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Online-SVR-compensated nonlinear generalized predictive control for hypersonic vehicles

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Abstract This paper is concerned with the problem of hypersonic vehicle (HSV) attitude control system in uncertain flight conditions. The problem can be expressed as the adaptive robust control for a class of uncertain nonlinear systems. Based on the description of the aerodynamic structure and the model of flight control system of a certain kind of HSV, the ideal nonlinear generalized predictive control (NGPC) law based on uncertain nonlinear model is raised first to optimize the receding-horizon criterion of tracking errors. Then the online support vector regression (SVR) is employed to identify the uncertain item in the ideal control law. It is the compensating part of the controller. In addition, the stability of the close-loop system is analyzed using the Lyapunov method. The developed control strategy is well-implemented in this HSV attitude control system, and the simulation results compared with both nominal NGPC and RBF neural network disturbance observer show the good robustness and disturbance attenuation ability of this strategy and demonstrate the efficiency of online SVR algorithm.

Keywords hypersonic vehicle, flight control, nonlinear generalized predictive control, online support vector regression

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1 Introduction

Hypersonic vehicle (HSV) is one of the most important weapons in modern warfare. The research of HSV has attracted great attention in aircraft control field. Due to its multi-mission profiles, super attitude maneuvers, complicated flight conditions, and large mach number, the flight control system of hypersonic vehicle must have strong nonlinearity, coupling and fast-variability and be impacted by strong external disturbance and internal uncertainty. It is a great challenge for control law design. In the existing literature to date, some researchers proposed different control methods to meet the requirements for high reliability, high precision, high maneuverability, and high adaptability under strong uncertainties. Snell [1] proposed the dynamic inverse control method for super-maneuverable aircraft. Respectively, Zhu [2] and Huang [3] designed a direct adaptive trajectory linearization controller and an adaptive terminal sliding mode controller for hypersonic aerospace vehicle. A robust neural adaptive control strategy of hypersonic aircraft was raised in [4].

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Predictive control is one of the most promising control methods in engineering. Much research work has been done to extend predictive control to nonlinear systems and robust adaptive control area. A generalized predictive control using recurrent fuzzy neural networks was proposed in [5] to be implemented in industrial processes. The predictive control law based on genetic algorithm has been studied in [6]. The nonlinear generalized predictive control (NGPC) strategy, raised in [7] and used there to design an autopilot for a missile, is an optimal control strategy which is based on the mechanism model. However, if the system has internal uncertainty, modeled error and external disturbance, the performance of the system which is controlled by NGPC based on the nominal model will be degraded. Many adaptive methods were proposed to estimate the compound disturbances and cancel the uncertainties, such as nonlinear disturbance observer, fuzzy systems, neural networks, etc. [2, 8–14]. These methods undoubtedly have solved many problems with complicated models, but sometimes they perform poorly in practice due to local minimum and over-fitting problems [8].

Support vector machine (SVM) [15] based on the clou of structural risk minimization has high generalization ability and global optimization property. It is an excellent method for predictive control [16–18]. Support vector regression (SVR) fits a continuous-valued to data in a way that shares many of the advantages of SVM classification [19]. An online SVR that efficiently updatas a trained SVR function whenever a sample is added to or removed from the training set has been developed in [19, 20]. It is more efficient than conventional batch SVR to identify uncertain models. Both the k-fold cross-validation error and the LOO error [21] of online SVR are less than the ones of batch algorithm.

In this paper, we proposed an online-SVR-compensated nonlinear generalized predictive control method for the attitude stabilization system of hypersonic vehicle. The NGPC which has good dynamical performance and optimality of tracking error is designed as the primary control law, while the online SVR is used to estimate the compound disturbances of systems. The rest of the paper is organized as follows. Firstly, the flight control model of HSV is raised in section 2. Then, for a class of uncertain systems, the control method proposed in this paper is analyzed systematically in section 3. This original work is necessary and convincing. Moreover, the controllers are designed for HSV flight control system in section 4 and the demonstrating simulations are implemented in section 5. Section 6 summaries this work.

2 HSV flight control model

The proposed hypersonic vehicle, whose aerodynamic model is winged-cone configuration, has two triangle wings, on which there are conventional, independently controllable, trailing edge elevons, a single vertical tail which has a full span rudder, and two small canards which are deployed only at subsonic speeds.

Attitude stabilization is the basic requirement for flight tasks. The HSV attitude control model which is simplified from the six-degree-of-freedom and twelve-state kinematic equations [22] can be written as affine nonlinear equations, given by

$$\begin{cases} \dot{\Omega} = f_s(\Omega) + g_s(\Omega)\omega_c + D_s(\Omega, \omega_c, d_s), \\ y_s = \Omega. \end{cases} \tag{1}$$

$$\begin{cases} \dot{\omega} = f_f(\omega) + g_f(\omega)M_c + D_f(\omega, M_c, d_f), \\ y_f = \omega, \end{cases} \tag{2}$$

where $\Omega = \begin{bmatrix} \alpha & \beta & \mu \end{bmatrix}^{\mathrm{T}}$, the states of slow-loop, is a vector of attitude angles which are angle of attack, sideslip angle, and flight-path roll angle, respectively; $\omega = \begin{bmatrix} p & q & r \end{bmatrix}^{\mathrm{T}}$ is the body-axis angular rates and the fast-loop states; $M_c = \begin{bmatrix} l_{ctrl} & m_{ctrl} & n_{ctrl} \end{bmatrix}^{\mathrm{T}}$ is a vector of control moments which includes roll, pitching, and yaw control moments. The detailed expressions of each variable can be found in [11]. (1) and (2) are slow-loop equations and fast-loop equations, respectively. In addition,

$$f_s(\Omega) = \begin{bmatrix} f_\alpha & f_\beta & f_\mu \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

$$f_f(\omega) = \begin{bmatrix} f_p & f_q & f_r \end{bmatrix}^{\mathrm{T}}, \tag{4}$$

$$g_s(\Omega) = \begin{bmatrix} -\tan\beta\cos\alpha & 1 & -\tan\beta\sin\alpha \\ \sin\alpha & 0 & -\cos\alpha \\ \sec\beta\cos\alpha & 0 & \sec\beta\sin\alpha \end{bmatrix}, \tag{5}$$

$$g_f(\omega) = J_I^{-1} = \mathrm{diag}\{I_{xx}^{-1}, I_{yy}^{-1}, I_{zz}^{-1}\}, \tag{6}$$

where $J_I = \mathrm{diag}\{I_{xx}, I_{yy}, I_{zz}\}$ is the inertia matrix which is a nonlinear function of mass of HSV and the elements in it denote roll, pitch, and yaw moments of inertia respectively. The concrete expressions of $f_s(\Omega)$ and $f_f(\omega)$ are omitted here and can also be found in [11].

Considering the complicated flight environment, the uncertainty and disturbance should not be omitted. Therefore, the compound disturbances upon each loop, D_s and D_f, are mentioned. In detail,

$$D_s(\varOmega, \omega_c, d_s) = \Delta f_s(\varOmega) + \Delta g_s(\varOmega)\omega_c + d_s, \tag{7}$$

$$D_f(\omega, M_c, d_f) = \Delta f_f(\omega) + \Delta g_f(\omega)M_c + d_f, \tag{8}$$

where $\Delta f_s(\varOmega)$, $\Delta g_s(\varOmega)$, $\Delta f_f(\omega)$, and $\Delta g_f(\omega)$ denote the uncertainties of the system; d_s and d_f are external disturbances upon slow-loop and fast-loop, respectively.

The objective of HSV attitude control system is to design the control moment M_c, which will be assigned to engine thrust, T_c, and the control surface deflection vector command, δ_c, by a corresponding algorithm to guarantee that the attitude states $\varOmega$ can track the guidance command $\varOmega_c$ stably and robustly. Meanwhile, the vector $\delta_c = \begin{bmatrix} \delta_e & \delta_a & \delta_r \end{bmatrix}^{\mathrm{T}}$ denotes the deflection angles of left elevon, right elevon, and rudder in order.

3 Online-SVM-compensated NGPC system

3.1 Nonlinear generalized predictive control

From (1) and (2) we can know that the attitude control system of HSV is a typical uncertain nonlinear system. For systematical analysis, a general model for this affine nonlinear uncertain MIMO system is considered in this paper and given by

$$\begin{cases} \dot{x} = f(x) + \Delta f(x) + (g_1(x) + \Delta g_1(x))u + g_2(x)d, \\ y = h(x), \end{cases} \tag{9}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ and $d \in \mathbb{R}^l$ are state vector, control input, system output and disturbance, respectively. The functions $f(x) \in \mathbb{R}^n$ and $h(x) \in \mathbb{R}^m$ are assumed to be continuously differentiable for a sufficient number of times. $g_1(x) \in \mathbb{R}^{n\times m}$ and $g_2(x) \in \mathbb{R}^{n\times l}$ are continuous functions of state vector. $\Delta f(x)$ and $\Delta g_1(x)$ that are also assumed to be continuous of x denote the system uncertainty, which contains structural and modeling errors.

After combining the uncertainty and disturbance together, the nonlinear system can be rewritten as

$$\begin{cases} \dot{x} = f(x) + g_1(x)u + D(x, u, d), \\ y = h(x), \end{cases} \tag{10}$$

where $D(x, u, d) = \Delta f(x) + \Delta g_1(x)u + g_2(x)d \in \mathbb{R}^n$ is called compound disturbance.

The following assumptions are imposed on the nonlinear system (10):

(A1) All system states are available, and the output and the reference signal are sufficiently many times continuously differentiable with respect to time.

(A2) The zero dynamics are stable.

(A3) The vector relative degree is $\{\rho_1, \rho_2, \ldots, \rho_m\}$, and $\rho_1 = \rho_2 = \cdots = \rho_m = \rho$.

(A4) The compound disturbance relative degree is the same as the vector relative degree. It means that $L_D L_f^i h(x) = 0$, $0 \leqslant i < \rho - 1$.

It is easy to demonstrate that the dynamic equations of HSV which are given by (1) and (2) satisfy these four assumptions. More details about this can be found in the section of controller design.

If the nonlinear system (9) satisfies assumptions (A1) – (A4), the predictive control performance index is given by

$$J = \frac{1}{2}\int_0^T \hat{e}^{\mathrm{T}}(t+\tau)\hat{e}(t+\tau)d\tau, \tag{11}$$

where $\hat{e}(t+\tau) = \hat{y}(t+\tau) - y_r(t+\tau)$, and T is the predictive time. $\hat{y}(t+\tau)$ and $y_r(t+\tau)$ are the predicted output and reference output in the future time τ, respectively.

Definition 3.1. If the control input in the moving time frame, $\hat{u}(t+\tau)$, satisfies

$$\frac{d^r \hat{u}(t+\tau)}{d\tau} \neq 0, \quad \frac{d^k \hat{u}(t+\tau)}{d\tau} = 0, \quad k > r, \quad \tau \in [0, T], \tag{12}$$

then the control order of the NGPC system is said to be r. The definition of control order is proposed by Chen et al. [7].

Firstly, we predict the future output $y(t+\tau)$ at the time τ via Taylor series expansion. Considering the control order defined above, and the input relative degree and disturbance relative degree which are assumed in assumptions (A3) and (A4), repeatedly differentiating it up to $\rho + r$ times, and omitting the Peano Redundancy in its Taylor expansion at the time t, the predicted output $\hat{y}(t+\tau)$ can be derived.

In the same way, the reference output at the time τ can be approximated by the Taylor expansion of $y_r(t+\tau)$ at the time t. In order to minimize the performance index (11), the necessary condition is

$$\left.\frac{\partial J}{\partial u}\right|_{u=\hat{u}_p} = 0. \tag{13}$$

Solving it yields the nonlinear generalized predictive control law of the system, given by

$$\hat{u}_p = -G(x)^{-1}(KM_\rho + F(x) - y_r^{[\rho]} + \Delta(x,u,d)), \tag{14}$$

where

$$G(x) = L_{g_1} L_f^{\rho-1} h(x) = \begin{bmatrix} L_{g_{1,1}} L_f^{\rho-1} h(x) & L_{g_{1,2}} L_f^{\rho-1} h(x) & \cdots & L_{g_{1,m}} L_f^{\rho-1} h(x) \end{bmatrix} \in \mathbb{R}^{m\times m}, \tag{15}$$

in which, $g_{1,i}$ is the ith column vector of g_1, and i =1, 2, …, m. It should be noted that $G(x)$ is an invertible matrix due to assumption (A3). $K \in \mathbb{R}^{m\times m\rho}$ is a matrix determined by relation degree ρ and the chosen variables, predictive time T and control order r. The detail can be found in [11]. Furthermore,

$$M_\rho = \begin{bmatrix} h(x) - y_r \\ L_f^1 h(x) - y_r^{[1]} \\ \vdots \\ L_f^{\rho-1} h(x) - y_r^{[\rho-1]} \end{bmatrix} \in \mathbb{R}^{m\rho}, \tag{16}$$

$$F(x) = L_f^\rho h(x) \in \mathbb{R}^m, \tag{17}$$

$$\Delta(x,u,d) = L_D L_f^{\rho-1} h(x) \in \mathbb{R}^m. \tag{18}$$

Apparently, $\Delta(x,u,d)$ cannot be directly solved due to the unknown compound disturbance D. Thus, $\hat{u}_p$ is only an ideal controller and cannot be realized. On the other hand, if we only use the control law of nominal system, which omits $\Delta(x,u,d)$ in (14), it will not meet the control requirement when the disturbance is high enough. To handle this problem, an observer should be designed to estimate the compound disturbance. In this paper, we use online SVR to identify the uncertain item $\Delta(x,u,d)$.

3.2 Online support vector regression

3.2.1 *Online ε-SVR*

Given a training set $T_r = \{(X_1, Y_1), (X_2, Y_2), \dots, (X_l, Y_l)\} \in (\mathbb{R}^n \times \mathbb{Y})^l$, where $X_i \in \mathbb{R}^n$, $Y_i \in \mathbb{Y} = \mathbb{R}$, i =1, 2,..., l, we construct a linear regression function,

$$Y = r(X) = (\omega_r \cdot \Phi(X)) + b, \tag{19}$$

on a feature space $\mathbb{F}$. Here, $\omega_r \in \mathbb{F}$ is a vector, and $\Phi(X)$ maps X to a vector in $\mathbb{F}$. According to ε-SVR [21] algorithm, ω_r and b are obtained by solving a primal optimization problem:

$$\begin{aligned} \min_{\omega_r, b} \quad & \frac{1}{2}\|\omega_r\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*), \\ \text{s.t.} \quad & (\omega_r \cdot \Phi(X_i)) + b - Y_i \leqslant \varepsilon + \xi_i, Y_i - (\omega_r \cdot \Phi(X_i)) - b \\ & \qquad \leqslant \varepsilon + \xi_i^*, \xi_i, \xi_i^* \geqslant 0, \quad i = 1, 2, \dots, l. \end{aligned} \tag{20}$$

In this optimization problem, $C > 0$ is a parameter to penalize data points whose Y-values differ from $r(X)$ more than ε. Here, $\varepsilon > 0$ is the chosen coefficient of accuracy. ξ_i and ξ_i^* are slack variables.

Then, the dual optimization problem of (20) can be derived by introducing the corresponding Lagrangian function and is expressed as

$$\begin{aligned} \min_{\alpha^{(*)}} \quad & \frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j)K(X_i, X_j) + \varepsilon\sum_{i=1}^{l}(\alpha_i^* + \alpha_i) - \sum_{i=1}^{l}Y_i(\alpha_i^* - \alpha_i), \\ \text{s.t.} \quad & \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0, \quad 0 \leqslant \alpha_i, \alpha_i^* \leqslant C, \quad i = 1, 2, \dots, l. \end{aligned} \tag{21}$$

Here $\alpha^{(*)} = [\ \alpha_1 \quad \alpha_1^* \quad \alpha_2 \quad \alpha_2^* \quad \cdots \alpha_l \quad \alpha_l^*\ ]^{\mathrm{T}}$ is defined as a Lagrangian multiplier vector and $K(X_i, X_j)$ is a kernel function. Using the solutions of this convex quadratic programming, the decision function (19) can be written as

$$Y = r(X) = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)K(X_i, X) + b. \tag{22}$$

Furthermore, the detailed calculation of b which depends on the solution $\alpha^{(*)}$ can be found in [21].

Now, we propose the incremental algorithm [19] of online ε-SVR. For each training sample X_i, i =1, 2,..., l, a coefficient difference θ_i and a margin function $h(X_i)$ are respectively defined as

$$\theta_i = \alpha_i - \alpha_i^*, \tag{23}$$

$$h(X_i) = r(X_i) - Y_i = \sum_{j=1}^{l}\theta_i K(X_j, X_i) + b - Y_i. \tag{24}$$

According to KKT conditions [21] and these two definitions, the samples in training set T_r can be classified into three subsets which are error support vector set $E_r = \{X_i | |\theta_i| = C, |h(X_i)| \geqslant \varepsilon\}$, margin support vector set $S_r = \{X_i | 0 < |\theta_i| < C, |h(X_i)| = \varepsilon\}$, and remaining sample set $R_r = \{X_i | \theta_i = 0, |h(X_i)| \leqslant \varepsilon\}$. The objective of online SVR is to update the regression function whenever a new sample X_c is added to the training set T_r. In other words, the values of θ_c and $h(X_c)$ are gradually changed by incremental algorithm. The new sample should be added to one of the three subsets maintaining the KKT conditions consistent. There can be two different possibilities:

Case 1: the new sample error $|h(X_c)| < \varepsilon$. Then, the sample can be added to the remaining sample set R_r without changing other samples.

Case 2: when $|h(X_c)| \geqslant \varepsilon$. In the more complicated situation, θ_c and $h(X_c)$ changes until the new sample reaches subset E_r or subset S_r, and other samples must be influenced by these variations. Some of them could exit from their original set to another one. The detailedly discussed samples moving principle and incremental algorithm can be found in [20].

3.2.2 *Online SVR identification for uncertainty*

Considering uncertain nonlinear system (10) controlled by NGPC law (14), the uncertain item $\Delta(x,u,d)$ which is identified by online SVR can be estimated as $\hat{\Delta}(x,u) = \begin{bmatrix} \hat{\Delta}_1 & \hat{\Delta}_2 & \cdots & \hat{\Delta}_m \end{bmatrix}^{\mathrm{T}}$. For i =1, 2, …, m,

$$\hat{\Delta}_i = Y_i = r_i(X) = \sum_{j=1}^{l}(\alpha_j^* - \alpha_j)K_i(X_j, X) + b_i \tag{25}$$

is the observed value of the ith element of $\Delta(x,u,d)$ by online SVR. It means that there are m online SVRs designed for each control channel. In (25), $X = \begin{bmatrix} x^{\mathrm{T}} & u^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{m+n}$ is the input vector, and $Y_i \in \mathbb{R}$ is the identified regression output. The training set of the ith SVR should be denoted as $T_{ri} = \{(X_1, Y_{i1}), (X_2, Y_{i2}), \ldots, (X_j, Y_{ij}), \ldots\}$, where $X_j = X(j)$ is the value of X at time j, while $Y_{ij} = Y_i(j+1)$ is the value of Y_i at time $j+1$. Moreover, $K_i(X_j, X)$ is the kernel function chosen for the ith SVR. In this paper, the kernel functions of each online SVR are all chosen as Gaussian radial basis kernel

$$K(X, X') = \exp\left(\frac{-\|X - X'\|^2}{\sigma^2}\right), \tag{26}$$

where σ, the kernel radius, is a designed parameter. The success of SVR depends heavily on the model selection. Gaussian kernel function that many researchers attach importance to has peculiar property and broad application because of its separability and property of localization. That is why we use it here. In addition, selecting parameters in SVR is also very important.

Here, we discuss the generalization ability of the online ε-SVR. The leave-one-out (LOO) error is a useful tool to assess the algorithm. One of the most popular approaches is to select the kennel and the parameters by minimizing the bound of LOO error [23].

Lemma 3.2. The LOO error of ε-SVR stated in (22) satisfying

$$R_{\mathrm{LOO}}(T_r) \leqslant \sum_{t=1}^{l} |r(X_t) - Y_t - (\alpha_t^* - \alpha_t)(R^2 + K(X_t, X_t))|, \tag{27}$$

where $R^2 = \max\{K(X_i, X_j) | i, j = 1, 2, \ldots, l\}$.

The proof of this lemma was raised in [23]. We can know that the observing errors of online SVRs used in each control channel are bounded. For i =1, 2, …, m, there exists a positive constant $\overline{\zeta}_i$ satisfying

$$|\Delta_i - \hat{\Delta}_i| \leqslant \overline{\zeta}_i. \tag{28}$$

In general, the norm of $\Delta(x,u,d) - \hat{\Delta}(x,u)$ is bounded as well. The bound is labeled as $\overline{\zeta}$, i.e.,

$$\|\Delta(x,u,d) - \hat{\Delta}(x,u)\| \leqslant \overline{\zeta}. \tag{29}$$

$\|\cdot\|$ here is Euclid norm.

3.3 Close-loop system analysis

As analyzed in subsection 3.1 and subsection 3.2, the online-SVR-compensated NGPC law should be written as

$$u = -G(x)^{-1}(KM_\rho + F(x) - y_r^{[\rho]} + \hat{\Delta}(x,u)). \tag{30}$$

It can be divided into two parts: $u = u_p + u_r$, where

$$u_p = -G(x)^{-1}(KM_\rho + F(x) - y_r^{[\rho]}) \tag{31}$$

is called nonlinear generalized predictive control law, and

$$u_r = -G(x)^{-1}\hat{\Delta}(x,u) \tag{32}$$

is the compensating control law.

Theorem 3.3. Consider an uncertain nonlinear MIMO system (10) satisfying assumptions (A1) — (A4). The NGPC law is given by (30) based on online SVRs expressed in (22), and the control order r is well chosen to make each element of the vector $H(S)$ given below a Hurwitz polynomial. That is,

$$H(S) = S^{\rho} + K_{\rho-1}S^{[\rho-1]} + \cdots + K_0, \tag{33}$$

where, $S^i = \begin{bmatrix} s^i & s^i & \cdots & s^i \end{bmatrix} \in \mathbb{R}^m$, i =1, 2, $\cdots$, ρ, and s here is a Laplacian. Then the tracking error of the closed-loop system is uniformly ultimately bounded.

Proof. Differentiating the output to ρ times with respect to time yields

$$y^{[\rho]}(t) = L_f^{\rho}h(x) + L_{g_1}L_f^{\rho-1}h(x)u + L_D L_f^{\rho-1}h(x). \tag{34}$$

Substituting (30) into (34), the error equations of the system can be written as

$$e^{[\rho]} = y^{[\rho]} - y_r^{[\rho]} = -KM_{\rho} - \hat{\Delta}(x,u) + \Delta(x,u,d). \tag{35}$$

Let $K = \begin{bmatrix} K_0 & K_1 & \cdots & K_{\rho-1} \end{bmatrix} \in \mathbb{R}^{m\times m\rho}$, in which $K_i \in \mathbb{R}^{m\times m}$, i =1, 2,. . ., $\rho - 1$. Then

$$e^{[\rho]} + K_{\rho-1}e^{[\rho-1]} + \cdots + K_0 e + \hat{\Delta}(x,u) - \Delta(x,u,d) = 0. \tag{36}$$

As stated in subsection 3.1, we can know that the matrix K depends on ρ, T, and r. The late design parameter should be suitably chosen to make each polynomial element of the vector $H(S)$ expressed by (33) be a Hurwitz polynomial.

Let $E = \begin{bmatrix} e^{\mathrm{T}} & e^{[1]^{\mathrm{T}}} & \cdots & e^{[\rho-1]^{\mathrm{T}}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{m\rho}$. Then the error equations of the close-loop system can be written as

$$\dot{E} = AE + B(\Delta(x,u,d) - \hat{\Delta}(x,u)), \tag{37}$$

where

$$A = \begin{bmatrix} O_m & I_m & O_m & \cdots & O_m \\ O_m & O_m & I_m & \cdots & O_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_m & O_m & O_m & \cdots & I_m \\ -K_0 & -K_1 & -K_2 & \cdots & -K_{\rho-1} \end{bmatrix} \in \mathbb{R}^{m\rho\times m\rho}, \tag{38}$$

$$B = \begin{bmatrix} O_m & O_m & \cdots & I_m \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{m\rho\times m}, \tag{39}$$

and I_m here is also an m-dimension identity matrix, while O_m is an m-dimension zero matrix.

It is obvious that A is a Hurwitz matrix, so that for any given positive definite symmetric matrix Q, i.e., $Q = Q^{\mathrm{T}} > 0$, there exists an exclusive positive definite symmetric matrix P satisfying

$$A^{\mathrm{T}}P + PA = -Q. \tag{40}$$

Choose the Lyapunov function

$$V = \frac{1}{2}E^{\mathrm{T}}PE. \tag{41}$$

Differentiating it yields

$$\begin{aligned} \dot{V} &= \frac{1}{2}(\dot{E}^{\mathrm{T}}PE + E^{\mathrm{T}}P\dot{E}) \\ &= \frac{1}{2}E^{\mathrm{T}}(A^{\mathrm{T}}P + PA)E + E^{\mathrm{T}}PB(\Delta(x,u,d) - \hat{\Delta}(x,u)) \\ &= -\frac{1}{2}E^{\mathrm{T}}QE + E^{\mathrm{T}}PB(\Delta(x,u,d) - \hat{\Delta}(x,u)). \end{aligned} \tag{42}$$

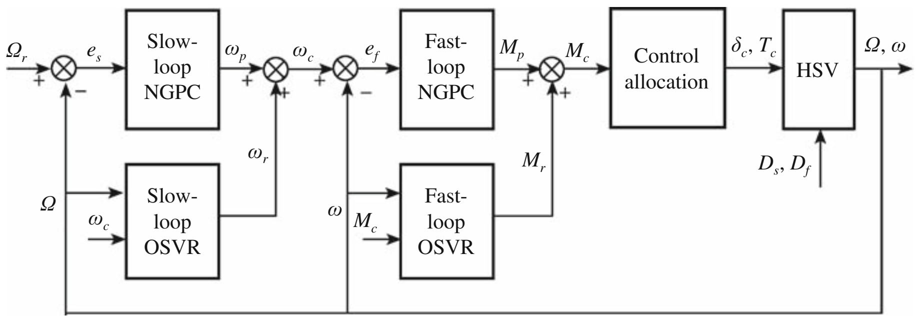

Figure 1 Online-SVR-compensated NGPC flight control system.

Furthermore, using conclusion (29) drawn from Lemma 3.2 and the property of consistent norm, we have

$$\dot{V} \leqslant -\frac{1}{2}\lambda_{\min}(Q)\|E\|^2 + \overline{\zeta}\lambda_{\max}(P)\|B\|\|E\|, \tag{43}$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of matrix Q, and $\lambda_{\max}(P)$ is the maximum one of P. By calculation, $\|B\| = 1$. So, when

$$\|E\| > \frac{2\overline{\zeta}\lambda_{\max}(P)}{\lambda_{\min}(Q)} \tag{44}$$

is satisfied, we will obtain the result that $\dot{V}$ is negative. Therefore, the error vector E of the close-loop system error is uniformly ultimately bounded.

Remark 3.4. The stability of close-loop system is independent of the predictive time T since it does not affect A expressed by (38) as Hurwitz or not. It only depends on the relative degree ρ and the choice of control order r. It can be proved that the system is always stable when the difference between ρ and r is less than four [7].

4 Design of HSV flight control system

4.1 Flight control system structure

The structure of online-SVR-compensated NGPC flight control system is shown in Figure 1. The controllers should be designed for both slow-loop and fast-loop. $\omega_c = \omega_p + \omega_r$ is the controller of slow-loop, and also the command of fast-loop. Meanwhile, $M_c = M_p + M_r$ is the control law for fast-loop.

In addition, three same command filters are designed for each component of the guidance commands Ω_c to smooth the changing of attitude angles, and Ω_r are the outputs of them. The transfer function of each command filter is given by

$$\frac{X_r(s)}{X_c(s)} = \frac{5}{s+5}, \tag{45}$$

where the corresponding signals x_r and x_c are reference model states and external input commands, respectively.

4.2 Controllers design

(1) and (2) show that both the two loops have well-defined vector relative degree. Assumptions (A1)–(A4) are satisfied for each loop, and there are $\rho_s = 1$ and $\rho_f = 1$. The control order is chosen as $r_s = r_f = 0$ in this paper. Furthermore, the slow-loop NGPC law and fast-loop NGPC law are derived as

$$\omega_p = -G_s^{-1}(K_s M_{\rho_s} + F_s - \Omega_r^{[\rho_s]}) = -g_s^{-1}(-K_s e_s + f_s - \dot{\Omega}_r), \tag{46}$$

$$M_p = -G_f^{-1}(K_f M_{\rho_f} + F_f - \omega_r^{[\rho_f]}) = -g_f^{-1}(-K_f e_f + f_f - \dot{\omega}_r), \tag{47}$$

where $e_s = \Omega_r - \Omega$ and $e_f = \omega_r - \omega$ are error vectors. Here, we have

$$K_s = \frac{3}{2T_s} I_3, \tag{48}$$

$$K_f = \frac{3}{2T_f} I_3, \tag{49}$$

where T_s and T_f are predictive time of corresponding loop, and I_3 is a three-dimension identity matrix.

In addition, the compensating control laws are

$$\omega_r = -G_s^{-1}\hat{\Delta}_s = -g_s^{-1}\hat{\Delta}_s, \tag{50}$$

$$M_r = -G_f^{-1}\hat{\Delta}_f = -g_f^{-1}\hat{\Delta}_f, \tag{51}$$

where $\hat{\Delta}_s$ and $\hat{\Delta}_f$ are the observed values of the uncertain item by the online SVRs in slow-loop and fast-loop, respectively. Obviously six online ε-SVRs in total are to be constructed. The outputs of them are calculated by (22) depending on the choice of the kernel function (26) and the optimal solutions of problem (21). The particular implementation of these online SVRs will not be given due to the limitation of space.

It should be noted that the arguments of all the variables are omitted in this section.

5 Simulation analysis

It is assumed that the vehicle is carrying out a hypersonic flight with the velocity of 2380 m/s (the Mach number is round about 7) and fight height of 27 km. The initial flight states of HSV are set as: the initial attitude angles are $\alpha(0) = 1.0°$, $\beta(0) = 2.5°$, and $\mu(0) = 3°$; the initial attitude angular velocity vector is $\omega(0) = \begin{bmatrix} p(0) & q(0) & r(0) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$.

The guidance commands chosen as $\alpha_c = 3.0°$, $\beta_c = 0°$, and $\mu_c = 0°$ are filtered by the command filter (45) to signals α_r, β_r, and μ_r. The predictive time of the two loops are both set as 0.4 s, i.e., $T_s = T_f$=0.4 s. In addition, the control surface deflection is limited by $|\delta_{e,a,r}| \leqslant 30°$.

In general, the model of aircraft's disturbance moments can be expressed by sinusoidal function forms [24]. In this simulation, the disturbance moments upon these three body-axes of hypersonic vehicle are supposed as follows:

$$\begin{cases} d_1 = 2 \times 10^5 (\sin 3t + 0.2) \mathrm{N \cdot m}, \\ d_2 = 2 \times 10^5 (\sin 5t - 0.3) \mathrm{N \cdot m}, \\ d_3 = 2 \times 10^5 \sin 4t \mathrm{N \cdot m}. \end{cases} \tag{52}$$

It is obvious that they are the external disturbances of fast-loop that can be defined as

$$d_f = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}^{\mathrm{T}}. \tag{53}$$

Besides, it is assumed that there are $\sin t \cdot 50\%$ and $-\sin t \cdot 40\%$ time-variant uncertainties in both aerodynamic coefficients and aerodynamic moment coefficients, respectively.

The selecting of kernel function and its parameters is based on experience. From Lemma 3.2, it can be known that the up bound of LOO error of SVR depends on the choice of kernel function, i.e. the selecting of kernel radius in (26). To make $R_{\mathrm{LOO}}(T_r)$ in (27) as small as possible, the parameters of each online-SVR in the simulation are chosen as: $C = 10$, $\varepsilon = 0.1$, and $\sigma = 0.2$.

Figure 2 shows the control performance of the nominal NGPC without compensating control law. When there is no uncertainties and disturbances in the system, i.e., $D = 0$, as shown by the solid lines, the control performance, no matter whether the dynamic process or the stable process or the control surfaces deflection, are good enough. However, when in the presence of uncertainties and disturbances which are described above, i.e., $D \neq 0$, the dashed lines show that the closed-loop system is no longer stable. Hence, the HSV is sensitive to the parameter uncertainty and external disturbance in the flight process and the nominal NGPC cannot satisfy the precision and robustness requirements.

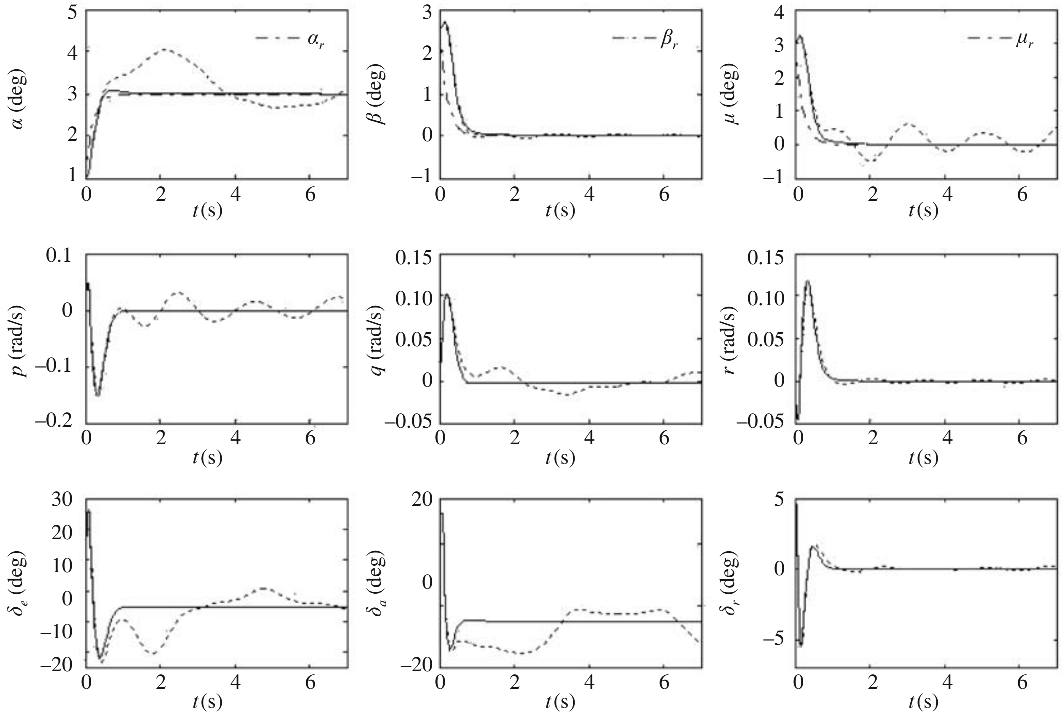

Figure 2 Control performance of nominal NGPC. —— $D=0$; ------ $D \neq 0$.

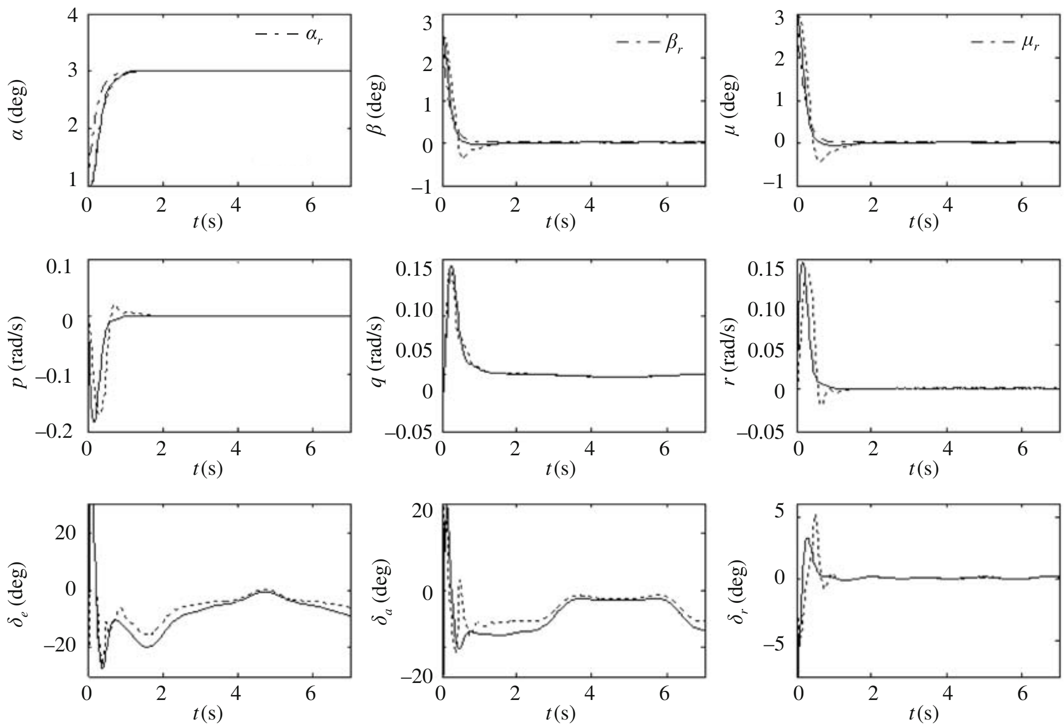

Figure 3 Control performance of online-SVR-compensated NGPC. —— OSVR; ------ RBFDO.

Therefore, the online SVR-compensated NGPC law is also simulated in this paper. The solid lines in Figure 3 show the tracking performance of the attitudes and angular velocities and the control surface deflection of this method in the assumed uncertain conditions. In addition, the control results of NGPC with RBF neural network disturbance observer (RBFDO) [13] are also provided in Figure 3 and shown as the dash lines. The structure of the neural networks for both the fast-loop and slow-loop is described

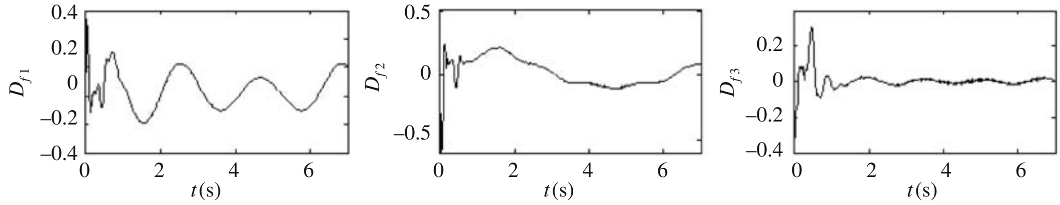

Figure 4 Observed values of the compound disturbances in fast-loop.

as 3-10-3 and there are 33 weights to be learned, while the number is only 6 for SVR design. As it shows, online SVR-compensated NGPC possesses a stronger adaptability, and obtains a higher precision and better dynamic performance. The advantages of this method have been demonstrated.

Moreover, Figure 4 gives the approximation performance of the compound disturbances D_{f1}, D_{f2}, and D_{f3} in each channel of fast-loop using the online SVR.

As the parameters of SVR is chosen well, the estimating ability of compound disturbance and the final control performance are both good enough.

6 Conclusions

As we know, there exist parameter uncertainties and external disturbances in a hypersonic flight. Hence, adaptiveness and robustness are critical requirements for the attitude stabilization control of HSV. This paper presents a systematic method for designing an adaptive roubust controller for a class of uncertain nonlinear systems that satisfy assumptions (A1) – (A4). The so-called online-SVR-compensated NGPC strategy is applied to HSV flight control. The performance of NGPC deteriorates much under strong uncertainties, so a new robust learning method based on online ε-SVR is proposed to counteract the negative effects of compound disturbances. The avoidance of curse of dimensionality, the use of kernel function, and the sparsity make SVR be an excellent algorithm for model identification. Simulation effort validates that the proposed control method for HSV flight control system has excellent performance and higher disturbance repellency over RBFDO. The control strategy developed in this paper is easy to design and implement and the design parameters are transparent.

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