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Design of robust output-feedback repetitive controller for class of linear systems with uncertainties

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Abstract Repetitive control, which adds a human-like learning capability to a control system, is widely used in many fields. This paper deals with the problem of designing a robust repetitive-control system based on output feedback for a class of plants with time-varying structured uncertainties. A continuous-discrete two-dimensional hybrid model is established that accurately describes the features of repetitive control so as to enable independent adjustment of the control and learning actions. A sufficient condition for the robust stability of the repetitive-control system is given in terms of a linear matrix inequality. The condition is then used to obtain the parameters of the repetitive controller. Finally, a numerical example demonstrates the effectiveness of the method.

Keywords learning, output feedback, repetitive control, robust control, linear matrix inequality

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1 Introduction

People often master a skill through repetition. By repeating the same action, a person gradually comes to understand the essential points, finally achieving great efficiency and precision. This is a process of learning and gradual progress. An investigation of the process reveals two main characteristics: 1) the same action is performed; and 2) the action currently being performed is based on the action performed in the previous repetition.

Inoue et al. [1] devised a new control strategy called repetitive control that adds a human-like learning capability to a control system. A repetitive-control system is different from other types of control systems in that it possesses a self-learning capability. For example, Inoue et al. [2] designed a control system for supplying power to the magnet of a proton synchrotron that tracks a given periodic reference input, namely the excitation current. After self-learning for 16 periods, the relative tracking precision reached 10^{-4}. This high precision was unobtainable by any other control method at that time. So the theory of repetitive control immediately received a great deal of attention; and it is now widely used in many fields from aerospace to public welfare systems.

From the standpoint of control theory, the self-learning mechanism of a control system involves embedding an internal model of a period signal in a repetitive controller. This theoretically guarantees

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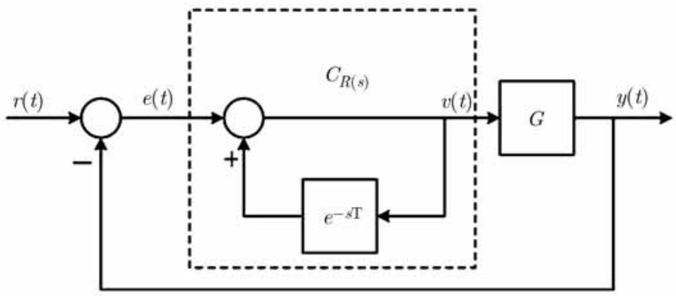

Figure 1 Configuration of basic repetitive control system.

gradual improvement, which is a chief characteristic of the human learning process, and finally provides precise tracking for any periodic reference input. Figure 1 shows the configuration of a repetitive-control system.

In the figure, $r(t)$ is a periodic reference input with a period of T, $G(s)$ is the compensated plant, and s is a Laplace operator. The part enclosed by the dotted line is the repetitive controller; it contains a pure delay with a positive-feedback loop. A repetitive-control system carries out *learning* in the following way: the control input, $v(t-T)$, of the last period is added to the control input, $v(t)$, of the present period by means of a pure-delay positive-feedback line so as to regulate the current control input. This allows the system to eliminate any tracking error and gradually provide very precise control.

The repetitive-control system in Figure 1 is a neutral-type delay system with an infinite number of poles. That makes the stability difficult to analyze and a controller hard to design. As pointed out in [3], a repetitive-control system can be stabilized only when the relative degree of the plant is zero. To stabilize a plant when the relative degree is not zero, the controller must be modified by the insertion of a low-pass filter, $q(s)$, into the time-delay feedback line. This reorganization of the system makes it a retarded-type delay system, and the stabilizing condition is much laxer than for a repetitive-control system. However, the laxness comes at the cost of tracking precision at high frequencies; that is, there is a trade-off between stability and steady-state tracking error. Since a repetitive-control system for a plant with a relative degree of zero is very difficult to stabilize and exhibits limited control performance, discussion of the design of such systems is theoretically significant.

A close examination of repetitive control shows that it actually involves two independent types of actions: continuous control within each repetition period and discrete learning between periods. Since, from the standpoint of system design, it is difficult to stabilize a repetitive-control system, all design methods developed so far focus mainly on stability; that is, they do not accurately describe what actually happens or thoroughly investigate the essence of the control and learning actions; they only consider the overall results in the time domain. As a result, they impose not only very strict requirements on the plant, but also a limit on how much control performance can be improved [4, 5].

A design method based on a two-dimensional (2D) continuous-discrete hybrid model of repetitive control was described in [6]. It employs 2D system theory [7, 8]; and unlike other methods, it enables independent adjustment of the control and learning actions. However, the whole state of the plant is needed for the design of the controller, which unfortunately is unavailable in many practical applications. A design method that employs only the output of a plant is more practical.

It is difficult to design a static output-feedback controller by conventional repetitive-control design methods. This paper addresses the problem of designing a robust repetitive-control system based on static output feedback for a class of linear systems with a relative degree of zero and with time-varying structured uncertainties. First, a robust repetitive-control system configuration based on output feedback is presented. With this configuration in mind, a 2D continuous-discrete hybrid model is established that reflects the characteristics of repetitive control. Next, a sufficient condition for robust stability in the form of a linear matrix inequality (LMI) is derived by combining a 2D Lyapunov functional with the structural singular value decomposition of the output matrix. The control gains are easily computed with the Matlab toolbox. Finally, a numerical example demonstrates the validity of the method.

Throughout this paper, $\mathbb{R}^n$ denotes n-dimensioned Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; I means an identity matrix with appropriate dimension; $X > 0$ (< 0) indicates that the matrix X is positive (negative) definite; and

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}.$$

2 Problem description

This paper considers a repetitive-control system with the configuration in Figure 2. The SISO linear plant with a relative degree of zero and with time-varying structured uncertainties is

$$\begin{cases} \dot{x}(t) = [A + \delta A(t)]x(t) + [B + \delta B(t)]u(t), \\ y(t) = Cx(t) + Du(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the plant; and $u(t), y(t) \in \mathbb{R}$ are the control input and output, respectively. The assumption that the relative degree of the plant is zero implies that $D \neq 0$. Assume that the uncertainty of the plant is given by

$$[\delta A(t), \quad \delta B(t)] = MF(t)[N_0, \quad N_1], \tag{2}$$

where M, N_0, and N_1 are known constant matrices; and $F(t) \in \mathbb{R}^{n\times n}$ is a real unknown and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$F^{\mathrm{T}}(t)F(t) \leqslant I, \quad \forall t > 0. \tag{3}$$

The static output-feedback control law is

$$u(t) = k_e v(t) + k_y y(t), \quad k_e, k_y \in \mathbb{R}, \tag{4}$$

where k_e is the feedforward gain of the repetitive controller and k_y is the output-feedback gain. For the system in Figure 2, the design problem can be stated as follows:

Design suitable static control gains k_e and k_y that robustly stabilize the repetitive-control system under control law (4) and provide a steady-state tracking error of zero.

We make the following assumption, which is true for many control engineering problems:

Assumption 1. The uncertainties $\delta A(t)$ and $\delta B(t)$ satisfy

$$\delta A(T + t) = \delta A(t), \ \delta B(T + t) = \delta B(t), \ \forall \ t > 0. \tag{5}$$

(1) and (4) yield

$$u(t) = \frac{k_e}{1 - k_y D} v(t) + \frac{k_y C}{1 - k_y D} x(t).$$

Since

$$v(t) = e(t) + v(t - T),$$

$v(t)$ contains both the control result for the current period ($e(t)$) and the effect of learning in the previous period ($v(t - T)$). So, directly changing the control gains, k_e and k_y, does not independently adjust the control and learning actions. To do that, and thereby dramatically improve system performance, we present an accurate 2D description of the repetitive-control system in Figure 2.

First, we divide the infinite interval $[0, +\infty)$ into an infinite number of finite intervals, $[kT, (k+1)T)$ ($k = 0, 1, \ldots$). Then, for any $t \in [0, +\infty)$, there exists an interval $[kT, (k + 1)T)$ such that

$$t = kT + \tau, \ \tau \in [0, T).$$

This allows us to write the variable $\xi(t)$ in the time domain as

$$\xi(t) = \xi(kT + \tau) := \xi(k, \tau),$$

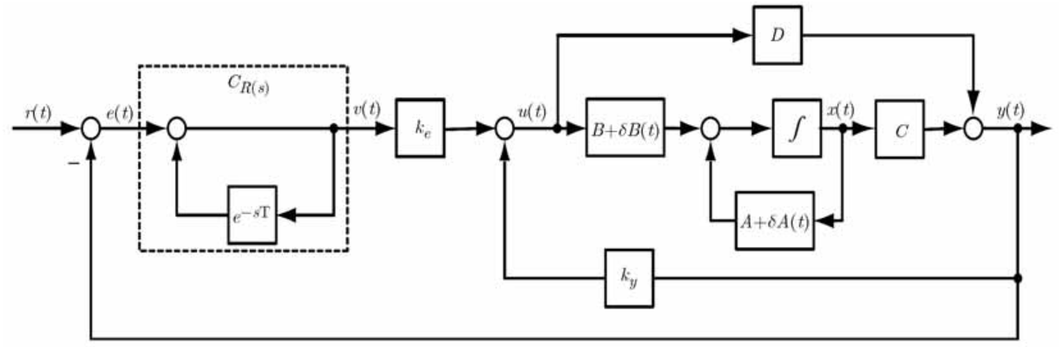

Figure 2 Configuration of repetitive control system based on the static output-feedback

and

$$\Delta\xi(t) := \Delta\xi(k,\tau) = \xi(k,\tau) - \xi(k-1,\tau).$$

For a given periodic reference input $r(t)$, the tracking error is

$$e(t) = e(k,\tau) = r(k,\tau) - y(k,\tau).$$

Taking the change in the state variables of the system (Figure 2) between the present and previous periods into account yields the following continuous-discrete 2D hybrid model:

$$\begin{cases} \Delta\dot{x}(k,\tau) = [A+\delta A(k,\tau)]\Delta x(k,\tau) + [B+\delta B(k,\tau)]\Delta u(k,\tau), \\ e(k,\tau) = e(k-1,\tau) - C\Delta x(k,\tau) - D\Delta u(k,\tau). \end{cases} \tag{6}$$

And the 2D description of the control law (4) is

$$\Delta u(k,\tau) = k_1 C\Delta x(k,\tau) + k_2 e(k-1,\tau), \tag{7}$$

where

$$k_1 = -\frac{k_e - k_y}{1+(k_e-k_y)D}, \quad k_2 = \frac{k_e}{1+(k_e-k_y)D}. \tag{8}$$

Unlike control law (4), which applies to the time domain, control law (7) in the 2D domain can be used to independently adjust the control behavior ($\Delta x(k,\tau)$) within one period and the learning process ($e(k-1,\tau)$) between periods by tuning k_1 and k_2 in (7). Note that the relationships between the control gains in Figure 2 and in (7) are

$$k_e = \tilde{k}_1 + \tilde{k}_2 \quad \text{and} \quad k_y = \tilde{k}_2, \tag{9}$$

where

$$\tilde{k}_1 = -\frac{k_1}{1+Dk_1}, \; \tilde{k}_2 = \left(1 - \frac{k_1 D}{1+Dk_1}\right)k_2 + \frac{k_1\tilde{k}_1}{1+k_1 D}. \tag{10}$$

3 Design of robust repetitive controller based on output feedback

Based on the above description, the problem of designing a repetitive-control system with the configuration in Figure 2 is formulated as the problem of stabilizing the continuous-discrete 2D system (6). So, we derive a sufficient stability condition for the closed-loop system of 2D system (6) under control law (7) by constructing a 2D Lyapunov functional and using it in combination with 2D system stability theory and the structural singular value decomposition of the output matrix.

Assume that the singular value decomposition of matrix Π is

$$\Pi = U[S, \quad 0]V^{\mathrm{T}},$$

where S is a semi-positive definite matrix, and U and V are unitary matrices.

The following lemmas are employed in the derivation of the robust-stability condition for the continuous-discrete 2D system.

Lemma 1 (see [9]). Let $\Pi \in \mathbb{R}^{p\times n}$, where $\mathrm{rank}(\Pi) = p$. For matrix $X \in \mathbb{R}^{n\times n}$, there exists a matrix, $\bar{X} \in \mathbb{R}^{p\times p}$, such that $\Pi X = \bar{X}\Pi$ holds if and only if X can be decomposed into

$$X = V \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{X}_{22} \end{bmatrix} V^{\mathrm{T}},$$

where V is the unitary matrix in the above singular value decomposition form of Π, $\bar{X}_{11} \in \mathbb{R}^{p\times p}$, and $\bar{X}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$.

Lemma 2 (see [10]). Let $\Omega_0(x)$ and $\Omega_1(x)$ be two quadratic matrix functions over $\mathbb{R}^n$, and let $\Omega_1(x) \leqslant 0$ for all $x \in \mathbb{R}^n - \{0\}$. Then, $\Omega_0(x) < 0$ holds for all $x \in \mathbb{R}^n - \{0\}$ if and only if there exists an $\varepsilon \geqslant 0$ such that

$$\Omega_0(x) - \varepsilon\Omega_1(x) < 0, \quad \forall x \in \mathbb{R}^n - \{0\}.$$

Lemma 3 (Schur complement [11]). For the real matrix $\Sigma = \Sigma^{\mathrm{T}}$, the following assertions are equivalent:
1. $\Sigma = \left[\begin{smallmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{smallmatrix}\right] < 0$;
2. $\Sigma_{11} < 0,\ \Sigma_{22} - \Sigma_{12}^{\mathrm{T}}\Sigma_{11}^{-1}\Sigma_{12} < 0$;
3. $\Sigma_{22} < 0,\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^{\mathrm{T}} < 0$.

Substituting the control input (7) into the continuous-discrete 2D system (6) yields the following representation of the closed-loop repetitive-control system:

$$\begin{bmatrix} \Delta\dot{x}(k,\tau) \\ e(k,\tau) \end{bmatrix} = \begin{bmatrix} A + Bk_1C & Bk_2 \\ -C - Dk_1C & 1 - Dk_2 \end{bmatrix} \begin{bmatrix} \Delta x(k,\tau) \\ e(k-1,\tau) \end{bmatrix} + \begin{bmatrix} M \\ 0 \end{bmatrix} \Gamma(k,\tau), \tag{11}$$

where

$$\begin{cases} \Gamma(k,\tau) = F(k,\tau)\Upsilon\eta, \\ \Upsilon = \begin{bmatrix} N_0 + N_1k_1C, & N_1k_2 \end{bmatrix}, \\ \eta^T = \begin{bmatrix} \Delta x^{\mathrm{T}}(k,\tau), & e^{\mathrm{T}}(k-1,\tau) \end{bmatrix}. \end{cases} \tag{12}$$

It is clear from (3) that

$$\Gamma^{\mathrm{T}}(k,\tau)\Gamma(k,\tau) = \eta^{\mathrm{T}}\Upsilon^{\mathrm{T}}F^{\mathrm{T}}(k,\tau)F(k,\tau)\Upsilon\eta \leqslant \eta^{\mathrm{T}}\Upsilon^{\mathrm{T}}\Upsilon\eta. \tag{13}$$

We obtain the following theorem from the above lemmas.

Theorem 1. If there exist symmetrical positive definite matrices X_{11}, X_{22}, and X_2, together with arbitrary matrices W_1 and W_2, such that the LMI

$$\begin{bmatrix} \Phi_{11} & BW_2 & M & \Phi_{14} & \Phi_{15} \\ * & -X_2 & 0 & \Phi_{24} & W_2^{\mathrm{T}}N_1^{\mathrm{T}} \\ * & * & -I & 0 & 0 \\ * & * & * & -X_2 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{14}$$

holds, where the the structural singular value decomposition of the output matrix is $C = U[S,\ \ 0]V^{\mathrm{T}}$, and

$$X_1 = V \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^{\mathrm{T}},$$

$$\Phi_{11} = AX_1 + X_1A^{\mathrm{T}} + BW_1C + C^{\mathrm{T}}W_1^{\mathrm{T}}B^{\mathrm{T}},$$

$$\Phi_{14} = -X_1 C^{\mathrm{T}} - C^{\mathrm{T}} W_1^{\mathrm{T}} D^{\mathrm{T}},$$
$$\Phi_{15} = C^{\mathrm{T}} W_1^{\mathrm{T}} N_1^{\mathrm{T}} + X_1 N_0^{\mathrm{T}},$$
$$\Phi_{24} = X_2 - W_2^{\mathrm{T}} D^{\mathrm{T}},$$

then system (6) with uncertainties (2), (3), and (5) is asymptotically stable under control law (7). Furthermore, the feedback gains are

$$k_1 = W_1 U S X_{11}^{-1} S^{-1} U^{\mathrm{T}} \text{ and } k_2 = W_2 X_2^{-1}. \tag{15}$$

Proof. Choose the Lyapunov functional candidate to be

$$V(k,\tau) = V_1(k,\tau) + V_2(k,\tau), \tag{16}$$

where

$$V_1(k,\tau) = \Delta x^{\mathrm{T}}(k,\tau) P_1 \Delta x(k,\tau),$$
$$V_2(k,\tau) = P_2 e^2(k,\tau),$$

and $P_1 = X_1^{-1}$, $P_2 = X_2^{-1}$.

Along the time trajectory of (11), we have

$$\frac{dV_1(k,\tau)}{d\tau} = \Delta\dot{x}^{\mathrm{T}}(k,\tau) P_1 \Delta x(k,\tau) + \Delta x^{\mathrm{T}}(k,\tau) P_1 \Delta\dot{x}(k,\tau) = \bar{\eta}^{\mathrm{T}} \Psi_1 \bar{\eta}, \tag{17}$$

$$\begin{aligned}\Delta V_2(k,\tau) &= e^{\mathrm{T}}(k,\tau) P_2 e(k,\tau) - e^{\mathrm{T}}(k-1,\tau) P_2 e(k-1,\tau)\\ &= \bar{\eta}^{\mathrm{T}} \Psi_2 \bar{\eta} - e^{\mathrm{T}}(k-1,\tau) P_2 e(k-1,\tau),\end{aligned} \tag{18}$$

and the increment of $V(k,\tau)$ is

$$\delta V = \frac{dV_1(k,\tau)}{d\tau} + \Delta V_2(k,\tau) = \bar{\eta}^{\mathrm{T}} \left(\Psi_1 + \Psi_2 + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \bar{\eta}, \tag{19}$$

where

$$\bar{\eta}^{\mathrm{T}} = \begin{bmatrix} \Delta x^{\mathrm{T}}(k,\tau), & e^{\mathrm{T}}(k-1,\tau), & \Gamma^{\mathrm{T}}(k,\tau) \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} \Omega_{11} & P_1 B k_2 & P_1 M \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix},$$

$$\Omega_{11} = (A^{\mathrm{T}} + C^{\mathrm{T}} k_1^{\mathrm{T}} B^{\mathrm{T}}) P_1 + P_1 (A + B k_1 C),$$

$$\Psi_2 = \begin{bmatrix} -C^{\mathrm{T}} - C^{\mathrm{T}} k_1 D^{\mathrm{T}} \\ 1 - k_2^{\mathrm{T}} D^{\mathrm{T}} \\ 0 \end{bmatrix} P_2 \begin{bmatrix} -C^{\mathrm{T}} - C^{\mathrm{T}} k_1 D^{\mathrm{T}} \\ 1 - k_2^{\mathrm{T}} D^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}}.$$

(19) yields

$$\delta V - \left[\Gamma^{\mathrm{T}}(k,\tau)\Gamma(k,\tau) - \bar{\eta}^{\mathrm{T}} \begin{bmatrix} \Upsilon^{\mathrm{T}} \\ 0 \end{bmatrix} [\Upsilon, \;\; 0] \bar{\eta} \right] = \bar{\eta}^{\mathrm{T}} \Lambda \bar{\eta}, \tag{20}$$

where

$$\Lambda = \Psi_1 + \Psi_2 + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -I \end{bmatrix} + \begin{bmatrix} \Upsilon^{\mathrm{T}} \\ 0 \end{bmatrix} [\Upsilon, 0].$$

According to Lemma 3, $\Lambda < 0$ is equivalent to the LMI

$$\Theta = \begin{bmatrix} \Theta_{11} & P_1Bk_2 & P_1M & \Theta_{14} & \Theta_{15} \\ * & -P_2 & 0 & \Theta_{24} & (N_1k_2)^{\mathrm{T}} \\ * & * & -I & 0 & 0 \\ * & * & * & -P_2 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (21)$$

where

$$\begin{aligned} \Theta_{11} &= (A^{\mathrm{T}} + C^{\mathrm{T}}k_1^{\mathrm{T}}B^{\mathrm{T}})P_1 + P_1(A + Bk_1C), \\ \Theta_{14} &= -C^{\mathrm{T}}P_2 - C^{\mathrm{T}}k_1^{\mathrm{T}}D^{\mathrm{T}}P_2, \\ \Theta_{15} &= (N_0 + N_1k_1C)^{\mathrm{T}}, \\ \Theta_{24} &= P_2 - k_2^{\mathrm{T}}D^{\mathrm{T}}P_2. \end{aligned}$$

Pre- and post-multiplying Θ by $\mathrm{diag}\{X_1, X_2, I, X_2, I\}$ yields the following LMI, which is equivalent to (21):

$$\begin{bmatrix} H_{11} & Bk_2X_2 & M & H_{14} & H_{15} \\ * & -X_2 & 0 & H_{24} & H_{25} \\ * & * & -I & 0 & 0 \\ * & * & * & -X_2 & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (22)$$

where

$$\begin{aligned} H_{11} &= X_1(A^{\mathrm{T}} + C^{\mathrm{T}}k_1^{\mathrm{T}}B^{\mathrm{T}}) + (A + Bk_1C)X_1, \\ H_{14} &= -X_1C^{\mathrm{T}} - X_1C^{\mathrm{T}}k_1^{\mathrm{T}}D^{\mathrm{T}}, \\ H_{15} &= X_1(N_0 + N_1k_1C)^{\mathrm{T}}, \\ H_{24} &= X_2 - X_2k_2^{\mathrm{T}}D^{\mathrm{T}}, \\ H_{25} &= X_2(N_1k_2)^{\mathrm{T}}. \end{aligned}$$

Since $X_1 = V[\begin{smallmatrix} X_{11} & 0 \\ 0 & X_{22} \end{smallmatrix}]V^{\mathrm{T}}$, from Lemma 1, we find that there exists $\bar{X}_1 = USX_{11}S^{-1}U^{\mathrm{T}}$ such that

$$CX_1 = \bar{X}_1C, \quad (23)$$

and

$$\bar{X}_1^{-1} = USX_{11}^{-1}S^{-1}U^{\mathrm{T}}. \quad (24)$$

Substituting (23), (24), $k_1 = W_1USX_{11}^{-1}S^{-1}U^{\mathrm{T}}$, and $k_2 = W_2X_2^{-1}$ into (22) yields LMI (14).

From Lemma 2 and the Lyapunov stability theorem, we can conclude that, if LMI (14) holds, then

$$\delta V < 0, \quad \forall \bar{\eta}^{\mathrm{T}} \neq 0.$$

The following algorithm, which is based on Theorem 1, gives the parameters of the repetitive controller in Figure 2.

Design Algorithm:

Step 1. Find a feasible solution to LMI (14).
Step 2. Use (15) to calculate k_1 and k_2.
Step 3. Use (10) to calculate $\tilde{k}_1$ and $\tilde{k}_2$.
Step 4. Use (9) to calculate k_e and k_y.

Remark 1. Theorem 1 provides an LMI-based sufficient stability condition for continuous-discrete 2D system (6) under control law (7). The algorithm just given is used to directly apply it to the design of the static-output-feedback repetitive-control system in Figure 2. To the best of our knowledge, this is the first result on robust static-output-feedback-based repetitive control.

In addition, we can derive a sufficient stability condition for the nominal continuous-discrete 2D system

$$\begin{cases} \Delta\dot{x}(k,\tau) = A\Delta x(k,\tau) + B\Delta u(k,\tau), \\ e(k,\tau) = e(k-1,\tau) - C\Delta x(k,\tau) - D\Delta u(k,\tau) \end{cases} \tag{25}$$

under control law (7).

Corollary 1. If there exist symmetrical positive definite matrices X_{11}, X_{22}, and X_2, together with arbitrary matrices W_1 and W_2, such that the LMI

$$\begin{bmatrix} \Xi_{11} & BW_2 & \Xi_{13} \\ * & -X_2 & \Xi_{23} \\ * & * & -X_2 \end{bmatrix} < 0 \tag{26}$$

holds, where the singular value decomposition of the output matrix is $C = U[S, \quad 0]V^{\mathrm{T}}$ and

$$\begin{aligned} X_1 &= V \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^{\mathrm{T}}, \\ \Xi_{11} &= AX_1 + BW_1C + X_1A^{\mathrm{T}} + C^{\mathrm{T}}W_1^{\mathrm{T}}B^{\mathrm{T}}, \\ \Xi_{13} &= -X_1C^{\mathrm{T}} - C^{\mathrm{T}}W_1^{\mathrm{T}}D^{\mathrm{T}}, \\ \Xi_{23} &= X_2 - W_2^{\mathrm{T}}D^{\mathrm{T}}, \end{aligned}$$

then the nominal continuous-discrete 2D system (25) is asymptotically stable under control law (7). Furthermore, the feedback gains are

$$k_1 = W_1USX_{11}^{-1}S^{-1}U^{\mathrm{T}} \quad \text{and} \quad k_2 = W_2X_2^{-1}. \tag{27}$$

4 Numerical example

This section considers the problem of designing a controller for a DC motor driven manipulator with a PI regulator. The control input is the voltage applied to the armature, and the output is the rotational torque of the manipulator. The dynamics of the motor in the state space can accurately be described using form (1).

Assume that the parameters of uncertain plant (1) are

$$A = \begin{bmatrix} -1.6 & -0.04 \\ 5 & -0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 10 & 0 \end{bmatrix}, \quad D = 1,$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad N_0 = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.001 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0 \\ 0.005 \end{bmatrix}, \quad F(t) = \begin{bmatrix} \sin\frac{2\pi}{10}t & 0 \\ 0 & \sin\frac{2\pi}{10}t \end{bmatrix}.$$

Consider the problem of tracking the reference input

$$r(t) = \sin\frac{2\pi}{10}t + 0.5\sin\frac{4\pi}{10}t + 0.5\sin\frac{6\pi}{10}t.$$

The algorithm in section 3 was used to design a robust repetitive-control law for the system in Figure 2. More specifically, solving the feasibility problem for LMI (14) and the equations in (15) yield $k_1 = -0.5993$ and $k_2 = 0.6875$. Substituting those values into (10) and (9) yields $k_e = 1.7156$ and $k_y = 0.2201$.

The simulation results in Figure 3 show that the system enters the steady state in the 9th period and that the steady-state tracking error is zero.

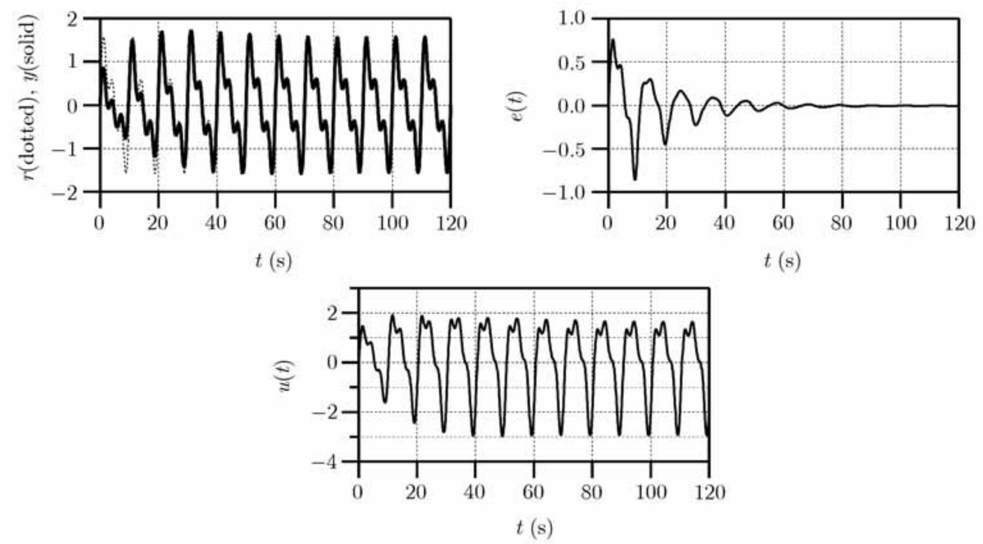

Figure 3 Simulation results for uncertain plant under control law (4).

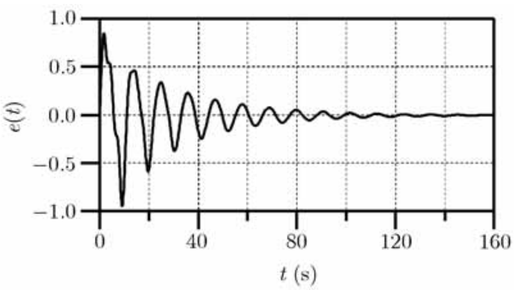

Figure 4 Simulation results for optimal state-feedback repetitive control for nominal plant.

For comparison, the optimal state-feedback repetitive-control law $u(t) = k_e v(t) + k_p x(t)$ was also designed by the following conventional design method for repetitive control.

First, choose the performance index to be

$$J = \int_0^{+\infty} (\Delta x^{\mathrm{T}}(t) Q \Delta x(t) + \Delta u^2(t)) dt.$$

Then, the feedback gains $k_e = 3.5$, $k_p = [0.1061, \ -1.6503]$ are obtained by solving the optimal control problem.

The simulation results in Figure 4 show that, for the nominal plant, the system entered the steady state in the 16th period. Clearly, our new 2D static output-feedback repetitive-control method is superior. It provides not only robust stability for a plant with structured uncertainties, but also satisfactory tracking performance. Furthermore, our control system configuration requires only information on the output of the plant. It is also simpler than a conventional full-state-feedback structure and is much easier to implement. That makes it very practical.

5 Conclusions

This paper presents the configuration of a robust static output-feedback repetitive-control system and a design method for a class of linear systems with a relative degree of zero and with time-varying structured uncertainties. First, a continuous-discrete 2D hybrid model is established that exploits the special features of a repetitive-control system. Next, the singular value decomposition of the output matrix is used, and the problem of designing a controller is converted into the problem of designing a robust stabilizing controller for a continuous-discrete 2D system. Then, an LMI-based stability condition for the closed-loop system is derived using 2D system stability theory and LMIs. The control gains of the repetitive-control law are obtained from a feasible solution to the LMI. Simulation results demonstrate the validity of the method.

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