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Diversity order of multiuser two-way relay networks with beamforming over Nakagami-m fading channels

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Abstract The diversity performance of multiuser two-way relay networks is investigated over Nakagami-m fading channels. We consider an amplify-and-forward relay network with beamforming at the base station and all the mobile stations, and derive closed-form lower and upper bounds on the outage probabilities of two unidirectional links, respectively. Moreover, the exact expression for the diversity order and the asymptotic expressions for the outage probabilities of two unidirectional links are given too. Finally, simulation results are presented to verify our formulations.

Keywords two-way relay, multiuser, beamforming, diversity order

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1 Introduction

Refs. [1, 2] have derived the outage probability and diversity order of multiuser one-way relay networks and have shown that the combination of multiuser diversity (MUD) and cooperative diversity can be realizable. Recently, ref. [3] investigated the average rates of the amplify-and-forward (AF) multiuser two-way relay network (MU-TWRN) consisting of a base station (BS), a fixed relay (R), and multiple mobile stations (MSs). But in an MU-TWRN, the achievable diversity order will be limited by the BS-R link due to the fact that there are no direct BS-MS links in this two-way network [4]. In this paper, in order to improve the achievable diversity order, using beamforming [5,6] to the BS and MSs in an AF MU-TWRN, we try to derive closed-form lower and upper bounds on the outage probabilities over Nakagami-m fading channels, and to obtain the exact expression for the achievable diversity order.

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2 System and channel models

Consider an AF MU-TWRN consisting of a BS S_0, a relay R, and K MSs $\{S_1, S_2, \ldots, S_K\}$, where all the equipments operate in half-duplex mode. It is assumed that the BS S_0 and MS S_k $(1 \leqslant k \leqslant K)$ are equipped with L_0 and L_k antennas, respectively, whereas the relay has a single antenna.

We assume that the MUD scheduling selects an MS S_k $(1 \leqslant k \leqslant K)$ to exchange information with the BS S_0. In the first time slot, S_0 and S_k weight their scalar symbols by their respective beamforming vectors, and then transmit their vector signals simultaneously to the relay. The received scalar signal at the relay R is given by

$$y_R = \boldsymbol{h}_0^{\dagger}\boldsymbol{w}_0 x_0 + \boldsymbol{h}_k^{\dagger}\boldsymbol{w}_k x_k + n_R, \tag{1}$$

where x_0 and x_k are the transmitted scalar symbols from S_0 and S_k, respectively, $\boldsymbol{h}_0$ is the $L_0 \times 1$ channel vector between S_0 and R, $\boldsymbol{h}_k$ is the $L_k \times 1$ channel vector between S_k and R, $(\cdot)^{\dagger}$ denotes conjugate transpose, n_R is the additive white Gaussian noise (AWGN) signal with zero mean and variance N_0, $\boldsymbol{w}_0 = \frac{\boldsymbol{h}_0}{\|\boldsymbol{h}_0\|_{\mathrm{F}}}$ and $\boldsymbol{w}_k = \frac{\boldsymbol{h}_k}{\|\boldsymbol{h}_k\|_{\mathrm{F}}}$ are the beamforming vectors at S_0 and S_k [5], respectively, and $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. In the second time slot, the relay R scales the received signal by

$$G = \sqrt{\frac{P_R}{\|\boldsymbol{h}_0\|_{\mathrm{F}}^2 P_0 + \|\boldsymbol{h}_k\|_{\mathrm{F}}^2 P_k + N_0}}, \tag{2}$$

and broadcasts it, where $P_0 = \mathbb{E}\left[|x_0|^2\right]$, $P_k = \mathbb{E}\left[|x_k|^2\right]$, and $\mathbb{E}[\cdot]$ denotes the expectation operator. Next, the received vectors at S_k and S_0 are weighted by their respective beamforming vectors and can be written as

$$z_k = \boldsymbol{w}_k^{\dagger}\left(\boldsymbol{h}_k G y_R + \boldsymbol{n}_k\right), \tag{3}$$

$$z_0 = \boldsymbol{w}_0^{\dagger}\left(\boldsymbol{h}_0 G y_R + \boldsymbol{n}_0\right), \tag{4}$$

respectively, where $\boldsymbol{n}_k$ and $\boldsymbol{n}_0$ are AWGN vectors satisfying $\mathbb{E}\left[\boldsymbol{n}_k\boldsymbol{n}_k^{\dagger}\right] = N_0\boldsymbol{I}_{L_k}$ and $\mathbb{E}\left[\boldsymbol{n}_0\boldsymbol{n}_0^{\dagger}\right] = N_0\boldsymbol{I}_{L_0}$, $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. After the self-interference parts are subtracted [4], the received signal-to-noise ratios (SNRs) of the links $S_0 \to R \to S_k$ and $S_k \to R \to S_0$ are given by

$$\varGamma_1^k = \frac{\gamma_0\gamma_R X Y_k}{\gamma_0 X + (\gamma_R + \gamma_k)Y_k + 1}, \tag{5}$$

$$\varGamma_2^k = \frac{\gamma_k\gamma_R X Y_k}{(\gamma_R + \gamma_0)X + \gamma_k Y_k + 1}, \tag{6}$$

respectively, where $X = \|\boldsymbol{h}_0\|_{\mathrm{F}}^2$, $Y_k = \|\boldsymbol{h}_k\|_{\mathrm{F}}^2$, $\gamma_0 = \frac{P_0}{N_0}$, $\gamma_k = \frac{P_k}{N_0}$, $\gamma_R = \frac{P_R}{N_0}$.

The MUD scheduling selects the MS S_{k^*} $(1 \leqslant k^* \leqslant K)$ that maximize $\varGamma_1^k$ and $\varGamma_2^k$ simultaneously [3]. Specifically, both $\varGamma_1^k$ and $\varGamma_2^k$ are strictly increasing functions with respect to Y_k, thus k^* should be selected as $k^* = \arg\max_{1\leqslant k\leqslant K} Y_k$. Without loss of generality, we assume that $\gamma_k = \gamma_R = \gamma_0$. Then the received SNRs of the links $S_0 \to R \to S_{k^*}$ and $S_{k^*} \to R \to S_0$ are given by $\varGamma_1 = \frac{\gamma_0^2 XY}{\gamma_0 X + 2\gamma_0 Y + 1} \approx \frac{XY}{X+2Y}\gamma_0$, $\varGamma_2 = \frac{\gamma_0^2 XY}{2\gamma_0 X + \gamma_0 Y + 1} \approx \frac{XY}{2X+Y}\gamma_0$, respectively, where $Y \triangleq \max_{1\leqslant k\leqslant K} Y_k$.

Assumiung that $\boldsymbol{h}_0$ and $\boldsymbol{h}_k$ are channel vectors with Nakagami-m fading entries, the probability density function (PDF) of $\|\boldsymbol{h}_k\|_{\mathrm{F}}^2$ $(0 \leqslant k \leqslant K)$ is given by [6,7]

$$f_{\|\boldsymbol{h}_k\|_{\mathrm{F}}^2}(x) = \frac{m_k^{L_k m_k} x^{L_k m_k - 1}}{\varGamma(L_k m_k)\varOmega_k^{L_k m_k}} \exp\left(-\frac{m_k x}{\varOmega_k}\right)\varepsilon(x), \tag{7}$$

where m_k is the fading parameter of the channel between S_k and R, $\varOmega_k = \frac{\mathbb{E}\|\boldsymbol{h}_k\|_{\mathrm{F}}^2}{L_k}$, $\varepsilon(\cdot)$ denotes the unit step function, and $\varGamma(\cdot)$ is the Gamma function (eq. (8.310.1) in [8]).

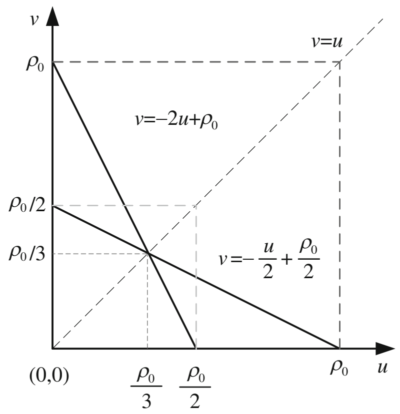

Figure 1 Integral area to determine bounds on outage probabilities.

3 Bounds on outage probabilities

The outage probabilities of the links $S_0 \to R \to S_{k^*}$ and $S_{k^*} \to R \to S_0$ are given by $P_{\rm out}^1 = \Pr\{\varGamma_1 < \gamma_{\rm th}\}$, $P_{\rm out}^2 = \Pr\{\varGamma_2 < \gamma_{\rm th}\}$, respectively, where $\gamma_{\rm th}$ is a certain threshold, and $\Pr\{\cdot\}$ symbolizes probability. Moreover, the overall outage probability [9] of the network can be defined as $P_{\rm out}^{\rm overall} = \Pr\{\min(\varGamma_1, \varGamma_2) < \gamma_{\rm th}\}$.

For $i = 1, 2$, it clearly holds that $\varGamma_{\min} \triangleq \min\{\varGamma_1, \varGamma_2\} \leqslant \varGamma_i \leqslant \max\{\varGamma_1, \varGamma_2\} \triangleq \varGamma_{\max}$. So we have

$$P_{\rm out}^{\rm LB} \triangleq \Pr\{\varGamma_{\max} < \gamma_{\rm th}\} \leqslant P_{\rm out}^i \leqslant \Pr\{\varGamma_{\min} < \gamma_{\rm th}\} \triangleq P_{\rm out}^{\rm UB}. \tag{8}$$

Let $U \triangleq \frac{1}{X}$, $V \triangleq \frac{1}{Y}$. From eq. (7), the cumulative distribution functions of U and V can be obtained as follows:

$$F_U(u) = \left[1 - \frac{\gamma\left(L_0 m_0, \frac{m_0}{\varOmega_0 u}\right)}{\varGamma(L_0 m_0)}\right] \varepsilon(u), \tag{9}$$

$$F_V(v) = \left[1 - \prod_{k=1}^{K} \frac{\gamma\left(L_k m_k, \frac{m_k}{\varOmega_k v}\right)}{\varGamma(L_k m_k)}\right] \varepsilon(v), \tag{10}$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function (eq. (8.350.1) in [8]). According to the integral area shown in Figure 1, we can obtain

$$\begin{aligned} P_{\rm out}^{\rm LB} &= \Pr\{\max\{\varGamma_1, \varGamma_2\} < \gamma_{\rm th}\} \\ &= \Pr\left\{V > -2U + \rho_0, V > -\frac{U}{2} + \frac{\rho_0}{2}\right\} \\ &> 1 - \Pr\{U < \rho_0, V < \rho_0\} = 1 - F_U(\rho_0)F_V(\rho_0), \end{aligned} \tag{11}$$

$$\begin{aligned} P_{\rm out}^{\rm UB} &= \Pr\{\min\{\varGamma_1, \varGamma_2\} < \gamma_{\rm th}\} \\ &= 1 - \Pr\left\{V < -2U + \rho_0, V < -\frac{U}{2} + \frac{\rho_0}{2}\right\} \\ &< 1 - \Pr\left\{U < \frac{\rho_0}{3}, V < \frac{\rho_0}{3}\right\} = 1 - F_U\left(\frac{\rho_0}{3}\right) F_V\left(\frac{\rho_0}{3}\right), \end{aligned} \tag{12}$$

where $\rho_0 \triangleq \frac{\gamma_0}{\gamma_{\rm th}}$. Let $P(c) = 1 - F_U(\frac{\rho_0}{c})F_V(\frac{\rho_0}{c})$. Then $P(1) < P_{\rm out}^{\rm LB} \leqslant P_{\rm out}^i \leqslant P_{\rm out}^{\rm UB} < P(3)$.

Let $f_U(u)$ and $f_V(v)$ denote the PDFs of U and V. Then it is easy to find that $f_U(u)$ (or $f_V(v)$) approaches zero fast with an increase in u (or v), so we can obtain a close approximation to the overall outage probability as follows:

$$P_{\rm out}^{\rm overall} = P_{\rm out}^{\rm UB} \approx 1 - \Pr\left\{U < \frac{\rho_0}{2}, V < \frac{\rho_0}{2}\right\} = P(2). \tag{13}$$

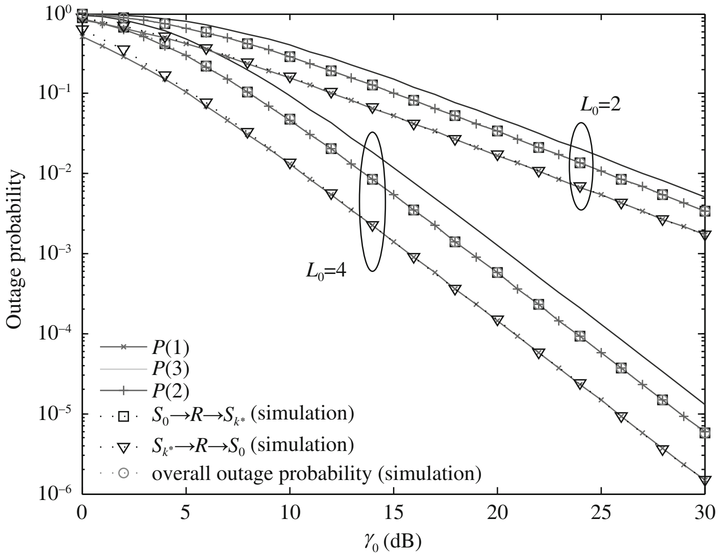

Figure 2 Outage probabilities of AF MU-TWRN with beamforming.

4 Diversity order

From (eqs. (8.351.2) and (9.210.1) in [8]), we know that $\gamma(s,x) \approx \frac{x^s}{s}$ as $x \to 0$. So the asymptotic expression for $P(c)$ at high SNR ($\gamma_0 \to \infty$) can be derived as follows:

$$P(c) \approx A(c) \times \gamma_0^{-L_0 m_0} + B(c) \times \gamma_0^{-\sum_{k=1}^{K} L_k m_k} \approx \Theta(c) \times \gamma_0^{-G_d}, \tag{14}$$

where $A(c) = \frac{(cm_0\gamma_{\rm th})^{L_0 m_0}}{L_0 m_0 \Gamma(L_0 m_0)\Omega_0^{L_0 m_0}}$, $B(c) = \prod_{k=1}^{K} \frac{(cm_k\gamma_{\rm th})^{L_k m_k}}{L_k m_k \Gamma(L_k m_k)\Omega_k^{L_k m_k}}$, and

$$G_d = \min\left\{L_0 m_0, \sum_{k=1}^{K} L_k m_k\right\}, \tag{15}$$

$$\Theta(c) = \begin{cases} A(c), & \text{if } L_0 m_0 < \sum_{k=1}^{K} L_k m_k, \\ B(c), & \text{if } L_0 m_0 > \sum_{k=1}^{K} L_k m_k, \\ A(c) + B(c), & \text{if } L_0 m_0 = \sum_{k=1}^{K} L_k m_k. \end{cases} \tag{16}$$

Thus, for $i = 1, 2$, we have $\Theta(1) \times \gamma_0^{-G_d} < P_{\rm out}^i < \Theta(3) \times \gamma_0^{-G_d}$ at high SNR. So both the diversity orders of two unidirectional links are G_d.

In fact, the simulation results in Figure 2 show that $P(1)$ and $P(2)$ agree excellently with the "worse" and "better" direction at moderate and high SNR, respectively. So $\Theta(1) \times \gamma_0^{-G_d}$ and $\Theta(2) \times \gamma_0^{-G_d}$ present the exact asymptotic expressions for the outage probabilities of the "worse" and "better" directions, respectively.

5 Simulation results

In this section, computer simulations are performed to validate the analytical results. Without loss of generality, we set $K = 4$, $m_0 = m_1 = \cdots = m_K = 0.5$, $\Omega_0 = 0.7^{-3}$, $\Omega_1 = \cdots = \Omega_K = 0.3^{-3}$,

$L_1 = \cdots = L_K = 2$, $\gamma_{\rm th} = 10$. Figure 2 shows the outage probabilities of AF MU-TWRNs with beamforming for $L_0 = 2$ and $L_0 = 4$.

In Figure 2, we can find that the simulation results are consistent with the derived diversity order G_d, and the lower and upper bounds $P(1)$ and $P(3)$ are parallel with the outage probabilities of two unidirectional links at high SNR. Furthermore, the lower bound $P(1)$ agrees well with the outage probability of the "worse" direction at moderate and high SNR. Besides, $P(2)$ provides a very good approximation to the overall outage probability, and is almost in line with the outage probability of the "better" direction at all SNR.

6 Conclusions

In this letter, the diversity order of AF MU-TWRNs with beamforming over Nakagami-m fading channels is determined, and the asymptotic expressions in concise closed-form for the outage probabilities of two unidirectional links are derived too.

Acknowledgements

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