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Sparse representation and blind source separation of ill-posed mixtures

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Abstract Bofill *et al*. discussed blind source separation (BSS) of sparse signals in the case of two sensors. However, as Bofill *et al*. pointed out, this method has some limitation. The potential function they introduced is lack of theoretical basis. Also the method could not be extended to solve the problem in the case of more than three sensors. In this paper, instead of the potential function method, a K-PCA method (combining K-clustering with PCA) is proposed. The new method is easy to be used in the case of more than three sensors. It is easy to be implemented and can provide accurate estimation of mixing matrix. Some criterion is given to check the effect of the mixing matrix A. Some simulations illustrate the availability and accuracy of the method we proposed.

Keywords: ill-posed mixture, blind source separation, sparse representation, PCA, K-mean clustering.

1 Introduction

Blind source separation (BSS) aims at gaining the estimation of unknown source signals without information of the channel. In mathematical words, the mixture of source signals is

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t), t = 1, \cdots, T, \tag{1}$$

where $\boldsymbol{s}(t) = (s_1(t), \cdots, s_n(t))^T$ is the vector of source signals, $\boldsymbol{x}(t) = (x_1(t), \cdots, x_m(t))^T$ is the vector of observed signals. Matrix A denotes the unknown $m \times n$ mixing matrix. The vector of source signals is the unknown vector $\boldsymbol{s}(t)$. BSS obtains the estimation of the source signals up to scales and permutation multiplications. Blind in BSS means that the information of the source signals and the mixing matrix (or the channel) are unknown.

Wide applications of BSS make itself a hot spot in signal processing and neural networks. It has developed rapidly for not more than twenty years. Solvability and algorithm mechanism of BSS are concerned as the basic theory[1−4]. Algorithms are proposed with different abilities of BSS and computational complexities[5−13]. In the previous results, ill-posed mixtures are seldom discussed. There are not effective methods to deal with ill-posed BSS problems, such as model (1) in $n > m$ case. People keep on researching, because the existing methods, such as JADE algorithm, ICA algorithms and blind extraction, cannot separate all the source signals.

In practice, many signals are sparse. Or in a suitable transformation domain, such as, Fourier or wavelet transformation domains, they can be represented as sparse signals[14]. Thus, sparse representation is applied to BSS and many novel results are reported. Especially, some ill-posed BSS examples were concerned using sparse representation. To solve some difficult problems via sparse representation is a new approach and has gained widely attention[14−21].

Bofill *et al.*[14] discussed a BSS problem with two sensors and six source signals. The paper is a pioneer in this research. However, as the authors pointed out, the potential function method proposed in the paper has some limitation. It needs to decide too many parameters. Also, there is no theory to support the decision of the parameters. The parameters are decided by experiments. Thus, this method is difficult to be extended. How to improve Bofill's method? It is one of the important problems. In this paper, instead of the potential function method, K-PCA method is proposed. It combines K-clustering with PCA. The K-PCA method can overcome the difficulty that Bofill encountered and can be applied to the case of more than three sensors.

2 Features and representation of sparse signals

What is a sparse signal? As depicted in ref. [17] intuitively, a sparse signal is the signal that is or almost is zero at most sampling points. In other words, it has larger magnitudes at few sampling points. Thus, at almost all sampling points, only one of the source signals values nonzero (priori to other signals). Usually, the signals are called to be sufficiently sparse. Eq. (1) can be written as

$$\boldsymbol{x}(t) = A\boldsymbol{s}(t) = \boldsymbol{a}_1 \cdot s_1(t) + \cdots + \boldsymbol{a}_n \cdot s_n(t). \tag{2}$$

If all the signals are sufficiently sparse, at a sampling point t, we assume that $s_i(t)$ takes priori value (i.e. the other signals have sufficient small values or are equal to zeros at this sampling point). It follows that eq. (2) approximates to the following expression:

$$\boldsymbol{x}(t) = \boldsymbol{a}_i \cdot s_i(t) \Rightarrow \frac{x_1(t)}{a_{1i}} = \cdots = \frac{x_m(t)}{a_{mi}} = s_i(t). \tag{3}$$

Obviously, eq. (3) gives a line. The sampling points of the source $s_i(t)$, at which $s_i(t)$ valued priori magnitudes, give a line. The direction vector $(a_{1i}, \cdots, a_{mi})^T$ of the line is just the ith column of the mixing matrix A. Assume that each column vector of A has norm 1, that is $\sum_{j=1}^{m} a_{ji}^2 = 1, i = 1, \cdots, n$ [15]. If every $m \times m$ sub-matrix of the mixing matrix A is invertible, every source signal will determine a line. Therefore, source signals $s_1(t), \cdots, s_n(t)$ determine n lines. For example, in ref. [14] two mixed signals are obtained from six sparse source signals. The scatter plot is given in Fig. 1. The mixing matrix is

$$A=\begin{pmatrix} 0.7071 & 0.9659 & 0.2588 & 0.9659 & 0.7071 & 0.2588 \\ 0.7071 & 0.2588 & -0.9659 & -0.2588 & -0.7071 & 0.9659 \end{pmatrix}. \tag{4}$$

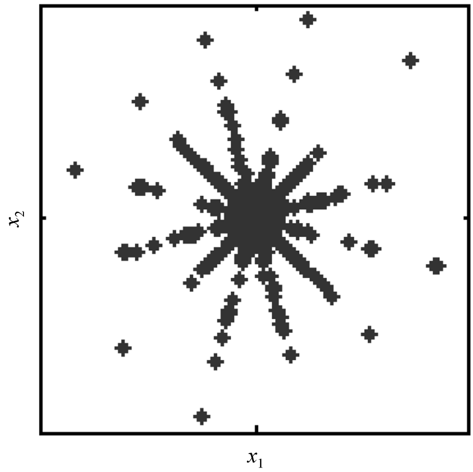

Fig. 1. Scatter plot of the six sparse signals.

From Fig. 1, one can see that the six sparse signals give six lines. Since the column vectors of the mixing matrix make the directions of the lines in the scatter plot, we can estimate the mixing matrix A via line clustering. We will discuss it later.

Signals in time domain may have no ideal sparse property. We use short-time Fourier transformation in Hanning window to transfer the signals into frequency domain. Like the method given in ref. [14], we do BSS in transformation domain. In the rest of this paper, t is frequency bin and $\boldsymbol{x}(t), t=1,\cdots,T$ are the sampling points in the transformation domain.

3 A new algorithm for BSS of sparse signals

If there exists noise, model (1) becomes $X = AS + V$. From ref. [14], BSS of sparse signals is to solve an optimization problem (5), when the signals are obey Laplacian distribution.

$$\min_{A,S} \frac{1}{2\sigma^2}\|AS-X\|^2+\sum_{i,t}|s_i(t)|, \tag{5}$$

where σ^2 is the variance of the noise V. The first term is the sum of squared error, while the second term is the penalty under the assumption of independent Laplacian source signals. Problem (5) is a multi-variables optimization problem. It is difficult to solve it directly. If A was given in ahead, problem (5) becomes

$$\min_{s(t)} \frac{1}{2\sigma^2}\|A\mathbf{s}(t)-\mathbf{x}(t)\|^2+\sum_{i}^{n}|s_i(t)|, \quad t=1,\cdots,T. \tag{6}$$

In the case of noise free, problem (6) is reduced into

$$\begin{cases} \min\limits_{\mathbf{s}(t)} \sum\limits_{i}^{n} \left|s_i(t)\right|, \\ s.t.: A\mathbf{s}(t) = \mathbf{x}(t),\ t = 1, \cdots, T. \end{cases} \tag{7}$$

Up to now, two-step approach is a main and effective approach to solve BSS problem of sparse signals[14−17]. This approach is divided into two steps: one is to estimate the mixing matrix A, the other is to solve the source signals $\boldsymbol{s}(t)$ based on the estimation of A. Problem (7) can be separated into T problems at each t, which are easier to be solved. The kernel is how to estimate the mixing matrix. Estimate of the mixing matrix A affects accuracy of the BSS problem directly. Lower accuracy of the estimate would cause failure of BSS. On the other hand, mixing matrix is unknown. How can we check the accuracy of the estimate of mixing matrix A? It is a problem.

3.1 Bofill's method to estimate the mixing matrix A

In ref. [14], Bofill proposed a method to estimate the mixing matrix A based on the potential-function. The authors gave an example of BSS with only two observed signals. The potential function is defined as

$$\Phi(\theta, \lambda) = \sum_t l_t \phi\left(\lambda\left(\theta - \theta_t\right)\right), \tag{8}$$

where $l_t = \sqrt{\left(x_1^t\right)^2 + \left(x_2^t\right)^2}$, $\theta_t = \tan^{-1}\left(x_2^t / x_1^t\right)$ and

$$\phi(\alpha) = \begin{cases} 1 - \dfrac{\alpha}{\pi/4}, & |\alpha| < \pi/4. \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Since there are only two observed signals $\boldsymbol{x}(t) = \left(x_1^t, x_2^t\right)$, $t = 1, \cdots, T$, Bofill *et al.* took $\theta_k = \pi/2K + k\pi/K$, $k = 1, \cdots, K$ and obtained $\phi\left(\theta_k, \lambda\right), k = 1, \cdots, K$. They recognized that the local maximum of the potential function is one of the column vectors of the mixing matrix. By this reason, the estimate of A was obtained.

Bofill's method is not reasonable since it is not proved theoretically. Also, this method is not convenient to be used. Parameter K could be taken almost arbitrary, and parameter λ may be taken up to users. How could the so-called "Heuristic" parameter be taken to suit a problem of BSS? There is no theory or empiricism to support this method. The parameter λ could be selected differently in each case. Moreover, the method does not suit the case of more than three observed signals. Therefore, the uncertainty makes Bofill's method difficult to estimate the mixing matrix A accurately. The conclusion is that a reasonable method for estimating the mixing matrix needs to be developed.

3.2 New method to estimate the mixing matrix A: K-PCA method

From the above discussion, to estimate the mixing matrix A for BSS can be realized by line clustering in the scatter plot of the observed signals. For simplicity, the observed signals $\boldsymbol{x}(t), t=1,\cdots,T$ should be normalized in scale and transformed in mirroring direction.

(i) Normalization.

$$\boldsymbol{x}(t)=\boldsymbol{x}(t)/\|\boldsymbol{x}(t)\|,$$

where $\|\cdot\|$ denotes Euclidean norm.

(ii) Transform in mirroring direction. A straight line can be depicted by two directions. For example, a straight line in 3-dimensional space can be depicted by direction vector $(1,2,3)$ or another vector $(-1,-2,-3)$. To depict the direction of the line by unique vector, we take mirroring mapping of the direction vectors to the positive side of the plane or sphere. For example, map vector $(-1,-2,-3)$ to $(1,2,3)$. Later, we always map the normalized lower half unit circle to the upper half unit circle in mirroring direction.

Denote $\boldsymbol{x}'(t), t=1,\cdots,T$ as the normalized signal of $\boldsymbol{x}(t), t=1,\cdots,T$. Normalization maps the points (except the origin) to the half unit circle (sphere or hyper-sphere) shown in Fig. 2(b). Each straight line maps to a unique point on the half unit circle. Thus, solving the mixing matrix A is changed into clustering the points $\boldsymbol{x}'(t), t=1,\cdots,T$ on the half unit circle. It is a standard problem of "non-spectral clustering" [22]. To this end, we use K-mean clustering based on Euclidean distance.

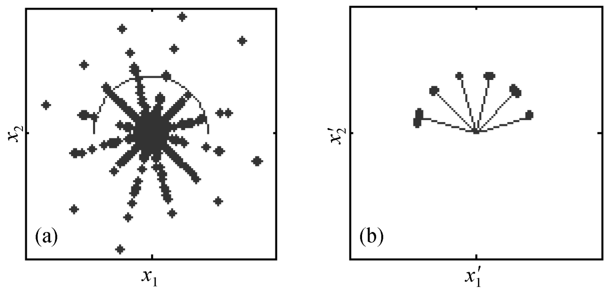

Fig. 2. Normalization of the observed signal (project on half unit circle).

Although K-mean clustering can realize classification of the T observed points $\boldsymbol{x}'(t), t=1,\cdots,T,$ it lacks accuracy of estimating direction of the lines (accordingly, the columns of matrix A) and robustness. For scatter plot in Fig. 2, there are 2 observed signals $(x_1(t), x_2(t)), t=1,\cdots,T.$ Normalize and map them into the upper half unit circle in Fig. 2(b). K-mean clustering classifies $(x_1'(t), x_2'(t)), t=1,\cdots,T$ into six classes. We obtain the estimate of mixing matrix $\hat{A}$ as (they are six centers of clustering)

$$\hat{A}=\begin{pmatrix} 0.7317 & 0.9775 & 0.2464 & 0.9095 & 0.6269 & 0.2419 \\ 0.6594 & 0.1341 & -0.9601 & -0.3892 & -0.7683 & 0.9582 \end{pmatrix}.$$

The scatter plots of the six classes are illustrated in Fig. 3. From the scatter plots, one can see that clustering gives six lines. The estimate $\hat{A}$ is different from A given by eq. (4). The second, fourth and fifth plots show errors. To enhance the accuracy, we use principle component analysis (PCA) to estimate mixing matrix A.

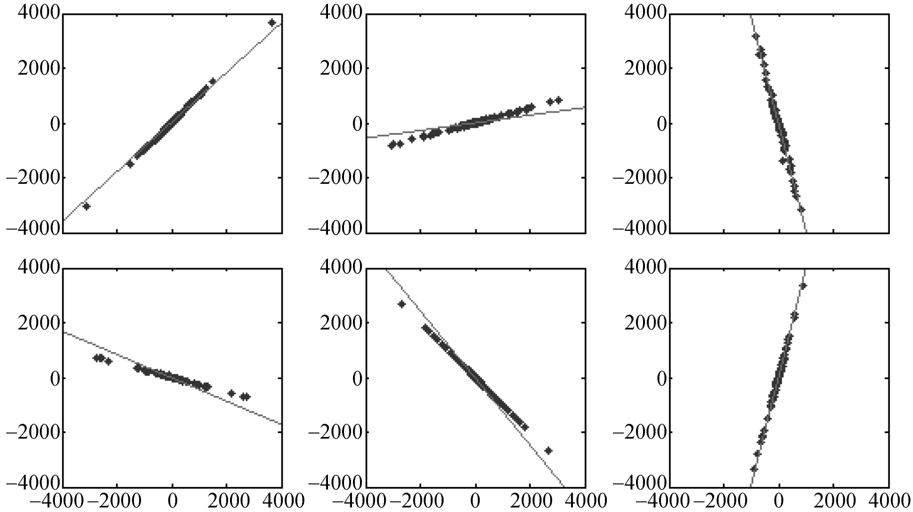

Fig. 3. Directions of lines estimated by clustering.

PCA is a classical statistical analysis[22]. It can discover the relationship of variables. Geometrically, observed signal vectors $\boldsymbol{z}(t)=(z_1(t),\cdots,z_m(t))^T$, $t=1,\cdots,T$ are T points in m-dimensional space. So, PCA of the observed signal vector $\boldsymbol{z}(t)$ is just the eigenvalue decomposition of $m\times m$ covariance matrix Σ. Without loss of generality, we assume that m eigenvalues satisfy $\hat{\lambda}_1 \geqslant \hat{\lambda}_2 \geqslant \cdots \geqslant \hat{\lambda}_m \geqslant 0$ and $\hat{\boldsymbol{a}}_1,\hat{\boldsymbol{a}}_2,\cdots,\hat{\boldsymbol{a}}_m$ are the corresponding eigenvectors. The eigenvector $\hat{\boldsymbol{a}}_1=(\hat{a}_{11},\hat{a}_{21},\cdots\hat{a}_{m1})^T$ derived from the maximal eigenvalue $\hat{\lambda}_1$ can indicate the linear relationship of the variables in vector $\boldsymbol{z}(t)$. It means that $\hat{\boldsymbol{a}}_1=(\hat{a}_{11},\hat{a}_{21},\cdots\hat{a}_{m1})^T$ is a good estimate of the lines in $\boldsymbol{z}(t)$. Doing PCA respectively to each signal of the six classes (each signal in Fig. 3), we obtain six eigenvectors. The six vectors make an estimate $\tilde{A}$ of mixing matrix A as

$$\tilde{A}=\begin{pmatrix} 0.7072 & 0.9664 & 0.2542 & 0.9656 & 0.7059 & 0.2592 \\ 0.7070 & 0.2569 & -0.9672 & -0.2601 & -0.7084 & 0.9658 \end{pmatrix}.$$

The relationship between each direction in scatter plot and the direction of the according column vector in $\tilde{A}$ is given in Fig. 4. From Figs. 3 and 4, we see that $\tilde{A}$ is more accurate than $\hat{A}$. By PCA, the estimate of the mixing matrix A becomes much more accurate.

3.3 *Trust degree criterion of the estimate of mixing matrix A*

It is important to check how accurate the estimate of mixing matrix A is, since A is unknown. In this section, we wish to give some criterion to check the trust of the estimate.

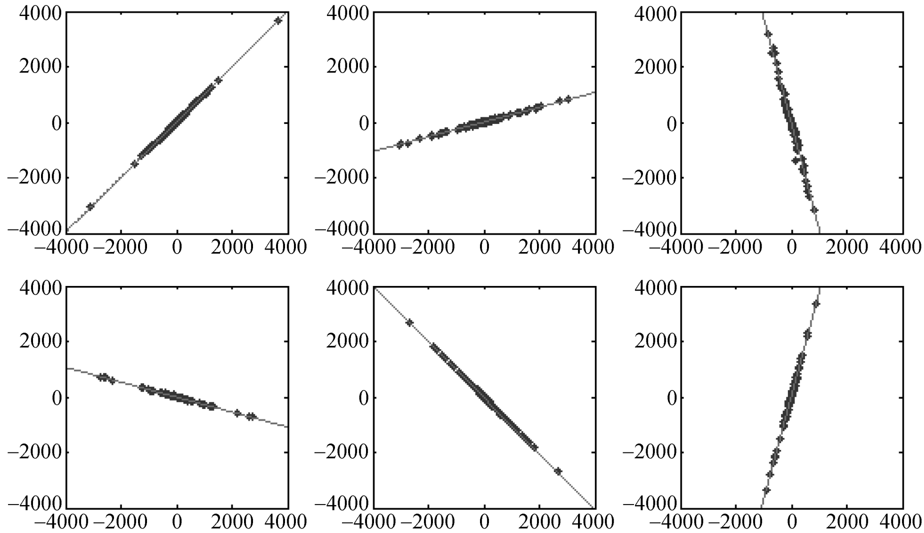

Fig. 4. Directions of lines given by PCA.

From ref. [22], the covariance matrix Σ of the signal vector $z(t)=\left(z_1(t),\cdots,z_m(t)\right)^T$ is a positive definite matrix. The *T* observed points $z(t), t=1,\cdots,T$ give an (hyper-) ellipse in scatter plot. The center of it is at $\bar{z}$. The main axis is with the direction of Σ. Each axis is in the positive proportion of $\sqrt{\hat{\lambda}_i}, i=1,\cdots,m$ respectively. If variables $z_1(t),\cdots,z_m(t)$ are collinear strongly, the maximal eigenvalue $\hat{\lambda}_1$ will be sufficiently greater than (or much more greater than) the rest eigenvalues $\hat{\lambda}_2,\cdots,\hat{\lambda}_m$. In this case, the (hyper-)ellipse becomes very thin and approximates to a line. Otherwise, if collinear relationship of the vectors $z_1(t),\cdots,z_m(t)$ is not strong enough, the maximal eigenvalue $\hat{\lambda}_1$ is not remarkably greater than $\hat{\lambda}_2,\cdots,\hat{\lambda}_m$. In this case, the ellipse becomes almost a sphere.

Next, we define an "intensity of line" to check whether the *T* samples of the signal vector $z(t), t=1,\cdots,T$ approximately lay on a line, or how they approximate a line.

Definition. Let covariance matrix of *m*-dimensional signal vectors $z(t), t=1,\cdots,T$ be Σ. Suppose that the *m* eigenvalues of Σ satisfy $\hat{\lambda}_1 \geqslant \hat{\lambda}_2 \geqslant \cdots \geqslant \hat{\lambda}_m \geqslant 0$. The linear intensity $Q(z(t))$ for vector $\mathbf{z}(t)$ is defined as

$$Q(z(t)) = 1 - \frac{\sqrt{\hat{\lambda}_2}}{\sqrt{\hat{\lambda}_1}}. \tag{10}$$

It is obvious to see that $0 \leqslant Q(z(t)) \leqslant 1$. When samples $z(t), t=1,\cdots,T$ tend to lie on a line, we have $Q(z(t)) \to 1$. Otherwise, it approaches to 0. Key problem of BSS for sparse signals is that the scatter plots of the mixed signals appear as lines. BSS for sparse signals would failure if the plots of $\boldsymbol{x}(t)$ were not lines. Thus, the linear intensity should be checked.

The T sampling points of observed signal vector $\boldsymbol{x}(t)$ are classified into n classes. Each class is according to a source signal. Denote every class of the sampling points is $\boldsymbol{x}^i(t), t=1,\cdots,T^i, i=1,\cdots,n$. If source signals are sufficiently sparse and the mixing matrix A is estimated accurately, each class of sampling points $\boldsymbol{x}^i(t), t=1,\cdots,T^i, i=1,\cdots,n$ appears as a line. The linear intensity of this class will satisfy $Q(\boldsymbol{x}^i(t)) \to 1$. Contrarily, when $Q(\boldsymbol{x}^i(t))$ is 0 approximately, we judge that the clustering is not accurate or the source signals are not sufficiently sparse. It may indicate that the estimate of mixing matrix A may be unsure. Therefore, linear intensity is used to judge the trust degree of the estimate of mixing matrix A. Then, reliability of BSS can be judged. From simulation, BSS will achieve a good effect when $Q(\cdot) > 0.6$.

3.4 Algorithms of BSS for sparse signals

A BSS algorithm is proposed in the following procedures:

1. If the signals are not sufficiently sparse, do Fourier transform (e.g., windowed Fourier transform) or wavelet transform of the signals. Generally, this procedure can transfer the signals in to sparse signals in some transform domain. Without loss of generality, we suppose that the observed signals $\boldsymbol{x}(t), t=1,\cdots,T$ are sufficiently sparse.

2. Normalize the observed signals $\boldsymbol{x}(t), t=1,\cdots,T$ by use the method given in section 3.2. It becomes $\boldsymbol{x}'(t), t=1,\cdots,T$.

3. Do K-clustering of $\boldsymbol{x}'(t), t=1,\cdots,T$ based on Euclidean distance. It gives n classes of $\boldsymbol{x}'(t)$. Accordingly, $\boldsymbol{x}(t), t=1,\cdots,T$ is divided into n classes $\boldsymbol{x}^i(t)$, $t=1,\cdots,T^i, i=1,\cdots,n$. Each class is according to a source signal, respectively.

4. Do PCA of $\boldsymbol{x}^i(t), t=1,\cdots,T^i, i=1,\cdots,n$ respectively. According to the ordered eigenvalues we have n eigenvectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n$. Then the estimate of the mixing matrix is $\hat{A} = (\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n)$.

5. Solve the optimization problem by linear programming

$$\begin{cases} \min\limits_{\boldsymbol{s}(t)} \sum\limits_i |s_i(t)|, \\ s.t.: \hat{A}\boldsymbol{s}(t) = \boldsymbol{x}(t), t=1,\cdots,T. \end{cases}$$

Taking transform

$$s_i^+(t) = \begin{cases} s_i(t), & s_i(t) > 0, \\ 0, & s_i(t) \leqslant 0, \end{cases} \qquad s_i^-(t) = \begin{cases} -s_i(t), & s_i(t) < 0, \\ 0, & s_i(t) \geqslant 0, \end{cases} \tag{11}$$

where $s_i^+(t)$ and $s_i^-(t)$ are nonnegative, $s_i(t)=s_i^+(t)-s_i^-(t)$, $|s_i(t)|=s_i^+(t)+s_i^-(t)$, thus the above problem is transferred into a linear programming

$$\begin{cases} \min\limits_{s^+(t),s^-(t)} \sum\limits_i \left(s_i^+(t)+s_i^-(t) \right) \\ \text{s.t.}: (\hat{A},-\hat{A}) \begin{pmatrix} \boldsymbol{s}^+(t) \\ \boldsymbol{s}^-(t) \end{pmatrix} = \boldsymbol{x}(t),\ t=1,\cdots,T. \end{cases} \tag{12}$$

6. Solve linear programming (12) to give n sparse source signal in the transform domain $\hat{\mathbf{s}}(t)=(\hat{s}_1(t),\cdots,\hat{s}_n(t))^T, t=1,\cdots,T$.

7. Take inverse Fourier transform or inverse wavelet transform to reconstruct the source signals.

8. Check the trust degree of the result. For classes $\boldsymbol{x}^i(t), t=1,\cdots,T^i, i=1,\cdots,n$, we compute linear intensities $Q(\boldsymbol{x}^i(t)), i=1,\cdots,n$, respectively. If $Q(\boldsymbol{x}^i(t)), i=1,\cdots,n$ are large, the trust degree of BSS is high. Otherwise, the trust degree of BSS is low.

4 Simulations and discussion

To check effectiveness of the results, some simulations are given here, in different cases.

4.1 Index of performances

To check the performance of the BSS algorithm, we use the correlative coefficient of the source signals and the separated signals. From probability and statistics, the correlative coefficient will approximate to 1 or −1, if the algorithm is effective. Otherwise, it will approximate to 0. Moreover, we use the SNR S/N proposed in Bofill's paper[14] to check the performance. The *S/N* is defined as

$$S/N = -10\log\frac{\|\hat{s}-s\|^2}{\|s\|^2}, \tag{13}$$

where $\hat{s}$ is the estimate of s. We adjust the scales of $\hat{s}$ and s so as to have the same energy. Then we compute *S/N* by formula (13). Big S/N means good effect of BSS. Generally, an algorithm of BSS is effective when $S/N \geqslant 25\,\text{dB}$.

4.2 Simulations

Since signals in time domain may be not sparse ideally, we use frame Hanning windowed Fourier transform[14] for sound signals in the following three simulations. Also, we do BSS in a transform domain for the real and image parts of the transformed signals.

Simulation 1. BSS of four voice signals from three mixed signals.

The four voice signals are taken from http://personals.ac.upc.edu/pau/shpica/instant.html (see Fig. 5). The sampling rate is 22050 Hz. There are 65536 sampling points.

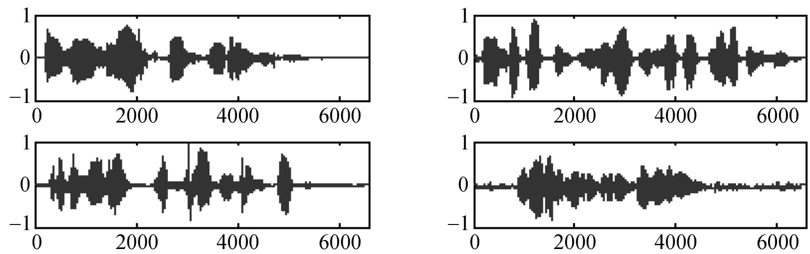

Fig. 5. Source signals.

Mixing matrix is obtained by MATLAB command rand (‘state’, 8); A = rand(3,4)–0.5 (see Fig. 6).

$$A=\begin{pmatrix} -0.4660 & -0.2186 & 0.1009 & 0.2584 \\ -0.0450 & 0.4909 & 0.3829 & 0.1737 \\ -0.3714 & -0.2444 & 0.4658 & -0.3423 \end{pmatrix}.$$

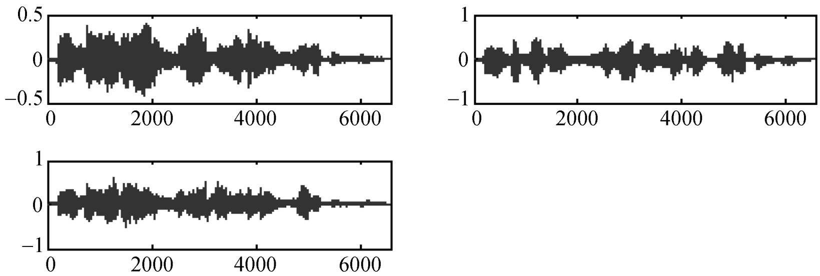

Fig. 6. Mixed signals.

As in ref. [14], we take Fourier transform of three mixed signals in piecewise. To each piece with $L=2048$ we multiple a Hanning window. The overlap between the continuous pieces measured by length $d=614$. Normalize A into $\tilde{A}$ using the method proposed in section 3.2.

$$\tilde{A}=\begin{pmatrix} 0.7798 & 0.3703 & 0.1650 & 0.5585 \\ 0.0753 & -0.8316 & 0.6263 & 0.3753 \\ 0.6215 & 0.4139 & 0.7619 & -0.7398 \end{pmatrix}.$$

Then, we obtain the estimate of A as

$$\hat{A}=\begin{pmatrix} -0.4543 & 0.7472 & 0.3642 & 0.2018 \\ -0.4887 & 0.0476 & -0.8499 & 0.6597 \\ 0.7449 & 0.6629 & 0.3810 & 0.7239 \end{pmatrix}.$$

Based on this estimate $\hat{A}$, we obtain the reconstruct signals shown in Fig. 7.

The intensity of the four lines can be computed as $Q(\boldsymbol{x}^1(t))=0.6139$, $Q(\boldsymbol{x}^2(t))=0.7884$, $Q(\boldsymbol{x}^3(t))=0.7826$, $Q(\boldsymbol{x}^4(t))=0.7716$. Big intensities of the lines cause the clear lines in the scatter plot. Then, the estimate of A is accurate. Comparing corresponding column vectors of $\tilde{A}$ and $\hat{A}$, we can see the accuracy. On the other hand, SNRs between source signals and separated signals S/N are listed as 26.1741, 26.3476, 29.2017 and 29.5396 dB. The correlative coefficient matrix is

$$\begin{pmatrix} 0.1276 & -0.9659 & -0.0771 & -0.0732 \\ -0.0746 & -0.1202 & -0.9630 & 0.0686 \\ 0.1444 & 0.0993 & -0.0991 & 0.9745 \\ -0.9705 & 0.0372 & 0.1164 & -0.0298 \end{pmatrix}.$$

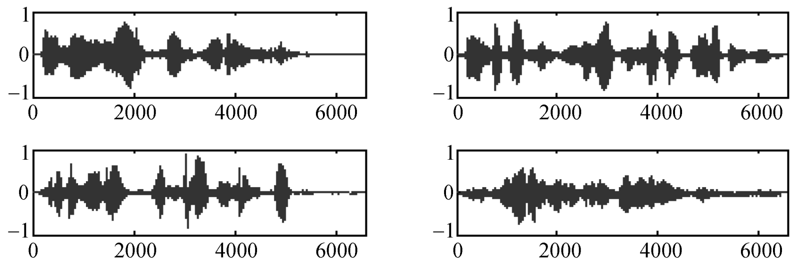

Fig. 7. Separated signals.

Every index illustrates the good effective of our method.

Simulation 2. Small linear intensity will cause failure of BSS.

Using the same source signals and same mixing matrix as them in Simulation 1, we do this simulation. We use $d = 1024$ to replace the d selected in Simulation 1. The parameter $L = 2048$ and Hanning window are the same as in Simulation 1. In this case, different parameters cause less sparse transformed signals. This then causes worse estimate of A and effect of BSS. In this case, the linear intensities are 0.5425, 0.7893, 0.6521 and 0.5457. The estimate $\hat{A}$ of mixing matrix A is

$$\hat{A} = \begin{pmatrix} -0.2616 & 0.3821 & -0.4579 & 0.5396 \\ 0.6422 & -0.8131 & -0.5500 & 0.3753 \\ 0.7205 & 0.4392 & 0.6985 & 0.7536 \end{pmatrix}.$$

The correlative coefficient matrix between source signals and separated signals can be computed as

$$\begin{pmatrix} -0.0011 & -0.4555 & 0.4024 & -0.7681 \\ 0.0851 & -0.8624 & -0.0271 & -0.0044 \\ 0.8958 & -0.1501 & 0.0640 & 0.6314 \\ -0.2404 & 0.0760 & -0.9005 & 0.0269 \end{pmatrix}.$$

Comparing $\hat{A}$ with $\tilde{A}$ in Simulation 1, we found that the estimate of mixing matrix is not accurate in this simulation. The SNRs S/N are also small, they are 7.1217, 13.2593, 14.4531 and 17.5622 dB. The separated signals are shown in Fig. 8. All of Fig. 5, Fig. 8, the coefficient matrix and the SNRs S/N show the worse result of BSS in this simulation. In many cases, the sparsity of signals is worse than this simulation. Then, the result of BSS will be not satisfied. This can be checked by compute the linear intensity.

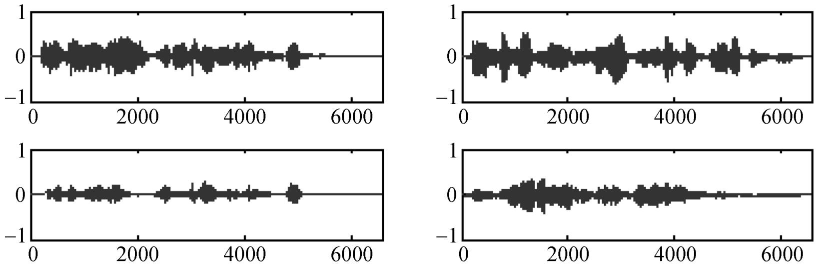

Fig. 8. Separated signals.

Simulation 3. Effect of BSS from over-estimate/under-estimate of the source number.

Practically, the number of source signals may not be known. Generally, this number could not be determined exactly (it is an open problem that all the sparse representation-based methods faced). This simulation will show that under-estimate of the source number affects BSS negatively. Contrarily, over-estimate of the source number may not affect BSS.

Preconditions of this simulation are the same as that in Simulation 1. Also, we use the same method as in Simulation 1. There are four source signals. They should be classified correctly into four classes by clustering. If the clustering gives 3 classes (less than the correct number), 3 source signals are separated. The corresponding estimate $\hat{A}$ of mixing matrix A is obtained.

$$\hat{A} = \begin{pmatrix} 0.5393 & -0.4535 & 0.3828 \\ 0.3740 & -0.5066 & -0.8273 \\ 0.7545 & 0.7333 & 0.4110 \end{pmatrix}.$$

The three linear intensities are $Q(\boldsymbol{x}^1(t)) = 0.5451$, , $Q(\boldsymbol{x}^2(t)) = 0.5958$, $Q(\boldsymbol{x}^3(t)) = 0.7447$. We found two of them are very small. Then, the SNRs S/N are also small. They are 3.5320, 14.1039 and 9.4660 dB. The correlative coefficient matrix is

$$\begin{pmatrix} 0.6600 & 0.3361 & -0.3943 \\ 0.0000 & -0.0126 & -0.7981 \\ 0.7543 & 0.3801 & -0.4341 \\ -0.0113 & -0.8627 & 0.1252 \end{pmatrix}.$$

This worse effect implies that the BSS is not achieved.

If clustering over-estimates the source number as 5, it classifies 5 classes. The estimate $\hat{A}$ of mixing matrix A is given as

$$\hat{A} = \begin{pmatrix} 0.7557 & 0.3676 & -0.2187 & -0.4532 & 0.1959 \\ 0.0608 & -0.8334 & 0.6007 & -0.5421 & 0.6484 \\ 0.6521 & 0.4126 & 0.7690 & 0.7076 & 0.7357 \end{pmatrix}.$$

So, 5 source signals are separated as in Fig. 9. The linear intensities are $Q(\boldsymbol{x}^1(t)) = 0.8076$, $Q(\boldsymbol{x}^2(t)) = 0.8271$, $Q(\boldsymbol{x}^3(t)) = 0.5797$, $Q(\boldsymbol{x}^4(t)) = 0.6632$, $Q(\boldsymbol{x}^5(t)) = 0.8111$.

The four big intensities among them give an almost accurate estimate of mixing matrix. The SNRs S/N are 23.3221, 28.7958, 3.3280, 30.2371 and 17.4069 dB.

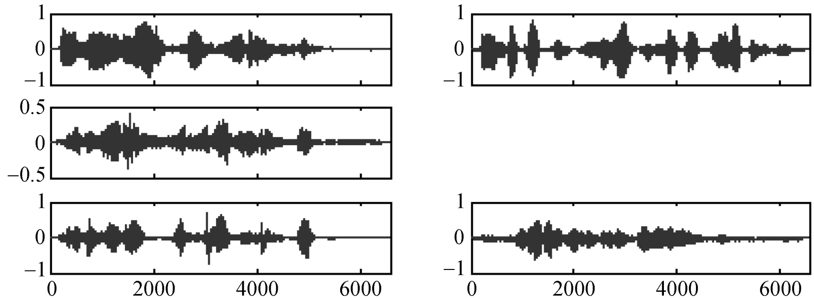

Fig. 9. Separated signals.

The correlative coefficient matrix is

$$\begin{pmatrix} -0.9536 & -0.0325 & 0.0853 & 0.0784 & -0.0766 \\ -0.1213 & -0.9710 & 0.0716 & -0.1100 & 0.1025 \\ 0.1607 & -0.0611 & 0.6487 & 0.0195 & 0.9100 \\ 0.0877 & -0.0135 & -0.5304 & -0.9697 & 0.0615 \end{pmatrix}.$$

From linear intensity, SNR S/N and correlative coefficient matrix, we can obtain that the third column in matrix $\hat{A}$ is not what we need. The corresponding signal is redundant. In Fig. 9, the signal in the second row is redundant. Therefore, we conclude that over-estimate can cause success of BSS while under-estimate causes failure of BSS.

5 Conclusion

This paper proposed a method to estimate the mixing matrix of BSS model for sparse source signals. It drives an effective BSS. Our method is easy to be implemented. Comparing with the method proposed in ref. [14], our new method suits the case of more than three observed signals. By computing linear intensities of the lines, one can check the reliability of BSS. The sparsity of the source signals extremely affects accuracy of BSS. If the sparse precondition is not satisfied, one can do transforms, such as Fourier transform or wavelet transform, first, and then, do BSS in the transform domain. However, it is really difficult to seek a suitable transform domain to give a sparse representation of the signals. It is a problem to be studied further.

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