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Pinning weighted complex networks with heterogeneous delays by a small number of feedback controllers

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Weighted complex dynamical networks with heterogeneous delays in both continuous-time and discrete-time domains are controlled by applying local feedback injections to a small fraction of network nodes. Some generic stability criteria ensuring delay-independent stability are derived for such controlled networks in terms of linear matrix inequalities (LMIs), which guarantee that by placing a small number of feedback controllers on some nodes the whole network can be pinned to some desired homogenous states. In some particular cases, a single controller can achieve the control objective. It is found that stabilization of such pinned networks is completely determined by the dynamics of the individual uncoupled node, the overall coupling strength, the inner-coupling matrix, and the smallest eigenvalue of the coupling and control matrix. Numerical simulations of a weighted network composing of a 3-dimensional nonlinear system are finally given for illustration and verification.

complex dynamical network, linear matrix inequality (LMI), weighted network, pinning control, heterogeneous delay

1 Introduction

Complex dynamical networks are ubiquitous in nature, man-made systems, and human societies[1,2], which have become a focal subject for study in recent years. Research on complex dynamical networks is pervading all kinds of sciences today, including physics, chemistry, biology, information technology, mathematics, sociology, etc. In particular, the discovery of some remarkable characteristics of complex networks, such as the small-world effect[3] and scale-free property[4,5], has led to dramatic advances in this active research area. Its impact on modern engineering and technology is prominent and far-reaching.

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In manipulating various networks, feedback pinning is a common technique for control[6], stabilization, and synchronization[7−10]. Wang and Chen[11−14] introduced a uniform model of complex dynamical network by considering dynamical elements of a network as nodes, and exploited a pinning control technique for scale-free chaotic dynamical networks, where local feedback injections are applied to a small portion of nodes so as to control the entire network. In ref. [15], Li et al. studied the stabilization of random networks and scale-free networks by using specifically and randomly selective feedback pinning schemes, respectively, applied also to a tiny fraction of network nodes. Some basic assumptions of previous work[12−15] are that the nodes in the network are coupled symmetrically with the same coupling strength and there are no coupling delays in the network. However, in many circumstances these simplifications do not match the peculiarities of real networks with satisfaction. For instance, the WWW[16,17], metabolic networks, and citation networks[18,19] are all directed graphs, whose coupling matrices are asymmetric. In addition, some phenomena, such as the diversity of the predator-prey interactions in food webs[20,21], different capabilities of transmitting electric signals in neural networks[22,23], and unequal traffic on the Internet[24] or of the passengers in airline networks[25,26], explain the existence of weighted wirings[27]. Additionally, due to the limited speeds of transmission and spreading, as well as traffic congestions, signals traveling through a network are often associated with time delays, which is very common in biological and physical networks. In refs. [28—31], homogenous time-delay complex networks (i.e., all the delays are the same) are considered; however, heterogeneous (i.e., unequal and non-commensurate) time-delay complex networks are of practical importance and have some special difficulties technically.

Motivated by the above discussions, the important pinning control problem is revisited for a weighted complex dynamical network with inhomogeneous delays, where there is little research on this important subject. The main contribution of this paper is to develop a general approach to stabilize such a network onto some desired homogenous states by injecting only a small number of local feedback controllers. Some simple feedback controllers are designed and some generic stability criteria are derived for multi-time-delay networks in both continuous-time and discrete-time settings, respectively. In particular, it will be shown that the stabilization of such networks is completely determined by the dynamics of each uncoupled node, the overall coupling strength, the inner-coupling matrix, and the smallest eigenvalue of the coupling and control matrix, by using a decoupling technique. Also, the main differences of the effect of the coupling strength on network stabilization and destabilization between delayed networks and time-invariant networks without delays are discussed.

The rest of this paper is organized as follows. The design of local stabilizing controllers of weighted complex dynamical networks with heterogeneous delays in both continuous-time and discrete-time domains are discussed in section 2, with some stability conditions derived based on Lyapunov stability theory and LMI criterion. Some simulated examples for weighted dynamical networks pinned by specifically selective pinning scheme are compared for illustration and verification in section 3. Finally, section 4 concludes the investigation and proposes some further work.

2 Pinning control of weighted complex dynamical networks with heterogeneous delays

In this section, the problem of how to pin a weighted complex dynamical network with heterogeneous delays to its desired homogenous state is investigated. Some stability criteria of such networks in both continuous-time and discrete-time settings are established in terms of Lyapunov stability theory and LMI criterion.

2.1 Continuous-time networks

Consider a new yet generic weighted complex dynamical network with heterogeneous delays consisting of N identical nodes with a diffusive coupling, where each node is an m-dimensional dynamical system, described by

$$\dot{x}_i = f(x_i) - \frac{a}{k_i^{\beta_w}} \sum_{j=1}^{N} L_{ij} \Gamma \overline{x_j(t-\tau)}, \quad i = 1, \cdots, N, \tag{1}$$

where $\overline{x_j(t-\tau)} = [x_{j1}(t-\tau_1), \cdots, x_{jm}(t-\tau_m)]^T$; $\tau_i > 0$ $(i = 1, \cdots, m)$ are the time delays. Here, all the nodes have the same time delay vector; however, different delay entries of the ith node have different values.

The function $f(\cdot)$, describing the local dynamics of the nodes, is continuously differentiable and capable of producing various rich dynamical behaviors. $x_i = (x_{i1}, \cdots, x_{im})^T \in R^m$ represents the state vector of the ith node, and the overall coupling strength a is positive; k_i is the out-degree of node i and β_w is a tunable weight parameter. Also, the real matrix $L = (L_{ij})_{i,j=1}^N$ is the usual (symmetric) Laplacian matrix with diagonal entries $L_{ii} = k_i$ and off-diagonal entries $L_{ij} = -1$ if node i and node j are connected by a link, and $L_{ij} = 0$ otherwise. $\Gamma \in R^{m \times m}$ is a constant matrix linking coupled variables.

Furthermore, the parameter $\beta_w = 0$ recovers that the network is unweighted and undirected, and the condition $\beta_w \neq 0$ corresponds to a network with weighted configuration and bidirectional[32].

For convenient analysis, we let

$$b_{ij} = -L_{ij} \big/ k_i^{\beta_w}, \tag{2}$$

then, the network (1) can be rewritten as

$$\dot{x}_i = f(x_i) + a \sum_{j=1}^{N} b_{ij} \Gamma \overline{x_j(t-\tau)}, \tag{3}$$

where $B = (b_{ij})_{i,j=1}^N$ is a coupling matrix, accounting for the topology of the network.

In matrix notation, eq. (2) can be written as $B = -K^{-\beta_w} L$, where $K = diag\{k_1, \cdots, k_N\}$ is the diagonal matrix of degree. Using matrix identities[32], we have that the spectrum of asymmetric matrix B is equal to the spectrum of the symmetric matrix $W = -K^{-\beta_w/2} L K^{-\beta_w/2}$, which is negative semi-define. It follows that asymmetric matrix B has real eigenvalues and can be diagonalizable. In this model, we suppose that network (3) is connected in the sense of having no isolated clusters, which means that the coupling matrix B is irreducible. In sum, B is an irreducible and negative semi-define matrix with zero-row sum.

For the latter use, some lemmas required in this paper are first given in the following.

Lemma 1. If $B = (b_{ij}) \in R^{N \times N}$ is defined in eq. (2), $D = \mathrm{diag}(d_1, \cdots, d_N)$ with $d_i \geqslant 0$

($i = 1,\cdots,N$) and $D \neq 0$, then $C = B–D$ is negative definite.

A proof of Lemma 1 is given in Appendix.

Lemma 2[33]**.** Suppose that a symmetric matrix is partitioned as

$$H = \begin{pmatrix} H_1 & H_2 \\ H_2^T & H_3 \end{pmatrix},$$

where H_1 and H_3 are square. The matrix H is positive definite if and only if H_1 is positive definite and $H_3 - H_2^T H_1^{-1} H_2 > 0$.

Lemma 3. For two vectors a and b, and for any positive-definite matrix X, the following inequality holds

$$-2a^T b \leqslant \inf_{X>0} \left\{ a^T X a + b^T X^{-1} b \right\}.$$

This lemma is a well known result in linear algebra[34].

The objective here is to stabilize network (3) onto a desired homogenous state[7]:

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t), \text{ as } t \to \infty, \tag{4}$$

where $s(t)$ can be an equilibrium point, a periodic orbit, or even a chaotic orbit, satisfying $\dot{s}(t) = f(s(t))$.

To achieve the goal (4), feedback pinning controllers are applied onto a small portion δ $(0 < \delta << 1)$ of nodes in network (3). Without loss of generality, let the first l nodes be selected to be pinned, where l is the integer part of the real number δN.

Thus, the controlled network can be described as

$$\dot{x}_i = f(x_i) + a\sum\nolimits_{j=1}^{N} b_{ij} \Gamma \overline{x_j(t-\tau)} + u_i, \quad i = 1,\cdots,N, \tag{5}$$

with the local negative feedback controllers designed by

$$u_i = -ad_i \Gamma(\overline{x_i(t-\tau)} - \overline{s(t-\tau)}), \quad i = 1,\cdots,N, \tag{6}$$

where $d_i = d > 0$ for $i = 1,\cdots,l$ and $d_i = 0$ for $i = l+1,\cdots, N$.

Let the errors be

$$e_i(t) = x_i(t) - s(t), \quad i = 1,\cdots,N. \tag{7}$$

Linearizing the controlled network (5) at the state $s(t)$ leads to

$$\dot{E} = EJ^T(s(t)) + aC\overline{E(t-\tau)}\Gamma^T, \tag{8}$$

where $J(s(t)) \in R^{m\times m}$ is the Jacobian matrix of f evaluated at $s(t)$, and

$$E^T = [e_1,\cdots,e_N] \in R^{m\times N},$$

$$\overline{E(t-\tau)}^T = [\overline{e_1(t-\tau)},\cdots,\overline{e_N(t-\tau)}] \in R^{m\times N},$$

$$C = B - D,$$

with control gain matrix $D = \mathrm{diag}(d_1,\cdots,d_N)$, and $\overline{e_j(t-\tau)} = [(e_{j1}(t-\tau_1),\cdots, \ e_{jm}(t-\tau_m)]^T$. For simplicity, we name C the “coupling and control” matrix in the sequel.

From Lemma 1, it follows that C is negative definite and can be diagonalizable, so all of its eigenvalues are strictly negative, denoted as

$$0 > \lambda_{\max}(C) = \lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_N, \tag{9}$$

with their corresponding (generalized) eigenvectors

$$\Phi = [\phi_1, \cdots, \phi_N] \in R^{N\times N}, \tag{10}$$

satisfying

$$C\phi_k = \lambda_k \phi_k, \quad k = 1, \cdots, N.$$

By expressing each column E on the basis Φ, one has

$$E = \Phi\eta. \tag{11}$$

Then, eq. (8) can be expanded into the following equation:

$$\dot{\eta}_k = J(s(t))\eta_k + a\lambda_k \Gamma \overline{\eta_k(t-\tau)}, \quad k = 1, \cdots, N, \tag{12}$$

where $$\eta_k(t) = [\eta_{k1}(t), \cdots, \eta_{km}(t)]^T \in R^m,$$

and

$$\overline{\eta_k(t-\tau)} = [(\eta_{k1}(t-\tau_1), \cdots, \eta_{km}(t-\tau_m))]^T \in R^m.$$

To this end, the local stability problem of the ($N\times m$)-dimensional system (5) is converted into the stability problem of the N independent m-dimensional linear systems (12).

The following theorem characterizing a sufficient condition for system (5) to be locally exponentially stable about the homogenous state $s(t)$.

Theorem 1. If there exist two symmetric positive-definite matrices $P \in R^{m\times m}$, $Q=\mathrm{diag}\{q_1^2, \cdots, q_m^2\}$ and a positive constant $\gamma > 0$, such that the following LMI holds:

$$\begin{bmatrix} J^T(s(t))P + PJ(s(t)) + Q + \gamma I_m & a\lambda_N P\Gamma \\ a\lambda_N \Gamma^T P & -Q \end{bmatrix} < 0, \tag{13}$$

then the controlled network (5) is locally exponentially stable in the sense of eq. (4) for arbitrary constant time delay $\tau_i > 0$ ($i = 1, \cdots, m$). Here, λ_N is the smallest eigenvalue of C and $I_m \in R^{m\times m}$ is an identity matrix.

Proof. From the Schur Complement (Lemma 2), the LMI (13) is equivalent to

$$J^T(s(t))P + PJ(s(t)) + Q + \gamma I_m + a^2\lambda_N^2 P\Gamma Q^{-1}\Gamma^T P < 0.$$

Therefore, there exists a small constant $\varepsilon > 0$, such that the following inequation holds

$$J^T(s(t))P + PJ(s(t)) + \varepsilon P + QE_\tau + a^2\lambda_N^2 P\Gamma Q^{-1}\Gamma^T P < 0, \tag{14}$$

where $E_\tau = \mathrm{diag}\{e^{\varepsilon\tau_1}, \cdots, e^{\varepsilon\tau_m}\}$.

Construct a Lyapunov-Krasovskii function

$$V(\eta_k(t)) = \eta_k^T(t)P\eta_k(t)e^{\varepsilon t} + \sum_{s=1}^m \int_{t-\tau_s}^t q_s^2 \eta_{ks}^2(\mu) e^{\varepsilon(\mu+\tau_s)} d\mu, \tag{15}$$

which is positive-definite.

The time derivative of $V(\eta_k(t))$ along the trajectories of the controlled network (5) is

$$\dot{V}(\eta_k(t)) = \eta_k^T(t)(J^T(s(t))P + PJ(s(t)) + \varepsilon P + QE_\tau)\eta_k(t)e^{\varepsilon t}$$
$$+ 2a\lambda_k \overline{\eta_k(t-\tau)}^T \Gamma^T P\eta_k(t)e^{\varepsilon t} - \overline{\eta_k(t-\tau)}^T Q\overline{\eta_k(t-\tau)}e^{\varepsilon t}. \tag{16}$$

Let

$$\rho_1 = -\overline{\eta_k(t-\tau)}, \tag{17}$$

$$\rho_2 = a\lambda_k \Gamma^T P\eta_k(t), \tag{18}$$

and $X = Q > 0$.

From Lemma 3, one has

$$-2\rho_1^T\rho_2 \leqslant \inf_{Q>0}\left\{\rho_1^T Q\rho_1 + \rho_2^T Q^{-1}\rho_2\right\}, \tag{19}$$

implying that

$$2a\lambda_k \overline{\eta_k(t-\tau)}^T \Gamma^T P\eta_k(t) \leqslant \overline{\eta_k(t-\tau)}^T Q\overline{\eta_k(t-\tau)} + a^2\lambda_k{}^2\eta_k^T(t)P\Gamma Q^{-1}\Gamma^T P\eta_k(t). \tag{20}$$

From eqs. (16) and (20), one has

$$\begin{aligned}\dot{V}(\eta_k(t)) &\leqslant \eta_k^T(t)(J^T(s(t))P + PJ(s(t)) + \varepsilon P + QE_\tau + a^2\lambda_k{}^2 P\Gamma Q^{-1}\Gamma^T P)\eta_k(t)e^{\varepsilon t} \\ &\leqslant \eta_k^T(t)(J^T(s(t))P + PJ(s(t)) + \varepsilon P + QE_\tau + a^2\lambda_N{}^2 P\Gamma Q^{-1}\Gamma^T P)\eta_k(t)e^{\varepsilon t}.\end{aligned} < 0. \tag{21}$$

From the Lyapunov stability theory, the controlled network (5) is locally exponentially stable in the sense of eq. (4). The proof is thus completed.

Theorem 1 gives a sufficient condition, inequality (13), for the existence of $P, Q \in R^{m\times m}$ and a positive constant $\gamma > 0$ that can stabilize the controlled network (5). By making some further simplifications, the following constructive corollary can be obtained.

Corollary 1. The controlled network (5) is locally exponentially stable in the sense of eq. (4) if there exists a symmetric positive-definite matrix $P \in R^{m\times m}$ and a constant $\gamma_0 > 1$ such that

$$\begin{bmatrix} J^T(s(t))P + PJ(s(t)) + \gamma_0 I_m & a\lambda_N P\Gamma \\ a\lambda_N \Gamma^T P & -I_m \end{bmatrix} < 0, \tag{22}$$

for arbitrary constant time delay $\tau_i > 0$ ($i = 1,\cdots,m$).

Another heterogeneous delay complex network is considered as

$$\dot{x}_i = f(x_i) + a\sum\nolimits_{j=1}^N b_{ij}\Gamma\overline{\overline{x_j(t-\tau_j)}}, \quad i = 1,\cdots,N, \tag{23}$$

where $\overline{\overline{x_j(t-\tau_j)}} = [x_{j1}(t-\tau_{j1}),\cdots,x_{jm}(t-\tau_{jm})]^T$; *f*, x_i, a, b_{ij} and Γ have the same meanings as those in network (3). The sole difference is that in eq. (23) a different node *j* has a different time-delay vector $[\tau_{j1},\cdots,\tau_{jm}]^T$.

Use the following time-delay state feedback controllers:

$$u_i = -ad_i\Gamma(\overline{\overline{x_i(t-\tau_i)}} - \overline{\overline{s(t-\tau_i)}}), \quad i = 1,\cdots,N, \tag{24}$$

where d_i is the same as that in eq. (6).

Then, the controlled network is

$$\dot{x}_i = f(x_i) + a\sum_{j=1}^{N} b_{ij}\Gamma\overline{x_j(t-\tau_j)} - ad_i\Gamma(\overline{x_i(t-\tau_i)} - \overline{s(t-\tau_i)}), \quad i = 1,\cdots,N . \tag{25}$$

Let the errors be

$$e_i(t) = x_i(t) - s(t), \quad i = 1,\cdots,N . \tag{26}$$

Similar to eqs. (8)—(12), system (25) can be reformulated as

$$\dot{\eta}_k = J(s(t))\eta_k + a\lambda_k\Gamma\overline{\eta_k(t-\tau)}, \quad k = 1,\cdots,N , \tag{27}$$

where $\overline{\eta_k(t-\tau)} = [(\eta_{k1}(t-\tau_{k1}),\cdots,\eta_{km}(t-\tau_{km})]^T \in R^m$. Then, we have the following result.

Theorem 2. If there exist two symmetric positive-definite matrices $P \in R^{m\times m}$, $Q = diag\{q_1^2,\cdots,q_m^2\}$ and a positive constant $\gamma > 0$, such that the following LMI holds

$$\begin{bmatrix} J^T(s(t))P + PJ(s(t)) + Q + \gamma I_m & a\lambda_N P\Gamma \\ a\lambda_N \Gamma^T P & -Q \end{bmatrix} < 0, \tag{28}$$

then the controlled network (25) is locally exponentially stable in the sense of (4) for any fixed delay $\tau_{ks} > 0$ ($k = 1,\cdots,N$; $s = 1,\cdots,m$), where all notations are as above.

Construct a Lyapunov-Krasovskii function

$$V(\eta_k(t)) = \eta_k^T(t)P\eta_k(t)e^{\varepsilon t} + \sum_{s=1}^{m}\int_{t-\tau_{ks}}^{t} q_s^2\eta_{ks}^2(\mu)e^{\varepsilon(\mu+\tau_{ks})}d\mu . \tag{29}$$

Theorem 2 can be easily proved in a way similar to Theorem 1.

Corollary 2. The controlled network (25) is locally exponentially stable in the sense of eq. (4) if there exists a symmetric positive-definite matrix $P \in R^{m\times m}$ and a constant $\gamma_0 > 1$ such that

$$\begin{bmatrix} J^T(s(t))P + PJ(s(t)) + \gamma_0 I_m & a\lambda_N P\Gamma \\ a\lambda_N \Gamma^T P & -I_m \end{bmatrix} < 0, \tag{30}$$

for any fixed delay $\tau_{ks} > 0$ ($k = 1,\cdots,N$; $s = 1,\cdots,m$), where all notations are as the above.

2.2 Discrete-time networks

In this subsection, the above-obtained results are extended to discrete-time networks of the form

$$x_i(n+1) = f(x_i(n)) + a\sum_{j=1}^{N} b_{ij}\Gamma\overline{x_j(n-\tau)}, \quad i = 1,\cdots,N , \tag{31}$$

where $\overline{x_j(n-\tau)} = [x_{j1}(n-\tau_1),\cdots,x_{jm}(n-\tau_m)]^{\mathrm{T}}$; f, x_i, a, b_{ij} and Γ have the same meanings as those in network (3), with the only difference that τ_i ($i = 1,\cdots,m$) is a positive integer here.

The objective, once again, is to stabilize network (31) onto a homogenous state

$$x_1(n) = x_2(n) = \cdots = x_N(n) = s(n), \text{ as } n \to \infty , \tag{32}$$

where $s(n) \in R^m$ is a solution of an isolated node, satisfying $s(n+1) = f(s(n))$.

Similarly, the pinning control strategy is applied to a small fraction of nodes in network (31). Suppose that the first l nodes are selected to be pinned.

Design the local negative feedback controllers as

$$u_i(n) = -ad_i\Gamma(\overline{x_i(n-\tau)} - \overline{s(n-\tau)}) . \tag{33}$$

Here, similarly, $d_i = d > 0$ for $i = 1,\cdots,l$ and $d_i = 0$ for $i = l+1,\cdots,N$.

Then, the controlled network is

$$x_i(n+1) = f(x_i(n)) + a\sum_{j=1}^{N} b_{ij}\Gamma\overline{x_j(n-\tau)} - ad_i\Gamma(\overline{x_i(n-\tau)} - \overline{s(n-\tau)}) , \quad i = 1,\cdots,N . \tag{34}$$

Let

$$e_i(n) = x_i(n) - s(n) , \quad i = 1,\cdots,N . \tag{35}$$

Linearizing the controlled network (34) about $s(n)$, one has

$$E(n+1) = E(n)J^T(s(n)) + aC\overline{E(n-\tau)}\Gamma^{\mathrm{T}} , \tag{36}$$

where $J(s(n)) \in R^{m\times m}$ is the Jacobian matrix of f evaluated at $s(n)$, and

$$E^T(n) = [e_1(n),\cdots,e_N(n)] \in R^{m\times N} ,$$

$$\overline{E(n-\tau)}^T = [\overline{e_1(n-\tau)},\cdots,\overline{e_N(n-\tau)}] \in R^{m\times N} ,$$

$$\overline{e_j(n-\tau)} = [(e_{j1}(n-\tau_1),\cdots,e_{jm}(n-\tau_m)]^{\mathrm{T}} ,$$

and $C = B - D$ with $D = \mathrm{diag}(d_1,\cdots,d_N)$.

Similarly, one can obtain

$$\eta_k(n+1) = J(n)\eta_k(n) + a\lambda_k\Gamma\overline{\eta_k(n-\tau)} , \quad k = 1,\cdots,N , \tag{37}$$

where $e_i(n) = \Phi\eta_i(n)$, $i = 1,\cdots,N$ and Φ is defined in eq. (10).

Theorem 3. If there exist two symmetric positive-definite matrices $P \in R^{m\times m}$ and Q = $\mathrm{diag}\{q_1^2,\cdots,q_m^2\}$, such that the following LMI holds:

$$\begin{bmatrix} J^T(s(n))PJ(s(n)) - P + Q & a\lambda_N J^T(s(n))P\Gamma \\ a\lambda_N\Gamma^T PJ(s(n)) & a^2\lambda_N^2\Gamma^T P\Gamma - Q \end{bmatrix} < 0 , \tag{38}$$

then the controlled network (34) is locally asymptotically stable in the sense of eq. (32) for arbitrary constant time delay $\tau_i > 0$ ($i = 1,\cdots,m$). Here, λ_N is the smallest eigenvalue of C.

Constructing a Lyapunov function

$$V(\eta_k(n)) = \eta_k^T(n)P\eta_k(n) + \sum_{s=1}^{m}\sum_{\sigma=n-\tau_s}^{n-1} q_s^2\eta_{ks}^2(\sigma) ,$$

where $\eta_k(n) = [\eta_{k1}(n),\cdots,\eta_{km}(n)]^{\mathrm{T}}$.

Theorem 3 can be easily proved in a way similar to Theorem 1.

Similarly, one has the following result.

Corollary 3. The controlled network (34) is locally asymptotically stable in the sense of eq.

(32) if there exists a symmetric positive-definite matrix $P \in R^{m \times m}$ such that

$$\begin{bmatrix} J^T(s(n))PJ(s(n)) - P + I_m & a\lambda_N J^T(s(n))P\Gamma \\ a\lambda_N \Gamma^T PJ(s(n)) & a^2\lambda_N^2 \Gamma^T P\Gamma - I_m \end{bmatrix} < 0, \tag{39}$$

where $I_m \in R^{m \times m}$ is an identity matrix.

3 Simulation study

As an application of the above-obtained theoretical criteria, pinning control of a weighted complex dynamical network composing of a 3-dimensional nonlinear system is simulated and discussed in this section.

Consider a complex dynamical network, in which each node is a 3-dimensional nonlinear system described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -x_1 + x_1 x_2 \\ -2x_2 \\ -3x_3 \end{pmatrix}, \tag{40}$$

with one equilibrium point $\bar{x} = [0,0,0]^{\mathrm{T}}$.

Using $\Gamma = diag(1,1,1)$, the whole dynamical network is described by

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} -x_{i1} + x_{i1}x_{i2} + a\sum_{j=1}^{N} b_{ij}x_{j1}(t-\tau_1) \\ -2x_{i2} + a\sum_{j=1}^{N} b_{ij}x_{j2}(t-\tau_2) \\ -3x_{i3} + a\sum_{j=1}^{N} b_{ij}x_{j3}(t-\tau_3) \end{pmatrix}, \quad i = 1,2,\cdots,N. \tag{41}$$

The objective is to stabilize the network (41) onto the originally equilibrium point $\bar{x}$, by applying feedback pinning control to a much smaller number of nodes.

In the following, some simulation results of the complex dynamical networks with delay vectors $[\tau_1, \tau_2, \tau_3]^{\mathrm{T}} = [2,3,4]^{\mathrm{T}}$ are presented with different coupling configurations. The node initial values of the network are in the uniform distribution on the interval (−1, 1).

Figure 1 shows the stabilization process of controlling a 50-node weighted coupled network corresponding to the weight parameter β_w=0.1, in which only the “smallest” node is pinned, which has out-degree 2. Design the feedback gain d_1=d=0.5 (i.e., only a single node to be pinned), according to Theorem 1, choose a = 0.02, γ=1.6, and use the MATLAB LMI Toolbox; there exist two positive-definite matrices,

$$P = \mathrm{diag}(42.7232, 24.1114, 16.1494),$$

$$Q = \mathrm{diag}(43.5299, 47.1963, 47.3467),$$

such that inequality (13) holds.

Remark 1. It is interesting to note that the stabilization condition for complex dynamical

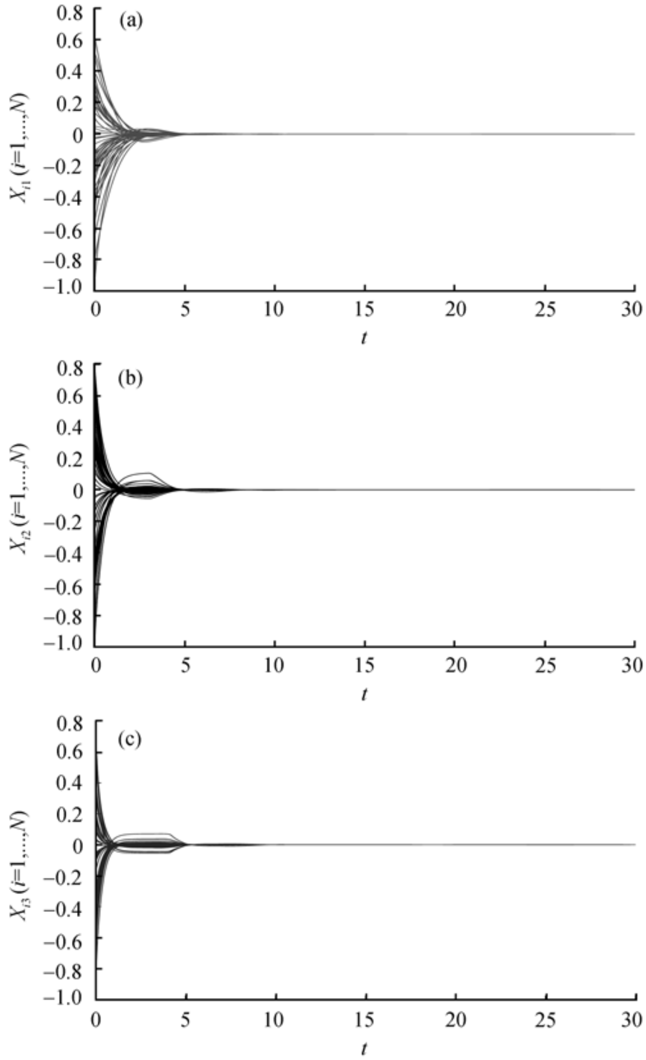

Figure 1 Specifically pinning the smallest node of out-degree 2 in a 50-node weighted network with a single controller: a=0.02, d=0.5. (a) $\boldsymbol{x}_{i1}$; (b) $\boldsymbol{x}_{i2}$; (c) $\boldsymbol{x}_{i3}$.

networks with delays is quite different from that for continuous complex dynamical networks without delays. The main differences are twofold:

1) For complex delayed dynamical networks, it is different from the intuition that sufficiently strong coupling will lead a network to stabilize; too strong a coupling may actually jeopardize the stability of the complex delayed networks. However, for a time-invariant continuous complex dynamical network without delays, it is simpler to achieve stabilization, where only a proper value of the coupling strength is selected to satisfy the stability condition.

2) From the condition inequality (13), it follows that the overall coupling strength has not only a lower bound, but also an upper bound. However, for a continuous-time time-invariant complex dynamical network, the coupling strength ensuring the network stabilization only has a lower bound, which means that the coupling strength is required to be large enough. Therefore, the re-

sult of significantly less local controllers are needed by the specifically selective pinning scheme than that required by the randomly pinning scheme for controlling scale-free time-invariant networks[15] does not directly apply to the delayed networks.

On the other hand, one may obtain some hints from Figure 2 about destabilization. One way to achieve destabilization is to keep the network size constant and the topology unchanged, but to vary the overall coupling strength, such that it does not satisfy the condition inequality (13).

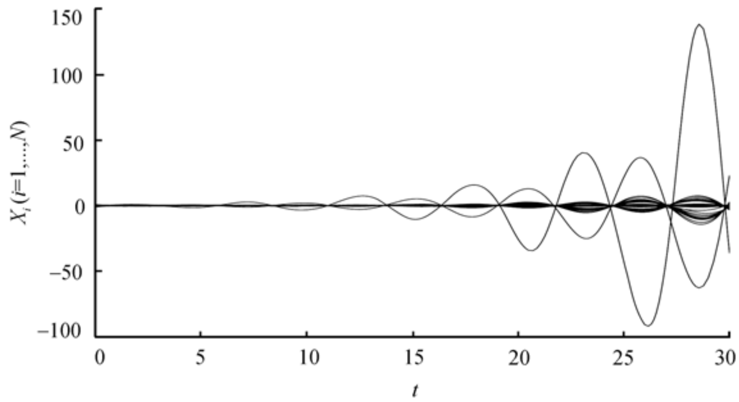

Figure 2 Specifically pinning the smallest node of out-degree 2 in the same network with a single controller. a=0.15, d=0.5.

4 Conclusions

In this paper, the stabilization problem of weighted complex dynamical networks with heterogeneous delays has been investigated by pinning a small fraction of nodes with delay negative feedback controllers. The pinned nodes can control other nodes through the networked connections dynamically. Here, the placement of the local controllers is affected by the delays, the topology of the network particularly the overall coupling strength and the inner-coupling matrix. For this reason, some interesting pinning phenomena have been explained, such as pinning one single node can achieve stabilization to the homogenous state of the network.

Several delay-independent stability theorems have been established for heterogeneous delayed network models subject to pinning control, which were not being studied elsewhere before. For each controlled network, the decoupling technique is used to convert the stabilization problem of the whole network into the stabilization of its sub-networks, making the stability analysis much easier.

Moreover, in numerical simulations, it was found that too large coupling strength may lead to destabilization instead. This finding is useful in that it accelerates the desired stabilization and/or destabilization of a network, which is particularly meaningful from an engineering point of view and is useful for engineering design.

It can be foreseen that pinning the proposed weighted complex dynamical networks with heterogeneous delays will be useful for the current studies of general complex dynamical networks. In this paper, some delay-independent stability results are obtained. However, delay-dependent stabilization criteria, which are less conservative, and effects of time delays on different kinds of networks deserve more attention in further studies.

Appendix Proof of Lemma 1

Proof. Let $\overline{\lambda}_1,\cdots,\overline{\lambda}_N$ be the eigenvalues of B in the decreasing order. From the definition of eq. (2), $\overline{\lambda}_1=0$ and $\overline{\lambda}_i<0$ for $i=2,\cdots,N$. Denote $\overline{\xi}_i$, $i=1,\cdots,N$ are the N eigenvectors of B, with $\overline{\xi}_1=[1,\ 1,\cdots,1]^T$ corresponding to $\overline{\lambda}_1=0$. Then, any nonzero vector $Z\in R^N$ can be expressed by $Z=\sum_{i=1}^{N}\alpha_i\overline{\xi}_i$ for some constants α_i, $i=1,\cdots,N$. Additionally, $D\neq 0$. Without loss of generality, assume $d_j>0$ for some j. It is clear that $\overline{\xi}_1^T D\overline{\xi}_1 \geqslant d_j$. Thus, in either the case when $\alpha_1\neq 0$, $\alpha_2=\cdots=\alpha_N=0$ or the case when $\alpha_i\neq 0$ for $i=2,\cdots,N$, one always has

$$Z^T(B-D)Z=Z^TBZ-Z^TDZ=\sum_{i=2}^{N}\overline{\lambda}_i\alpha_i^2\overline{\xi}_i^T\overline{\xi}_i-Z^TDZ<0,$$

for nonzero vector Z. The proof is thus completed.

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