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Fuzzy tracking control design for hypersonic vehicles via T-S model

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Abstract The main focus of this paper is on designing a T-S fuzzy controller for the hypersonic vehicle. The longitudinal dynamics of the vehicle are studied using time-scale decomposition to reduce the complexity of T-S modeling. The dynamic inversion with PI control technique is applied for the slow dynamics to derive the flight path angle command and throttle setting by taking the pilot altitude and velocity command as its inputs. The T-S fuzzy controller is designed for the fast dynamics to derive the elevator deflection to track the flight path angle command. The discrepancy between the T-S model and real vehicle model is considered by using sliding mode control for the system stability. Simulation results are included to show the effectiveness of the controller.

Keywords fuzzy control, T-S model, sliding mode control, hypersonic vehicle

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1 Introduction

There is intensive research on the hypersonic flight as it represents a promising technology for feasible and reliable access to space. The main challenge of the control law design for the hypersonic vehicle is due to the high complexity of the equations of motion and the difficulty in measuring and estimating the aerodynamic parameters of the vehicle. Therefore, a reasonable tradeoff should be made between performance in handling system nonlinearity and controller complexity.

Linearized models of the hypersonic vehicles simplifying the nonlinear model facilities the controller design. Several results are available in the literatures that consider control solutions for a linearized version of a generic hypersonic vehicle model linearized about a specific trim condition, for instance, the application of H^∞ and μ-synthesis method [1], implicit model following control [2] and linear output feed-back control [3]. As can be expected, controlling hypersonic vehicles is a difficult task, even after linearization of the dynamics.

When considering nonlinear control method, the feedback linearization techniques are then applied to design a nonlinear controller [4]. Combining with adaptive sliding mode control strategy [5, 6], this approach achieves a good tracking performance, but leads to a complicated high-order Lie derivatives due to the model complexity. Furthermore, it is not possible to perform a robustness analysis when

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model uncertainty is taken into consideration. For the nonlinear dynamics of hypersonic vehicle, the back-stepping method provides an efficient way to design the robust adaptive controller [7, 8].

The T-S fuzzy model has been proved to be a good representation for a certain class of nonlinear dynamic systems. The stabilization problem for the systems represented in T-S fuzzy models has been well addressed [9–12]. More recent researches pay attention to the tracking controller design based on T-S fuzzy models [13, 14]. In general, there must be a discrepancy between the T-S model and the real physical model. Model inaccuracies have an adverse effect on the nonlinear dynamic system. Sliding mode control is known to be an efficient control technique applicable to systems with modeling uncertainties [15–17]. In [18], the authors proposed a sliding mode control based on T-S fuzzy model for a class of perturbed nonlinear systems to solve the tracking problem.

In this paper, a fuzzy tracking controller is developed for the hypersonic vehicle based on its T-S fuzzy model. To minimize the design effort and model complexity, we break the original system dynamics into slow dynamics and fast dynamics by time scale decomposition. The slow dynamics consist of the altitude and velocity dynamics and the fast dynamics include the rest of the states. For the slow dynamics, taking the pilot commanded altitude and velocity as inputs, the dynamic inversion plus PI control method is used to derive the flight path angle command and throttle setting. And then, feeding the flight path angle command into the fast dynamics, one can design a T-S fuzzy controller to derive the elevator deflection. The time scale decomposition makes T-S fuzzy modeling easily, and the modeling inaccuracies are compensated by sliding mode control.

2 Hypersonic vehicle model

The hypersonic vehicle under study is a conical vehicle, which is taken from the NASA langley hypersonic vehicle simulation model. The dynamic equations of airspeed, altitude, flight path angle, angle of attack and pitch rate for the longitudinal dynamics of a generic hypersonic vehicle, cruising at a Mach number of 15 and at an altitudes of 110000 ft, are given as follows [19]:

$$\dot{V} = \frac{T(V,\beta)\cos\alpha - D(V,\alpha)}{mV} - \frac{\mu\sin\gamma}{r^2}, \tag{1}$$

$$\dot{h} = V\sin\gamma, \tag{2}$$

$$\dot{\gamma} = \frac{L(V,\alpha) + T(V,\beta)\sin\alpha}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{Vr^2}, \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma}, \tag{4}$$

$$\dot{q} = M_{yy}(V,\alpha,q,\delta_E)/I_{yy}, \tag{5}$$

where, $T(V,\beta)$, $D(V,\alpha)$, $L(V,\alpha)$ and $M_{yy}(V,\alpha,q,\delta_E)$ represent thrust, drag, lift-force and pitching moment respectively, which are defined as

$$\begin{aligned} L(V,\alpha) &= \overline{q}SC_L, \\ D(V,\alpha) &= \overline{q}SC_D, \\ T(V,\beta) &= \overline{q}SC_T, \\ M_{yy}(V,\alpha,q,\delta_E) &= \overline{q}S\overline{c}[C_M(\alpha) + C_M(\delta_E) + C_M(q)], \end{aligned}$$

where $\overline{q} = \rho V^2/2$ is the dynamic pressure and ρ is air density. The force and moment coefficients are

$$\begin{aligned} C_L &= 0.6203\alpha, \\ C_D &= 0.6450\alpha^2 + 0.0043378\alpha + 0.003772, \\ C_T &= \begin{cases} 0.02576\beta, & \text{if} \quad \beta < 1, \\ 0.0224 + 0.00336\beta, & \text{if} \quad \beta > 1, \end{cases} \\ C_M(\alpha) &= 0.035\alpha^2 + 0.036617\alpha + 5.3261 \times 10^{-6}, \end{aligned}$$

$$C_M(q) = (\bar{c}/2V)q(-6.796\alpha^2 + 0.3015\alpha - 0.2289),$$
$$C_M(\delta_E) = c_e(\delta_E - \alpha),$$

where V is the velocity, h is the altitude, γ is the flight path angle, α is the attack angle, q is the pitch rate, and m, I_{yy}, μ and r represent the mass of aircraft, moment of inertia about pitch axis, gravity constant and the radial distance from center of the earth. δ_E is elevator deflection and β is the throttle setting.

The model is composed of five state variables $x = [V \ \ h \ \ \gamma \ \ \alpha \ \ q]^{\mathrm{T}}$, and two control inputs $u = [\delta_E \ \ \beta]^{\mathrm{T}}$, whereas the output to be controlled is selected as $y = [V \ \ h]^{\mathrm{T}}$. The design objective is to track the desired altitude and velocity command $y_d = [V_d \ \ h_d]$ and achieve good performance in command responses.

3 Controller design

In order to facilitate the fuzzy tracking controller design for hypersonic vehicle, the nonlinear dynamics (1)–(5) should be transformed into a regular form with full relative degree with respect to the outputs and their time derivatives. However, tedious analysis and high-order Lie derivatives (see [4]) make it difficult for T-S system modeling and fuzzy controller design. In this paper, we employe the time scale decomposition to break the system into fast and slow dynamics to reduce the modeling complexity, and meanwhile, the proposed fuzzy tracking scheme can guarantee the system stability with desired control performance.

The starting point is the time-scale decomposition of the dynamic equations (1)–(5) into functional subsystems, namely, the slow dynamics (the velocity and the altitude subsystem), and the fast dynamics (flight-path angle, the angle of attack and the rotational pitch rate dynamic subsystem). When dealing with the slow dynamics, it is assumed that fast-dynamics states are at their equilibrium condition, and when dealing with the fast dynamics, it is assumed that states of the slow dynamics remain constant.

Next, the control inputs for the slow dynamic subsystems, γ_d and β, are designed to track the pilot command inputs $y_d = [V_d \ \ h_d]$ for the altitude and velocity subsystems, respectively. They are the direct inversion of the altitude and velocity dynamics with proportional plus integral compensator to minimize the steady-state error.

Finally, we use the T-S Fuzzy model to represent the fast dynamics. The intermediate command γ_d is fed into the fast dynamics, where the elevator deflection, δ_E is derived by a T-S fuzzy controller.

3.1 Dynamic inversion with PI control of slow dynamics

For the velocity subsystem, one can transform (1) into a standard form as follows:

$$\dot{V} = f_V(\gamma, \alpha, V, h) + g_V(\alpha, V)\beta. \tag{6}$$

We design the throttle setting as

$$\beta = -1/g_V(\alpha, V)\left[f_V(\gamma, \alpha, V, h) + k_{1V}(V - V_d) + k_{2V}\int(V - V_d)\mathrm{d}t - \dot{V}_d\right], \tag{7}$$

where $k_{1V}, k_{2V} > 0$ are constants.

Note that (2) is analytic and do not have any uncertainties because it is a kinematic equation. The exact analytical inverse may be used,

$$\gamma_d = \arcsin\left[\frac{-k_{1h}(h - h_d) - k_{2h}\int(h - h_d)\mathrm{d}t + \dot{h}_d}{V}\right], \tag{8}$$

where $k_{1h}, k_{2h} > 0$ are constants. Thus, the altitude command is converted into the flight path angle command to simplify the T-S fuzzy modeling.

3.2 T-S fuzzy control of fast dynamics

According to the definition of flight path angle γ, attack angle α and pitch angle θ, we have $\theta = \alpha + \gamma$. Define the state, output and control variables of fast dynamics $x_f = [x_1, x_2, x_3]^{\mathrm{T}}$, $x_1 = \gamma$, $x_2 = \theta$, $x_3 = q$, $y_f = x_1 = \gamma$. $u_f = \delta_E$. In order to derive T-S model of the fast dynamics conveniently, the thrust term $T(V, \beta)\sin\alpha$ in (3) can be neglected since it is generally much smaller than lift. The fast dynamic equations can be rewritten as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, x_3, V, h), \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= f_3(x_1, x_2, x_3, V) + g_3(x_1, x_2, V)u_f, \\ y_f &= x_1, \end{aligned} \tag{9}$$

where $f_1(x_1, x_2, x_3, V, h) = L(V, \alpha)/(mV) - (\mu - V^2 r\cos\gamma)/(Vr^2)$, $f_3(x_1, x_2, x_3, V) = M_N/I_{yy}$, $g_3(x_1, x_2) = M_{\delta_E}/I_{yy}$, $M_{yy}(V, \alpha, q, \delta_E) = M_N + M_{\delta_E}\delta_E$, $u_f = \delta_E$. The control objective is to design a T-S fuzzy controller $u_f = \delta_E$, which makes $\gamma \to \gamma_d$, further, $h \to h_d$.

To design the T-S fuzzy controller, we transform the fast dynamic equations into a regular form. Let $y_{fd} = [y_{1d}\ \ y_{2d}\ \ y_{3d}]^{\mathrm{T}} = [\gamma_d\ \ \dot{\gamma}_d\ \ \ddot{\gamma}_d]^{\mathrm{T}}$.

Define

$$\begin{aligned} e_1 &= y_f - y_{1d}, \\ e_2 &= \dot{e}_1 = \dot{y}_f - \dot{y}_{1d} = \dot{x}_1 - \dot{y}_{1d} = E_{11}(x_f, V, h), \\ e_3 &= \dot{e}_2 = \dot{E}_{11}(x_f, V, h) = E_2(x_f, V, h). \end{aligned} \tag{10}$$

Solving the following nonlinear equations,

$$\phi_1(x_f, V, h) = \begin{bmatrix} y_f - y_{1d} \\ E_1(x_f, V, h) \\ E_2(x_f, V, h) \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \tag{11}$$

we can transform the fast dynamic equation (9) with state variable x_f into a regular form with state variable $e = [e_1\ \ e_2\ \ e_3]^{\mathrm{T}}$, when $\phi_1(x_f, V, h)$ are diffeomorphism. The feasibility of the transformation has been demonstrated in [4]. Then, (10) can be rewritten as follows:

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= e_3, \\ \dot{e}_3 &= a(e) + b(e)u_f. \end{aligned} \tag{12}$$

Note that the tracking problem of the fast dynamic system (9) has been converted into a regulation problem of the system (12). The fast dynamics of the hypersonic vehicle (12) or (9) can be represented by T-S fuzzy model, which is described by fuzzy If-Then rules and will be employed to deal with the control design problem for the fast dynamic system.

$$\begin{aligned} \text{Rule } i: \quad & \text{If } z_1(t) \text{ is } F_{i1},\ \ z_2(t) \text{ is } F_{i2},\ \ \ldots\ \ z_r \text{ is } F_{ir}, \\ & \text{Then } \dot{e} = A_i(t)e + B_i(t)u_f + d_i(t), \quad i = 1, 2, \ldots, m, \end{aligned} \tag{13}$$

where $z_j(t), j = 1, 2, \ldots, r$ are measurable variables of system, $F_{ij},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, r$ are fuzzy sets described by membership functions $\mu_{F_{ij}}(z(t))$, and the center of the membership function is the operating point. Besides, $A_i(t) \in R^{3\times 3}$, $B_i(t), d_i(t) \in R^3$ are appropriate matrices and vectors, respectively, which includ the time-varied trajectory y_{fd}. By using the center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy model (13) can be written as (see [18])

$$\dot{e} = A(t)e + B(t)u_f + d(t), \tag{14}$$

where $A(t) = \sum_{i=1}^{m} \mu_i(z(t))A_i(t)$, $B(t) = \sum_{i=1}^{m} \mu_i(z(t))B_i(t)$, $d(t) = \sum_{i=1}^{m} \mu_i(z(t))d_i(t)$, in which, $\mu_i(z(t)) = \Pi_{j=1}^{r}\mu_{F_{ij}}(z(t))/\sum_{i=1}^{m}\Pi_{j=1}^{r}\mu_{F_{ij}}(z(t))$ is the normalized fuzzy membership function for the ith fuzzy inference rule, and $\sum_{i=1}^{m} \mu_i(z(t)) = 1$.

Because of the decomposition of the original vehicle system, one can obtain the T-S fuzzy model (14) easily. However, there must be a discrepancy between the T-S model (14) and real vehicle model due to the modeling error and unknown uncertainties. In this paper, a lumped perturbation with a known upper bound is introduced to represent this discrepancy. The T-S fuzzy model is modified,

$$\dot{e} = A(t)e + B(t)u_f + d(t) + \Delta f(e, u_f), \tag{15}$$

where $\Delta f(e, u_f)$ represents the lumped perturbation of parameter variation, external disturbances. It satisfies

$$||\Delta f(e, u_f)|| \leqslant \rho_0 + \rho_1||e|| + \rho_2||u_f||, \tag{16}$$

where $\rho_0, \rho_1, \rho_2 > 0$ are constants. It is assumed that $\Delta f(e, u_f)$ belongs to a matched uncertainty.

We present a sliding mode control method to stabilize the T-S fuzzy system with the perturbation $\Delta f(e, u_f)$, so that both tracking error e and the system state x_f are bounded.

In general, sliding mode control design follows two standard steps [15–17]: 1) a sliding surface is designed such that the closed-loop system during sliding mode exhibits desired dynamic behavior; and 2) a discontinuous control law is employed to force the system states to remain on the surface.

Define the sliding surface,

$$s = Ce, \tag{17}$$

where $C = [c_1 \;\; c_2 \;\; c_3] \in R^{1\times 3}$, in which $c_i, i = 1, 2, 3$ is a positive constant. The vector C is chosen such that $CB(t)$ is nonsingular.

Calculating the time derivative of s defined in (17) and using (15), we have

$$\dot{s} = CA(t)e + CB(t)u_f + Cd(t) + C\Delta f(e, u_f). \tag{18}$$

We now design the control scheme that derives the state trajectories of the system in (15) onto the sliding surface $s = 0$ and the system remains in it thereafter. In order to fulfill the sliding condition, the control input u is divided into two parts. One is the equivalent control input u_{eq}, and the other is robust control input u_s, i.e.,

$$u_f = u_{eq} + u_s, \tag{19}$$

where

$$\begin{aligned} u_{eq} &= -[CB(t)]^{-1}[CA(t)e + Cd(t)], \\ u_s &= \begin{cases} -[CB(t)]^{-1}[\eta_1 + \eta_2 + ||C||(\rho_0 + \rho_1||e||)]\dfrac{s}{||s||}, & s \neq 0, \\ 0, & s = 0, \end{cases} \end{aligned} \tag{20}$$

where $\eta_1 > 0$ is a design constant, and $\eta_2 = \dfrac{k||[CB]^{-1}||||C||}{1 - k||[CB]^{-1}||||C||}[\eta_1 + ||C||(||Ae|| + ||d|| + \rho_0 + \rho_1||e||)]$. Choose $\rho_2 \leqslant k \leqslant 1/(||[CB]^{-1}||||C||)$. And, η_2 can be used to cancel the adverse effect of the control input uncertainty. The equivalent control u_{eq} can be solved from the equation $\dot{s} = 0$ under the condition $\Delta f(e, u_f) = 0$. If the upper bound of the perturbation is unknown in prior, the adaptive law can be designed for the on-line estimation [18, 20]. For the stability analysis of the fast dynamics, see Appendix located at the end of this paper.

4 Numerical simulation

This section presents the simulation results for proposed control approach. The control performance is tested in simulation on the longitudinal flight control of the generic hypersonic aircraft cruising at an altitude of 110000 ft and Mach number 15 (15060 ft/s). The parameters of simulation model are taken

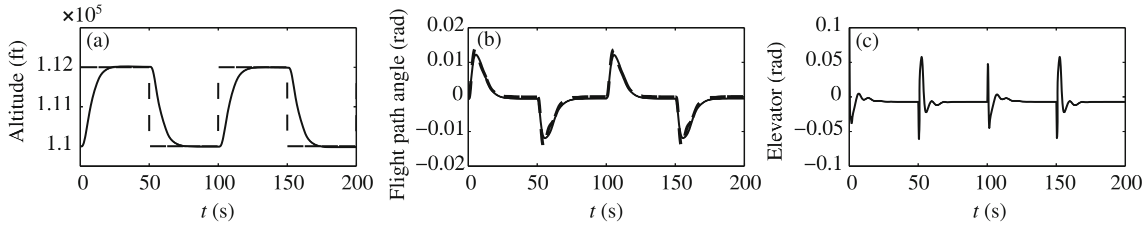

Figure 1 (a) Altitude; (b) flight path angle tracking (γ_d: dashed line; γ: solid line); and (c) elevator deflection.

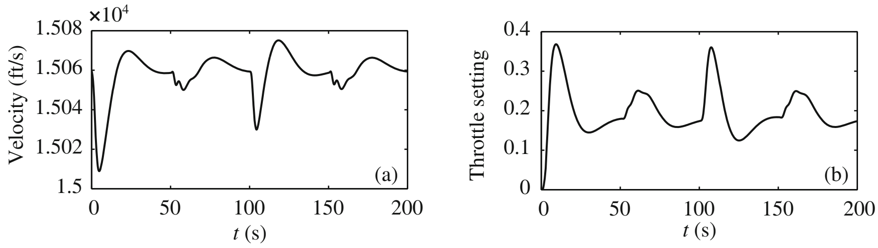

Figure 2 (a) Velocity tracking; (b) throttle setting.

from [5]. In simulation study, the engine dynamics, i.e., the throttle setting are modeled by a second order system,

$$\ddot{\beta} = -2\xi\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c. \tag{21}$$

Thus, the control input are elevator deflection δ_E and throttle setting β_c.

The control objective is to utilize the proposed control scheme so that the outputs V and h can track the desired trajectory. The velocity output should maintain at 15060 ft/s, and meanwhile, third-order linear command filters are used to generate the differentiable altitude command,

$$\frac{h_d(s)}{h_c(s)} = \frac{1}{(s+1)^2(3s+1)},$$

where h_c is square-wave signal with magnitude 2000 ft and periodic 100 s and s is Laplace operator.

The state variables $e = [e_1, e_2, e_3]^{\mathrm{T}}$ of the fuzzy system are constrained within $[-0.01, 0.01]$. In the simulation, the fuzzy membership functions are defined for each variable as follows:

$$\begin{aligned}
\mu_{F_{1j}} &= 1/\{1+\exp[\overline{\zeta}_j(z+\overline{v}_j)]\},\\
\mu_{F_{2j}} &= \exp[-(z/v_j)^2],\\
\mu_{F_{3j}} &= 1/\{1+\exp[-\overline{\zeta}_j(z-\overline{v}_j)]\},
\end{aligned}$$

where the parameters $\overline{\zeta}_j$, $\overline{v}_j$, v_j are chosen such that the input spaces of fuzzy logic system can be divided evenly. In this paper, by linearizing (12) at 27 operating points of $[e_{1i}, e_{2i}, e_{3i}] = [0.01, 0.01, 0.01]$, $[0, 0.01, 0.01], \ldots, [-0.01, -0.01, -0.01], i = 1, 2, \ldots, 27$ and using the fuzzy rules (13), we can obtain the T-S fuzzy model of fast dynamics of the hypersonic vehicle. Then, the tracking problem of the fast dynamics is converted into a regulation problem. The control objective is to regulate $e \to 0$ as $t \to \infty$.

The sliding surface is chosen as $s = 0.5e + 0.5\dot{e} + \ddot{e}$. The designed parameters used in this simulation are selected as follows, $k_{1V} = 0.1$, $k_{2V} = 0.002$, $k_{1h} = 0.3$, $k_{2h} = 0.003$, $\eta_1 = 1$, $k = 0.5$, $\rho_0 = 2$, $\rho_1 = 2$, $\rho_2 = 0.5$.

Figure 1 shows that the designed elevator deflection for the fast dynamics can track the commanded flight path angle in a good performance. And, the commanded flight path angle is used to make the slow dynamic subsystem follow the pilot altitude command successfully. The direct inversion with PI control strategy can maintain the velocity at the neighborhood of 15060 ft/s as shown in Figure 2. The control performance of θ, q and α are given in Figure 3.

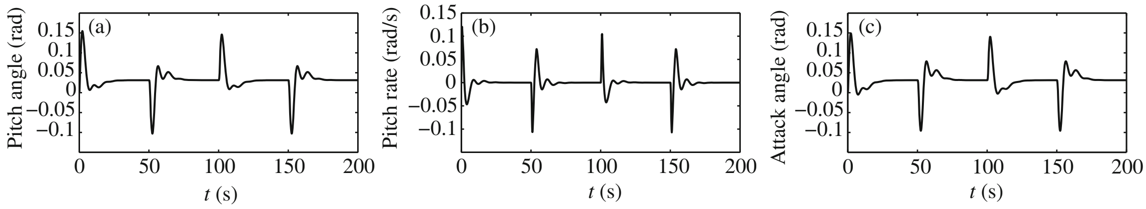

Figure 3 Control performance of θ (a), q (b) and α (c).

5 Conclusions

In this paper, the tracking problem of the hypersonic vehicle has been investigated. The controller design follows an approach that uses the time scale decomposition of the longitudinal vehicle dynamics into the velocity, altitude/flight path angle and attack angle/pitch rate subsystems. The issue of model discrepancy when performing T-S fuzzy modeling is addressed by using sliding mode control. Simulation results have verified the effectiveness of the proposed design.

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Appendix

Choose a Lyapunov function as $\overline{V} = 1/2s^2$ to prove the controller can maintain the sliding mode. Differentiating $\overline{V}$ with respect to time and using (15) yield

$$\dot{\overline{V}} = s\dot{s} = s[CA(t)e + CB(t)u_f + Cd(t) + C\Delta f(e, u_f)].$$

If $s \neq 0$, substituting control law (19) into the previous equation under the bounding relations in (16), one obtains

$$\begin{aligned}
\dot{\overline{V}} &\leqslant ||s||||C\Delta f(e, u_f)|| - ||s||[\eta_1 - \eta_2 - ||C||(\rho_0 + \rho_1||e||)] \\
&\leqslant (-\eta_1 - \eta_2)||s|| + ||C||||s||[\rho_0 + \rho_1||e|| + \rho_2||u_f|| - (\rho_0 + \rho_1||e||)] \\
&\leqslant -\eta_1||s|| + ||s||[-\eta_2 + \rho_2||C||||[CB]^{-1}||(||CAe|| + ||Cd|| + \eta_1 + \eta_2 + ||C||(\rho_0 + \rho_1||e||))] \\
&\leqslant -\eta_1||s|| + ||s||[-\eta_2(1 - k||C||||[CB]^{-1}||) + k||C||||[CB]^{-1}||(\eta_1 + ||C||(||Ae|| + ||d|| + \rho_0 + \rho_1||e||))] \\
&\leqslant -\eta_1||s|| < 0.
\end{aligned}$$

This implies that s will approach to zero in finite time.