

An efficient ranging method based on Chinese remainder theorem for RIPS measurement

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Abstract The radio interferometric positioning system (RIPS) measures the phase difference of the interference signal to provide high accuracy and at the same time maintain simple hardware configuration for wireless sensor networks. However, it suffers from phase ambiguity problem because of the periodicity of phase, which makes it hard to determine the actual distance difference only from a single phase measurement. To solve the problem, RIPS makes multiple measurements at different frequencies so as to determine the distance difference from multiple phases. However, this is a computationally intensive searching process and not suitable for energy-constrained wireless sensor nodes. In this paper, we introduce the Chinese remainder theorem (CRT) to RIPS to solve the phase ambiguity problem. Meanwhile, we utilize some properties of the coefficients in the CRT algorithm to avoid the over-sensitivity of the traditional CRT, which increases the robustness of the algorithm. We apply this robust CRT algorithm to the ranging process which calculates the distance difference directly from a closed-form equation and therefore reduces the response time and the energy consumption of the ranging procedure.

Keywords ranging, phase ambiguity, Chinese remainder theorem, robustness, energy efficiency, wireless sensor networks

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1 Introduction

Position information of nodes is very important to wireless sensor networks (WSN). In many occasions, the sensor information will have little value if it is not combined with the position information. In addition, location information is the prerequisite of many routine methods in WSN. By far, the most widely discussed localization techniques in the literature can be classified into two categories: range-free and range-based [1]. Range-free contains those positioning methods which do not need the actual distance between nodes. They are usually based on connectivity or coverage, etc. The range-based approach, which measures the relative distance between nodes directly, usually provides higher localization accuracy than the range-free approach. These distance measurements required by the range-based approach can be implemented by using the received signal strength indicator (RSSI), time of arrival (TOA), time difference of arrival (TDOA) or angle of arrival (AOA). However, these distance measurement methods will face many practical challenges when applied to the resource-limited wireless sensor nodes. TDOA

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usually requires extra acoustic actuators/detectors such as ultrasonic sensors and has limited range [2, 3]; AOA requires an antenna array on the node, which is not feasible for size-limited and energy-constrained wireless sensor nodes. TOA usually transmits wide-band signal to provide high accuracy synchronization clock, which consumes more power, besides, it is usually implemented on UWB system, so the range is also very limited; ranging techniques based on RSSI do not need additional devices but only result in low accuracy.

The radio interferometric positioning system (RIPS [4]) and its derivative system [5–9] achieve high localization accuracy and a long range while maintaining the simplicity of the hardware. The main advantage of the system is that it does not require any additional sensors other than the radio transceivers used in wireless communication but still provides an average localization error as small as 3 cm and a range of 160 m [4]. The system utilizes the frequency interference, by measuring the relative phase offset of the interference signal at multiple frequencies, to obtain a combination of distances among four nodes rather than the direct distance between two nodes. By measuring different combinations of the distances and utilizing genetic algorithm, it is possible to find out the location of the nodes in 3D [4]. Furthermore, if three of the four nodes involved in a RIPS measurement are anchor nodes, which means their locations are already known, we can use hyperbolic positioning method to calculate the position of the fourth node directly [10].

The RIPS locates the nodes with the phase measurement, which actually measures the distance by the wave length of the radio frequency (RF) signal. The wavelength of the RF signal is very small, so that the system yields high accuracy. Similar approaches have been applied to the global positioning system (GPS) for high resolution positioning, such as the real time kinematic (RTK) technique in differential GPS [11, 12]. However, although phase measurement provides high accuracy, it inevitably introduces the problem of integer ambiguity. To be more specific, the phase measured by the nodes is periodic, so the measured phase can only be a certain value within the principal value interval of 2π . In other words, no matter how far the actual distance is, the distance directly converted from the measured phase is always the residue distance which is the actual distance modulo wavelength, and the integer wavelength information is ignored.

In order to solve the phase ambiguity problem, RIPS measures the phase offset at different frequencies, namely measures the same unknown distance with different wavelengths and gets the corresponding residual distances. By searching for the possible integer combinations of wavelengths, it finally finds the distance with minimum mean square error. If this computationally intensive searching algorithm is implemented on the energy-restrained wireless sensor nodes, it certainly becomes a heavy burden of the nodes, and the response time of the localization process will be increased. In short, how to obtain the distance from phase quickly and efficiently becomes a key problem in localization system based on phase measurement.

In fact, the Chinese remainder theorem (CRT) is an effective tool to solve the problem. It is an analytic procedure for calculating the unambiguous integer dividend from the remainders which are produced from integer division by several prime numbers. By analogy with the phase ambiguity problem here, it is not hard to find out that the dividend in the theorem corresponds to the distance we need to estimate; the prime divisors correspond to different wavelengths; the remainders correspond to the fraction remainder parts of the distance measured by different wavelengths. The problem of applying CRT is that it is too sensitive to noise. Even a small amount of noise in phase measurement can result in an enormous error. Unfortunately, phase measurement is not perfect and is inevitably affected by noise. For this reason, some papers propose several robust CRT algorithms which reduce the sensitivity to noise and apply them to the velocity and range estimation in pulse Doppler radar and continuous wave radar system [13, 14]. Inspired by those related research work, we present an efficient ranging method based on robust CRT, which avoids the time and energy-consuming search process in RIPS and calculates the distance difference in a closed form equation.

The rest of the paper is arranged as follows. In the next section we first set up the model of the problem, and present a brief introduction to the CRT algorithm meanwhile show its over-sensitivity to noise. Then we extend the condition of the traditional CRT to non-prime divisors, and present our robust

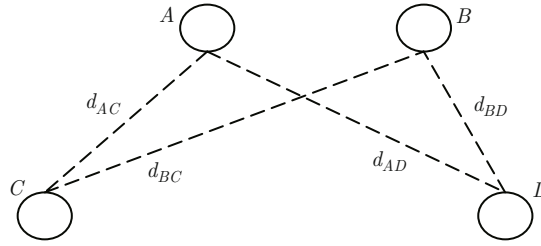


Figure 1 Without directly measuring the distance, RIPS obtains the distance difference $d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}$ from multiple phase measurements.

phase to distance difference estimation method. Simulation results are demonstrated in section 3, followed by the conclusion in section 4.

2 Ranging method based on Chinese remainder theorem

2.1 Problem description

Four nodes are involved in a single RIPS measurement, as illustrated in Figure 1. Two nodes, say node A and node B , transmit sine wave at slightly different frequencies to obtain an interference signal which is received by the other two nodes (node C and node D). The phase difference between the two received signals on each node is a function of the distances between the four nodes [4]. Let θ be this phase difference of the two received signals. Then we have

$$\theta = 2\pi \frac{d_{AD} - d_{BD} + d_{BC} - d_{AC}}{\lambda_{\text{carrier}}} \pmod{2\pi}, \tag{1}$$

where λ_{carrier} is the wavelength of carrier signal.

After figuring out the combination of distances such as d_{ABCD} , etc., RIPS utilizes genetic algorithm to determine the position of each node. In addition, if nodes A, B , and C are anchor nodes whose positions are known, we can further calculate distance $d_{AD} - d_{BD}$, and $d_{CD} - d_{AD}$ with which two hyperbolas are determined and the position of node D is the intersection point of the two hyperbolas.

Obviously, it is not possible to determine the actual distance difference d_{ABCD} from the measured phase difference merely by a single RIPS measurement, because the phase is always modulo 2π . The process of converting phase difference into distance difference is the focus of this paper.

In order to solve phase ambiguity, we can measure the same distance with multiple frequencies so that there are more constrains to the equation. To be more specific, if we measure the same distance difference d by k different wavelengths $\lambda_i (i = 1, \dots, k)$ (i.e. the frequency of the signal is $f_i = c/\lambda_i$), we can get the corresponding phase $\theta_i \in [0, 2\pi]$ which can be further converted to the fraction part of the distance $\gamma_i = \lambda_i \cdot \theta_i / (2\pi)$. This process can be expressed as

$$d = n_i \lambda_i + \gamma_i, \tag{2}$$

where n_i is the unknown integer part of the i th wavelength. Ranging is the process of estimating d from a set of γ_i .

The simplest but the most inefficient approach is to search every possible combination of the k integer n_i . Each combination consists of k distance values. When the k distances are consistent with each other or their mean square error is small enough, the actual distance is obtained as demonstrated in Figure 2. However, this procedure is computationally inefficient. To be more specific, assume that the maximum distance is d_{max} , and then the integer part n_i of wavelength λ_i has $\lfloor d_{\text{max}}/\lambda_i \rfloor + 1$ possible values (including zero), where $\lfloor \cdot \rfloor$ denotes round down to the nearest integer. There are $N_{\text{comb}} = \prod_{i=1}^k (\lfloor d_{\text{max}}/\lambda_i \rfloor + 1)$ combinations for k wavelengths, and each combination consists of k multiplications, so the total number of multiplications is

$$N_{\text{mul}} = k N_{\text{comb}}. \tag{3}$$

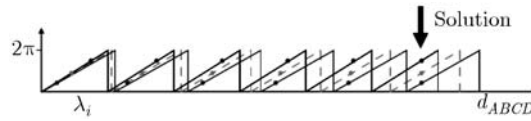


Figure 2 Distance estimation from phase measurement on multiple wavelengths.

For example, if the searching range is 100 m and 3 frequencies are used in 2.4 GHz band, around 1.5×10^9 multiplications are needed. The searching process is obviously a heavy burden to the wireless sensor nodes which have very limited computational capacity and energy.

2.2 A brief introduction to Chinese remainder theorem (CRT)

CRT is an analytical process of calculating dividend from remainders. Assume the unknown integer dividend is x . Dividing it by k prime numbers $P_i > 1$ ($i = 1, 2, \dots, k$), we get the k corresponding remainders m_i :

$$x \equiv m_i \pmod{P_i}, i = 1, \dots, k. \quad (4)$$

If $x < M = \prod_{i=1}^k P_i$, x can be determined uniquely with the following equation:

$$x \equiv \left(\sum_i^k \delta_i m_i \right) \pmod{M}, \quad (5)$$

where

$$\delta_i = Q_i q_i, \quad (6)$$

$$Q_i = M/P_i \quad (7)$$

and q_i is the modular inverse of Q_i , i.e. $Q_i q_i \equiv 1 \pmod{P_i}$. Modular inverse can be calculated by extended Euclidean algorithm or other improved algorithms [15].

The coefficient δ_i in (5) has the following properties:

Property 1.

$$\sum_{i=1}^k \delta_i \equiv 1 \pmod{M}. \quad (8)$$

Proof. For any given $P_i, i = 1, \dots, k$, let $m_i = 1$. It is easy to verify that $x = 1$ is one solution to the problem in (4). According to CRT, $x = 1$ is the only solution in $x < M = \prod_{i=1}^k P_i$, and we also have

$$\sum_{i=1}^k \delta_i m_i = \sum_{i=1}^k \delta_i \equiv x = 1 \pmod{M}.$$

Usually, the value of δ_i is far larger than 1, so the sensitivity of (5) is very high. However, Property 1 tells us that the sum of δ_i modulo M is very small. According to Property 1, it is easy to get the following property.

Property 2. If each value of m_i is constant, namely $m_i = m$, then

$$x \equiv \sum_{i=1}^k \delta_i m \pmod{M} \equiv m \pmod{M}. \quad (9)$$

Now we discuss the sensitivity to noise of CRT here. If there is some error in measurement, say the error of m_i is Δm_i , the final estimation error of x is

$$\Delta x \equiv \sum_{i=1}^k \delta_i \Delta m_i \pmod{M}, \quad (10)$$

δ_i is so large that even a small number of errors in phase measurement will be amplified greatly by δ_i , resulting in a very large error of distance estimation. However, there are exceptions: if the errors on each wavelength are the same, $\Delta m_i = \Delta m$, by Property 2, we have

$$\Delta x \equiv \sum_{i=1}^k \delta_i \Delta m \pmod{M} \equiv \Delta m \pmod{M}. \tag{11}$$

Thus the sensitivity to noise is decreased significantly.

We can demonstrate the problem of applying traditional CRT to the issue of converting phase into distance by the following example. First of all, since the theorem is discussed in the field of integer, but the phase or distance is a real number, we have to quantize the measured phase to integer. Assuming the quantization step is $u = 0.1$ mm, we choose $P_1 = 1223, P_2 = 1229, P_3 = 1231$, (the corresponding frequencies are 2.4530, 2.4410, 2.4370 GHz, respectively) to perform phase measurement. If we measure the distance of $d = 51$ m and assume the phase measurement is perfect, then the quantized fraction parts of the distance difference are $m_i = \{9, 1194, 366\}$. Utilizing (5), we can directly calculate $x = 510000$, and the estimated distance difference $d = m \cdot u = 51$ m. However, the practical challenge in CRT is that the phase measurement is noisy. If we introduce a small perturbation to the above noiseless phase measurement, say only changing m_2 to 1195 which is equivalent to $360/P_2 = 0.29$ degrees in phase error, and utilize (5) again, then we get an estimation of $d \approx 77$ km, which is far away from the actual distance.

Noise in phase is inevitable in practical measurement. In order to apply CRT to the noisy measurement, we have to reduce its sensitivity to noise. According to Property 2, if the three errors in phase measurement are the same, then the theorem is not sensitive to noise. For example, when $\Delta m_i = \Delta m = 1$, i.e. $m_i = \{10, 1195, 367\}$, the final estimation of the distance is $\hat{d} = 51.0001$ m.

The following question is how to utilize these properties of the coefficients to avoid the over-sensitivity of traditional CRT. For this purpose, we first extend the condition of traditional CRT to non-prime divisors and then make the error on every wavelength have the same value by exploiting some of the properties of the extended CRT.

2.3 Extended CRT

If the divisors L_i are not relative-prime numbers, suppose they have the greatest common divisor $C \geq 1$. Then L_i can be expressed as $L_i = CP_i, (P_i, P_j) = 1, i \neq j$, where (\cdot) denotes the greatest common divisor. $\{L_i\}$ have the least common multiple of $N = CM = C \prod_{i=1}^k P_i$. The extended CRT on non-relative-prime divisors condition is stated as follows:

Given $x \equiv m_i \pmod{L_i}, i = 1, 2, \dots, k, x < N, x$ has a unique solution:

$$x \equiv Cx_0 + r \pmod{N}, \tag{12}$$

where

$$x_0 \equiv \sum_{i=1}^k \delta_i b_i \pmod{M}, \tag{13}$$

$$b_i = \lfloor m_i / C \rfloor. \tag{14}$$

$\lfloor \cdot \rfloor$ denotes rounding down to the nearest integer. And r is the common remainder of m_i modulo C [14].

$$r = m_i - b_i \cdot C. \tag{15}$$

When examining the extended CRT, it is not hard to find out that since the coefficients δ_i of b_i are large, the theorem has a high sensitivity to the error of b_i but a low sensitivity to that of r . If each b_i has the same error, utilizing Property 2 we can reduce the sensitivity to error. Therefore, the problem is further transformed into the one of how to make the errors on each b_i equal.

According to (15), we have

$$b_i = \frac{m_i - m_1}{C} + b_1. \tag{16}$$

Although this is obtained under the ideal noiseless condition, it provides another approach of calculating b_i from b_1 . Actual measurement result is inevitably contaminated by noise. Assuming $m_i = m_{i0} + \Delta m_i$, where m_{i0} is the ideal value and Δm_i is the error, (16) is approximate to

$$b_i \approx \frac{(m_{i0} + \Delta m_i) - (m_{10} + \Delta m_1)}{C} + b_1 = \frac{m_{i0} - m_{10}}{C} + b_1 + \frac{\Delta m_i - \Delta m_1}{C}, \quad (17)$$

where b_1 is calculated from (14), $b_1 = \lfloor m_1/C \rfloor = b_{10} + \Delta b_1$.

If the term $\frac{\Delta m_i - \Delta m_1}{C}$ does not introduce extra error, then b_i and b_1 will have the same error. Besides, the noise makes b_i not be integers any more in (17), while the CRT persists to perform in the field of integer. A simple solution is to round b_i to the nearest integer. That is,

$$b_i \approx \text{round}\left(\frac{m_i - m_1}{C}\right) + b_1 = \frac{m_{i0} - m_{10}}{C} + b_1 + \text{round}\left(\frac{\Delta m_i - \Delta m_1}{C}\right). \quad (18)$$

When the following condition is satisfied:

$$\left| \frac{\Delta m_i - \Delta m_1}{C} \right| < \frac{1}{2}, \quad (19)$$

we have $\text{round}\left(\frac{\Delta m_i - \Delta m_1}{C}\right) = 0$ such that

$$b_i = \frac{m_{i0} - m_{10}}{C} + b_1 = b_{i0} + \Delta b_1. \quad (20)$$

Eq. (20) shows that every b_i has the same error as b_1 . According to Property 2, if the condition of (19) is satisfied, the final error on x_0 will be $\Delta b_1 \pmod{M}$, rather than $\sum_{i=1}^k \delta_i \Delta b_i \pmod{M}$.

As for the remainder part r in (12), because the measured value m_i is contaminated by noise, (15) no longer holds. In other words, the remainder of each m_i modulo C is not the same. We average the remainders of m_i modulo C to reduce the impact of noise, that is,

$$r = \sum_{i=1}^k r_i/k, \quad (21)$$

where

$$r_i = m_i - b_i C. \quad (22)$$

Substituting m_i, b_i into (21), we obtain

$$r = \frac{\sum_{i=1}^k ((m_{i0} + \Delta m_i) - (b_{i0} + \Delta b_i)C)}{k} = r_0 + \overline{\Delta m} + \overline{\Delta b}C, \quad (23)$$

where $r_0 = m_{i0} - b_{i0}C$, $i = 1, \dots, k$ is the common remainder of m_{i0} modulo C , $\overline{\Delta m} = \sum_{i=1}^k \Delta m_i/k$, and $\overline{\Delta b} = \sum_{i=1}^k \Delta b_i/k$. Usually $C \ll \delta_i$, so the error in r contributes little to the final error.

2.4 Estimation of the distance difference

To sum up, the proposed distance difference estimation procedure is stated as follows:

Known conditions: the k noisy phases ϕ_i which are obtained from the measurement of an unknown distance difference d by k different wavelengths $\lambda_i (i = 1, 2, \dots, k)$. The measured phase ϕ_i can be converted into the corresponding fraction distance $\gamma_i = \frac{\phi_i}{2\pi} \lambda_i$.

Solving distance difference d .

Step 1: Quantize real distance value to integer number. Let u be quantization step. Then the integer corresponding to the fraction distance is $m_i = \text{round}(\gamma_i/u)$ while the integer corresponding to the wavelength is $L_i = \text{round}(\lambda_i/u)$. Let $(P_i, P_j) = 1, i \neq j, C \geq 1$, so that the maximum unambiguous period is $N = CM = C \prod_{i=1}^k P_i$, meaning that the unknown distance difference d can be uniquely determined if d is defined within a certain unambiguous period by a priori information. For example, the solution of d is unique if $d < Nu$.

Step 2: Utilize (14) to obtain b_1 , and then calculate the rest of $b_i (i = 2, 3, \dots, k)$ from (18).

Step 3: Utilize (13) to calculate x_0 . Notice that the modular inverse in δ_i is only related to P_i , so it can be determined in advance. Consequently, we can make a look-up table of the coefficients δ_i according to the pre-determined wavelength to reduce the computational burden of the processor and increase real-time performance of the system.

Step 4: Calculate r_i and r from (22) and (21), respectively.

Step 5: Substitute C, x_0, r into (12) to obtain x . The estimation of d is

$$\hat{d} = x \cdot u. \quad (24)$$

For example, we choose three wavelengths which are $\lambda_i = \{0.1150, 0.1200, 0.1250\}$ m respectively, and the quantization step is 0.1 mm. Then we have $L_i = \{1150, 1200, 1250\}$ and the maximum unambiguous distance is 69 m. Assume the unknown distance difference is 50 m. Then the ideal remainder converted from the measured phase would be $m_{i0} = \{900, 800, 0\}$. Suppose the errors added to the remainders are $\Delta m_i = \{10, -12, 8\}$, which are equivalent to errors of $\{3.1^\circ, -3.8^\circ, 2.3^\circ\}$ degrees in phase measurement. So we have the noisy remainders $m_i = \{910, 788, 8\}$. The final estimation result by the proposed method is $\hat{d} = 50.0002$ m with an error of only $\hat{d} - d = 0.0002$ m.

Compared with the searching approach mentioned in subsection 2.1, the proposed ranging method needs only $3k$ multiplications/divisions for each estimation (for each wavelength $\lambda_i, i = 1, \dots, k$, the calculation of b_i, x_0 and r_i needs one multiplication respectively). So it greatly simplifies the computational complexity of ranging process.

2.5 Error analysis

From the above discussion, we know that if all b_i have the same error value, the final estimation result will have a normal error, which means it does not suffer from over-sensitivity. Here we analyze the possibility of normal error. The possibility of having the same error value among b_i and b_1 is $P\{|\frac{\Delta m_i - \Delta m_1}{C}| < \frac{1}{2}\} = p$. Suppose Δm_i is i.i.d. and the possibility of normal error of the final estimation is $P_c = p^{k-1}$, where k is the number of wavelengths used in measurement. We assume $\Delta m_i \sim N(0, \sigma^2)$, and then we have $(\Delta m_i - \Delta m_1) \sim N(0, 2\sigma^2)$. The possibility of the same error among b_i is

$$p = \frac{1}{2\sigma\sqrt{\pi}} \int_{-C/2}^{C/2} \exp\left(-\frac{x^2}{4\sigma^2}\right) dx = 2\Phi\left(\frac{C}{2\sqrt{2}\sigma}\right) - 1, \quad (25)$$

where $\Phi(x)$ is the distribution function of the standard normal distribution. p increases with C/σ . If $k = 10, C = 50, \sigma = 6, p = 0.9968$, the possibility of normal error is $P_c = 0.9716$.

3 Simulation experiments

3.1 The distribution of estimation error

The following simulation demonstrates the distribution of estimation error with Gaussian noise. We choose wavelengths to be $\lambda_i = \{0.1150, 0.1200, 0.1250\}$ m, and quantization step to be 0.1 mm, so that $C=50, P_i=\{23, 24, 25\}$, and the maximum unambiguous range is 69 m. The unknown distance difference d is uniformly distributed in $[0, 69]$ m. Assuming Δm_i obeys normal distribution $N(0, \sigma^2)$, we have normal estimation error if condition (19) is satisfied. For example, we choose $\sigma=10$, and perform 100000 trials of simulations. We obtain the histogram of the estimation error $\Delta x = \hat{x} - x$, as shown in Figure 3.

If condition (19) is not satisfied, over-sensitive error occurs, as the side lobes shown in Figure 3(a). If Δm_i is constrained to satisfy (19), we have the distribution of error shown in Figure 3(b). The error has zero-mean value, and the estimation is unbiased. The variance of error is affected by σ , so we will discuss the influence of the noise in phase measurement.

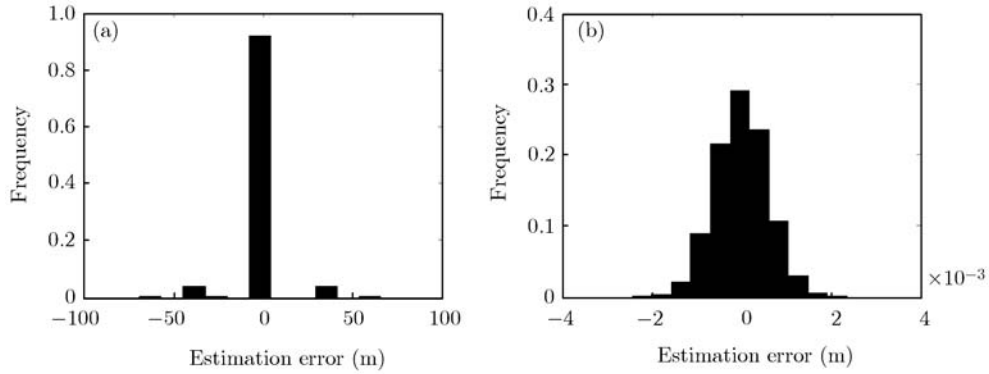


Figure 3 Statistical histogram of the estimation error Δx . (a) Over-sensitive error occurs if Δm_i is too large; (b) Δx has zero mean and a small variance if Δm_i satisfies condition (19).

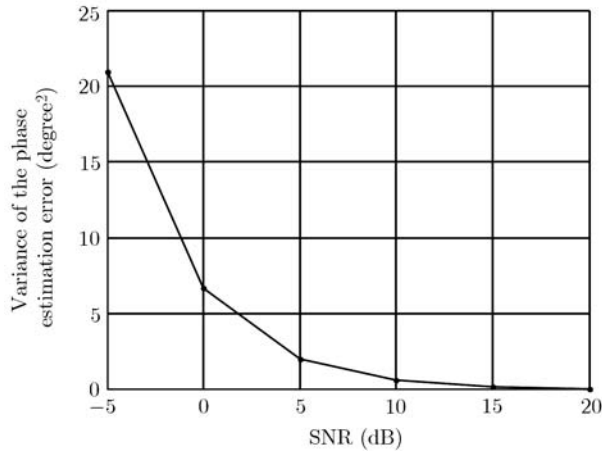


Figure 4 Phase estimation error vs. SNR.

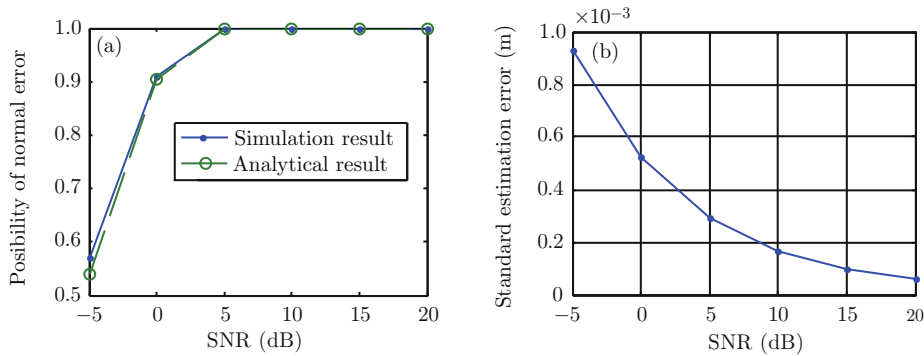


Figure 5 The possibility of normal error increases with SNR, while the standard estimation error decreases with SNR. (a) Possibility of normal error; (b) standard estimation error.

3.2 The possibility of normal estimation error

There are many methods to estimate phase. One classic way is to perform correlation between two co-frequency complex signals, which is unbiased. The phase estimation accuracy increases with the signal to noise ratio (SNR). In the simulation, we assume that the frequency of signal is 500 Hz, its duration is 10 ms, sampling rate is 50 KHz, and the phase difference between two signals is uniformly distributed in $[0, 2\pi]$. Figure 4 demonstrates the relation between variance of the phase estimation error ϕ_e^2 and SNR.

We put these variances on different SNR into the proposed ranging algorithm. The standard variance σ in the simulation is determined by the following equation:

$$\sigma = \sqrt{\phi_e^2 L_{\max} / 360}, \tag{26}$$

where $L_{\max} = \max(L_1, L_2, \dots, L_k)$.

We calculate the possibility of normal error under different SNR, and the standard estimation error when condition (19) is satisfied. They are shown in Figure 5(a) and (b), respectively.

If SNR is low, the possibility of normal error is small. The possibility of normal error increases with SNR, and so does the accuracy. Figure 5(a) verifies the correctness of (25). Meanwhile, the standard estimation error decreases with SNR when the error is normal, as shown in Figure 5(b).

4 Conclusions

We propose an efficient ranging method for RIPS based on CRT in this paper. Compared with the methods used in RIPS, it reduces the response time and energy consumption of the ranging procedure by closed-form calculation rather than the searching procedure originally proposed in RIPS. Simulation experiments show its validity. Under the condition of frequency synchronization between transmitter and receiver, SNR of 10 dB, the ranging error is submillimeter using only three frequencies.

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