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Optimal power allocation for cognitive relay networks: amplify-and-forward versus selection relay

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Abstract We consider a cognitive relay network which is defined by a source, a destination, and cognitive relay nodes and primary user nodes. In this network, a source is assisted by cognitive relay nodes which allow coexisting with primary user nodes by imposing severe constraints on the transmission power so that they operate below the noise floor of primary user nodes. In this paper, we mainly study the power allocation strategies of this system to minimize the outage probability subject to total and individual power constraints for cognitive relay nodes and subject to interference constraints for primary user nodes. A relay transmission scheme, namely amplify-and-forward (AF), is considered. We first present an optimal power allocation (OPA) scheme to minimize the system outage probability, based on instantaneous channel state information among the source, destination and relay nodes and mean channel gains between primary user nodes and cognitive relays. Next, we propose a selection AF scheme (S-AF) where the single best relay is chosen to assist in the transmission and study power allocation strategy for S-AF to minimize outage probability. The analytical power allocation strategies have been validated through numerical simulations. The results indicate that the AF with OPA and S-AF with OPA have significantly better outage behavior and average throughput than AF and S-AF schemes respectively. We also find that S-AF with OPA achieves a higher throughput, and hence lower outage probability than AF with OPA.

Keywords cognitive radio (CR), relay networks, power allocation, outage probability

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1 Introduction

With the rapid growth of wireless communication, the demand for radio spectrum is expected to grow rapidly in the near future. However, radio spectrum is a limited resource and it is already very crowded. It seems that it is difficult to accommodate more wireless application within this limited resource. On the other hand, the licensed spectrum bands are underutilized due to the current inflexible spectrum allocation policy. This point of view is supported by recent studies of the FCC [1]. The allocation of

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*Corresponding author (email: ywjtylg9124@126.com) the spectrum currently is regulated. The spectrum bands are licensed and the sharing of bands is not allowed. Because this inflexible spectrum allocation policy will result in the underutilization of overall spectrum, more flexible alternatives for better utilization of the spectrum should be found out.

To deal with the conflicts between spectrum scarcity and spectrum under-utilization, cognitive radio [2] has been considered as an efficient approach to improving the spectrum utilization by spectrum sharing between primary users (PUs) and secondary users (SUs). The sharing could be either in the form of opportunistic overlay of idle bands in the licensed spectrum or in the form of an underlay fashion by allowing an SU to coexist with the PU as long as the quality of service (QoS) of the PU is not degraded by the interference caused by the SU [3]. However, in overlay scenario, to precisely detect a spectrum hole which is not being used is a very tough job. Moreover, this approach may not fully utilize the characteristics of co-channel interference in cellular environment and/or wideband signaling such as wideband code-division multiple access (WCDMA) and ultra-wideband (UWB) that can coexist with other systems because of low inter-user interference. Different from spectrum overlay, the underlay approach does not necessarily rely on detection and exploitation of spectrum white space, but rather on how much transmission power of SUs so as not to cause any harmful interference to the active PUs.

On the other hand, relay communications have emerged as a powerful spatial diversity technique that can improve the performance over conventional point-to-point transmissions, which has received more attentions [4, 5]. Relay networks, where a source node is assisted by intermediate nodes, offer a significant performance gain advantage. Different relay protocols have been investigated in [6]. One of the most popular cooperative protocols is amplify-and-forward (AF), in which the relay simply amplifies received signal from the source and retransmits it to the destination.

Inspired by these two techniques of wireless networks, i.e., cognitive radios and relay networks, in this paper, we propose a *cognitive relay network*, combining the idea of cognitive radio with the relay networks, which is defined by a source node, a destination node and a group of cognitive relay nodes and primary user nodes. A source is assisted by cognitive relay nodes which allow coexisting with primary user nodes subject to transmitted power constraints so that they operate below the noise floor of primary users. We derive the optimal power allocation strategies among the cognitive relay nodes to achieve the minimum system outage probability with total and individual power constraints for relays and interference constraints for primary user nodes when the AF protocol is used. Additionally, we also introduce a selection-based relay scheme, selection amplify-and-forward (S-AF), in which only one “best” node is chosen as a relay. Meanwhile, the optimal power allocation algorithm for S-AF among the relay and source nodes is also designed to minimize the outage probability.

Optimal power allocation in AF networks has been studied in [7, 8]. Most of these solve for the optimal power division between the source and relay nodes to maximize capacity or minimize outage probability. However, the extension of these algorithms to cognitive relay networks is not obvious. Recently, the combination of relaying and cognitive radio techniques has been investigated [9–11]. Ref. [9] studied the approximate outage probability of a repetition-based cognitive relay network in the high signal-to-noise ratio (SNR) regime under overlay fashion. Their main emphasis was to analyze the achievable diversity order of the system subject to cognitive relays being able to acquire spectrum. In contrast to the standard relay scenario, cognitive relays will only transmit if they are successful in obtaining spectrum. Ref. [10] considered the same cognitive relay network architecture described in [9] and derived exact expressions for the outage probability of the repetition-based scheme and selection cooperation. Meanwhile, refs. [9] and [10] also derived the outage probability under imperfect spectrum acquisition. However, to the best of our knowledge, recent research on the combination of relaying and cognitive radio techniques mainly focus on opportunistic cognitive relay network in which cognitive nodes will only relay information for the source node if they are successful in obtaining spectrum unused by a primary user node. We propose for the first time cognitive relay networks where cognitive relay node is designed to allow the coexistence with PU and study optimal power allocation strategies of this system to minimize the outage probability subject to transmitted power constraints of cognitive relays and interference constraints of primary users. Ref. [11] proposed an ad-hoc cognitive radio concept, in which transceivers with small power and multi-hop communications are used for expanding the service area by using SDR (software defined radio) terminal.

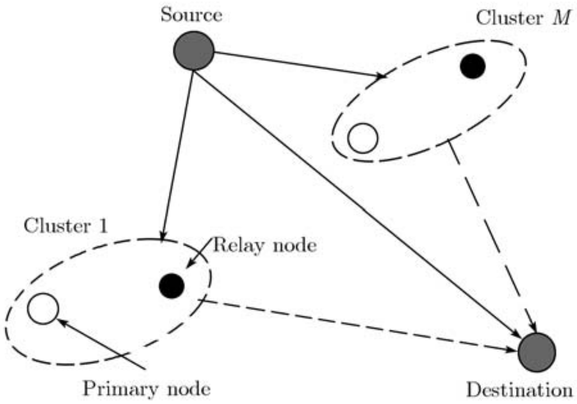

Figure 1 Proposed cognitive relay network model.

The rest of this paper is organized as follows. Section 2 briefly describes the system model and analyzes its outage performance when the AF cooperative diversity scheme is used. Section 3 presents the interference constraints for primary user nodes and the formulation for power allocation problem. Section 4 derives the optimal power allocation among the cognitive relay nodes to achieve the minimum system outage probability. Section 5 presents the S-AF scheme. Meanwhile, the optimal power allocation algorithm for S-AF among the relay and the source is also designed in this section. Performance evaluation and comparisons are given in section 6, and finally our conclusions are offered in section 7.

2 Cognitive relay network with multinode AF relays

2.1 System model

In this paper, the suggested cognitive relay network model is defined by a source, a destination, M cognitive relay nodes and M primary user nodes, as depicted in Figure 1. The cognitive relay nodes are cognitive (secondary) users. The cognitive node relays information for the source node "s" in underlay approach which allows the coexistence with a primary user node by imposing severe constraints on the transmission power so that it operates below the noise floor of primary user node. We assume that the transmission of primary users is orthogonal, either through time or frequency division. A cognitive relay node is only allowed to coexist with a primary user node, shown as dashed ellipse in Figure 1, called a cluster. We assume that clustering has been done by upper layers, based on their geographical location. Therefore, the transmission of the relay nodes to the destination is also orthogonal because the acquired time slots or spectra from different primary user nodes are non-overlapping. All M relays help the source and $(M+1)$ orthogonal channels are used for transmission. For orthogonal channels, the destination receives $(M+1)$ copies of the source symbol from the source and the relay nodes with no interference among each other. For convenience, we assume that the cognitive relay network is time division and so each relay node is assigned one of M time slots in each information packet. All links associated with the primary user nodes are modeled as flat fading channels and the channel fade coefficients for different links are assumed to be statistically independent.

The communication consists of two transmission phases. In the first phase, the source broadcasts to its destination and relay nodes, shown as real line in Figure 1. During the second phase, the relay node simply amplifies the received signal and retransmits it to the destination node, shown as dashed line in Figure 1. Therefore, during the first phase, the received signals at the destination and the ith relay are given by, respectively,

$$y_{s,d} = \sqrt{E_s} h_{s,d} x + n_{s,d}, \tag{1}$$

$$y_{s,i} = \sqrt{E_s} h_{s,i} x + n_{s,i}, \quad i = 1, \ldots, M, \tag{2}$$

where x denotes the transmitted signal from the source node. $h_{s,d}$ and $h_{s,i}$ are channel coefficients of the source-destination and source-relay channels, including the effect of shadowing, channel loss and fading. E_s is the average energy transmitted in this time slot. Assuming all the time slots have unit duration, E_s can be considered as the transmission power. $n_{s,d}$ and $n_{s,i}$ are additive circularly symmetric white Gaussian noise in the corresponding channels, with variance $N_{s,d}$ and $N_{s,i}$, respectively, i.e., $n_{s,d} \sim CN(0, N_{s,d})$ and $n_{s,i} \sim CN(0, N_{s,i})$.

During the second phase, the received signal at the destination through the ith relay is represented as

$$y_{i,d} = \eta_i h_{i,d} y_{s,i} + n_{i,d}, \tag{3}$$

where $h_{i,d}$ is the channel coefficient between the ith relay and the destination and η_i denotes the amplification factor of the ith relay.

We assume that the relay terminal normalizes the received signal by a factor of $\sqrt{\mathrm{E}\{|y_{s,i}|^2\}}$ (so that the average energy is unity). Thus, the signal transmitted from the ith relay is

$$x_i = \frac{y_{s,i}}{\sqrt{\mathrm{E}\{|y_{s,i}|^2\}}} = \frac{\sqrt{E_s} h_{s,i} x + n_{s,i}}{\sqrt{E_s |h_{s,i}|^2 + N_{s,i}}}, \tag{4}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator. Note that the ith relay is assumed to have perfect knowledge of E_s, $|h_{s,i}|^2$ and $N_{s,i}$ associated with the source-relay link.

Based on eq. (4), the signal received by the destination from the ith relay node can be rewritten as

$$y_{i,d} = \sqrt{E_i} h_{i,d} x_i + n_{i,d} = \sqrt{\frac{E_s E_i}{E_s |h_{s,i}|^2 + N_{s,i}}} h_{i,d} h_{s,i} x + \widetilde{n}_{i,d}, \tag{5}$$

where E_i denotes the transmitted power at the ith relay. $\widetilde{n}_{i,d}$ is the equivalent noise term in $y_{i,d}$. It can be easily shown that $\widetilde{n}_{i,d} \sim CN(0, \widetilde{N}_{i,d})$ with $\widetilde{N}_{i,d} = N_{i,d} + \frac{E_i |h_{i,d}|^2 N_{s,i}}{E_s |h_{s,i}|^2 + N_{s,i}}$.

2.2 Outage probability analysis

The equivalent signal-to-noise ratio (SNR) of the channel through the ith relay, γ_i is expressed as (see [12]),

$$\gamma_i = \frac{\frac{E_s |h_{s,i}|^2}{N_{s,i}} \frac{E_i |h_{i,d}|^2}{N_{i,d}}}{\frac{E_s |h_{s,i}|^2}{N_{s,i}} + \frac{E_i |h_{i,d}|^2}{N_{i,d}} + 1}. \tag{6}$$

When the maximal ratio combining (MRC) at the destination is applied with perfect knowledge of the instantaneous channel gains $|h_{s,i}|^2$, $|h_{i,d}|^2$ and $|h_{s,d}|^2$, the received SNR at the MRC output becomes $\gamma_{\mathrm{MRC}}^{\mathrm{AF}} = \gamma_s + \sum_{i=1}^{M} \gamma_i$, where $\gamma_s = E_s |h_{s,d}|^2 / N_{s,d}$ is the SNR due to the source transmission. Therefore, the average mutual information between the source and the destination can be given by

$$I_{\mathrm{AF}} = \frac{1}{M+1} \log_2 \left[1 + \gamma_s + \sum_{i=1}^{M} \gamma_i \right]. \tag{7}$$

Assuming all the channels are Rayleigh distributed, $|h_{s,i}|^2$, $|h_{i,d}|^2$ and $|h_{s,d}|^2$ are exponentially distributed random variables.

Outage probability is one of the most commonly used performance measures in wireless systems. For a general communication system, the outage occurs when the mutual information I_{AF} falls below a target value R. Therefore, for a given transmission rate R, the outage probability is defined as $P_{\mathrm{out}}^{\mathrm{AF}} = \Pr\{I_{\mathrm{AF}} < R\}$, where $\Pr\{\cdot\}$ denotes the probability.

It is extremely difficult to directly compute the exact outage probabilities for the AF scheme for arbitrary SNR because the probability density function (PDF) of I_{AF} is hard to obtain. However, the outage probability for the AF scheme in the high SNR regime can be approximated as (see [13]),

$$P_{\mathrm{out}}^{\mathrm{AF}}(\bar{\boldsymbol{E}}, R) \sim \frac{\lambda_0 \prod_{i=1}^{M} (\lambda_{s,i} + \lambda_{i,d})}{(M+1)} \left(\frac{2^{(M+1)R} - 1}{1/N_0} \right)^{M+1}, \tag{8}$$

where λ_0, $\lambda_{s,i}$ and $\lambda_{i,d}$ are the exponential distribution parameters of $E_s|h_{s,d}|^2$, $E_s|h_{s,i}|^2$ and $E_i|h_{i,d}|^2$ respectively and $\bar{\boldsymbol{E}} = [E_s, E_1, E_2, \ldots, E_M]^{\mathrm{T}}$ is the power vector. Note that we assume that all the noise variances are equal, i.e., $N_{s,d} = N_{s,i} = N_{i,d} = N_0$.

Let E_T be the total power and $E_T = E_s + \sum_{i=1}^{M} E_i$. Define the SNR as $SNR = E_T/N_0$. Since the term *diversity order* is defined as the negative slope of a plot of log-outage versus SNR in decibels $d_{\mathrm{out}}^{\mathrm{AF}} \triangleq \lim_{SNR\to\infty} \frac{-\log P_{\mathrm{out}}^{\mathrm{AF}}(SNR,R)}{\log(SNR)} = M+1$, the system achieves full diversity order of $(M+1)$.

3 Constraints and formulation of power allocation problem

We are interested in finding optimal power allocation solutions subject to interference constraints for primary user nodes and subject to total and individual power constraints for cognitive relays. A cognitive relay node would adjust its transmitted power so that the interference temperature limit at the corresponding primary user node is not violated and its QoS requirements are satisfied. We consider the case where only mean channel gains from cognitive relay nodes to corresponding primary user nodes are available while instantaneous channel gains for the relay-destination links and the source-destination link are available at the destination. It may be possible to estimate instantaneous channel gains among the source, relays and destination while it is difficult to estimate instantaneous channel gains from cognitive relay node to primary user node. However, by exploiting pilot signal transmitted from primary user, the cognitive relay node can estimate the mean channel gain from primary user to itself. Due to the reciprocal characteristic of the wireless channel, these mean channel gains would be equal to the mean channel gains from cognitive relay node to primary user node. We assume that the mean channel gains are averaged over short-term fading, such that only the effects of long-term shadowing and path-loss are reflected in these gains.

3.1 Interference constraints for PUs

Let $g_{i,p}$ denote the instantaneous channel gain from the ith relay to the corresponding primary user node "p" and $\overline{g}_{i,p}$ be its mean channel gain. Then, the channel gain $g_{i,p}$ can be decomposed into

$$g_{i,p} = a_{i,p} \cdot \overline{g}_{i,p}, \tag{9}$$

where $\overline{g}_{i,p}$ is the local average of $g_{i,p}$ and $a_{i,p}$ represents short-term fading with mean value normalized to one.

Also, let $\xi_i^{(p)}$ be the maximum interference limit tolerable at primary user node i. Then, the interference constraints of the ith primary user node can be written as

$$\theta_i = g_{i,p}E_i \leqslant \xi_i^{(p)}, \quad i = 1, 2, \ldots, M. \tag{10}$$

As mentioned before, the instantaneous channel gains $g_{i,p}$ may be difficult to estimate in practice. We assume that only the mean channel gain $\overline{g}_{i,p}$ can be estimated by processing the pilot signal from the corresponding primary user node. With only mean channel gains, interference constraints can be set in an *average sense* as follows:

$$\overline{\theta}_i = \overline{g}_{i,p}E_i \leqslant \beta\xi_i^{(p)} \tag{11}$$

for some $\beta < 1$. Since the instantaneous interference level $\theta_i = g_{i,p}E_i$ may exceed the tolerable limit $\xi_i^{(p)}$, and therefore, violate the absolute interference constraint, we define a constraint on the *violation probability* as follows:

$$\Pr[\theta_i > \xi_i^{(p)} | \overline{\theta}_i \leqslant \beta\xi_i^{(p)}] \leqslant \delta_i^{(p)}, \tag{12}$$

where $\delta_i^{(p)}$ denotes the maximum interference violation probability allowed for the ith primary user node.

Based on eq. (9), eq. (10) becomes $\theta_i = g_{i,p}E_i = a_{i,p} \cdot \overline{g}_{i,p}E_i$, where the short-term fading gain $\sqrt{a_{i,p}}$ is assumed to be Rayleigh distributed.

Then, the PDF of $\theta_i = g_{i,p}E_i$ can be calculated as

$$f_{\theta_i}(x) = \frac{1}{\overline{g}_{i,p}E_i}\mathrm{e}^{-x/\overline{g}_{i,p}E_i}. \tag{13}$$

Hence, the violation probability for the *average-sense* interference constraint is evaluated as

$$\begin{cases} \Pr[\theta_i > \xi_i^{(p)} | \overline{\theta}_i \leqslant \beta\xi_i^{(p)}] = \exp\left[-\dfrac{\xi_i^{(p)}}{\overline{g}_{i,p}E_i}\right], \\ \text{subject to} \quad \overline{\theta}_i \leqslant \beta\xi_i^{(p)}. \end{cases} \tag{14}$$

The parameter β should be determined *a priori* to meet the constraint on violation probability.

To observe the interaction between the interference limit $\xi_i^{(p)}$ and the achieved signal-to-interference radio (SIR) at the ith primary user node, the SIR at the ith primary user node, $\mu_i^{(p)}$, should satisfy

$$\mu_i^{(p)} = \frac{P_r}{\xi_i^{(p)}} \geqslant \gamma_{th}^{(p)}, \tag{15}$$

where P_r denotes received power from primary transmitter and $\gamma_{th}^{(p)}$ is the target SIR of primary user nodes. Note that background noise is ignored. Then the tolerable interference limit $\xi_i^{(p)}$ is obtained from eq. (15) for given P_r and $\gamma_{th}^{(p)}$.

3.2 Problem formulation

We are interested in finding optimal power allocation solutions among source and relay nodes to maximize mutual information I_{AF} subject to limits on both sum and individual transmitted power constraints for relays and subject to the tolerable interference limit for primary user nodes. This is equivalent to obtaining the optimal power allocation strategy that minimizes the system outage probability. The ith cognitive relay node has a power budget of $E_i^{\max}$ while total power available is limited to E_T. Specifically, we will solve the following optimization problem:

$$\max_{\boldsymbol{E}} \quad I_{\mathrm{AF}} = \frac{1}{M+1}\log_2\left[1+\gamma_s+\sum_{i=1}^{M}\gamma_i\right], \tag{16}$$

subject to

$$E_s + \sum_{i=1}^{M} E_i \leqslant E_T,\ E_s \leqslant E_s^{\max},\ E_i \leqslant E_i^{\max}, \tag{17}$$

$$\overline{g}_{i,p}E_i \leqslant \beta\xi_i^{(p)}, \quad i = 1, 2, \ldots, M, \tag{18}$$

where $E_i^{\max}$ is maximum transmission power for the source.

4 Solutions of optimal power allocation problem

In this section, we present a solution approach for the power allocation problem described in the previous section.

To simplify eq. (16), let $a_0 = |h_{s,d}|^2/N_0$, $a_i = |h_{s,i}|^2/N_0$ and $b_i = |h_{i,d}|^2/N_0$. Then, I_{AF} can be rewritten as

$$\begin{aligned} I_{\mathrm{AF}} &= \frac{1}{M+1}\log_2\left[1+E_s a_0+\sum_{i=1}^{M}\frac{E_s E_i a_i b_i}{E_s a_i + E_i b_i + 1}\right] \\ &= \frac{1}{M+1}\log_2\left[1+E_s\left(\sum_{i=0}^{M}a_i\right)-\sum_{i=1}^{M}\frac{E_s^2 a_i^2 + E_s a_i}{E_s a_i + E_i b_i + 1}\right], \end{aligned} \tag{19}$$

$$[\boldsymbol{\overline{E}}]_{\text{opt}} = \arg \max_{\substack{E_s+\sum_{i=1}^{M}E_i\leqslant E_T\\ E_s\leqslant E_s^{\max}, E_i\leqslant E_i^{\max}\\ \overline{g}_{i,p}E_i<\beta\xi_i^{(p)}}} \left[E_s\left(\sum_{i=0}^{M}a_i\right)-\sum_{i=1}^{M}\frac{E_s^2a_i^2+E_sa_i}{E_sa_i+E_ib_i+1}\right]. \tag{20}$$

Since $\log_2(x)$ is a monotonically increasing function of x, the equivalent modified optimization problem can be given by eq. (20)

Solving optimization problem eq. (20) in closed-form expression appears to be difficult. But if we relax the problem to one with a fixed pre-determined E_s, then the new optimization problem

$$[E_1\cdots E_M]_{\text{opt}} = \arg \min_{\substack{\sum_{i=1}^{M}E_i\leqslant E_r, 0\leqslant E_i\leqslant E_i^{\max}\\ \overline{g}_{i,p}E_i<\beta\xi_i^{(p)}}} \left[\sum_{i=1}^{M}\frac{E_s^2a_i^2+E_sa_i}{E_sa_i+E_ib_i+1}\right] \tag{21}$$

has a closed-form solution, where $E_r = E_T - E_s$ is the remaining power to be allocated to relays. The relaxed problem eq. (21) is equivalent to having the source node transmit at some reasonable power, and then optimally allocating the remaining power among the relay nodes.

Since the objective function in eq. (21) is a strictly decreasing and convex function of E_i, the optimal point must be on the boundary. This can also be easily verified by the Karush-Kuhn-Tucker (KKT) conditions [14], and thus we have the following theorem.

Theorem 1. The optimal power allocation among the cognitive relay nodes to maximize I_{AF}, given a fixed transmit power E_s for the source node, with total and individual power constraints for relays and interference constraints for primary user nodes, is

$$E_i = \left(\sqrt{\frac{E_s^2a_i^2+E_sa_i}{b_i\lambda}}-\frac{E_sa_i+1}{b_i}\right)_0^{\min\{E_i^{\max},\beta\xi_i^{(p)}/\overline{g}_{i,p}\}}, \tag{22}$$

where λ is chosen to meet the total power constraint of the cognitive relay nodes and $(x)_l^u = l(x<l) = x(l\leqslant x\leqslant u), u(x>u)$.

Proof. See Appendix 1.

After computing the optimal solutions, the destination node would then forward the resulting transmit power levels to the cognitive relay nodes. If the ith relay node is assigned zero power, it means that the corresponding time slot is idle.

5 Selection amplify-and-forward scheme

In the previous section, we analyzed the optimal power allocation among relays for the AF scheme. However, with the increase of the number of M, this will produce adverse effect on average throughput, because every relay node can only transmit in a slot with duration $1/(M+1)$ of the entire block. In the following, to overcome this shortcoming for AF transmission, we therefore introduce selection amplify-and-forward (S-AF) relaying where the transmission is divided into only two orthogonal time slots. Within the first time slot, it implements the data-sharing phase of AF. However, during the second phase, the relaying phase of S-AF contains only one slot, in which only a single relay node selected by the destination amplifies and forwards its received signal to the destination.

If the ith relay is chosen by the destination for relaying, the average mutual information between the source and the destination for S-AF scheme is given by

$$I_S = \frac{1}{2}\log_2[1+\gamma_s+\gamma_i]. \tag{23}$$

By selecting only one cooperative partner, selection cooperation avoids the problem of bandwidth expansion inherent in the repetition-based protocol (the pre-log term is $1/2$ compared to $1/(M+1)$).

The relay with the largest equivalent SNR, i.e., $\max_i \gamma_i$, is selected for the purpose of relaying. Therefore, the "best" relay is determined by not only the channel power gains of relay-destination links, but also that of source-relay links.

The destination node needs only to select and notify the best relay for S-AF, instead of computing and feeding back the power allocated to every relay node. Therefore the complexity of S-AF is lower than that of AF. Meanwhile because S-AF only repeats information once whereas AF repeats M times, S-AF actually has a higher average throughput.

For a given transmission rate R, the outage probability for S-AF scheme is $P_{\text{out}}^S = \Pr(I_S < R)$. Similarly, as P_{out}^S is complicated, it is difficult to find the solution. Rather than solve it directly, we have the following high SNR approximation:

$$P_{\text{out}}^S \sim \frac{\lambda_0 \prod_{i=1}^{M}(\lambda_{s,i} + \lambda_{i,d})}{M+1} \left(\frac{2^{2R}-1}{1/N_0} \right)^{M+1}. \tag{24}$$

We conclude that from the preceding analysis, that for S-AF relaying scheme, we also observe the full diversity order of $(M+1)$.

In the following, we will find the optimal power allocation strategies between source and selected relay to maximize I_S. Firstly, we consider the optimal power allocation between the source and single relay node.

In a single relay network, assume that the total transmission energy is E_T. The source node uses ρE_T for its transmission and the rest $(1-\rho)E_T$ is for the relay node i. Denote $A_0 = a_0 E_T$, $A_i = a_i E_T$ and $B_i = b_i E_T$, where a_0, a_i and b_i are defined in previous section. When the MRC at the destination is applied, the received SNR at the destination can be rewritten as

$$\gamma_{\text{MRC},i}^S = A_0\rho + \frac{A_i B_i \rho(1-\rho)}{A_i\rho + B_i(1-\rho) + 1}. \tag{25}$$

In order to maximize instantaneous SNR, $\gamma_{\text{MRC},i}^S$, our goal is thus to choose the optimal transmission power ratio ρ^{opt} subject to interference constraints for primary user nodes. In such scenario, the optimization problem can be formulated as

$$\begin{cases} \rho^{\text{opt}} = \arg \max\limits_{0 \leqslant \rho \leqslant 1} \left\{ A_0\rho + \frac{A_i B_i \rho(1-\rho)}{A_i\rho + B_i(1-\rho)+1} \right\}, \\ \text{subject to} \quad (1-\rho)E_T \overline{g}_{i,p} \leqslant \beta \xi_i^{(p)}. \end{cases} \tag{26}$$

The result is given in the following theorem.

Theorem 2. The optimal power ratio for the source node in a single relay node, with interference constraints for primary user nodes, is

$$\rho^{\text{opt}} = \begin{cases} 1, & D_i < 0, \\ \min\{F_i, 1\}, & D_i > 0 \ \& \ F_i > G_i, \\ \arg\max\{A_0, \gamma_{\text{MRC},i}^S(\rho = G_i)\}, & D_i > 0 \ \& \ F_i < G_i, \end{cases} \tag{27}$$

where $A_0 = \gamma_{\text{MRC},i}^S(\rho = 1)$, $D_i = A_i B_i + A_0 B_i - A_0 A_i$, $C_i = D_i A_i B_i (A_i+1)(B_i+1)$, $F_i = \frac{B_i+1}{B_i-A_i} - \frac{\sqrt{C_i}}{D_i(B_i-A_i)}$ and $G_i = 1 - \frac{\beta\xi_i^{(p)}}{E_T \overline{g}_{i,p}}$. It should be noted that A_i and B_i are generally not equal, since they are realizations of two independent exponential distributions.

Proof. See Appendix 2.

Therefore, based on Theorem 2, the optimal power allocation for S-AF, called OPA S-AF, can be listed as

1. For each relay node i, compute the optimal power ratio ρ_i^{opt} using eq. (27).
2. Compute the received SNR with optimal power ratio using relay node i as
   $$\gamma_{\text{MRC},i}^{S,\text{opt}} = A_0\rho_i^{\text{opt}} + \frac{A_i B_i \rho_i^{\text{opt}}(1-\rho_i^{\text{opt}})}{A_i\rho_i^{\text{opt}} + B_i(1-\rho_i^{\text{opt}})+1}.$$
3. Choose the relay node with the largest $\gamma_{\text{MRC},i}^{S,\text{opt}}$ for relaying.
4. Notify the selected relay and the source of their transmission power level.

Table 1 The system parameters

$\alpha = 3$	$R = 1$
$E_s = 1$	$E_r = M$
$E_i^{\max} = 2$	$\gamma_{th}^{(p)} = 3.5$
$\sigma_i^{(p)} = 0.001$	$P_r \sim \mathcal{U}(1, 2)$

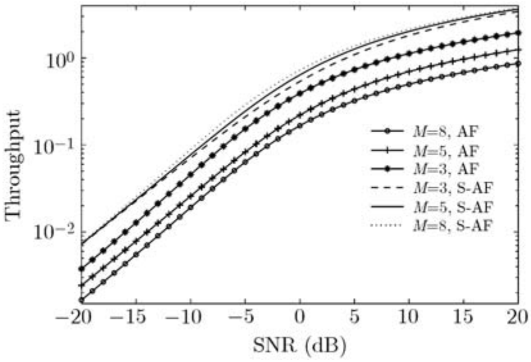

Figure 2 Throughput comparison between S-AF and AF.

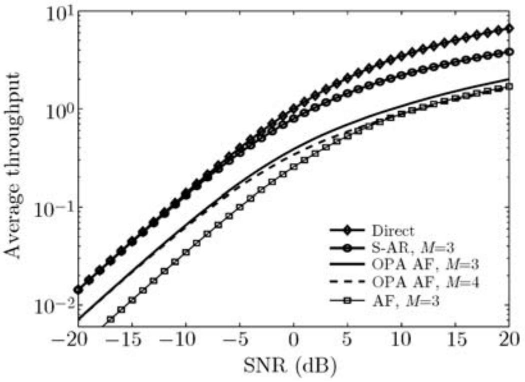

Figure 3 Throughput comparison between AF and S-AF in proposed cognitive relay network model. Cases with direct transmission and OPA AF are shown for comparison.

6 Performance evaluation

In this section, we present numerical results to support our analysis. Assume that all link lengths are normalized with respect to the distance between the source node and the destination node. We consider a cognitive relay network consisting of a source and a destination, and M cognitive relay nodes and M primary user nodes that are distributed in a 1×1 square area, according to the uniform distributed, where the source and destination nodes are located at $(-0.5, 0)$ and $(0.5, 0)$, respectively. We assume that we have $1/\lambda_{sd} = 1/d_{sd}^{\alpha}$, $1/\lambda_{sr_i} = 1/d_{sr_i}^{\alpha}$ and $1/\lambda_{r_id} = 1/d_{r_id}^{\alpha}$, where d_{AB} is the distance between nodes A and B, and α is the path-loss exponent. Throughout the numerical results, the system parameters are listed in Table 1.

In Table 1, $\mathcal{U}(a, b)$ represents uniformly distributed in $[a, b]$. Assume that the mean channel gains of the relay-primary links are uniformly distributed in $[-30, 0]$ dB. Meanwhile, $E_T = 2$ is used in the simulations of S-AF.

In order to illustrate the factor M impacts on the performance, numerical results are given for different values of M, $M = 3$, 5 and 8, as shown in Figure 2. From the figure we can see that the throughput of AF actually decreases with the increase in the number of M. This is because the more relay nodes, the lower TDMA factor of $1/(M + 1)$. However, the throughput of S-AF increases with the increase of M. Meanwhile, we find that, for a given M, the throughput of S-AF is larger than that of AF. This is consistent with the results we analyzed in section 5.

Figure 3 compares the average throughput of AF and S-AF schemes in suggested cognitive relay network. Numerical results are obtained by averaging over 1000 channel realizations in the following simulations. It is seen from the figure that direct transmission, in which the source transmits information to the destination directly without help from the relays, has greater average throughput compared to AF and S-AF schemes, but has a far poorer outage probability (diversity order of 1, not $(M + 1)$). As expected, optimal power allocation on AF (OPA AF) can improve average throughput than AF for a given M. Meanwhile, we observe that, for OPA AF scheme, when SNR is very small, the average throughput of OPA AF for different M is almost the same. This reveals that SNR is the bottleneck which limits the performance of the system when SNR is very small. With the increase of SNR, the average throughput decreases with the increase of the number M. For a specified SNR, i.e., 10 dB, the average throughput is approximately 1.1 bps for OPA AF compared with 0.9 bps for AF when $M = 3$. Besides, S-AF achieves much larger average throughput than OPA AF.

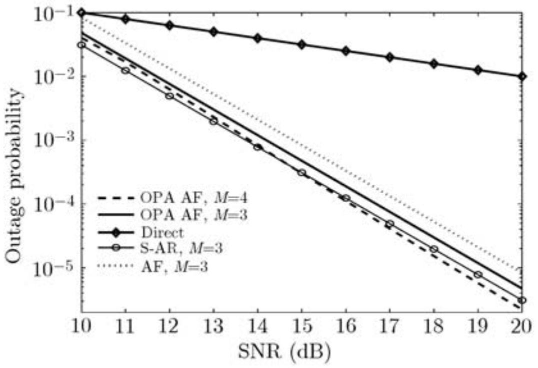

Figure 4 Outage probability comparison between AF and S-AF in suggested system model. Cases with direct transmission and OPA AF are shown for comparison.

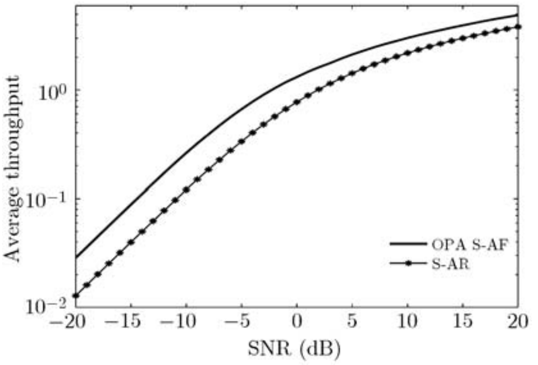

Figure 5 Average throughput comparison between S-AF and OPA S-AF, $M = 3$.

In order to carry out an outage probability comparison between AF and S-AF in suggested system model, simulations are given, as shown in Figure 4. From Figure 4, we find that the outage probability of OPA AF is less than that of AF for a given M. Meanwhile, the outage probability of OPA AF decreases with the increase of M. This means that a diversity gain of M is achieved. Additionally, the outage probability of S-AF is less than that of OPA AF for a given M. In order to carry out a comparison, a plot for direct transmission is also provided. The results indicate that better outage performance can be achieved using AF and S-AF than the direct transmission.

Figure 5 compares the average throughput of S-AF with and without OPA for the case M=3. From Figure 5, we observe that the average throughput of S-AF system is greatly improved by optimal power allocation. Particularly, the average throughput is approximately 3 bps for OPA S-AF compared with 2 bps for S-AF when SNR=10 dB. This verifies that power allocation has a significant impact on the S-AF scheme.

7 Conclusions

In this paper, the optimal power allocation strategies to minimize the system outage probability under both the transmitted power constraints of cognitive relay nodes and interference power constraints of primary user nodes in proposed cognitive relay network model have been studied. Firstly, we have derived optimal power allocation among the relay nodes to minimize the outage probability when AF cooperative protocol is used. Then, a selection AF with optimal power allocation has also been analyzed. The numerical results have been presented to validate the proposed power allocation strategies. The results indicated that S-AF has better outage probability and average throughput than the AF with and without OPA. We also found that S-AF with OPA achieves a higher throughput, and hence lower outage probability than AF with OPA.

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Appendix

1 Optimal power allocation on AF (OPA AF)

It can be easily shown that the cost function in eq. (21) is convex in $\{E_1, \ldots, E_M\}$ over the convex feasible set defined by the linear power constraints. Firstly, without individual power constraints for relays and interference constraints for primary user nodes, the Lagrangian of this optimization problem can be written as

$$L(E_i, \lambda) = \sum_{i=1}^{M} \frac{E_s^2 a_i^2 + E_s a_i}{E_s a_i + E_i b_i + 1} + \lambda\left(\sum_{i=1}^{M} E_i - E_r\right), \tag{A1}$$

where λ is the Lagrange multiplier.

We take the first derivative of $L(E_i, \lambda)$ with respect to E_i and obtain

$$\frac{dL(E_i, \lambda)}{dE_i} = \frac{-b_i(E_s^2 a_i^2 + E_s a_i)}{(E_s a_i + E_i b_i + 1)^2} + \lambda. \tag{A2}$$

Setting eq. (A2) at 0, we have $E_i = (\sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{b_i \lambda}} - \frac{E_s a_i + 1}{b_i})$.

Since the power allocation among relays can only be non-negative, we can use the KKT conditions to verify that optimal power allocation is

$$E_i = \left(\sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{b_i \lambda}} - \frac{E_s a_i + 1}{b_i}\right)^+, \tag{A3}$$

where $(a)^+ = \max(a, 0)$.

Note that the ith relay node is allowed to transmit if and only if $\sqrt{(E_s^2 a_i^2 + E_s a_i)/b_i \lambda} > (E_s a_i + 1)/b_i$.

Now consider the individual transmitted power constraints for relays and interference constraints for primary user nodes. The maximum power available at the ith relay is

$$\min\{E_i^{\max}, \beta \xi_i^{(p)} / \overline{g}_{i,p}\}. \tag{A4}$$

By this means, it can be guaranteed that not only each PU experiences an interference power of at most $\beta \xi_i^{(p)}$, but also the maximum transmitted power available at each relay does not exceed $E_i^{\max}$.

Since the objective function in eq. (21) is a monotonically decreasing and convex function of E_i, the optimal point must be on the boundary. Therefore, the optimal power allocation is

$$E_i = \left(\sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{b_i \lambda}} - \frac{E_s a_i + 1}{b_i}\right)_0^{\min\{E_i^{\max}, \beta\xi_i^{(p)}/\overline{g}_{i,p}\}}. \tag{A5}$$

2 Optimal power allocation on S-AF (OPA S-AF)

First ignoring the interference constraints for primary user nodes, we take the first derivative of the objective function in eq. (26) with respect to ρ and set it to zero. Then we obtain

$$D_i(B_i - A_i)\rho^2 - 2D_i(B_i + 1)\rho + (A_0 B_i + A_i B_i + A_0)(B_i + 1) = 0, \tag{A6}$$

where $D_i = A_0 B_i + A_i B_i - A_0 A_i$.

The discriminant of above equation is

$$\Delta = 4 D_i A_i B_i (B_i + 1)(A_i + 1). \tag{A7}$$

Therefore, the above equation has no real number solution when $D_i < 0$. In this case, a dollar square distance in eq. (A6) never changes sign; therefore it is monotone function. Due to the fact that

$$\gamma^S_{\mathrm{MRC},i}(\rho = 1) = A_0 > 0 = \gamma^S_{\mathrm{MRC},i}(\rho = 0). \tag{A8}$$

Eq. (25) is a monotonically increasing function. Therefore, ρ^{opt} is equal to 1 in such scenario.

If $D_i > 0$, the equation has two real solutions. It is easy to show that one solution is always outside the regime (0, 1). Ignoring this solution, we get

$$\rho^{\mathrm{opt}} = \frac{B_i + 1}{B_i - A_i} - \frac{\sqrt{C_i}}{D_i(B_i - A_i)}, \tag{A9}$$

where $C_i = D_i A_i B_i (B_i + 1)(A_i + 1)$.

If ρ^{opt} is outside (0, 1), KKT condition guarantees that the optimal solution lies on the boundary. In this case $\rho^{\mathrm{opt}} = 1$. Now consider the interference constraint for primary user nodes. In order to avoid harmful interference to primary user, ρ^{opt} also satisfies $\rho^{\mathrm{opt}} \geqslant 1 - \frac{\beta\xi_i^{(p)}}{E_T \overline{g}_{i,p}}$.

Let $F_i = \frac{B_i+1}{B_i-A_i} - \frac{\sqrt{C_i}}{D_i(B_i-A_i)}$ and $G_i = 1 - \frac{\beta\xi_i^{(p)}}{E_T \overline{g}_{i,p}}$.

Under $D_i > 0$, if $F_i > G_i$, according to the KKT condition, we have

$$\rho^{\mathrm{opt}} = \min\{F_i, 1\}. \tag{A10}$$

If $F_i < G_i$, since the objective function of eq. (26) is a monotone function in the regime $[G_i, 1]$, the optimal point must be on the boundary. Therefore, in this case we have

$$\rho^{\mathrm{opt}} = \arg\max\{\gamma^S_{\mathrm{MRC},i}(\rho = 1), \gamma^S_{\mathrm{MRC},i}(\rho = G_i)\}. \tag{A11}$$

Theorem 2 is proved.