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Dynamic multi-objective intelligent optimal control toward wastewater treatment processes

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Wastewater treatment plays a crucial role in alleviating water shortages and protecting the environment from pollution. Due to the strong time variabilities and complex nonlinearities within wastewater treatment systems, devising an efficient optimal controller to reduce energy consumption while ensuring effluent quality is still a bottleneck that needs to be addressed. In this paper, in order to comprehensively consider different needs of the wastewater treatment process (WTTP), a two-objective model is to consider a scope, in which minimizing energy consumption and guaranteeing effluent quality are both considered to improve wastewater treatment efficiency. To efficiently solve the model functions, a grid-based dynamic multi-objective evolutionary decomposition algorithm, namely GD-MOEA/D, is designed. A GD-MOEA/D-based intelligent optimal controller (GD-MOEA/D-IOC) is devised to achieve tracking control of the main operating variables of the WTTP. Finally, the benchmark simulation model No. 1 (BSM1) is applied to verify the validity of the proposed approach. The experimental results demonstrate that the constructed models can catch the dynamics of WWTP accurately. Moreover, GD-MOEA/D has better optimization ability in solving the designed models. GD-MOEA/D-IOC can achieve a significant improvement in terms of reducing energy consumption and improving effluent quality. Therefore, the designed multi-objective intelligent optimal control method for WWTP has great potential to be applied to practical engineering since it can easily achieve a highly intelligent control in WTTP.

wastewater treatment processes, evolutionary algorithms (EAs), multi-objective optimization, performance functions

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1 Introduction

During recent decades, frustrating environmental change has appeared due to the rapid economic development and excessive pursuit of industrialization. Among all the environmental problems, water pollution, which is regarded as one of the important factors endangering people's health, has been attracted special attention. It is an urgent task to remove water pollutants and improve water quality $[1-3]$ $[1-3]$. Hence, devising an advanced and effective controller towards was-

Several related works were conducted to investigate the control of EC in WWTPs. A data-driven fuzzy controller was designed for dissolved oxygen control [\[7\]](#page-10-5). A proportionalintegral intelligent controller was proposed to reduce the EC

tewater treatment processes (WWTPs) was considered [[4,](#page-10-2)[5](#page-10-3)]. An unavoidable problem is that it is difficult to guarantee the effluent quality in the conventional WWTPs with nonlinear and time-varying while reducing energy consumption (EC) [\[6\].](#page-10-4) How to develop an advanced intelligent optimal controller to deal with the time-varying and complex WWTPs has been regarded as an important research direction in the field of eliminating water pollution and saving energy.

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of the system operation [\[8\]](#page-10-6). An aeration controller integrated by a feedback strategy and a feed forward strategy was developed, which could maintain the stability of the system while ensuring the dissolved oxygen concentration [\[9\]](#page-10-7). A model-free dissolved oxygen set-point autonomous tracking control method was devised, which utilized reinforcement learning technology to adjust dissolved oxygen concentration in real time according to the dynamic process of the wastewater treatment $[10]$. A pumping system based on data mining was applied to balance the wastewater flow rate and pumping energy [\[11\].](#page-10-9) The case study was adopted to calibrate the dynamic model and then more accurately predict pumping EC $[12]$. It is pretty confirmed that the intelligent optimal control for reducing EC will still be a hot topic of advanced control design.

Although many optimal controllers have been designed to reduce system EC, limited efforts have been devoted to improving effluent quality (EQ). An integrated fuzzy logic model was developed to enhance the prediction of key parameters in ref. [\[13\]](#page-10-11). A novel quantitative image analysis method was proposed to evaluate the main parameters of EQ in ref. [\[14\]](#page-10-12). An effective EQ controller was derived based on the relative gain array analysis in ref. $[15]$. A genetic algorithm-based deep belief network model was applied to improve the prediction accuracy of EQ in ref. [\[16\]](#page-10-14). Besides, in ref. [\[17\],](#page-10-15) a model-based optimal control strategy was adopted to improve the EQ.

To some extent, the above-described works have naturally inspired the idea of designing a control system that can strike a balance between ensuring EQ and reducing EC. Although the idea is very attractive, it is not an easy task but full of challenges and difficulties. To date, only a few attempts have been devoted in this way. A multi-criteria selection strategy was proposed to optimize the set-points of the control variables in order to reduce the operation cost and improve the EQ [\[18\].](#page-10-16) A locally weighted learning scheme was applied to WWTPs to realize the monitoring of the nonlinear process control [\[19\].](#page-10-17) A novel decision intelligent control method was presented in ref. [\[20\]](#page-10-18) to improve the EQ and reduce the operation costs. Note that those approaches take into account EC and EQ, but it is difficult for the above control methods to obtain appropriate set-points of the manipulated variables.

Evolutionary multi-objective optimization, as a branch of computational intelligence, has been developed rapidly in the past decades due to its fast convergence and powerful global exploration ability $[21-26]$ $[21-26]$. To overcome the problems of premature convergence and low search ability, ref. [\[27\]](#page-10-21) proposed an improved quantum-inspired differential evolution algorithm with multi-population mutation evolution and solution space transformation strategies. Large-scale optimization faces problems such as loss of diversity and low validity of solutions. Deng et al. [\[28\]](#page-10-22) designed a novel

method that integrates cooperative coevolution and hybrid mutation strategies to improve the optimization performance. The unique advantage of multi-objective optimization algorithms in obtaining the trade-off solutions of conflicting objective functions provides a new perspective for solving traditional engineering problems [[29–](#page-10-23)[33\]](#page-10-24).

Gate resource assignment determines the efficient connection between flights, Deng et al. [\[34\]](#page-10-25) devised an enhanced adaptive particle swarm optimization algorithm to solve the multi-objective gate assignment problem. The experimental results demonstrate that the designed approach can achieve efficient gate allocation. The multi-objective differential evolution algorithm was designed to solve the performance functions to obtain the appropriate set-points in ref. [\[35\].](#page-10-26) The experimental results show that the proposed method can effectively improve the performance of wastewater treatment on the premise of reducing EC. A multi-objective optimal control system was applied to WWTPs to achieve a balance between EQ and operating cost in ref. [\[36\]](#page-10-27). An evolutionary algorithm based on non-dominated sorting was developed for WWTPs, which decreased the EC and improved the EQ while reducing greenhouse gas emissions [\[37\]](#page-11-0). A genetic algorithm was proposed to obtain the optimal process parameters to minimize the EC while maintaining the EQ in ref. [\[38\].](#page-11-1) Note that the competition results were obtained by the aforementioned control systems.

Wastewater treatment is a complex dynamic process with non-linear characteristics. The models based on conventional reaction mechanisms are difficult to capture the real timevarying characteristics. In addition, the general multi-objective evolutionary algorithms face difficulties in solving dynamic multi-objective optimization problems (DMOPs) to track the changing Pareto optimal front efficiently and accurately. Dynamic multi-objective evolutionary algorithms (DMOEAs) have a fast convergence performance and can address diversity loss when the evolutionary environment changes. Therefore, based on the analysis of WWTP, it is worthwhile to design a dynamic multi-objective evolutionary algorithm to solve the performance functions of wastewater treatment with time-varying characteristics.

In this paper, a novel dynamic multi-objective intelligent optimal control method, built on a grid-based dynamic multiobjective evolutionary decomposition algorithm (GD-MOEA/D-IOC), is developed to improve the control performance of WWTPs. The novelties and contributions of the proposed GD-MOEA/D-IOC contain the following parts.

(1) A two-objective model based on considering the objective functions of the minimizing EC and guaranteeing effluent quality is constructed. The relevant process variables are extracted as the decision variables of the performance functions, so as to reflect the characteristics of WWTPs accurately.

(2) A novel DMOEA based on grid partitioning strategies

is proposed to efficiently solve the constructed model function to obtain reliable optimal solutions.

(3) A GD-MOEA/D-based intelligent optimal controller is designed to detail the control operation of WWTPs. Simulation results on the BSM1 further demonstrate that the designed intelligent control method can achieve remarkable improvements in control performance.

The rest of this paper is outlined as follows. Section 2 illustrates the basic characteristics concerning WWTPs and the core knowledge of MOEA/D. Dynamism handling techniques are also presented in this section. The details of the proposed GD-MOEA/D-IOC are presented in Section 3, together with the multi-objective operational indices optimization problem, the GD-MOEA/D algorithm, and the intelligent optimal control scheme. In Section 4, the effectiveness of our proposed GD-MOEA/D-IOC is validated by comparing it with other control methods. Finally, the conclusion is given in Section 5.

2 Related work

2.1 Benchmark study on WWTPs

Wastewater

Primary

treatment

Sludge

Dynamic wastewater treatment systems play an increasingly critical role in protecting the environment and recycling resources when coping with modern urban diseases. However, WWTPs are complex dynamic processes with non-linear characteristics, which contain a series of nitrification and denitrification reactions. It brings a tough challenge for the application of control strategies in the real dynamic wastewater treatment systems. An effective method to overcome this drawback is to devise a benchmark simulation model to verify the design of the control systems. Therefore, benchmark simulation model No. 1 (BSM1), as a typical benchmark platform for wastewater treatment is adopted in this paper. The basic schematic layout of the BSM1 plant is depicted in [Figure 1.](#page-2-0)

Pump

The BSM1 plant is integrated with five activated sludge reaction units consisting of 2 anoxic units followed by 3 aerated units. The two anoxic units mainly carry out denitrification chemical reactions, and nitrite nitrogen (NO_2^-) or nitrate nitrogen (NO_3) is reduced into N_2 by anaerobic bacteria. The nitrification reaction is mainly carried out in that three aerated units. The nitrifying bacteria convert ammonium nitrogen (NH₄⁺) into NO₂⁻ and NO₃⁻ under aerobic conditions. Note that the oxygen transfer coefficients of the third and fourth units are usually fixed, and the oxygen transfer coefficient of the fifth unit $(K_ia₅)$ is automatically adjusted by the controller according to the actual situation. The dissolved oxygen concentration of the fifth unit $(S_{0.5})$ plays a critical role in the performance of wastewater treatment. If the dissolved oxygen concentration is too low, it is not conducive to the nitrification reaction, so that the discharged water contains a large amount of NH_4^+ . In contrast, if the dissolved oxygen concentration is at a high level, part of the $O₂$ does not fully participate in the nitrification reaction, so that it flows into the first anoxic unit in the internal circulation process, which is not conducive to the denitrification operation of wastewater. In addition, the internal recycle flow rate (Q_a) is also a vital factor affecting the anaerobic reaction. Therefore, it is of great significance to design a reasonable oxygen transfer coefficient of the fifth unit and a proper internal recycle flow rate for improving the performance of wastewater treatment systems.

Furthermore, the fifth unit is followed by a secondary settler. The wastewater flows into the secondary settler after biochemical reaction, the upper clarified water in the secondary settler can be recycled for irrigation, and so on. One part of the sludge discharged from the lower of the secondary settler is buried or made into organic fertilizer after being dried, and the remaining part of the sludge flows into the first unit through external circulation. Note that the wastewater treatment is a complex, non-linear, and time-varying process with multiple variables, and the two performance indicators

Secondary settler

Effluent water

Blower

 $-K_1a_5$

Unit₅

Biochemical reactor

Unit₃

Unit₄

Unit₂

Unit1

of EC and EQ conflict with each other. Therefore, the intelligent optimal controller should be developed to balance the conflicted relationship and meet the requirements of reducing EC under the premise of ensuring EQ.

2.2 MOEA/D

As a branch of computational intelligence, a multi-objective evolutionary algorithm based on decomposition (MOEA/D) is a kind of evolutionary algorithm with low computational complexity, which can achieve good exploration and exploitation. MOEA/D applies decomposition and collaborative optimization strategies to solve the multi-objective optimization problems (MOPs). In order to facilitate the reader's understanding, we leverage a simple schematic diagram to explain this process, as shown in [Figure 2](#page-3-0). In each iteration of population evolution, MOEA/D first decomposes an MOP into *N* scalar optimization sub-problems (*N* represents the number of individuals in the population), and then uses the information of its adjacent subspaces to achieve cooperative optimization for each sub-problem so as to obtain the optimal solutions of the sub-problem.

The commonly utilized decomposition strategies include the weighted sum decomposition approach, the Tchebycheff decomposition approach, and the penalty-based boundary intersection (PBI) approach. In this paper, the PBI approach is used to decompose an MOP into *N* scalar optimization subproblems and each subproblem is defined as

$$
\min g^{\text{pbi}}(\mathbf{x}|\lambda, \mathbf{z}^*) = d_1 + \theta d_2, \n\text{subject to } x \in \Omega,
$$
\n(1)

where

$$
d_1 = \frac{\left\| \left(\mathbf{f}(\mathbf{x}) - \mathbf{z}^* \right)^T \lambda \right\|}{\left\| \lambda \right\|},\tag{2}
$$

$$
d_2 = \left\| \mathbf{f}(\mathbf{x}) - \left(\mathbf{z}^* + d_1 \frac{\lambda}{|\mathcal{U}|} \right) \right\|, \tag{3}
$$

and θ is the preset penalty factor. $\mathbf{z}^* = (z_1^*, \dots, z_M^*)^T$ denotes the reference point and *M* is defined as the number of objective functions. $\lambda = (\lambda_1, \lambda_2, ..., \lambda_M)^T$ is the weight vector, where λ_i satisfies $\lambda_i \geq 0$ and $\sum \lambda_i = 1$ *i M* $\sum_{i=1}^{\infty} \lambda_i = 1$. *L* is the projection of **f**(**x**) onto the corresponding weight vector λ , where **f**(**x**)= $(f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_M(\mathbf{x}))$ ^T. d_2 is the vertical distance between **f**(**x**) and *L*. d_1 is the straight distance from z^* to *L*. The iteration process continues until the number of iterations reaches a prespecified number.

When solving MOPs, MOEA/D can make the solution individuals in the population quickly and evenly distributed in the Pareto optimal front. This advantage of MOEA/D brings a new way for maintaining the dynamic balance of $K_{L}a_{5}$ and Q_{a} in WWTPs, so as to ensure that $S_{0,5}$ and the nitrate level in the second unit $(S_{NO₂)$ are maintained within a reasonable range, and also improve the overall performance of wastewater treatment plants.

2.3 Dynamism handling

The WWTP is an optimal control process with dynamic characteristics. EC and EQ form a two-objective dynamic optimization problem. Various algorithms are designed to solve DMOPs. A general framework of DMOEAs is pre-sented in [Algorithm 1.](#page-4-0) It can be clearly observed that when

[Figure 2](#page-3-0) (Color online) Basic schematic of MOEA/D.

Algorithm 1 A general framework of dynamic multiobjective evolutionary algorithm (DMOEA)

the environment change is detected, DMOEA algorithms are applied to generate the initial solutions in the new environment. These algorithms are classified as population-based, diversity-based, and prediction-based algorithms.

To enhance the optimization performance of evolutionary algorithms in dynamic environments, Zhou et al. [\[39\]](#page-11-2) designed a population-based prediction strategy. The proposed strategy takes the previously obtained center points and manifolds to predict the center points and manifolds in the new environment. Conventional DMOEAs suffer performance degradation under rapidly changing environments, Li et al. [\[40\]](#page-11-3) proposed a network-based population prediction strategy. When an environmental change is detected, a neural network is adopted to generate a subset of the initial individuals.

The population diversity strategy can effectively avoid the algorithm from being trapped into local optimal in the process of evolution $[41, 42]$ $[41, 42]$ $[41, 42]$ $[41, 42]$ $[41, 42]$. To improve prediction accuracy, Ruan et al. [\[43\]](#page-11-6) devised an algorithm that uses generated random individuals to increase the diversity of the population. To keep the balance of population diversity and convergence, Liang et al. [\[44\]](#page-11-7) developed a classification method for decision variables. The method classifies decision variables into different sets according to the evolutionary state, so that the different strategies can be used to optimize these variables and thus achieve a balance between population diversity and convergence.

Prediction-based approaches have gained particular attention in recent years [[45–](#page-11-8)[48\]](#page-11-9). To handle DMOPs efficiently, Jiang and Yang [\[49\]](#page-11-10) designed a steady-state and generational evolutionary algorithm to improve the performance of handling DMOPs. The algorithm predicts the initial individuals in a new environment according to the moving direction and movement stepsize. To achieve accurate tracking of the changing Pareto front, Zou et al. [\[50\]](#page-11-11) presented a special points-based predictive strategy that can eliminate useless individuals while maintaining population diversity and convergence. To overcome the data imbalance in the knowledge transfer process, Jiang et al. [\[51\]](#page-11-12) devised a knee point-based transfer learning method to generate highquality individuals. In conclusion, prediction-based DMOEAs are gradually becoming dominant techniques for solving DMOPs.

3 Dynamic multi-objective intelligent optimal control design

This section is mainly devoted to the construction of the dynamic multi-objective intelligent optimal control system for WWTPs. First, the performance functions are established by analyzing the relationship between the relevant variables of WWTPs. Second, the GD-MOEA/D algorithm is developed based on the MOEA/D framework. Furthermore, the integrated intelligent optimal controller is devised.

3.1 Design of performance functions

EQ and EC are two important principles to evaluate the performance of wastewater treatment control systems. Since wastewater treatment is a complex non-linear process with time-varying characteristics, the conventional EC and EQ models could not catch the nonlinear and dynamic characteristics of wastewater treatment processes accurately. The kernel function has become an effective method for solving nonlinear function modeling problems. The models of the multiple conflicting criteria, based on the adaptive kernel function modeling approach, are constructed to design the performance functions in accordance with the dynamic characteristics of WWTPs.

WWTPs comprise a plurality of biochemical reactions. Nitrification and denitrification are two important biochemical reactions that determine EQ and EC. Through the principal component analysis of the substances involved in the biochemical reaction process, the important process variables are extracted as the input variables of the model, while EC and EQ are used as the output variables of the model. The functional relationships between input variables and output variables are established by kernel functions. The constructed minimized dynamic performance function can be formulated as

Minimize
$$
\mathbf{f}(\mathbf{x},t) = (f_1(\mathbf{x},t), f_2(\mathbf{x},t))^T, \tag{4}
$$

where

$$
f_1(\mathbf{x}, t) = \sum_{j=0}^{r} \omega_{1j}(t) \cdot \exp\left(-\frac{\left\|\mathbf{x}(t) - c_{1j}(t)\right\|^2}{\sigma_{1j}^2(t)}\right),\tag{5}
$$

$$
f_2(\mathbf{x},t) = \sum_{j=0}^r \omega_{2j}(t) \cdot \exp\left(-\frac{\left\|\mathbf{x}(t) - c_{2j}(t)\right\|^2}{\sigma_{2j}^2(t)}\right),\tag{6}
$$

where $f_1(\mathbf{x},t)$ and $f_2(\mathbf{x},t)$ represent the mathematical model

expressions of EC and EQ in the dynamic control system of WWTPs, respectively. Let $\mathbf{x}(t)=[x_1(t), x_2(t)]^T$, where $x_1(t)$ and $x_2(t)$ are practical values of $S_{NO,2}$ and $S_{O,5}$ measured at time step *t*. Let $\omega_j(t) = [\omega_{1j}(t), \omega_{2j}(t)]^T$, where $\omega_{1j}(t)$ and $\omega_{2j}(t)$ represent the connection weights from $f_1(\mathbf{x},t)$ and $f_2(\mathbf{x},t)$ to the *j*th kernel function, respectively. Let $c_j(t) = [c_{1j}(t), c_{2j}(t)]$ ^T and $\sigma_j(t) = [\sigma_{1j}(t), \sigma_{2j}(t)]$ ^T represent the dynamic center vector and the dynamic width vector of the *j*th kernel function, respectively.

Note that by establishing the functional relationships between EC/EQ and the related process variables of WWTPs, the dynamic change of WWTPs can be more accurately captured. Then the dynamic tracking control of the $S_{0.5}$ and $S_{NO,2}$ is realized, and the optimal performance of WWTPs is improved on the whole.

3.2 Proposed GD-MOEA/D

The complex performance functions EC and EQ of WWTPs are time-varying. Although the conventional MOEA/D algorithm has shown superior performance in solving static MOPs, it is difficult to achieve satisfactory solutions when the MOEA/D is required to quickly track the moving Pareto optimal front of an MOP once the environmental changes occur. In order to make a rapid response to the time-varying characteristics of a MOP, and to predict the severity of the MOP in adjacent time more accurately, a grid-based dynamic MOEA/D, GD-MOEA/D for short, is proposed to solve EC and EQ to obtain the optimal values of $S_{0.5}$ and S_{NO2} .

The conventional dynamic multi-objective evolutionary algorithms regard the decision space as a whole and predict the change of the whole Pareto optimal set (POS) by predicting the change of the center point of the POS of a dynamic MOP. This brings a certain error to the prediction of the real POS change trend.

In order to more accurately predict the changing trend of POS, a grid strategy to divide the decision space into different sub-regions is designed in this paper. By analyzing the position change of all the individuals in the POS in each subregion, we can independently predict the changing trend of the individuals, thus improving the prediction accuracy to a certain extent. The center of the *j*th sub-region at time *t* is defined as

$$
C_{j}^{t} = \frac{1}{\left| \text{POS}_{j}^{t} \right|} \sum_{i=1}^{\left| \text{POS}_{j}^{t} \right|} x_{ji}^{t}, \tag{7}
$$

where $j \in \{1, 2, ..., n^2\}$, *n* is the number of decision variables. POS^t_j represents the *j*th subspace of POS at time *t*, POS^t_j represents the number of solutions in POS^t_j , and x_{ji}^t represents the *i*th solutions in POS_{j}^{t} .

The core ideas of the prediction method are described

as follows. Let $C^t = \left\{C_1^t, C_2^t, ..., C_{n^2}^t\right\}$ and $C^{t+1=}$ $\left\{C_1^{t+1}, C_2^{t+1}, \ldots, C_{n^2}^{t+1}\right\}$ be the sets of center individuals with different sub-regions at time step *t* and *t*+1, respectively. For any individual C_j^t in C_j^{t+1} , the proposed method first seeks the nearest center point to C_j^t in C_j^{t+1} , denoted by C_d^{t+1} ($d \in \{1,$ $2, ..., n^2$) through calculating the Euclidean distance. Then, the evolutionary direction of the individuals in the *d*th subregion is defined as

$$
\Delta C_d^{t+1,t} = C_d^{t+1} - C_j^t. \tag{8}
$$

The set of evolution directions predicted at time step *t*+1 is $C = (\Delta C_1^{t+1,t}, \Delta C_2^{t+1,t}, \dots, \Delta C_{n^2}^{t+1,t}).$ For an individual, x_d^{t+1} , which belongs to the *d*th sub-regions, its corresponding new individual is generated according to the following criterion:

$$
\mathbf{x}_{d}^{t+2} = \mathbf{x}_{d}^{t+1} + \Delta C_{d}^{t+1,t} + \vartheta^{t},
$$
\n(9)

where \mathcal{G}^t is a random number between zero and one. The method of generating new individuals in different sub-regions is summarized in Algorithm 2.

The developed GD-MOEA/D can well capture the dynamic characteristics of EQ and EC. The grid-based strategy can more accurately detect the changing intensity of individuals in different regions, so as to improve the accuracy of predicting the changing trend of EC and EQ in WWTPs. Thus, the tracking control performance of $S_{0.5}$ and $S_{N0.2}$ is improved in WWTPs.

3.3 Integrated intelligent optimal controller

The performance of the proposed grid-based dynamic multiobjective intelligent optimal control strategy is evaluated on a wastewater treatment plant. As depicted in [Figure 3](#page-6-0), the

[Figure 3](#page-6-0) (Color online) Grid-based dynamic multi-objective intelligent optimal control framework towards wastewater treatment processes, including the neural network model, the GD-MOEA/D algorithm, and the proportion integral differential controller.

proposed intelligent optimal controller mainly includes three modules. First, through the principal component analysis of the process variables of WWTPs, EC and EQ are determined as the output variables of the model, and $S_{0.5}$ and S_{NQ} are determined as the input variables. The functional relationships between input variables and output variables are established accurately through the kernel functions. Second, based on the dynamic functions of EC and EQ, GD-MOEA/D algorithm is developed. The algorithm obtains the optimal set-points of $S_{0,5}$ and $S_{NO,2}$ by solving a dynamic MOP composed of EC and EQ. In the iterative process, GD-MOEA/D employs the DE/rand/1 operator and the polynomial mutation operator for producing new solutions. Third, the established proportion integral differential (PID) controller realizes the dynamic variation of $K_L a_5$ and Q_a , S_0 that $S_{0,5}$ and $S_{N0,2}$ are maintained within a reasonable range. Algorithm 3 depicts the main program of the designed GD-MOEA/D-IOC.

It should be emphasized that the intelligent optimal controller can be expressed as

7: **Step 4** Tracking control of $S_{0,5}$ and $S_{N0,2}$ by the designed controller.

Algorithm 3 The overall GD-MOEA/D-IOC framework

^{1:} **Input**: Algorithm parameters

^{2:} **Output**: Control results

^{3:} Initialization parameters.

^{4:} **Step 1** Designing a multi-objective optimization function consisting of EC and EQ.

^{5:} **Step 2** Designing a grid-based dynamic multiobjective evolutionary decomposition algorithm: GD-MOEA/D.

^{6:} **Step 3** Employing GD-MOEA/D to find the optimal solutions of the multiobjective optimization problem consisting of EC and EQ.

$$
\mathbf{u}(t) = \mathbf{K}_p \mathbf{e}(t) + \mathbf{K}_i \int_0^t \mathbf{e}(t) dt + \mathbf{K}_d \frac{\mathrm{de}(t)}{\mathrm{d}t},
$$
 (10)

where $\mathbf{u}(t) = [Q_a(t), K_{L}a_5(t)]^T$, \mathbf{K}_p is the proportionality coefficient matrix, \mathbf{K}_i is the integral coefficient matrix, and \mathbf{K}_d is the derivative coefficient matrix. The error matrix **e**(*t*) defined as

$$
\mathbf{e}(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} x_{d1}(t) - x_1(t) \\ x_{d2}(t) - x_2(t) \end{bmatrix}.
$$
 (11)

Based on the above description, we can see that the established multi-objective function can capture the dynamic characteristics of WWTPs well, and the designed GD-MOEA/D algorithm can accurately predict the changing trend of the dynamic multi-objective function. The devised GD-MOEA/D provides an effective guarantee for obtaining the optimal set-points of $S_{O,5}$ and $S_{NO,2}$ and greatly improves the overall performance of the control system.

4 Application to the proposed wastewater treatment process

In this section, we verify the control performance of the devised GD-MOEA/D-IOC by means of the experiments on a wastewater treatment simulation platform. The experimental results are analyzed through the IAE performance metric and other important process variables in WWTPs. In order to further evaluate the control performance of the developed GD-MOEA/D-IOC in WWTPs, several other intelligent optimal controllers are used for comparison. Furthermore, the deeper insights of the control results of GD-MOEA/D-IOC and other controllers in WWTPs are given from the perspective of intelligent optimal algorithms.

4.1 Parameter settings and performance metric

The specific parameters of various multi-objective intelligent algorithms considered in comparative experiments are summarized in [Table 1.](#page-7-0) Furthermore, some important parameters employed in these algorithms are as follows.

(1) Population size: The population size is configured as 40 in all the algorithms.

(2) Stopping criterion: Each algorithm terminates after a prespecified number of generations. The total number of iterations is set to 50.

(3) Number of archive: The maximum number of archives is configured as 40.

To analyze experimental results clearly, in this paper, we apply the typical used the integral of absolute error (IAE) indicator to verify the effectiveness of the developed GD-MOEA/D-IOC. IAE is an indicator to evaluate the degree of excellence in the performance of control system, which is derived by integrating the absolute value of the difference

[Table 1](#page-7-0) Algorithmic parameter settings

Algorithm	Parameter		
AMODE	The scaling factor: $F=0.5$; The crossover fate: $C_r = 0.2$		
NSGA-II	The crossover probability: $p_r = 0.9$; The distribution index for crossover: η_c =20; The mutation probability: $p_m=1/n$; The distribution index for mutation: η_m =20		
MOGA	The crossover probability: 0.9; The probability of crossover: 10; The probability of mutation: 0.1; The mutation distribution index: 20		
GD-MOEA/D	The neighborhood size: 20; Differential evolution: $CR=1$ and $F=0.5$; Polynomial mutation: $n=20$ and $p_m=1/n$; MOEA/D: $T=20$; $n_r=2$; $\delta=0.9$		

between the desired and actual output of the control system. That is

$$
IAE(t) = \frac{1}{S_T} \sum_{i=1}^{2} \sum_{t=1}^{S_T} |e_i(t)|,
$$
\n(12)

where S_T is the total number of samples and $e_i(t)$ can be derived directly via [eq. \(11\).](#page-7-1)

4.2 Performance evaluation

To evaluate the control effectiveness of the designed GD-MOEA/D-IOC on the wastewater treatment simulation platform, three intelligent controllers including adaptive multi-objective differential evolution-based intelligent optimal controller (AMODE-IOC) [\[35\]](#page-10-26), non-dominated sorting genetic algorithm-II-based intelligent optimal controller (NSGA-II-IOC) [\[37\]](#page-11-0), multi-objective genetic algorithmbased intelligent optimal controller (MOGA-IOC) [\[38\]](#page-11-1), and a conventional PID controller are considered for performance comparison of WWTPs.

First, we evaluate the control performance of the developed GD-MOEA/D-IOC in the dry weather and tabulate the results of the related process variables in [Table 2.](#page-8-0) Clearly, the proposed GD-MOEA/D-IOC has achieved significant advantages over other comparison controllers. Let us first compare the proposed control strategy and PID. The proposed control strategy achieves the goal of minimizing EC and optimizing EQ. Specifically, the EC of GD-MOEA/ DIOC in WWTPs is reduced by 3.11%, while the EQ is improved by 13.92% compared with PID. In addition, the IAE value of our proposed controller is much smaller than that of PID, which shows that our proposed control strategy can achieve better tracking control. Subsequently, the proposed GD-MOEA/D-IOC is compared with the other three controllers. For AMODE-IOC, NSGA-II-IOC, and MO-GAIOC, the control results show that the EC of the proposed control strategy is reduced by 2.13%, 1.98%, and 1.81%,

[Table 2](#page-8-0) Performance comparison of different controllers in general BSM1 plant

Weather	Controller	ЕC (kW h)	EQ (kg poll.units)	IAE (mg/L)
Dry	GD-MOEA/D-IOC	3859	6492	0.092
	AMODE-IOC [35]	3943	7231	0.123
	NSGA-II-IOC [37]	3937	7214	0.100
	MOGA-IOC ^[38]	3930	7204	0.120
	PID	3983	7542	0.135
Storm	GD-MOEA/D-IOC	3923	7503	0.089
	AMODE-IOC [35]	4128	7551	0.123
	NSGA-II-IOC [37]	4169	7551	0.125
	MOGA-IOC ^[38]	4180	7633	0.111
	PID	4324	8051	0.142

while the EQ is improved by 10.22%, 10.01%, and 9.89%, respectively. Eventually, we find that the other three controllers (AMODE-IOC, NSGA-II-IOC, and MOGA-IOC) also have achieved excellent control performance compared with PID.

Second, we focus on the proposed GD-MOEA/D-IOC and the other four controllers in stormy weather. The data in [Table 2](#page-8-0) show that in stormy weather, the developed controller achieves the best control performance, and PID control effect is the worst. Specifically, compared with the PID controller, the EC of the proposed GD-MOEA/D-IOC is reduced by 9.27%, while the EQ is improved by 6.81%. In addition, compared with the PID controller, the EC of other three controllers (AMODE-IOC, NSGA-II-IOC, and MOGA-IOC) is decreased 4.53%, 3.58%, and 3.33%, while the EQ is increased 6.21%, 6.21%, and 5.19%, respectively. The performance metric IAE results show that the developed GD-MOEA/D-IOC can minimize the error between the optimal setting value and the actual value, which also shows that GD-MOEA/D-IOC can achieve satisfactory tracking control performance in the dynamic process of wastewater treatment.

Third, it should be emphasized that in different weather conditions, the control effect of the five comparative controllers presents a rule as follows. The control performance of GD-MOEA/D-IOC is the best, followed by the other three controllers (AMODE-IOC, NSGA-II-IOC, and MOGA-IOC) based on multi-objective optimization, and the PID control performance is the worst.

In our view, this performance mainly comes from the following two aspects.

First, compared with PID, the remaining four controllers are built on the basis of multi-objective optimization, and the multi-objective intelligent optimization algorithms have significant advantages in solving multiple conflicting objective functions. The multi-objective intelligent optimization algorithm is applied to WWTPs to realize the optimal control of $S_{0.5}$ and $S_{NO,2}$, thereby achieving the goal of reducing EC and improving EQ.

Second, wastewater treatment is a dynamic process with complex characteristics, the function models of EC and EQ established by neural networks have dynamic characteristics, and the conventional multi-objective algorithms can not effectively solve the dynamic multi-objective optimization problems efficiently. Particularly, the grid-based GD-MOEA/D divides the decision space into different subspaces, and predicts the changing trend of the Pareto subset in each subspace independently, so as to improve the prediction accuracy of the changing trend of the whole Pareto set to a certain extent. The grid strategy can quickly and accurately track the changing front of the Pareto set after detecting the environmental changes, and therefore, the appropriate solutions can be obtained.

Apart from the tabular display of process variables, we also provide the tracking control curves of $S_{0.5}$ and $S_{NO,2}$ under the dry weather and the stormy weather in Figures [4](#page-9-0) and [5,](#page-9-1) respectively. From tracking control curves of $S_{0.5}$ and $S_{NO,2}$, we can directly observe that the developed GD-MOEA/DIOC can effectively track and control $S_{0.5}$ and S_{NQ2} in WWTPs. The tracking control errors of $S_{0.5}$ and S_{NO2} in the dry weather and the storm weather are depicted in [Fig](#page-9-0)[ures 4](#page-9-0)(c) and [5](#page-9-1)(c). As can be viewed, the tracking control errors of $S_{0.5}$ and $S_{NO,2}$ are kept within ± 0.2 and 50.17 mg/L in most cases, respectively, which reflect the effectiveness of the developed GD-MOEA/D-IOC.

5 Conclusions

Focusing on reducing EC and improving effluent quality in WWTPs, a novel intelligent optimal controller dubbed GD-MOEA/D-IOC is devised to dynamically control $S_{0.5}$ and $S_{NO,2}$. In contrast to other methods, including AMODE-IOC, NSGA-II-IOC, MOGA-IOC, and PID controller, the developed GD-MOEA/D-IOC controller has a more efficient operational performance, which is attributed to the integration of functions extraction module, optimization algorithm design module, and control module. The functions extraction module establishes the functional relationships between the input variables and the output variables through a neural network model. In the optimization algorithm module, a grid-based GD-MOEA/D is designed, which can effectively predict the changes of the dynamic environment and obtain satisfactory optimal solutions. The PID controller realizes the control of $K_La₅$ by obtaining the error between the optimal set-value of $S_{0,5}$ and the actual measured value, and realizes the control of Q_a by obtaining the error between the optima set value of S_{NO2} and the actual measured value. The experimental results on a wastewater treatment plant demonstrate that the devised GD-MOEA/D-IOC is prominently superior to other controllers.

In this paper, the devised GD-MOEA/D-IOC has achieved

[Figure 4](#page-9-0) (Color online) Control results in the dry weather. (a) $S_{0.5}$; (b) $S_{N0,2}$; (c) errors.

[Figure 5](#page-9-1) (Color online) Control results in the storm weather. (a) $S_{0,5}$; (b) $S_{N0,2}$; (c) errors.

remarkable performance in WWTPs. Future works will be focused on the following two directions. One direction is to combine transfer learning technology with optimization algorithms to improve the prediction accuracy of the changing trend of the Pareto set by using knowledge sharing mechanism according to the time-varying characteristics of dynamic multi-objective functions. Another direction is to improve the robustness of the control system, so that it can guarantee the stable operation of the multi-objective intelligent optimal system in the wastewater treatment plant.

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