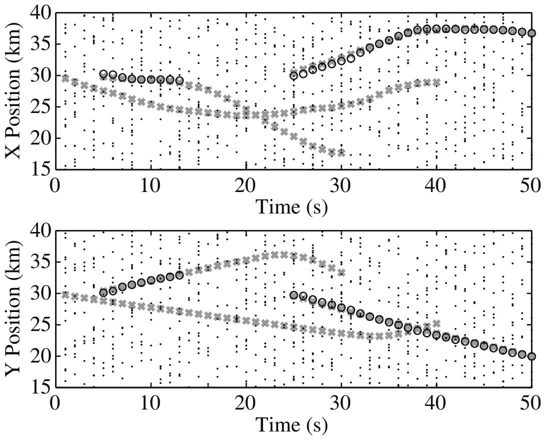

Figure 9 PHDF results ($\lambda_k = 15$).

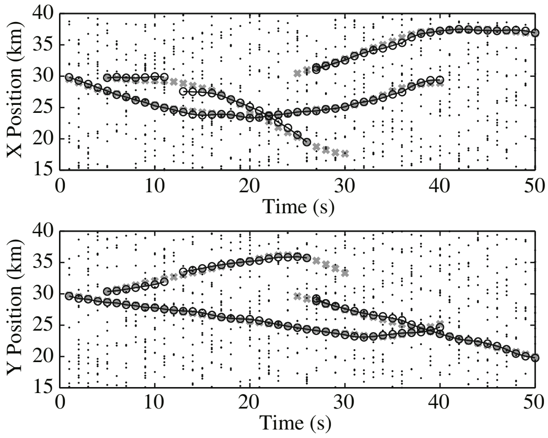

Figure 10 MMPHDF-DA results ($\lambda_k = 15$).

(4) Setting the average clutter per scan $\lambda_k = 0$ and the maximum range of interest $R_{\max} = 60$ km, Table 3 shows the average multiple targets miss distance (average error) and average running time per step of the PHDF and MMPHDF-DA when the probability of detection varies from 0.65 to 0.95.

Table 3 shows that:

(1) The average miss distance of the MMPHDF-DA is much less than that of the PHDF despite the probability of detection.

(2) Both of the average miss distance and the average running time of PHDF and MMPHDF-DA grow with the decrease of the probability of detection. This is because the processing time of the PHDF kernel based method grows linearly with the estimated number of targets, while both of the PHDF and MMPHDF are prone to obtain lots of false target estimates when the probability of detection is low.

Setting the maximum range of interest $R_{\max} = 60$ km, the probability of detection , and the average clutter per scan $\lambda_k = 15$, Figures 9 and 10 present the simulation results of the PHDF and the MMPHDF-DA, respectively. The crosses "×" are the true positions of targets, the dots "·" are the clutters, the small circles "o" are the estimates of target given by the PHDF, and the lines with circles "-o-" denote estimates of target tracks given by the MMPHDF-DA. Figures 9 and 10 demonstrate that the PHDF can not identify targets effectively, while the MMPHDF-DA gives a nice performance in dense clutters environment.

6 Conclusion

In this paper, the problem of multiple maneuvering targets tracking with range ambiguity is studied and a novel algorithm for joint range ambiguity resolving and multiple targets tracking based on the MMPHDF-DA is proposed. By extending the target state vector with the discrete PIN and modeling the dynamic equation with the discrete incremental variable of PIN, the problem of multiple maneuvering targets tracking with range ambiguity is converted into a hybrid state estimation problem. The proposed method can overcome the limitation of the Chinese Remainder Theorem and solve range ambiguity and multiple maneuvering targets tracking simultaneously. Simulation results demonstrate that the MMPHDF-DA algorithm can provide multiple targets tracks in the presence of clutters together with range ambiguity.

Compared with the PHDF, the average processing time of the proposed method has been reduced greatly, but it still cannot meet the real-time requirements of the dynamic system. As a result, the computation load is still a problem to be solved. A work will also involve the detection and tracking multiple weak targets with range ambiguous radar and the demonstration of the technique on real data.

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References

1 Mahler R. Data Fusion Handbook: Random Set Theory for Target Tracking and Identification. Boca Raton: CRC Press, 2001. 1–33
2 Mahler R P S. Multi-target Bayes filtering via first order multi-target moments. IEEE Trans Aerosp Electron Syst, 2003, 39: 1152–1178
3 Vo B N, Singh S, Doucet A. Sequential Monte Carlo methods for multi-target filtering with random finite sets. IEEE Trans Aerosp Electron Syst, 2005, 41: 1224–1245
4 Georgescu R, Willett P. The multiple model CPHD tracker. IEEE Trans Signal Process, 2012, 60: 1741–1751
5 Hong Y J, Kim D Y, Yoon K J. Efficient importance sampling function design for sequential Monte Carlo PHD filter. Signal Process, 2012, 92: 2315–2321
6 Lian F, Han C Z, Liu W F, et al. Joint spatial registration and multi-target tracking using an extended PM-CPHD filter. Sci China Inf Sci, 2012, 55: 501–511
7 Panta K, Clark D E, Vo B N. Data association and track management for the Gaussian mixture probability hypothesis density filter. IEEE Trans Aerosp Electron Syst, 2009, 45: 1003–1016
8 Vo B N, Ma W K. The Gaussian mixture probability hypothesis density filter. IEEE Trans Signal Process, 2006, 54: 4091–4104
9 Li W L, Jia Y M, Du J P, et al. Gaussian mixture PHD filter for multi-sensor multi-target tracking with registration errors. Signal Process, 2013, 93: 86–99
10 Ma W K, Vo B N, Singh S S, et al. Tracking an unknown time-varying number of speakers using TDOA measurements: a random finite set approach. IEEE Trans Signal Process, 2006, 54: 3291–3304
11 Wood T M, Yates C A, Wilkinson D A, et al. Simplified multitarget tracking using the PHD filter for microscopic video data. IEEE Trans Circ Syst Vid, 2012, 22: 702–713
12 Long Y L, Xu H, An W, et al. Track-before-detect for infrared maneuvering dim multi-target via MM-PHD. Chinese J Aeronautics, 2012, 25: 252–261
13 Maggio E, Taj M, Cavallaro A. Efficient multiple targets visual tracking using random finite sets. IEEE Trans Circ Syst Vid, 2008, 18, 1016–1027
14 Stimson G W. Introduction to Airborne Radar. 2nd ed. New Jersey: SciTech Publishing, 1998. 153–154
15 Liao H, Xia X G. A sharpened dynamic range of a generalized Chinese remainder theorem for multiple integers. IEEE Trans Inform Theory, 2007, 53: 428–433
16 Wang C, Yin Q Y, Wang W J. An efficient ranging method based on Chinese remainder theorem for RIPS measurement. Sci China Inf Sci, 2010, 53: 1233–1241
17 Jiang K, Li M. A novel algorithm for PD radar range resolving ambiguity. Fire Control Radar Technol, 2008, 37: 25–28
18 Liu Z L, Guo Y C, Zhang G Y, et al. Multiple models track algorithm for radar with high pulse-repetition frequency in frequency-modulated ranging mode. IET Radar Sonar Navig, 2007, 1: 1–7
19 Wang N, Wang G H, Zeng J Y, et al. Range ambiguity resolving of HPRF radar based on hybrid filter. Sci China Inf Sci, 2011, 54: 1534–1546
20 Wang N. Study on target tracking of airborne pulse Doppler radar on the condition of range ambiguity. Doctor Degree Thesis. Yantai: Naval Aeronautical and Astronautical University, 2011. 59–77
21 Ristic B, Arulampalam S, Gordon N J. Beyond the Kalman Filter. Artech House, 2004
22 He Y, Xiu J J, Zhang J W, et al. Radar Data Processing with Applications. 2nd ed. Beijing: Publishing House of Electronics Industry, 2009. 87–103
23 Hoffman J R, Mahler R P S. Multi-target miss distance via optimal assignment. IEEE Trans Syst, Man Cy B, 2004, 34: 327–336