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Data fusion for target tracking in wireless sensor networks using quantized innovations and Kalman filtering

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Abstract A novel networked data-fusion method is developed for the target tracking in wireless sensor networks (WSNs). Specifically, this paper investigates data fusion scheme under the communication constraint between the fusion center and each sensor. Such a message constraint is motivated by the bandwidth limitation of the communication links, fusion center, and by the limited power budget of local sensors. In the proposed scheme, each sensor collects one noise-corrupted sample, performs a quantizing operation, and transmits quantized message to the fusion center. Then the fusion center combines the received quantized messages to produce a final estimate. The novel data-fusion method is based on the quantized measurement innovations and decentralized Kalman filtering (DKF) with feedback. For the proposed algorithm, the performance analysis of the estimation precision is provided. Finally, Monte Carlo simulations show the effectiveness of the proposed scheme.

Keywords data fusion, target tracking, limited bandwidth, quantized innovation, Kalman filtering

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1 Introduction

Data fusion for estimation is a well studied topic in multi-sensor systems [1–5]. The estimation fusion theory has been widely applied to integrated navigation systems for maneuvering targets, such as airplanes, ships, cars and robots.

In [1,2], an efficient federated Kalman filtering method is developed for distributed multisensor systems based on rigorous information-sharing principles. Ref. [3] established a unified linear model and a general framework for centralized, distributed, and hybrid architectures. There, the optimal fusion rules are based on the best linear unbiased estimation (BLUE) and the weighted least squares (WLS). In [4], an optimal information fusion method is given for discrete time linear stochastic control systems with multiple sensors and correlated noises. Ref. [5] also presented an efficient iterative algorithm for distributed multisensor estimation fusion without any restrictive assumption on the noise covariance. However, those works, e.g.,

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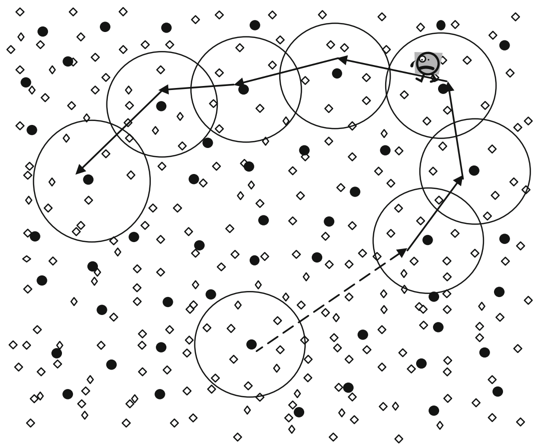

Figure 1 Target tracking in WSNs.

[1–5], only devised estimation fusion based on the local measurements or local state estimations, but without considering the communication bandwidth and the power limitations of sensor.

Recently, WSNs have attracted much attention due to their significant applications in environmental monitoring, intelligent transportation and space exploration, military surveillance, etc. [6–12]. The current and future wireless sensor networks usually deploy a large number of inexpensive sensors whose dynamic range, resolution, and power can be severely limited. Moreover, there can be physical limitations in the communication links from the sensors back to a central site (also known as fusion center). In such cases, local data compression is not only a necessity, but also an integral part of the design of sensor networks [6]. As a result, a main challenge in sensor network research is how to design optimal decentralized estimation fusion schemes in the presence of channel bandwidth and power limitations.

Figure 1 shows a typical tracking scenario in WSNs. The target walks inside the square monitored area of the WSNs. At each time step, one sensor or more, as represented by small square in the figure, is activated to observe the moving target. The big circles in the figure represent the regions in which the sensor nodes may be able to detect the target. The small black circles are the corresponding local fusion centers(LFC). As the target moves, the WSNs must maintain the tracking of the target.That is to say it is able to locate the target with a certain level of accuracy all the time by information exchanges and fusion among the sensor nodes.

The common way to reduce the energy consumption and bandwidth usage is to quantize WSNs data. Obviously, the quantization will result in the loss a large amount of information. Especially, quantizing sensor measurements can lead to large quantization noises when the observed values are large, which then leads to poor estimation accuracy. In [8,9], this limitation is overcome by developing an elegant distributed estimation approach based on quantizing the innovation to one bit (the so-called sign of innovation or SOI). In [10,11], the quantization filter is generalized to handle multiple levels quantized innovations (i.e. MLQI-KF). In [12], taking into account the quantization, encoding and transmitting of measurements in WSNs, Xu et al. studied the estimation fusion of dynamical stochastic processes based on severely quantized observations. It is worth noting that almost all of those works [8–12] focused on the designing of quantization/compression to save bandwidth and energy of the sensors, while they ignored the fact that the bandwidth and energy of local fusion center (LFC) are also limited.

In all of those algorithms [8–10], the estimated state and the associated state error covariance are necessary for the measurement innovations quantization and the next time step estimation. Then the estimated state and the associated state error covariance must be passed around the LFC. For example, in a d-dimensional system, there are $d(d+1)/2+d$ elements to be transmitted at each time step. The first term $d(d+1)/2$ is the number of transmitted elements of the state error covariance matrix, due to its symmetry property, and the second term d is the number of transmitted elements of the estimated state. The transmitted elements increases dramatically with the increasing system dimensions. Hence, in this paper, taking into account both severely quantized observations and the data compression in LFC, we study the estimation fusion of dynamical stochastic processes.

The main objective of the current study is to propose a novel data-fusion approach for target tracking in WSNs. The proposed scheme can reduce the transmitting bandwidth and energy cost, and still keep very high tracking accuracy. In this approach, the innovations quantization and decentralized information fusion technique are adopted. For convenience, we name the proposed data-fusion approach quantized innovation-decentralized Kalman filter(QI-DKF). The innovation quantizing technique is adopted to reduce the energy consumption and bandwidth usage from sensor nodes to LFC. The decentralized Kalman filter is used to produce a final estimate in LFC. Then, the LFC transmits the predicted measurements and the corresponding measure covariance matrix to corresponding sensor nodes. If the observation of sensor node is a scalar, then the transmitting bandwidth and energy cost in LFC can be reduced. The performance of the proposed fusion method is analyzed. The resulting method is simulated for target tracking scenario in WSNs.

The remainder of this paper is organized as follows. Problem statements including the modelling assumptions and preliminaries are given in Section 2. Section 3 introduces the quantization approach firstly. Then a novel estimation fusion method is developed based on the quantized innovations, conditional expectation property and DKF. The performance analysis is presented in Section 4. The simulation results are presented in Section 5. Finally, conclusions are given in Section 6.

Notation. We use $\sigma\{M_1^k\}$ to denote the σ-field generated by random variables $M_1^k = \{M_1, M_2, \ldots, M_k\}$. The probability density function (pdf) of X conditioned on $\sigma\{M_1^k\}$ is represented as $p(X|\sigma\{M_1^k\})$. The Gaussian pdf with mean $\mathrm{E}(X) = \mu$ and covariance matrix $\mathrm{Cov}(X) = \mathbb{C}$ is represented as $p(X) = \mathcal{N}(X; \mu, \mathbb{C})$. The probability mass function for a discrete random variable m is denoted by $\Pr(m)$. Estimators are represented using a hat, e.g., $\widehat{X}(k|\sigma\{M_1^k\}) = \mathrm{E}[X(k)|\sigma\{M_1^k\}]$. Finally, $(\cdot)^{\mathrm{T}}$ stands for transposition and $(\cdot)^{-1}$ stands for the matrix inverse.

2 Modelling assumptions and preliminaries

The sensor measurement is scalar in general WSNs, but for the universality of the proposed results, we consider the state estimation problem of general "vector state-vector measurement" case

$$X(k) = FX(k-1) + \mathit{\Gamma} w(k-1), \tag{1}$$

$$y_i(k) = H_i X(k) + v_i(k), \tag{2}$$

$$k = 0, 1, 2, \ldots,$$

where $X(k) \in \mathbb{R}^d$ is a state vector to be estimated at time $t_k = k\Delta t$, Δt is the time step-length of sample, F and $\mathit{\Gamma}$ are two matrices with suitable dimensions, H_i is the observation coefficient matrix. $H = \{(H_1)^{\mathrm{T}}, (H_2)^{\mathrm{T}}, \ldots, (H_N)^{\mathrm{T}}\}^{\mathrm{T}}$, $y_i(k) \in \mathbb{R}^r$ is the r-dim observation of the sensor i, $r \geqslant 1$, $i = 1, \ldots, N$, N is the number of the active sensors, $Y(\cdot) = \{y_1(\cdot)^{\mathrm{T}}, y_2(\cdot)^{\mathrm{T}}, \ldots, y_N(\cdot)^{\mathrm{T}}\}^{\mathrm{T}}$. $w(k) \in \mathbb{R}^d$ and $v(k) = \{v_1(k)^{\mathrm{T}}, v_2(k)^{\mathrm{T}}, \ldots, v_N(k)^{\mathrm{T}}\}^{\mathrm{T}}$ are uncorrelated Gaussian noises with zero mean and covariance matrix Q and R. In this paper, we consider the WSNs that the sensor measurements are independent of each other, i.e., $\mathbb{R} = \mathrm{diag}\{R_1, R_2, \ldots, R_N\}$. The initial value $X(0)$ with mean μ_0 and variance P_0 is independent of $w(k)$ and $v(k)$. In this paper, we assume that all the parameters of systems (1), (2) are known and the channel is perfect, i.e., without bit error, from sensors to the fusion center.

In order to affect digital communication in the bandwidth-limited WSNs, the observations are quantized. The quantizer is described mathematically by

$$m_i(k) = q_{L_i}(y_i(k)),\ k = 0, 1, 2, \ldots,\ i = 1, 2, \ldots, N, \tag{3}$$

where, $q_{L_i}(\cdot)$ denotes the nonlinear quantization mapping with L_i-levels. Then, the measurement $y_i(k)$, $i = 1, 2, \ldots, N$, is quantized, and thus $Y \in A$ implies $\{a_i \leqslant y_i < b_i, i = 1, 2, \ldots, N\}$. A majority of the following results do not depend on the fact that A is a hypercube. $M(k)$ denotes the vector $\{m_1(k), m_2(k), \ldots, m_N(k)\}^{\mathrm{T}}$.

Define the following events:

$$M(k) = \{a_i(k) \leqslant y_i(k) < b_i(k), i = 1, 2, \ldots, N\}, k = 1, 2, \ldots \tag{4}$$

and

$$M_1^k = \{M(1), M(2), \ldots, M(k)\}, 1 \leqslant k,$$
$$m_i(1:k) = \{m_i(1), m_i(2), \ldots, m_i(k)\}, 1 \leqslant k, \quad 1 \leqslant i \leqslant N.$$

If we had infinite bandwidth available, we could be able to communicate the observations $Y(k)$ error-free. Thus we have $M(k) = Y(k)$. In this case, we have a well-known linear Gaussian state estimation problem with multiple sensors whose estimator can be recursively obtained by the centralized Kalman filtering (CKF)[13] (Algorithm 1-A, 1-B).

Define the estimate $\hat{X}_{k|k} = \mathrm{E}[X_k|M_1^k]$ and the corresponding ECM $P_{k|k} = \mathrm{MSE}[\hat{X}_{k|k}|M_1^k] = \mathrm{E}[(X_k - \hat{X}_{k|k})(X_k - \hat{X}_{k|k})^{\mathrm{T}}|M_1^k]$.

Given $\hat{X}_{0|0}$ and $P_{0|0}$.

Algorithm 1-A. Prediction step.

Given: $\hat{X}_{k-1|k-1}$ and $P_{k-1|k-1}$,

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1}, \tag{5}$$

$$P_{k|k-1} = FP_{k-1|k-1}F^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}}, \tag{6}$$

$$\hat{y}_i(k|k-1) = H_i\hat{X}_{k|k-1}, \ i = 1, 2, \ldots, N. \tag{7}$$

we denote $\hat{Y}(k|k-1) = (\hat{y}_1(k|k-1)^{\mathrm{T}}, \hat{y}_2(k|k-1)^{\mathrm{T}}, \ldots, \hat{y}_N(k|k-1)^{\mathrm{T}})^{\mathrm{T}}$.

Algorithm 1-B. Correction step.

Receive: new observations $M_k = Y(k)$,

$$\widehat{X}_{k|k} = \mathrm{E}[X(k)|\sigma\{M_1^{k-1}, Y(k)\}] = \hat{X}_{k|k-1} + K_k(Y(k) - \hat{Y}(k|k-1)), \tag{8}$$

$$P_{k|k} = P_{k|k-1} - K_kS_kK_k^{\mathrm{T}}, \tag{9}$$

where

$$K_k = P_{k|k-1}S_k^{-1}, \tag{10}$$

$$S_k = HP_{k|k-1}H^{\mathrm{T}} + R. \tag{11}$$

If Algorithm 1 is adopted to process data in WSNs, then each sensor node must transmit its measurement to the corresponding local fusion center without any information loss. After receiving the all sensor's measurements, the fusion center combines the received measurements to produce a final estimate. Then, the fusion center sends the corresponding estimate $\widehat{X}_{k|k}$ and error covariance matrix $P_{k|k}$ to the next local fusion center, if the target moves to the next local tracking region.

Due to the bandwidth constraints from sensor nodes to the LFC and the LFC to other LFC and sensor nodes, we must reduce the traffic between sensor nodes and the LFC to save energy and satisfy the communication bandwidth constraints. In this paper, a nonlinear compression method, i.e. measurement quantization, is adopted to reduce the communication bandwidth from sensor nodes to the LFC. The distributed Kalman filtering (DKF) is adopted to reduce the energy consumption when the LFC transmit messages to the nearby sensor nodes and other LFC.

3 Data fusion using quantized innovations and DKF

In this section, we will investigate the estimation fusion method based on the multiple-level quantized innovations and DKF. It is worth noting that, those algorithms in [8–12] are based on the designing of quantization to save bandwidth and energy of the sensor notes, while they ignored that the bandwidth and energy of LFC are also limited. In order to further reduce the energy consumption of LFC and prolong the WSNs lifetime, we give a novel data fusion method based on quantized innovations and DKF for state estimation in WSNs.

3.1 Quantized innovations

We consider the linear systems (1), (2) in wireless sensor networks. The activated sensor makes an observation, and computes the innovation $\varepsilon_i(k) = y_i(k) - \hat{Y}_i(k|k-1)$, where the one-step predictor $\hat{Y}_i(k|k-1)$ of observation together with the innovation covariance $S_i(k)$ (see Eq. (20)) is received by the sensor from the LFC. Denote the normalized innovation by

$$\overline{\varepsilon}_i(k) = (S_i(k)^{1/2})^{-1}\varepsilon_i(k).$$

Then each component $\overline{\varepsilon}_i(k),\ i = 1, 2, \ldots, N$, of the normalized innovation $\overline{\varepsilon}(k)$ is quantized to produce an $L = (2l + 1)$-level quantized innovation $\varXi(k) = q_L(\overline{\varepsilon}(k))$. We consider a symmetric quantizer $\varXi_i(k) = q_L(\overline{\varepsilon}_i(k))$ for $\overline{\varepsilon}_i(k)$

$$\Xi_i(k) = \begin{cases} \xi_l, & \text{if } \overline{\varepsilon}_i(k) \in (\eta_l, +\infty); \\ \vdots & \\ \xi_2, & \text{if } \overline{\varepsilon}_i(k) \in (\eta_2, \eta_3]; \\ \xi_1, & \text{if } \overline{\varepsilon}_i(k) \in (\eta_1, \eta_2]; \\ 0, & \text{if } \overline{\varepsilon}_i(k) \in (-\eta_1, \eta_1]; \\ -\xi_1, & \text{if } \overline{\varepsilon}_i(k) \in (-\eta_2, -\eta_1]; \\ -\xi_2, & \text{if } \overline{\varepsilon}_i(k) \in (-\eta_3, -\eta_2]; \\ \vdots & \\ -\xi_l, & \text{if } \overline{\varepsilon}_i(k) \in (-\infty, -\eta_l]. \end{cases} \tag{12}$$

Thus, the normalized innovation $\overline{\varepsilon}(k)$ is quantized into a quantized innovation $\varXi(k) = \{\varXi_1(k), \varXi_2(k), \ldots, \varXi_N(k)\}^{\mathrm{T}}$.

Remark 1. Comparing the definition of $M(k)$ in Eq. (4) with $\varXi(k) = q_L(\overline{\varepsilon}(k))$, the difference is that the former gives only the range of $\overline{\varepsilon}(k)$, while the latter gives not only a range information of $\overline{\varepsilon}(k)$, but also a specific value. Clearly the definition of $\varXi_i(k)$ gives a one-one mapping from $M_i(k)$ to $\mathbb{R}$. Hence in following text we will not distinguish $M_i(k)$ and $\varXi_i(k)$. If we take the value of $\xi_j = \mathrm{E}[\overline{\varepsilon}_i(k))|\eta_j < \overline{\varepsilon}_i(k)) \leqslant \eta_{j+1}],\ j = 1, 2, \ldots, l_i$, i.e., $\varXi_i(k) = \mathrm{E}[\overline{\varepsilon}_i(k))|\eta_j < \overline{\varepsilon}_i(k)) \leqslant \eta_{j+1}]$, then the numerical integration (see Eq. (25)) can not be executed in the filtering algorithms. Thus the online computation cost can be reduced. More details will be given in Subsection 3.2.

After quantization, the quantized innovation $\varXi(k) = \{\varXi_1(k), \varXi_2(k), \ldots, \varXi_N(k)\}^{\mathrm{T}}$ is transmitted to the LFC by a dynamic transfer strategy [12]. Then, the fusion center will combine the received messages $\varXi(k)$ to estimate the state $X(k)$. Refs. [8–12] assume that the fusion center has enough bandwidth and energy to transmit the information. In fact, the energy and transmitting bandwidth of LFC are also limited. In order to get the proposed algorithm of this paper, we deduce a KF vision state estimator using quantized innovations in Subsection of 3.2.

3.2 Estimation fusion using quantized innovations and DKF

In this subsection, following the idea of Curry et al. [14], we first introduce a linear approximate MMSE filtering with single quantized innovation to get a local state estimation.

By Eq. (3), for $i = 1, 2, \ldots, N$, we have

$$\sigma\{M_1^{k-1}, y_i(k)\} \supset \sigma\{M_1^{k-1}, m_i(k)\},\ k = 1, 2, \ldots. \tag{13}$$

By the property of iterated conditional expectation [15], for the state variable $X(k)(k = 1, 2, \ldots)$, it holds that

$$\mathrm{E}[X(k)|\sigma\{M_1^{k-1}, m_i(k)\}] = \mathrm{E}[E[X(k)|\sigma\{M_1^{k-1}, y_i(k)\}]|\sigma\{M_1^{k-1}, m_i(k)\}]. \tag{14}$$

Hence, the expectation of $X(k)$ conditioned on quantized measurements $\{M_1^{k-1}, m_i(k)\}$ can be performed in two steps:

1) find $\mathrm{E}[X(k)|\sigma\{M_1^{k-1}, y_i(k)\}]$, which is the usual goal of estimation with unquantized measurements;

2) find the expectation of $\mathrm{E}[X(k)|\sigma\{M_1^{k-1}, y_i(k)\}]$ conditioned on the quantized measurements $\{M_1^{k-1}, m_i(k)\}$.

The first step, under the Gaussian assumption of prior pdf $p(X(k)|\sigma\{M_1^{k-1}\})$, can be performed (similar to the Algorithm 1 in a Gauss-Markov model) as

$$\begin{aligned}\widehat{X}_{k|k}^{i*} &= \mathrm{E}[X(k)|\sigma\{M_1^{k-1}, y_i(k)\}]\\ &= \hat{X}_{k|k-1} + K_i(k)(y_i(k) - \hat{y}_i(k|k-1))\\ &= \hat{X}_{k|k-1} + K_i(k)(\varepsilon_i(k)), \qquad (15)\\ P_{k|k}^{i*} &= P_{k|k-1} - K_i(k)S_i(k)K_i(k)^{\mathrm{T}}, \qquad (16)\end{aligned}$$

where $\hat{X}_{k|k-1}, P_{k|k-1}, K_i(k)$ and $S_i(k)$ can be obtained by (4),(5),(9) and (10) respectively.

The second step is to take the expectation of (15) conditioned on $\{M_1^{k-1}, m_i(k)\}$ to find the mean of $X(k)$ conditioned on ith quantized measurement $m_i(k)$

$$\begin{aligned}\hat{X}_{k|k}^{i} &= \mathrm{E}[X(k)|\sigma\{M_1^{k-1}, m_i(k)\}]\\ &= \mathrm{E}[\mathrm{E}[X(k)|\sigma\{M_1^{k-1}, y_i(k)\}]|\sigma\{M_1^{k-1}, m_i(k)\}]\\ &= \hat{X}_{k|k-1} + K_i(k)\mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]. \qquad (17)\end{aligned}$$

Correspondingly $P_{k|k}^{i} = \mathrm{E}[(X_k - \hat{X}_{k|k}^{i})(X_k - \hat{X}_{k|k}^{i})^{\mathrm{T}}|\sigma\{M_1^{k-1}, m_i(k)\}]$, where, by Eqs. (15) and (17),

$$\begin{aligned}X_k - \hat{X}_{k|k}^{i} &= X_k - \hat{X}_{k|k}^{i*} + (\hat{X}_{k|k}^{i*} - \hat{X}_{k|k}^{i})\\ &= X_k - \hat{X}_{k|k}^{i*} + K_i(k)(\varepsilon_i(k) - \mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]).\end{aligned}$$

Then

$$\begin{aligned}&\mathrm{E}[(X_k - \hat{X}_{k|k}^{i})(X_k - \hat{X}_{k|k}^{i})^{\mathrm{T}}|\sigma\{M_1^{k-1}, m_i(k)\}]\\ &= P_{k|k}^{i*} + K_i(k)(\varepsilon_i(k) - \mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}])(\varepsilon_i(k) - \mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}])^{\mathrm{T}}K_i(k)^{\mathrm{T}}\end{aligned}$$

and finally

$$\begin{aligned}P_{k|k}^{i} &= P_{k|k}^{i*} + K_i(k)\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]K_i(k)^{\mathrm{T}}\\ &= P_{k|k-1} - K_i(k)S_i(k)K_i(k)^{\mathrm{T}} + K_i(k)\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]K_i(k)^{\mathrm{T}}. \qquad (18)\end{aligned}$$

Thus, the local approximate MMSE filter with quantized measurement $m_i(k)$ is given by (15)–(18). The first step is directly obtained from the Kalman-type filter solution because of the Gauss-Markov property of the system (1), (2). In the second step, an important practical problem is to compute efficiently $\mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]$ in (17) and $\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]$ in (18). Despite this difficulty, this approach does provide one common point of departure for designing purposes, and approximations can be made depending on the specific method of quantization.

Now, we will discuss how to calculate $\mathrm{E}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]$ and $\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]$.

First, given $\hat{X}_{k-1|k-1}$ and $P_{k-1|k-1}$, we have

$$\begin{aligned}&\hat{y}_i(k|k-1) = H_i\hat{X}_{k-1|k-1},\ i = 1, 2, \ldots, N, \qquad (19)\\ &P_{k|k-1} = FP_{k-1|k-1}F^{\mathrm{T}} + \Gamma Q\Gamma^{\mathrm{T}}.\end{aligned}$$

Then we can obtain the covariance of innovation

$$\begin{aligned}S_i(k) &= \mathrm{Cov}[\varepsilon_i(k)|M_1^{k-1}] = \mathrm{E}[(y_i(k) - \hat{y}_i(k|k-1))(y_i(k) - \hat{y}_i(k|k-1))^{\mathrm{T}}|M_1^{k-1}]\\ &= H_iP_{k|k-1}H_i^{\mathrm{T}} + R_i. \qquad (20)\end{aligned}$$

Further, under the Gaussian assumption of the prior pdf $p(X(k)|\sigma\{M_1^{k-1}\}) = \mathcal{N}(X(k); X_{k|k-1}, P_{k|k-1})$, for the innovation $\varepsilon_i(k)$, we obtain

$$\begin{aligned}p(\varepsilon_i(k)|M_1^{k-1}) &= \int p(\varepsilon_i(k), X(k)|M_1^{k-1})\mathrm{d}X(k)\\ &= \int p(\varepsilon_i(k)|X(k))p(X(k)|M_1^{k-1})\mathrm{d}X(k)\\ &= \int \mathcal{N}(\varepsilon_i(k); H_iX(k) - H_iX_{k|k-1}, R)\mathcal{N}(X(k); X_{k|k-1}, P_{k|k-1})\mathrm{d}X(k)\\ &\approx \mathcal{N}(\varepsilon_i(k); 0, S_i(k)).\end{aligned}$$

Thus, for the normalized innovation $\overline{\varepsilon}_i(k) = (S_i(k)^{1/2})^{-1}\varepsilon_i(k)$, we have

$$p(\overline{\varepsilon}_i(k)|M_1^{k-1}) = \mathcal{N}(\overline{\varepsilon}_i(k); 0, I_{r\times r}), \tag{21}$$

where $I_{r\times r}$ is an r order identity matrix. Further,

$$\begin{aligned}p(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) &= \begin{cases}\dfrac{p(\overline{\varepsilon}_i(k)|M_1^{k-1})}{\Pr(m_i(k)|M_1^{k-1})}, & \text{if } Q_L(\overline{\varepsilon}_i(k)) = m_i(k);\\ 0, & \text{else}\end{cases}\\ &= \frac{\mathrm{e}^{-\frac{1}{2}\overline{\varepsilon}_i(k)^{\mathrm{T}}\overline{\varepsilon}_i(k)}}{(2\pi)^{\frac{1}{2}}\Pr(M_k|M_1^{k-1})} I_{[a^k,b^k)}(\overline{\varepsilon}_i(k)).\end{aligned}$$

By the property of conditional probability, we have

$$\begin{aligned}p(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) &= \begin{cases}\dfrac{p(\overline{\varepsilon}_i(k)|M_1^{k-1})}{\Pr(m_i(k)|M_1^{k-1})}, & \text{if } Q_L(\overline{\varepsilon}_i(k)) = m_i(k);\\ 0, & \text{else}\end{cases}\\ &= \frac{\mathrm{e}^{-\frac{1}{2}\overline{\varepsilon}_i(k)^{\mathrm{T}}\overline{\varepsilon}_i(k)}}{(2\pi)^{\frac{1}{2}}\Pr(m_i(k)|M_1^{k-1})} I_{[a^k,b^k)}(\overline{\varepsilon}_i(k)),\end{aligned} \tag{22}$$

where $[a^k, b^k)$ is the hypercube corresponding to $m_i(k)$,

$$\Pr(m_i(k)|M_1^{k-1}) = \int_{a^k}^{b^k} p(\overline{\varepsilon}_i(k)|M_1^{k-1})\mathrm{d}\overline{\varepsilon}_i(k) = \int_{a^k}^{b^k} \mathcal{N}(\overline{\varepsilon}_i(k); 0, I_{r\times r})\mathrm{d}\overline{\varepsilon}_i(k), \tag{23}$$

and

$$I_{[a^k,b^k)}(\overline{\varepsilon}_i(k)) = \begin{cases}1, & \text{if} \quad \overline{\varepsilon}_i(k) \in [a^k, b^k);\\ 0, & \text{else.}\end{cases} \tag{24}$$

Then

$$\begin{aligned}\mathrm{E}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) &= \int_{a^k}^{b^k} \overline{\varepsilon}_i(k)p(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))\mathrm{d}\overline{\varepsilon}_i(k)\\ &= \frac{\int_{a^k}^{b^k} \overline{\varepsilon}_i(k)p(\overline{\varepsilon}_i(k)|M_1^{k-1})\mathrm{d}\overline{\varepsilon}_i(k)}{\int_{a^k}^{b^k} p(\overline{\varepsilon}_i(k)|M_1^{k-1})\mathrm{d}\overline{\varepsilon}_i(k)}\\ &= \frac{\int_{a^k}^{b^k} \overline{\varepsilon}_i(k)\mathcal{N}(\overline{\varepsilon}_i(k); 0, I_{r\times r})\mathrm{d}\overline{\varepsilon}_i(k)}{\int_{a^k}^{b^k} \mathcal{N}(\overline{\varepsilon}_i(k); 0, 1)\mathrm{d}\overline{\varepsilon}_i(k)},\end{aligned} \tag{25}$$

and

$$\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) = \mathrm{E}(\overline{\varepsilon}_i(k)\overline{\varepsilon}(k)^{\mathrm{T}}|M_1^{k-1}, m_i(k))$$

$$
\begin{aligned}
&-\mathrm{E}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))\mathrm{E}^{\mathrm{T}}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) \\
=& \int_{a^k}^{b^k} \overline{\varepsilon}_i(k)\overline{\varepsilon}_i(k)^{\mathrm{T}} p(\overline{\varepsilon}(k)|M_1^{k-1}, m_i(k))\mathrm{d}\overline{\varepsilon}_i(k) \\
&-\mathrm{E}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))\mathrm{E}^{\mathrm{T}}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) \\
=& \frac{\int_{a^k}^{b^k} \overline{\varepsilon}_i(k)\overline{\varepsilon}_i(k)^{\mathrm{T}} \mathcal{N}(\overline{\varepsilon}_i(k); 0, I_{r\times r})\mathrm{d}\overline{\varepsilon}_i(k)}{\int_{a^k}^{b^k} \mathcal{N}(\overline{\varepsilon}_i(k); 0, I_{r\times r})\mathrm{d}\overline{\varepsilon}_i(k)} \\
&-\mathrm{E}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))E^{\mathrm{T}}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)). \qquad (26)
\end{aligned}
$$

Remark 2. Numerical computation of the above integrals has a heavy computational burden, but it is worth noting that the integrals are uniquely determined by its integral intervals, while the integral intervals are only related to the quantitative strategies. Therefore, these integrals can be calculated off-line. As described in Remark 1, we can take $\xi_j = \mathrm{E}[\overline{\varepsilon}_i(k))|\eta_j < \overline{\varepsilon}_i(k)) \leqslant \eta_{j+1}]$, $j = 1, 2, \ldots, l_i$, i.e., $\Xi_i(k) = \mathrm{E}[\overline{\varepsilon}_i(k))|\eta_j < \overline{\varepsilon}_i(k)) \leqslant \eta_{j+1}]$. Similarly, the covariance matrix of quantization error can also be calculated off-line for different quantitative results $m_i(k)$ of normalized innovation, denoted by $\Sigma_i(k)$.

Finally, by property of expectation and covariance, we have

$$
\mathrm{E}(\varepsilon_i(k)|M_1^{k-1}, m_i(k)) = S_i(k)^{1/2}\mathrm{E}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k)) = S_i(k)^{1/2}\Xi_i(k), \qquad (27)
$$

$$
\begin{aligned}
\mathrm{Cov}(\varepsilon_i(k)|M_1^{k-1}, m_i(k)) &= S_i(k)^{1/2}\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))(S_i(k)^{1/2})^{\mathrm{T}} \\
&= S_i(k)^{1/2}\Sigma_i(k)(S_i(k)^{1/2})^{\mathrm{T}}. \qquad (28)
\end{aligned}
$$

Thus, using (5)–(11), (15)–(28) and the distributed Kalman filter (DKF) with feedback [18], we can constitute the distributed Kalman filter based on multi-level quantized innovations (QI-DKF).

QI-DKF is implemented in four stages: Compute the value of $\mathrm{E}(\overline{\varepsilon}(k)|M_1^{k-1}, m_i(k))$ and $\mathrm{Var}(\overline{\varepsilon}(k)|M_1^{k-1}, m_i(k))$ using (27) and (28) for all possible values of $m_i(k)$ off-line.

Given $\hat{X}_{0|0}$ and $P_{0|0}$.

Algorithm 2-A. Prediction step (in LFC).

Given: $\hat{X}_{k-1|k-1}$ and $P_{k-1|k-1}$,

$$
\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1}, \qquad (29)
$$

$$
P_{k|k-1} = FP_{k-1|k-1}F^{\mathrm{T}} + \Gamma Q\Gamma^{\mathrm{T}}; \qquad (30)
$$

For $i = 1, 2, \ldots, N$;

$$
S_i(k) = H_iP_{k|k-1}H_i^{\mathrm{T}} + R_i, \qquad (31)
$$

$$
K_i(k) = P_{k|k-1}S_i(k)^{-1}, \qquad (32)
$$

$$
\hat{y}_i(k|k-1) = H_i\hat{X}_{k|k-1}, \qquad (33)
$$

Transmit the $\hat{y}_i(k|k-1)$ and $S_i(k)$ to corresponding active sensor nodes.

Algorithm 2-B. Measurement and quantization (in sensor nodes).

For $i = 1, 2, \ldots, N$,

Given: $\hat{y}_i(k|k-1)$ and $S_i(k)$;

Measure: $y_i(k)$,

Construct quantized innovations $m_i(k) = q_{L_i}(\overline{\varepsilon}_i(k))$ as in Subsection 3.1;

Transmit quantized innovations $m_i(k)$ to LFC.

Algorithm 2-C. Correction step (in LFC).

Receive: Quantized observations $m_i(k)$, $i = 1, 2, \ldots, N$,

According to $m_i(k)$, $i = 1, 2, \ldots, N$, take the corresponding values of $\Xi_i(k)$ and $\Sigma_i(k)$ as in Remark 2;

Compute $\mathrm{E}(\varepsilon_i(k)|M_1^{k-1}, m_i(k))$ and $\mathrm{Var}(\varepsilon_i(k)|M_1^{k-1}, m_i(k))$ using (27), (28);

$$
\hat{X}_{k|k}^i = \hat{X}_{k|k-1} + K_i(k)S_i(k)^{1/2}\Xi_i(k), \qquad (34)
$$

$$P^i_{k|k} = P_{k|k-1} - K_i(k)S_i(k)K_i(k)^{\mathrm{T}} + K_i(k)S_i(k)^{1/2}\Sigma_i(k)(S_i(k)^{1/2})^{\mathrm{T}}K_i(k)^{\mathrm{T}}. \tag{35}$$

Algorithm 2-D. Fusion step (in LFC).

The following fusion schema was suggested by Chong et al. [16].

$$P^{-1}_{k|k} = \sum_{i=1}^{N}(P^i_{k|k})^{-1} - (N-1)P^{-1}_{k|k-1}, \tag{36}$$

$$P^{-1}_{k|k}\hat{X}_{k|k} = \sum_{i=1}^{N}(P^i_{k|k})^{-1}\hat{X}^i_{k|k} - (N-1)P^{-1}_{k|k-1}\hat{X}_{k|k-1}. \tag{37}$$

Remark 3. 1) In the proposed fusion method, $\hat{y}_i(k|k-1)$ and $S_i(k)$ are transmitted to the corresponding active sensor nodes at each time step. In the case of scalar measurement, there are only $2N$ elements transmitted at each time step. The $\hat{X}_{k|k-1}$ and $P_{k|k-1}$ transmission occur only when the LFC has changed. However, those existing estimation fusion algorithm [8–12] transmit the $\hat{X}_{k|k-1}$ and $P_{k|k-1}$ to the corresponding active sensor nodes at each time step. In this case the number of transmitted elements is $d(d+1)/2+d$, where d is the dimension of state vector. Generally, the dimension d of target state is not less than four. However, it is enough to track a target with $N=3$ sensors. Hence, $2N \leqslant d(d+1)/2+d$ mostly. Thus, the proposed algorithm can save more communication bandwidth and energy of LFC than existing estimation fusion algorithm [8–12].

2) If there is no quantization for sensor innovation, i.e., $\varepsilon_i(k) = S_i(k)^{1/2}\Xi_i(k)$, then QI-DKF will degenerate into ordinary distributed Kalman filter (DKF) with feedback. Zhu et al. [13] point out that the distributed Kalman filtering fusion formula with feedback is exactly equivalent to the corresponding centralized Kalman filtering formula (Algorithm 1). Hence, the fusion algorithm is still global optimal, and the performance degradation of QI-DKF merely from quantization. Furthermore, In the study of the performance of QI-DKF, the filtering precision loss brought by quantized innovations only needs to be taken into consideration.

4 Performance analysis

In this section, we analyse the performance of the QI-DKF.

4.1 Analysis of estimated accuracy

In order to study the performance of QI-DKF, we first give the case without quantization vision QI-DKF, i.e. DKF (Algorithm 3).

Given $\hat{X}_{0|0}$ and $P_{0|0}$.

Algorithm 3-A. Prediction step (in LFC).

Given: $\hat{X}_{k-1|k-1}$ and $P_{k-1|k-1}$,

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1}, \tag{38}$$

$$P_{k|k-1} = FP_{k-1|k-1}F^{\mathrm{T}} + \Gamma Q\Gamma^{\mathrm{T}}; \tag{39}$$

For $i = 1, 2, \ldots, N$,

$$S_i(k) = H_iP_{k|k-1}H_i^{\mathrm{T}} + R_i, \tag{40}$$

$$K_i(k) = P_{k|k-1}S_i(k)^{-1}, \tag{41}$$

$$\hat{y}_i(k|k-1) = H_i\hat{X}_{k|k-1}, \tag{42}$$

Transmit the $\hat{y}_i(k|k-1)$ and $S_i(k)$ to corresponding active sensor nodes.

Algorithm 3-B. Measurement (in sensor nodes).

For $i = 1, 2, \ldots, N$,

Given: $\hat{y}_i(k|k-1)$;
Measure: $y_i(k)$,
Construct innovations $\varepsilon_i(k) = y_i(k) - \hat{y}_i(k|k-1)$;
Transmit innovations $\varepsilon_i(k)$ to LFC.
Algorithm 3-C. Correction step (in LFC).
Receive: Innovations $\varepsilon_i(k),\ i = 1, 2, \ldots, N$;
Compute

$$\hat{X}^i_{k|k} = \hat{X}_{k|k-1} + K_i(k)\varepsilon_i(k), \tag{43}$$
$$P^i_{k|k} = P_{k|k-1} - K_i(k)S_i(k)K_i(k)^{\mathrm{T}}. \tag{44}$$

Algorithm 3-D. Fusion step (in LFC).
The following fusion schema was suggested by Chong et al. [16].

$$\hat{P}^{-1}_{k|k} = \sum_{i=1}^{N}(\hat{P}^i_{k|k})^{-1} - (N-1)\hat{P}^{-1}_{k|k-1}, \tag{45}$$
$$\hat{P}^{-1}_{k|k}\hat{X}_{k|k} = \sum_{i=1}^{N}(\hat{P}^i_{k|k})^{-1}\hat{X}^i_{k|k} - (N-1)\hat{P}^{-1}_{k|k-1}\hat{X}_{k|k-1}. \tag{46}$$

Comparison of the ECM corrections for the DKF in (44) with those for the Algorithm 2 (QI-DKF) in (35) reveal that they are identical except for the third term in (35). The similarity is quantified by defining the ECM reduction per correction step [cf. (35)]

$$\begin{aligned}\Delta P^i(k) &:= P_{k|k-1} - P^i_{k|k}\\ &= K_i(k)S_i(k)K_i(k)^{\mathrm{T}} - K_i(k)\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]K_i(k)^{\mathrm{T}}.\end{aligned} \tag{47}$$

If we use $y_i(k)$ instead of $m_i(k)$ in the correction step, the ECM reduction will be [cf. (44)]

$$\Delta P^i(k) := P_{k|k-1} - P^i_{k|k} = K_i(k)S_i(k)K_i(k)^{\mathrm{T}}. \tag{48}$$

Comparing (47) with (48), we see that the ECM reduction achieved by the QI-DKF is less than by the DKF.

Similar to [10], the optimal quantizer is defined as the one that maximizes the average variance reduction, i.e.,

$$\begin{aligned}\{\eta^*_j(k)\}_{j=1}^L &:= \arg\max_{\{\eta_j(k)\}_{j=1}^L} \mathrm{E}_{m_i(k)}(\Delta P^i(k)|M_1^{k-1})\\ &= \arg\min_{\{\eta_j(k)\}_{j=1}^L} \mathrm{E}_{m_i(k)}(\mathrm{Cov}(\varepsilon_i(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}).\end{aligned} \tag{49}$$

An MSE distortion conditioned on M_1^{k-1} is adopted and the optimal quantizer of $\overline{\varepsilon}_i(k)$ is defined as

$$\{\eta^\dagger_j(k)\}_{j=1}^L := \arg\min_{\{\eta_j(k)\}_{j=1}^L} \mathrm{E}_{m_i(k)}(\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}). \tag{50}$$

It is easy to see [10] that the corresponding optimal thresholds are uniform, i.e., $\{\eta^*_i(k)\}_{i=1}^L = \{\eta^\dagger_i(k)\}_{i=1}^L$.

From Eq. (21), we know that the components $\overline{\varepsilon}_{il}(k), (i = 1, 2, \ldots, N,\ l = 1, 2, \ldots, r)$ of $\overline{\varepsilon}_i(k)$ are independence identically distributed (IID). Obviously, the quantization does not undermine this independent. In other words, the components $m_{il}(k), (i = 1, 2, \ldots, N,\ l = 1, 2, \ldots, r)$ of $m_i(k)$ are still IID. Hence, the covariance $\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))$ is a diagonal matrix, and

$$\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))$$

$$
= \begin{pmatrix} \mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^k, m_i(k)) & 0 & \dots & 0 \\ 0 & \mathrm{var}(\overline{\varepsilon}_{i2}(k)|M_1^k, m_i(k)) & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \mathrm{var}(\overline{\varepsilon}_{ir}(k)|M_1^k, m_i(k)) \end{pmatrix}. \tag{51}
$$

Furthermore, noting that $\mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{il}(k)|M_1^k)), (l = 1, 2, \dots, r)$ are equal, we have, without loss of generality,

$$
\mathrm{E}_{m_i(k)}(\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) = \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) \times I_{r\times r}. \tag{52}
$$

Hence, the optimization problem in (50) is equivalent to

$$
\{\tilde{\eta}_j^{\dagger}(k)\}_{j=1}^L := \arg \min_{\{\eta_j(k)\}_{j=1}^L} \mathrm{E}_{M_k}(var(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}), \tag{53}
$$

and for the the covariance of $\varepsilon_i(k)$ conditioned on $\{M_1^{k-1}, m_i(k)\}$, we have

$$
\begin{aligned}
&\mathrm{E}_{m_i(k)}(\mathrm{Cov}(\varepsilon_i(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) \\
&= \mathrm{E}_{m_i(k)}(S_i(k)^{1/2}\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1}, m_i(k))(S_i(k)^{1/2})^{\mathrm{T}}|M_1^{k-1}) \\
&= S_i(k)^{1/2}\mathrm{E}_{m_i(k)}(\mathrm{Cov}(\overline{\varepsilon}_i(k)|M_1^{k-1})|M_1^{k-1})(S_i(k)^{1/2})^{\mathrm{T}} \\
&= S_i(k)^{1/2} \times \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) \times I_{r\times r} \times (S_i(k)^{1/2})^{\mathrm{T}} \\
&= \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) \times I_{r\times r} \times S_i(k).
\end{aligned} \tag{54}
$$

Combining Eq. (47) with Eq. (52), we have

$$
\begin{aligned}
&\mathrm{E}_{m_i(k)}(\Delta P^i(k)|M_1^{k-1}) \\
:=\ &\mathrm{E}_{m_i(k)}(P_{k|k-1} - P_{k|k}^i|M_1^{k-1}) \\
=\ &K_i(k)S_i(k)K_i(i)^{\mathrm{T}} - K_i(k)\mathrm{E}_{m_i(k)}(\mathrm{Cov}[\varepsilon_i(k)|\sigma\{M_1^{k-1}, m_i(k)\}]|M_1^{k-1})K_i(k)^{\mathrm{T}} \\
=\ &K_i(k)S_i(k)K_i(i)^{\mathrm{T}} - \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}) \times I_{r\times r} \times K_i(k)S_i(k)K_i(i)^{\mathrm{T}} \\
=\ &(1 - \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1})) \times I_{r\times r} \times K_i(k)S_i(k)K_i(i)^{\mathrm{T}}.
\end{aligned} \tag{55}
$$

It means that QI-DKF exhibiting MSE performance is identical to those of DKF with MSE reduction multiplied by a factor of

$$
\overline{\alpha} = 1 - \mathrm{E}_{m_i(k)}(\mathrm{var}(\overline{\varepsilon}_{i1}(k)|M_1^{k-1}, m_i(k))|M_1^{k-1}). \tag{56}
$$

The optimization problem in (53) has a well-known solution given by the Lloyd-Max quantizer [17]. For the optimal normalized thresholds values and the corresponding values of Ξ and Σ ,one can refer to [17]. The corresponding $\overline{\alpha}$ obtained by using (56) and the maximal number u of transmitting bytes (Max u) are summarized in Table 1.

Remark 4. Table 1 reveals that, under the Gaussian and independent assumptions, with 2-bit communication constraint, the proposed fusion method can achieve the 95.60 percent performance of the decentralized Kalman filtering fusion, with 3-bit bandwidth, 98.93 percent performance achieved.

4.2 Energy model

The energy model used in this paper is based on [19]. Energy is consumed to be mainly during data communication; therefore, in this paper we only consider such energy. The energy to transmit b bits from sensor node s_j to s_k is $\mathrm{En}_t(s_j, s_k) = (e_t + e_d r_{jk}^{a_c})b$, where e_t and e_d are decided by the specifications of the transmitter of s_j, r_{jk} is the distance between sensor s_j and s_k, and a_c depends on the channel characteristics and is assumed to be known. The energy consumption on sensor s_k side for receiving data of b bits is $\mathrm{En}_r(s_k) = e_r b$, where e_r is decided by the specifications of the receiver of sensor node s_k. It is known that the communication operations dominate the energy consumption unlike to sensing and processing operations [20]. Hence, the communication energy consumption, both for transmitting and receiving data,

Table 1 $\overline{\alpha}$ Values

Quantization level	$L=3$	$L=7$	$L=15$
Max u (bit)	1	2	3
$\overline{\alpha}$	0.8098	0.9560	0.9893

Table 2 Typical parameters in the energy model

Parameters	α_c	e_t	e_d	e_r
Value	2	45×10^{-9} J/bit	10×10^{-12} J/bit·m^2	135×10^{-9} J/bit

$$\mathrm{En}(s_j,s_k)=(e_t+e_d r_{jk}^{a_c})b+e_r b=e_0+e_1 r_{jk}^{a_c} \tag{57}$$

is the focus of this paper, where $e_0=(e_t+e_r)b$ and $e_1=e_d b$. Typical parameters in the energy model are listed in Table 2 [19,20]. We present this communication energy consumption comparisons between our proposed QI-DKF algorithm and the standard MLQI-KF [10,11] algorithm in the simulation results in the following sections.

5 Simulation results

In this section, we consider a target tracking system in WSNs. The network is formed using randomly distributed ranging sensors. There are 200 sensors randomly located in a square region of about 400×400 m^2 and the position (x_i,y_i) of the ith sensor is known. It is assumed that there is no communication loss and that the sensors are synchronized.

The target (e.g., human, car, radiation source etc.) is considered as a point-object moving in a two-dimensional plane. Here, we consider a quite general, nonlinear motion model, the coordinated turn rate model [18]. This model assumes that the target moves at a nearly constant speed and unknown turn rate. We denote by $X(k)$ the state of the target. $X(k)$ represents the coordinates X_1,X_2, the velocities $\dot{X}_1,\dot{X}_2$, and the turn rate ϕ

$$X(k)=\{X_1(k),\dot{X}_1(k),X_2(k),\dot{X}_2(k),\phi(k)\}^{\mathrm{T}}. \tag{58}$$

The target motion in the Cartesian coordination system is modelled as

$$X(k)=\Phi(k-1)X(k-1)+\Gamma(k-1)w(k-1), \tag{59}$$

where the $w(k-1)$ is the process noise with covariance matrices of $\boldsymbol{Q}$. The system noise is a Gaussian noise $w(k)\sim\mathcal{N}(0,\mathrm{diag}[\varrho_1^2,\varrho_2^2,\varrho_\Phi^2])$, where $\varrho_1=\varrho_2=\varrho_\phi=0.1$. The state transition matrix $\Phi(k)$ and process noise coefficient matrices $\Gamma(k)$ can be written as respectively

$$\Phi(k)=\begin{pmatrix} 1 & \dfrac{\sin\phi(k)\Delta t}{\phi(k)} & 0 & \dfrac{1-\cos\phi(k)\Delta t}{\phi(k)} & 0 \\ 0 & \cos\phi(k)\Delta t & 0 & -\sin\phi(k)\Delta t & 0 \\ 0 & -\dfrac{1-\cos\phi(k)\Delta t}{\phi(k)} & 1 & \dfrac{\sin\phi(k)\Delta t}{\phi(k)} & 0 \\ 0 & \sin\phi(k)\Delta t & 0 & \cos\phi(k)\Delta t & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{60}$$

and

$$\Gamma(k)=\begin{pmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{61}$$

For all simulations, we take the following parameters. The total time T is 100 s and the time step is $\Delta t = 1$ s. The turn rate ϕ takes the value of -0.08 rad ($1 \leqslant k \leqslant 30$), 0.15 rad ($31 \leqslant k \leqslant 60$), -0.15 rad ($61 \leqslant k \leqslant 80$) and 0.1 rad ($81 \leqslant k \leqslant 100$). The target initial state is $X_0 = [40, 4, 110, 5, 0.08]^{\mathrm{T}}$. Here, we employ the tracking algorithm starting from $X_{0|0} = [50, 3, 90, 6, 0.08]^{\mathrm{T}}$ and $P_{0|0} = 3 \times Q$.

In order to compare the performance of QI-DKF with the performance of DKF for target tracking in WSNs, we consider the WSN with scaler measurements [18] under the same bandwidth constraining conditions. In this example, we consider the target tracking problem with 2-bit and 3-bit bandwidth restriction.

The measurement is the noisy relative distance between the sensors and the target (e.g., radar, acoustic sensors, sonar, etc.). Sensor i, located at position $X^i = (x_i, y_i)$ measures [18]

$$y_i(k) = h_i(X(k)) + v_i(k) = \sqrt{(X_1(k) - x_i)^2 + (X_2(k) - y_i)^2} + v_i(k). \tag{62}$$

The measurement noise is a Gaussian noise $v_i(k) \sim \mathcal{N}(0, 25)$. Linearizing (62) about a generic state prediction $\widehat{X}_{k|k-1}$ in a similar fashion to the extended Kalman filter (E)KF, one can obtain

$$y_i(k) \approx \frac{\partial h_i(X(k))}{\partial X(k)}|_{\widehat{X}_{k|k-1}} X(k) + y_i^0(k) + v_i(k), \tag{63}$$

where $y_i^0(k)$ is a function of $\widehat{X}_{k|k-1}$ and X^i. The linearized observations (62) together with (59), (60), and (61) are amenable to the QI-DKF algorithms.

Remark 5. In the algorithm running, the choice of LFC and sensor data is based on the nearest neighbor principle. Specifically, every five seconds, the LFC will activate the next LFC and three sensors nearest to the predicted location of target. The predicted measurements $y_i(k|k-1)$ and the corresponding elements of prediction measurement error covariance $S_i(k)$ are sent to them. Then, the sensor nodes approximatively calculate the quantized innovations using the quantization algorithm in Section 3, and return them to the next LFC. Thus, the fusion center can use the quantized innovations returned by the three sensor nodes for target state estimation.

In Figure 2 are shown the simulation results of MLQIKF[10] and QI-DKF with 2-bit bandwidth and 3-bit bandwidth, and the DKF [14,18] in target tracking. The simulation results are based on 400 Monte-Carlo runs. The criterion for comparison is the RMSE on the position and velocity of the target. In Figures 2(a), (b), the RMSEs of position along the X-axis and Y-axis are given respectively. In Figure 2, the estimation performance of Algorithm 2 is almost equivalent to that of DKF, but the energy consumption of the Algorithm 2 is greatly reduced (see Table 3). These simulation results confirm the correctness of the theoretical analysis in Section 4.

In Table 3, the communication energy consumption of DKF, MLQI-KF and QI-DKF in different time steps are shown. Here the communication energy consumption from sensor nodes to LFC and from LFC to corresponding sensor nodes are considered. The number of analog-to-digital conversion bits for intersensor communication is assumed to be $b = 32$ bits, and the number of bits used in quantizing innovations is assumed to be $b = 2$ or $b = 3$ bits. The QI-DKF consumes much less energy than the original DKF and MLOI-KF algorithm, as shown in Table 3.

Figure 2 and Table 3 show that the proposed algorithm can reduce a lot of energy losses and keep very high estimation precision.

Table 3 Energy consumption (Unit: J) for the corresponding algorithm

Time step	20	40	60	80	100
DKF	0.0018994	0.0038667	0.0058867	0.0079357	0.0099706
MLQI-KF 3-bit	0.0017734	0.0036109	0.0054973	0.007405	0.0092786
MLQI-KF 2-bit	0.0017693	0.0036021	0.0054845	0.007388	0.0092559
QI-DKF 3-bit	0.00096734	0.0019681	0.0029902	0.0040208	0.0050378
QI-DKF 2-bit	0.00096323	0.0019593	0.0029774	0.0040038	0.0050151

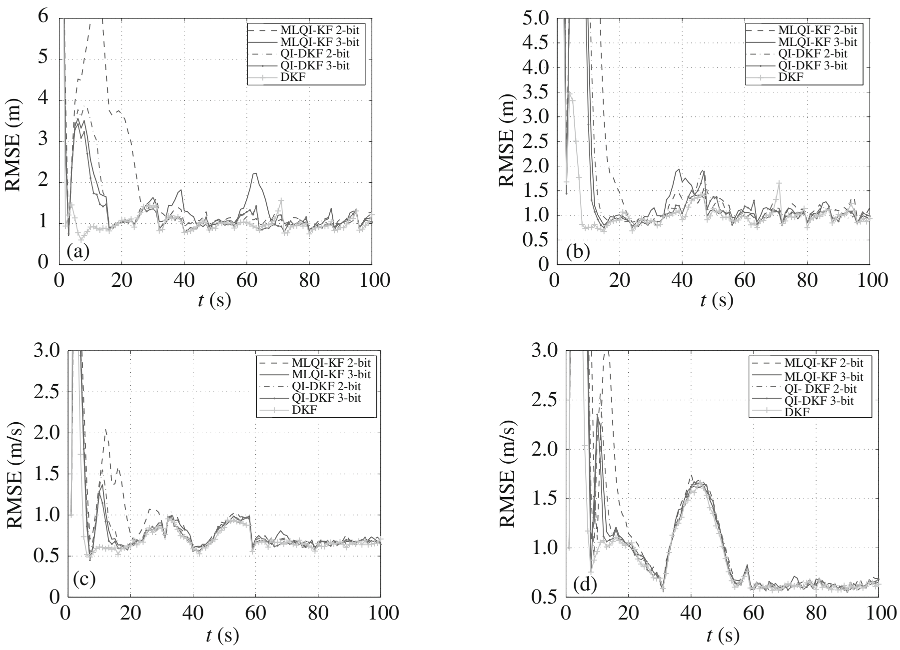

Figure 2 RMSE of the example. (a) RMSE of position along the X-axis; (b) RMSE of position along the Y-axis; (c) RMSE of velocity along the X-axis; (d) RMSE of velocity along the Y-axis.

6 Coclusion

The problem of estimation fusion with quantized innovations is considered in WSNs. The proposed data fusion method is based on quantized measurement innovations and decentralized Kalman filtering with feedback. The performance of resulting filter is discussed. Under the Gaussian and independent assumptions, performance analysis reveals that with 2-bit communication constraint, the proposed fusion method can achieve 95.60 percent performance of the decentralized Kalman filtering fusion; with 3-bit bandwidth, 98.93 percent performance is achieved. Finally, Monte Carlo simulations show that the proposed scheme reduces the energy consumption dramatically. At the same time, the proposed scheme still maintains very high tracking accuracy.

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