SCIENCE CHINA
Technological Sciences

• Article •

February 2022 Vol. 65 No. 2: 361–374
https://doi.org/10.1007/s11431-020-1803-4

Design and stability analysis of a generalized reduced-order active disturbance rejection controller

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Received November 9, 2020; accepted March 1, 2021; published online October 28, 2021

Disturbance and uncertainty rejection is a key objective in control system design, and active disturbance rejection control (ADRC) exactly provides an effective solution to this issue. To this end, this paper presents a generalized active disturbance rejection controller for a class of nonlinear uncertain systems with linear output. To be specific, a generalized reduced-order extended state observer (ESO) is proposed to reduce phase delay and complexity of the system, which can take full advantage of the system output. Also, this method includes the existing results with fewer assumptions, and can be applied to systems with any order measurable states or multiple states, even linear combination states. Furthermore, the stability of this approach is guaranteed and demonstrated through matrix transformation and Lyapunov method, and design examples and numerical simulations are given to show the effectiveness and practicability of the method.

generalized reduced-order ESO, active disturbance rejection control (ADRC), linear combination states, stability analysis, Gronwall-Bellman inequality

Citation: Wang Y S, Chen Z Q, Sun M W, et al. Design and stability analysis of a generalized reduced-order active disturbance rejection controller. Sci China Tech Sci, 2022, 65: 361–374, https://doi.org/10.1007/s11431-020-1803-4

1 Introduction

Uncertainty and feedback are the core issues in system control, and the main purpose of feedback is to deal with the impact of various uncertainties on system performance [1]. During this process, the observer plays an important role in designing the feedback controller through estimating system states and uncertainties. Therefore, designing an observer appropriately is crucially important.

The extended state observer, emerging as a novel and effective method to estimate states and uncertainties simultaneously, was first proposed by Han [2] in the 1990s and is the core of ADRC. ADRC inherits the advantages of PID, and is not model-based and featured with a strong capability to resist disturbance. For this method, the internal model uncertainties and external disturbances are regarded as the total disturbance, which will be estimated by ESO as a new state, and then be compensated by error feedback. Due to these attractive advantages, ADRC has received more and more attention in various fields since it was proposed. To simplify the designing process, Gao [3] generalized ADRC to the linear form in 2003 using parameterized bandwidth, greatly promoting its developments [4–7] and applications. So far, ADRC has been verified to have great potentials for time-delay [8–10], multivariable [11], coupled [12] and other complex systems and ADRC has been applied to many actual systems successfully, such as turbine [13], gasoline engines [14,15], power plant [16], observatory antenna [17] and robot [18,19].

The observer is such a kind of dynamic system that esti-

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mates state variables according to the system input and output [20]. Different from conventional observers [21], ESO regards the total disturbance as a new state to estimate, which will extend the system order by one as a result. It is not a good choice to design the full-order ESO when some states are measurable, which is the waste of model information and leads to phase delay, so the reduced-order observer (RESO) develops. For RESO, there have been some existing results. Tian [22] first proposed the RESO in 2007, and corresponding frequency response analysis was performed to quantify its performance and stability characteristics, but this method is limited to the case where the output and its continuous n-th order derivatives are measurable, so it is not universal. Xue [23] designed the RESO only using the single output, which is by far the most commonly used form. Teppa-Garran employed a method to obtain the RESO that does not depend on the output derivative [24], which is actually equivalent to [23], and developed the reduced-order version of the general ESO [25]. Although there is only one form (described in ref. [23] or [24]) of RESO that is generally recognized in the field of ADRC, its development and applications are very common; for example, refs. [26–28] considered the control performance, stability and parameters tuning of RESO. Refs. [29, 30] introduced the application of RESO to integrated missile guidance and two tanks multivariable level control systems. And a specific parameter selection method for the ADRC based on the RESO approach has been presented in ref. [31] for a fluid-driven hand rehabilitation device, which shows that if the parameter is properly selected, the ADRC based on the RESO has a better disturbance and noise rejection ability than that of the ADRC based on the ESO. Therefore, the RESO is of great importance in system control.

Based on the above discussion, a generalized reduced-order ADRC for a class of nonlinear uncertain systems with linear output is proposed. To be specific, the main contributions can be summarized as follows.

(1) The generalized RESO is applicable for more cases, such as any order measurable states and linear combination states, in which there are no additional restrictions and assumptions for the measurable state. And to the best of the authors' knowledge, the proposed RESO is not considered in the existing studies, which can make full use of the measurable information and reduce phase delay.

(2) The stability of the generalized RESO and corresponding ADRC is guaranteed, together with rigorous mathematical proof using Lyapunov method. And the theoretical analysis provides a sufficient feasible region to ensure the stability of the ADRC-based closed-loop system, which has the certain significance for the RESO design and application.

The remainder of this article is organized as follows. The problem formulation is introduced in Sect. 2. In Sect. 3, the design of ADRC with generalized RESO is presented. Sect. 4 shows the stability analysis of the generalized RESO and the corresponding ADRC. Numerical simulations are carried out to demonstrate the results in Sect. 5. Sect. 6 is the conclusion of the article.

2 Problem formulation

For the following nonlinear uncertain systems:

$$x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}, d) + b_0 u, \tag{1}$$

where x, u stand for the state and control input of the plant, respectively. b_0 is the known system parameter, and $f(x, \dot{x}, \cdots, x^{(n-1)}, d)$ (denoted as f simply) is the total disturbance, including the unknown internal uncertainties and external disturbances d.

Let $x_1 = x, x_2 = \dot{x}, \cdots, x_n = x^{(n-1)}, x_{n+1} = f(x, \dot{x}, \cdots, x^{(n-1)}, d)$, and suppose f is differentiable, which satisfies $\dot{x}_{n+1} = \dot{f}$. Then for measurable output states $\boldsymbol{Y} = \boldsymbol{c}\boldsymbol{X}$, plant (1) can be rewritten as the following state-space form by extending one order.

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}u + \boldsymbol{E}\dot{f}, \\ \boldsymbol{Y} = \boldsymbol{c}\boldsymbol{X}, \end{cases} \tag{2}$$

where $\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n & x_{n+1} \end{bmatrix}^{\mathrm{T}}$,

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{(n+1)\times(n+1)}, \boldsymbol{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \\ 0 \end{bmatrix}_{(n+1)\times 1}, \boldsymbol{E} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(n+1)\times 1},$$

and $\boldsymbol{c}$ is a known constant matrix.

ADRC is mainly composed of three parts: extended state observer (ESO), tracking differentiator (TD), error feedback (EF), and the structure diagram is shown in Figure 1.

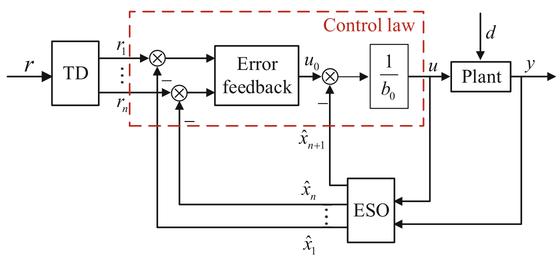


Figure 1 (Color online) Structure diagram of ADRC technique.

As the core of ADRC, ESO is an effective tool to estimate states and disturbance, and estimations will be employed to design error feedback.

RESO is proposed to reduce phase delay and complexity of the system. For the simple case $\boldsymbol{c} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1\times(n+1)}$, the RESO can be designed through introducing an auxiliary variable, which is by far the most commonly used form of RESO, and as the basis of RESO, the design process is introduced.

Since x_1 is measurable, it is not necessary to estimate x_1, then the RESO in ref. [23] is designed.

$$\begin{cases} \dot{\hat{x}}_2 = \hat{x}_3 + \beta_1(x_2 - \hat{x}_2), \\ \dot{\hat{x}}_3 = \hat{x}_4 + \beta_2(x_2 - \hat{x}_2), \\ \vdots \\ \dot{\hat{x}}_n = \hat{x}_{n+1} + b_0 u + \beta_{n-1}(x_2 - \hat{x}_2), \\ \dot{\hat{x}}_{n+1} = \beta_n(x_2 - \hat{x}_2), \end{cases} \tag{3}$$

$\hat{x}_i$ $(i = 2, \cdots, n+1)$ is the estimate of state x_i, and β_i $(i = 1, \cdots, n)$ is the observer gain. Considering x_2 is not measurable, which can be replaced by $\dot{x}_1$, RESO eq. (3) can be rewritten as

$$\begin{cases} \dot{\hat{x}}_2 = \hat{x}_3 + \beta_1(\dot{x}_1 - \hat{x}_2), \\ \dot{\hat{x}}_3 = \hat{x}_4 + \beta_2(\dot{x}_1 - \hat{x}_2), \\ \vdots \\ \dot{\hat{x}}_n = \hat{x}_{n+1} + b_0 u + \beta_{n-1}(\dot{x}_1 - \hat{x}_2), \\ \dot{\hat{x}}_{n+1} = \beta_n(\dot{x}_1 - \hat{x}_2). \end{cases}$$

Actually, it is not easy to get $\dot{x}_1$, so we make some changes to the equations.

$$\begin{cases} \dot{\hat{x}}_2 - \beta_1\dot{x}_1 = \hat{x}_3 - \beta_2 x_1 - \beta_1(\hat{x}_2 - \beta_1 x_1) - \beta_1^2 x_1 + \beta_2 x_1, \\ \dot{\hat{x}}_3 - \beta_2\dot{x}_1 = \hat{x}_4 - \beta_3 x_1 - \beta_2(\hat{x}_2 - \beta_1 x_1) - \beta_1\beta_2 x_1 + \beta_3 x_1, \\ \vdots \\ \dot{\hat{x}}_n - \beta_{n-1}\dot{x}_1 = \hat{x}_{n+1} - \beta_n x_1 + b_0 u - \beta_{n-1}(\hat{x}_2 - \beta_1 x_1) - \\ \qquad \beta_1\beta_{n-1}x_1 + \beta_n x_1, \\ \dot{\hat{x}}_{n+1} - \beta_n\dot{x}_1 = -\beta_n(\hat{x}_2 - \beta_1 x_1) - \beta_1\beta_n x_1. \end{cases}$$

Introducing the auxiliary variable $z_i = \hat{x}_i - \beta_{i-1}x_1$ $(i = 2, \cdots, n+1)$, the standard RESO is obtained.

$$\begin{cases} \dot{z}_2 = \begin{cases} -\beta_1 z_2 - \beta_1^2 x_1 - \beta_1 b_0 u, n = 1, \\ z_3 - \beta_1 z_2 + (\beta_2 - \beta_1^2)x_1, n > 1, \end{cases} \\ \dot{z}_3 = z_4 - \beta_2 z_2 + (\beta_3 - \beta_1\beta_2)x_1, \\ \vdots \\ \dot{z}_n = z_{n+1} - \beta_{n-1}z_2 + (\beta_n - \beta_1\beta_{n-1})x_1 + b_0 u, \\ \dot{z}_{n+1} = -\beta_n z_2 - \beta_1\beta_n x_1, \\ \hat{x}_i = z_i + \beta_{i-1}x_1. \end{cases} \tag{4}$$

Considering the measurable output states and estimated states, the control law is designed as

$$u = \frac{u_0 - \hat{x}_{n+1}}{b_0}, \tag{5}$$

where $u_0 = k_1(r - x_1) + \cdots + k_n(r^{(n-1)} - \hat{x}_n) + r^{(n)}$, r is the reference. k_i $(i = 1, \cdots, n)$ is the feedback gain, and

$$k_i = \frac{n!\omega_c^{n+1-i}}{(i-1)!(n+1-i)!}, i = 1, 2, \cdots, n, \tag{6}$$

is selected to make polynomial $s^n + k_n s^{n-1} + \cdots + k_1$ Hurwitz.

When the total disturbance $f(x, \dot{x}, \cdots, x^{(n-1)}, d)$ is accurately estimated, there exists $\hat{x}_{n+1} \approx f(x, \dot{x}, \cdots, x^{(n-1)}, d)$, and then plant (1) is equal to $x^{(n)} \approx u_0$, which is transformed into the standard integral form. Actually, control law (5) is a generalized PD (proportional-differential) controller based on the estimation error and output error.

However, when matrix $\boldsymbol{c}$ has a complex expression, it is not easy to find the auxiliary variables to design the RESO, so the generalized RESO and ADRC are developed in this situation.

3 ADRC with generalized RESO: design

For example, when other states x_i are measurable instead of only x_1, or the measurable variable is a linear combination of state x_i, such as $y = c_p x_p + c_q x_q$, it is necessary to design a generalized RESO using the most measurable information, rather than struggling to find auxiliary variables.

3.1 Design basis

Transform plant (2) into the following form:

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}u + \boldsymbol{E}\dot{f}, \\ \boldsymbol{y} = \boldsymbol{CX}, \end{cases} \tag{7}$$

where $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{E}$ are the same matrices as plant (2), $\boldsymbol{C}$ is the largest linearly independent group of matrix $\boldsymbol{c}$, and matrix $\boldsymbol{c}$ satisfies rank$(\boldsymbol{c}) = m, (m \le n)$. Clearly, $\boldsymbol{y}$ is the part of measurable output state $\boldsymbol{Y}$.

Introduce an auxiliary variable $\boldsymbol{w}$, which satisfies $\boldsymbol{w} = \boldsymbol{LX}$ [32], and $\boldsymbol{L}$ is a $(n+1-m) \times (n+1)$ matrix. Then we have

$$\begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{C} \end{bmatrix} \boldsymbol{X}. \tag{8}$$

Define $\boldsymbol{M} \triangleq \begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{C} \end{bmatrix}$, and $\boldsymbol{L}$ should be chosen appropriately to satisfy $\boldsymbol{M}$ is nonsingular, which will be discussed in Remark 1, so $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{y} \end{bmatrix}$.

Considering eqs. (7) and (8) gives

$$\begin{bmatrix} \dot{w} \\ \dot{y} \end{bmatrix} = M\dot{X} = M\left[AM^{-1}\begin{bmatrix} w \\ y \end{bmatrix} + Bu + E\dot{f}\right]$$
$$= MAM^{-1}\begin{bmatrix} w \\ y \end{bmatrix} + MBu + ME\dot{f}, \tag{9}$$

where

$$MAM^{-1} = \begin{bmatrix} T_{ww(n+1-m)\times(n+1-m)} & T_{wy(n+1-m)\times m} \\ T_{yw_{m\times(n+1-m)}} & T_{yy_{m\times m}} \end{bmatrix},$$
$$MB = b_0 \cdot M(:, n),$$
$$ME = M(:, n+1),$$

$M(:, n)$, $M(:, n+1)$ represent the n-th and (n+1)-th column of matrix M, respectively. Obviously, x_{n+1} is unmeasurable, so the elements in the last column of C are 0, which means the elements in the last m-row of ME are zero.

Then the generalized RESO is designed.

$$\dot{\hat{w}} = T_{ww}\hat{w} + T_{wy}y + LBu, \tag{10}$$

where $\hat{w}$ is the estimate of state w, so $\hat{X} = M^{-1}\begin{bmatrix} \hat{w} \\ y \end{bmatrix}$, $\hat{X}$ is the estimate of state X.

Considering eq. (7), there is

$$L\dot{X} = LAX + LBu + LEf. \tag{11}$$

From eq. (9),

$$\dot{w} = T_{ww}w + T_{wy}y + LBu + LE\dot{f} \tag{12}$$

is obtained, and substitute $w = LX$ to get

$$L\dot{X} = T_{ww}LX + T_{wy}CX + LBu + LE\dot{f}. \tag{13}$$

Subtracting eqs. (11) and (13) gives

$$LA - T_{ww}L = T_{wy}C. \tag{14}$$

So the relation among plant, RESO and linear transformation matrix are shown in eq. (14).

To analyze the stability of generalized RESO, letting $\tilde{w} = w - \hat{w}$ and combining eqs. (10) and (12) yields

$$\dot{\tilde{w}} = \dot{w} - \dot{\hat{w}} = T_{ww}\tilde{w} + LE\dot{f}. \tag{15}$$

Based on the existing results, to obtain a stable observer, T_{ww} must be Hurwitz whether $\dot{f}$ is bounded or satisfies the Lipchitz condition, and this will be discussed in Remark 2.

So far, the design basis of generalized RESO is completed. Clearly, there are two core issues, the one is to ensure that M is nonsingular, and the other one is to guarantee that T_{ww} is Hurwitz, which will be discussed in Remarks 1 and 2, respectively.

Before expounding these two issues, the following lemmas are given.

Lemma 1. (Theorem 2 in ref. [33]) Let A_0 and B_0 be $n \times n$ matrices with no common eigenvalues. Let a and b be vectors such that (A_0, a^{T}) is completely observable and (B_0, b) is completely controllable. Let M be the unique solution of $MA_0 - B_0M = ba^{\mathrm{T}}$. Then M is invertible.

Lemma 2. (Theorem 4 in ref. [33]) Let S_1 be a completely observable n-th order system with m independent outputs. Then an observer S_2 may be built for S_1 using only $n - m$ dynamic elements. Thus, the eigenvalues of the observer are essentially arbitrary.

Remark 1. There are two opposite ways to ensure that M is invertible.

The one is selecting appropriate parameters to guarantee the linear independence of row or column vectors according to the particularity of matrix C. But the form of designed RESO cannot be selected in advance, which means a designed M corresponds to a form of RESO .

The other one is to design the RESO in advance, and then derive the transformation matrix M according to Lemma 1, which is actually a solution of Sylvester equation [34]. But the measurable information (i.e., matrix C) cannot be sufficiently applied in this case, and it is difficult to calculate the invertible M, which may be nonexistent. This way means the designed RESO corresponds to a matrix M. The basic idea is shown below.

For system

$$\dot{x} = A_0x + D_0u.$$

The corresponding RESO is designed as the following form:

$$\dot{z} = B_0z + C_0x + G_0u.$$

To satisfy Lemma 1 that A_0, B_0 are square matrix with the same size, a linear combination of x is adjoined to z, which can be C in eq. (8), but not limited to C. Suppose that $z = Mx$, and we have

$$M\dot{x} = MA_0x + MD_0u,$$
$$M\dot{x} = B_0Mx + C_0x + G_0u,$$

with $G_0 = MD_0$, there is $MA_0 - B_0M = C_0$, where M is a square matrix and has the same size as A_0, B_0. Then an invertible M is obtained through choosing appropriate a, b based on Lemma 1. Comparing the two ways, the first is considered in this article which is more practical and convenient.

Remark 2. According to Lemma 2, the eigenvalues of RESO are essentially arbitrary if plant (7) is completely observable, which means

$$N = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^n \end{bmatrix}_{(n+1)m\times(n+1)}, \ \mathrm{rank}(N) = n+1.$$

Then we can always find appropriate matrix L to guarantee that T_{ww} is Hurwitz.

Then the two essential issues are solved, and an effective RESO is obtained.

Next, a simple way to construct matrix M is proposed.

Based on system (2), without loss of generality, assume

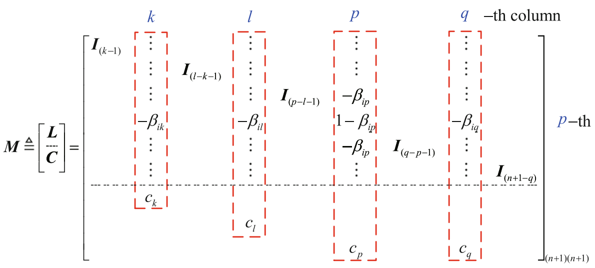
$$y = CX, \tag{16}$$

where

$$C = \begin{bmatrix} 0 & \cdots & c_k & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & c_l & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & c_p & \cdots & c_q & \cdots & 0 \end{bmatrix}_{3\times(n+1)}$$

is measurable, and $k < l < p < q \le n$ ($k, l, p, q \in N^+$) are the column numbers.

According to eq. (8), we can design matrix M, which is shown below together with some notes.



Some notes of M:

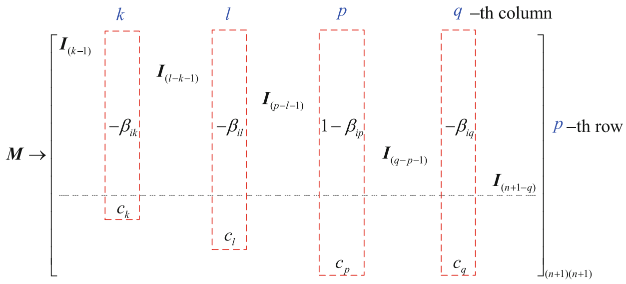
(1) Unfilled elements are zero;

(2) $\beta_{ik}, \beta_{il}, \beta_{ip}, \beta_{iq} \ge 0$ represent the gain in row i, column k, l, p, q of matrix M, respectively;

(3) $\beta_{ik}, \beta_{il}, \beta_{ip}, \beta_{iq} \ge 0$ are not zero at the same time;

(4) Matrix M is decided by measurable matrix C, and the gain exists in the column where matrix C has elements.

By using elementary column transformations of the matrix, matrix M is transformed into



Considering that a matrix is nonsingular if its column vector group is linearly independent, therefore M is invertible if and only if $\frac{c_p}{c_q} \neq \frac{1-\beta_{(p-2)p}}{-\beta_{(p-2)q}}$.

Based on eq. (9), it obtains

$$\begin{bmatrix} \dot{w} \\ \dot{y} \end{bmatrix} = MAM^{-1} \begin{bmatrix} w \\ y \end{bmatrix} + MBu + ME\dot{f},$$

where

$$MAM^{-1} = \begin{bmatrix} T_{ww(n-2)\times(n-2)} & T_{wy(n-2)\times 3} \\ T_{yw3\times(n-2)} & T_{yy3\times 3} \end{bmatrix},$$

$$MB = \begin{cases} b_0 \cdot \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}_{1\times(n+1)}, \\ \text{if } x_n \text{ is not measurable}, \\ b_0 \cdot \begin{bmatrix} -\beta_{1n} & \cdots & -\beta_{(n-2)n} & 0 & 0 & c_n \end{bmatrix}^{\mathrm{T}}, \\ \text{if } x_n \text{ is measurable}, \end{cases}$$

$$ME = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}_{1\times(n+1)}.$$

The generalized RESO is

$$\dot{\hat{w}} = T_{ww}\hat{w} + T_{wy}y + LBu,$$

where

$$LB = \begin{cases} b_0 \cdot \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}_{1\times(n-2)}, \\ \text{if } x_n \text{ is not measurable}, \\ b_0 \cdot \begin{bmatrix} -\beta_{1n} & -\beta_{2n} & \cdots & -\beta_{(n-2)n} \end{bmatrix}^{\mathrm{T}}_{1\times(n-2)}, \\ \text{if } x_n \text{ is measurable}. \end{cases}$$

Obviously, matrices A and C are known for a certain plant, so it is a problem to make T_{ww} Hurwitz if plant (7) is unobservable. Actually, it can be solved by making some changes for matrix A and total disturbance f, and this will be discussed below (Case 2).

3.2 Design examples

To show the efficiency of the proposed RESO, two representative cases are considered.

Case 1. Only $y = x_1$ is measurable.

According to the design basis, the matrix $\boldsymbol{M}$ is designed.

$$\boldsymbol{M} = \begin{bmatrix} -\beta_{11} & 1 & 0 & \cdots & 0 & 0 \\ -\beta_{21} & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & & \vdots & \\ -\beta_{(n-1)1} & 0 & 0 & \cdots & 1 & 0 \\ -\beta_{n1} & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

and

$$\boldsymbol{M}^{-1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 & \beta_{11} \\ 0 & 1 & \cdots & 0 & 0 & \beta_{21} \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \beta_{(n-1)1} \\ 0 & 0 & \cdots & 0 & 1 & \beta_{n1} \end{bmatrix}.$$

Then we can obtain

$$\boldsymbol{MAM}^{-1} = \begin{bmatrix} -\beta_{11} & 1 & 0 & \cdots & 0 & \beta_{21} - \beta_{11}^2 \\ -\beta_{21} & 0 & 1 & \cdots & 0 & \beta_{31} - \beta_{11}\beta_{21} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -\beta_{(n-1)1} & 0 & 0 & \cdots & 1 & \beta_{n1} - \beta_{11}\beta_{(n-1)1} \\ -\beta_{n1} & 0 & 0 & \cdots & 0 & -\beta_{11}\beta_{n1} \\ 1 & 0 & 0 & \cdots & 0 & \beta_{11} \end{bmatrix},$$

$$\boldsymbol{MB} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{ME} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

and RESO has the following form.

$$\begin{cases} \dot{\hat{\boldsymbol{w}}} = \begin{bmatrix} -\beta_{11} & 1 & 0 & \cdots & 0 \\ -\beta_{21} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -\beta_{(n-1)1} & 0 & 0 & \cdots & 1 \\ -\beta_{n1} & 0 & 0 & \cdots & 0 \end{bmatrix} \hat{\boldsymbol{w}} + \\ \begin{bmatrix} \beta_{21} - \beta_{11}^2 \\ \beta_{31} - \beta_{11}\beta_{21} \\ \vdots \\ \beta_{n1} - \beta_{11}\beta_{(n-1)1} \\ -\beta_{11}\beta_{n1} \end{bmatrix} y + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \\ 0 \end{bmatrix} u, n > 1, \\ \dot{\hat{w}} = -\beta_{11}\hat{w} - \beta_{11}^2 y - \beta_{11} b_0 u, n = 1, \end{cases} \tag{17}$$

where $\boldsymbol{T}_{ww}$ is Hurwitz if

$$\beta_{i1} = \frac{(n+1)!\omega_o^i}{i!(n+1-i)!}(\omega_o > 0), i = 1, 2, \cdots, n+1.$$

And the estimated states are

$$\hat{\boldsymbol{X}} = \boldsymbol{M}^{-1} \begin{bmatrix} \hat{\boldsymbol{w}} \\ y \end{bmatrix} = \begin{bmatrix} x_1 \\ \hat{w}_1 + \beta_{11}x_1 \\ \hat{w}_2 + \beta_{21}x_1 \\ \vdots \\ \hat{w}_{n-1} + \beta_{(n-1)1}x_1 \\ \hat{w}_n + \beta_{n1}x_1 \end{bmatrix}. \tag{18}$$

Comparing eq. (4) and eqs. (17) and (18), the generalized RESO has the same form as the standard RESO, and it also explains the effectiveness of the generalized RESO.

Then a more general case is considered in Case 2.

Case 2. The linear combination states are measurable.

Assume

$$\boldsymbol{y} = \begin{bmatrix} x_3 \\ 2x_2 + x_5 \end{bmatrix}.$$

According to Lemma 2, there is

$$\boldsymbol{A}^i = \begin{bmatrix} \boldsymbol{0}_{(n+1-i)\times i} & \boldsymbol{I}_{(n+1-i)\times(n+1-i)} \\ \boldsymbol{0}_{i\times i} & \boldsymbol{0}_{i\times(n+1-i)} \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & \boldsymbol{0} \\ 0 & 2 & 0 & 0 & 1 & \boldsymbol{0} \end{bmatrix}_{2\times(n+1)},$$

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \vdots \\ \boldsymbol{CA}^n \end{bmatrix}_{(n+1)m\times(n+1)}, \ \text{rank}(\boldsymbol{N}) = n < n+1,$$

so it does not guarantee that RESO has essentially arbitrary closed-loop poles. To solve this problem, make some changes to matrix $\boldsymbol{A}$ and total disturbance f.

Denote

$$\boldsymbol{A}_n = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{(n+1)\times(n+1)}, \dot{f}_n = \dot{f} - x_1,$$

and there is

$$\boldsymbol{A}_n^i = \begin{bmatrix} \boldsymbol{0}_{(n+1-i)\times i} & \boldsymbol{I}_{(n+1-i)\times(n+1-i)} \\ \boldsymbol{I}_{i\times i} & \boldsymbol{0}_{i\times(n+1-i)} \end{bmatrix}_{(n+1)\times(n+1)},$$

$$\boldsymbol{C}=\begin{bmatrix}0&0&1&0&0&\mathbf{0}\\0&2&0&0&1&\mathbf{0}\end{bmatrix}_{2\times(n+1)},$$

$$\boldsymbol{N}=\begin{bmatrix}\boldsymbol{C}\\\boldsymbol{CA}\\\vdots\\\boldsymbol{CA}^n\end{bmatrix}_{(n+1)m\times(n+1)},\ \mathrm{rank}(\boldsymbol{N})=n+1.$$

Therefore, the eigenvalues of generalized RESO are essentially arbitrary through changing matrix $\boldsymbol{A}$ and total disturbance f, and the estimated disturbance will include the item about x_1.

In this case, the matrix $\boldsymbol{M}$ is designed as

$$\boldsymbol{M}=\begin{bmatrix}1&0&-\beta_{13}&0&0&0&\cdots&0\\0&1&-\beta_{23}&0&-1&0&&0\\0&0&-\beta_{33}&1&0&0&\cdots&0\\0&0&-\beta_{43}&0&0&1&&0\\\vdots&&\vdots&&\vdots&&\ddots&\\0&0&-\beta_{(n-1)3}&0&0&0&\cdots&1\\0&0&1&0&0&0&\cdots&0\\0&2&0&0&1&0&\cdots&0\end{bmatrix}_{(n+1)(n+1)}$$

and $\boldsymbol{M}^{-1}$ is calculated.

$$\boldsymbol{M}^{-1}=\begin{bmatrix}1&0&0&0&\cdots&0&\beta_{13}&0\\0&\frac{1}{3}&0&0&&0&\frac{\beta_{23}}{3}&\frac{1}{3}\\0&0&0&0&\cdots&0&1&0\\0&0&1&0&&0&\beta_{33}&0\\0&-\frac{2}{3}&0&0&\cdots&0&-\frac{2\beta_{23}}{3}&\frac{1}{3}\\0&0&0&1&&0&\beta_{43}&0\\\vdots&0&\vdots&0&\ddots&0&\vdots&\vdots\\0&0&0&0&\cdots&1&\beta_{(n-1)3}&0\end{bmatrix},$$

so

$$\boldsymbol{MAM}^{-1}=\begin{bmatrix}\boldsymbol{T}_{ww_{(n-1)\times(n-1)}}&\boldsymbol{T}_{wy_{(n-1)\times2}}\\\boldsymbol{T}_{yw_{2\times(n-1)}}&\boldsymbol{T}_{yy_{2\times2}}\end{bmatrix},$$

$$\boldsymbol{MB}=\begin{bmatrix}0&\cdots&0&1&0&0&0\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{ME}=\begin{bmatrix}0&\cdots&0&0&1&0&0\end{bmatrix}^{\mathrm{T}},$$

where

$$\boldsymbol{T}_{ww}=\begin{bmatrix}0&\frac{1}{3}&-\beta_{13}&0&0&\cdots&0\\0&0&-\beta_{23}&-1&0&&0\\0&-\frac{2}{3}&-\beta_{33}&0&0&\cdots&0\\0&0&-\beta_{43}&0&1&&0\\\vdots&&\vdots&&&\ddots&\\0&0&-\beta_{(n-2)3}&0&0&&1\\1&0&-\beta_{(n-1)3}&0&0&&0\end{bmatrix},$$

$$\boldsymbol{T}_{wy}=\begin{bmatrix}\frac{\beta_{23}}{3}-\beta_{13}\beta_{33}&\frac{1}{3}\\1-\beta_{23}\beta_{33}-\beta_{43}&0\\-\beta_{33}^2-\frac{2\beta_{23}}{3}&\frac{1}{3}\\\beta_{53}-\beta_{43}\beta_{33}&0\\\vdots&\vdots\\\beta_{(n-1)3}-\beta_{(n-2)3}\beta_{33}&0\\\beta_{13}-\beta_{(n-1)3}\beta_{33}&0\end{bmatrix},$$

$$\boldsymbol{T}_{yw}=\begin{bmatrix}0&0&1&0&0&\cdots&0\\0&0&0&1&0&\cdots&0\end{bmatrix},\boldsymbol{T}_{yy}=\begin{bmatrix}\beta_{33}&0\\\beta_{43}+2&0\end{bmatrix}.$$

Specially, $\boldsymbol{MB}=\begin{bmatrix}0&0&0&0&0&1\end{bmatrix}^{\mathrm{T}}$ when $n=5$. The corresponding RESO is

$$\dot{\hat{\boldsymbol{w}}}=\boldsymbol{T}_{ww}\hat{\boldsymbol{w}}+\boldsymbol{T}_{wy}\boldsymbol{y}+\boldsymbol{LB}u,$$

where

$$\boldsymbol{LB}=\begin{cases}\begin{bmatrix}0&0&0&0\end{bmatrix}^{\mathrm{T}},n=5,\\\begin{bmatrix}0&\cdots&0&b_0&0\end{bmatrix}^{\mathrm{T}},n>5.\end{cases}$$

So the estimated states are

$$\hat{\boldsymbol{X}}=\boldsymbol{M}^{-1}\begin{bmatrix}\hat{\boldsymbol{w}}\\y\end{bmatrix}=\begin{bmatrix}\hat{w}_1+\beta_{13}x_3\\\frac{1}{3}(\hat{w}_2+\beta_{23}x_3+2x_2+x_5)\\x_3\\\hat{w}_3+\beta_{33}x_3\\\frac{1}{3}(2x_2+x_5-2\hat{w}_2-2\beta_{23}x_3)\\\hat{w}_4+\beta_{43}x_3\\\vdots\\\hat{w}_{n-1}+\beta_{(n-1)3}x_3\end{bmatrix}.$$

For plant (7), there is a differential relation among system states. So for the reference $\boldsymbol{r}$, there exists a scalar r such that $\lim_{t\to\infty}\boldsymbol{y}=\boldsymbol{r}$ is equivalent to $\lim_{t\to\infty}x_i=r^{(i-1)}(i=1,\cdots,n)$. Then the control objective can be transformed into $\lim_{t\to\infty}x_i=r^{(i-1)}(i=1,\cdots,n)$ when performing stability analysis.

Define $\boldsymbol{K}=\begin{bmatrix}k_1 & k_2 & \cdots & k_n & 1\end{bmatrix}, \boldsymbol{R}=\begin{bmatrix}r & \dot r & \ddot r & \cdots & r^{(n)}\end{bmatrix}^{\mathrm{T}}$, and consider the control law

$$u=\frac{1}{b_0}\boldsymbol{K}(\boldsymbol{R}-\hat{\boldsymbol{X}}),$$

where $k_i=\frac{n!\omega_c^{n+1-i}}{(i-1)!(n+1-i)!}, i=1,2,\cdots,n$ is also selected to make the polynomial $s^n+k_ns^{n-1}+\cdots+k_1$ Hurwitz.

4 ADRC with generalized RESO: analysis

Denote output error $e_i=r_i-x_i(r_i=r^{(i-1)})$, estimation error $\tilde x_i=x_i-\hat x_i$, it obtains

$$\begin{cases}\dot e_1=\dot r_1-\dot x_1=e_2,\\ \dot e_2=\dot r_2-\dot x_2=e_3,\\ \vdots\\ \dot e_{n-1}=\dot r_{n-1}-\dot x_{n-1}=e_n,\\ \dot e_n=\dot r_n-\dot x_n=-\boldsymbol{K}_n\boldsymbol{e}-\boldsymbol{K}\tilde{\boldsymbol{x}},\end{cases}$$

where $\boldsymbol{K}_n=[k_1\ \cdots\ k_n,], \boldsymbol{e}=\begin{bmatrix}e_1 & \cdots & e_n\end{bmatrix}^{\mathrm{T}}, \tilde{\boldsymbol{x}}=\begin{bmatrix}\tilde x_1 & \cdots & \tilde x_{n+1}\end{bmatrix}^{\mathrm{T}}$. Considering

$$\boldsymbol{X}=\boldsymbol{M}^{-1}\begin{bmatrix}\boldsymbol{w}\\ \boldsymbol{y}\end{bmatrix}, \hat{\boldsymbol{X}}=\boldsymbol{M}^{-1}\begin{bmatrix}\hat{\boldsymbol{w}}\\ \boldsymbol{y}\end{bmatrix},$$

therefore there is

$$\tilde{\boldsymbol{x}}=\boldsymbol{M}^{-1}\begin{bmatrix}\tilde{\boldsymbol{w}}\\ \boldsymbol{0}\end{bmatrix}=\boldsymbol{M}_o\tilde{\boldsymbol{w}}, \tag{19}$$

$\boldsymbol{M}_o$ are the first $(n+1-m)$ columns of $\boldsymbol{M}^{-1}$. Then combining eq. (15), we can get

$$\dot{\boldsymbol{E}}\triangleq\begin{bmatrix}\dot{\boldsymbol{e}}\\ \dot{\tilde{\boldsymbol{w}}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{E}_{A_{n\times n}} & \boldsymbol{E}_B\boldsymbol{M}_{o_{n\times(n+1)}}\\ \boldsymbol{0}_{(n+1-m)\times n} & \boldsymbol{T}_{ww_{(n+1-m)\times(n+1)}}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}\\ \tilde{\boldsymbol{w}}\end{bmatrix}+\begin{bmatrix}\boldsymbol{0}\\ \dot f\end{bmatrix}, \tag{20}$$

where

$$\boldsymbol{E}_A=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & & & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -k_1 & -k_2 & -k_3 & \cdots & -k_n\end{bmatrix}_{n\times n},$$

$$\boldsymbol{E}_B=\begin{bmatrix}0 & 0 & \cdots & 0 & 0\\ 0 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 0\\ -k_1 & -k_2 & \cdots & -k_n & -1\end{bmatrix}_{(n+1)\times(n+1)}.$$

Let

$$\boldsymbol{\Gamma}=\begin{bmatrix}\boldsymbol{E}_{A_{n\times n}} & \boldsymbol{E}_B\boldsymbol{M}_{o_{n\times(n+1-m)}}\\ \boldsymbol{0}_{(n+1-m)\times n} & \boldsymbol{T}_{ww_{(n+1-m)\times(n+1-m)}}\end{bmatrix},$$

since $\boldsymbol{E}_A$ and $\boldsymbol{T}_{ww}$ are both Hurwitz matrices, $\boldsymbol{\Gamma}$ is also a Hurwitz matrix [35, 36]. Besides, estimation error and output error satisfy

$$\boldsymbol{E}_o\triangleq\begin{bmatrix}\boldsymbol{e}\\ \tilde{\boldsymbol{x}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_o\end{bmatrix}\begin{bmatrix}\boldsymbol{e}\\ \tilde{\boldsymbol{w}}\end{bmatrix}=\boldsymbol{TE},$$

where $\boldsymbol{T}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_o.\end{bmatrix}$.

To show the stability of closed-loop system, the following assumptions are considered.

Assumption 1. The reference r and its derivatives satisfy

$$\left\|\begin{bmatrix}r & \dot r & \cdots & r^{(n-1)}\end{bmatrix}\right\|\le r_0,$$

where r_0 is a positive constant.

Assumption 2. The disturbance f is differential, and there exist two constants L, L_0 that satisfy

$$\left|\dot f(x,\dot x,\cdots,x^{(n-1)},d)\right|\le L\left\|\begin{bmatrix}x & \dot x & \cdots & x^{(n-1)}\end{bmatrix}\right\|+L_0.$$

Therefore, considering Assumptions 1 and 2, the derivative of total disturbance satisfies

$$\begin{aligned}\left|\dot f\right| &\le L\left\|\boldsymbol{x}_e-\boldsymbol{r}_e+\boldsymbol{r}_e\right\|+L_0\\ &\le L(\|\boldsymbol{e}\|+r_0)+L_0\\ &\le L(\|(\boldsymbol{e},\tilde{\boldsymbol{w}})\|+r_0)+L_0,\end{aligned}$$

where $\boldsymbol{x}_e=\begin{bmatrix}x & \dot x & \cdots & x^{(n-1)}\end{bmatrix}, \boldsymbol{r}_e=\begin{bmatrix}r & \dot r & \cdots & r^{(n-1)}\end{bmatrix}$.

Remark 3. Assumption 2 is the combination of Lipchitz condition and boundedness. It indicates that the ADRC controller can deal with disturbance in practice for which the derivative is Lipchitz or bounded; in other words, the rate of change for f is bounded or linearly increasing.

However, this is a sufficient result, and ADRC can deal with other complex situations that can not be expressed by formulas. Furthermore, it is almost impossible to build an absolutely accurate model for an actual plant, so there still exists a certain gap between theoretical results and practical applications. The relation between theory and practice for disturbance that ADRC can tackle is described in Figure 2.

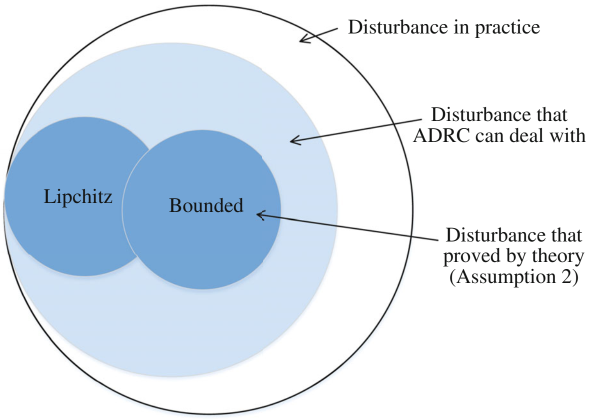


Figure 2 (Color online) Relation of disturbance rejection between theory and practice.

Theorem 1. Under Assumptions 1 and 2, the ADRC-based closed-loop system with a generalized RESO is bounded if $1-2\lambda_{\max}(\boldsymbol{P})L>0$ holds, and the error satisfies

$$\|\boldsymbol{E}_o\| \leq \max\left\{\frac{2\,\|\boldsymbol{T}\|\,\lambda^2_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\lambda_{\min}(\boldsymbol{P})(1-2\lambda_{\max}(\boldsymbol{P})L)}, \frac{\sqrt{\lambda_{\max}(\boldsymbol{P})}}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}\,\|\boldsymbol{T}\|\,\boldsymbol{E}_o(t_0)\right\},$$

where $\boldsymbol{E}_o$ includes both estimation error $\tilde{\boldsymbol{x}}$ and output error $\boldsymbol{e}$. Furthermore, as $t\to\infty$, the estimation error and output error are uniformly ultimately bounded with

$$\|\boldsymbol{E}_o\| \leq \frac{2\,\|\boldsymbol{T}\|\,\lambda^2_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\lambda_{\min}(\boldsymbol{P})(1-2\lambda_{\max}(\boldsymbol{P})L)},$$

where $\|\cdot\|$ represents the standard Euclidean norm, $\lambda_{\max}(\boldsymbol{P})$ and $\lambda_{\min}(\boldsymbol{P})$ are the maximum and minimum eigenvalues of matrix $\boldsymbol{P}$.

Proof of Theorem 1. Since $\boldsymbol{\Gamma}$ is Hurwitz, there exists a positive definite matrix $\boldsymbol{P}$ such that $\boldsymbol{P\Gamma}+\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{P}=-\boldsymbol{I}$.

With $\boldsymbol{E}=\begin{bmatrix}\boldsymbol{e}^{\mathrm{T}} & \tilde{\boldsymbol{w}}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, and for system $\dot{\boldsymbol{E}}=\boldsymbol{\Gamma E}$, choose Lyapunov function as follows:

$$V=\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}, \tag{21}$$

which has some properties

$$\lambda_{\min}(\boldsymbol{P})\left\|\boldsymbol{E}^2\right\| \leq V \leq \lambda_{\max}(\boldsymbol{P})\left\|\boldsymbol{E}^2\right\|, \tag{22}$$

$$\dot{V}=\frac{\partial V}{\partial \boldsymbol{E}}\boldsymbol{\Gamma E}=\boldsymbol{E}^{\mathrm{T}}(\boldsymbol{P\Gamma}+\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{E}=-\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}=-\|\boldsymbol{E}\|^2, \tag{23}$$

$$\left\|\frac{\partial V}{\partial \tilde{w}_{n+1-m}}\right\| \leq \left\|\frac{\partial V}{\partial \boldsymbol{E}}\right\| = \left\|2\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\right\| \leq 2\lambda_{\max}(\boldsymbol{P})\,\|\boldsymbol{E}\|. \tag{24}$$

Choose the same Lyapunov function V, and taking its time derivative along system (20) yields

$$\begin{aligned}\dot{V}&=\frac{\partial V}{\partial \boldsymbol{E}}\boldsymbol{\Gamma E}+\frac{\partial V}{\partial \tilde{w}_{n+1-m}}\dot{f}\\ &\leq -\|\boldsymbol{E}\|^2+2\lambda_{\max}(\boldsymbol{P})\,\|\boldsymbol{E}\|\,(L(\|\boldsymbol{E}\|+r_0)+L_0)\\ &=-(1-2\lambda_{\max}(\boldsymbol{P})L)\|\boldsymbol{E}\|^2+2\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)\,\|\boldsymbol{E}\|.\end{aligned} \tag{25}$$

Obviously, $1-2\lambda_{\max}(\boldsymbol{P})L>0$ must be satisfied to guarantee the system stability. According to eq. (22), $\frac{V}{\lambda_{\max}(\boldsymbol{P})} \leq \|\boldsymbol{E}\|^2 \leq \frac{V}{\lambda_{\min}(\boldsymbol{P})}$ is obtained, and substituting it into eq. (25) yields

$$\dot{V} \leq -\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{\lambda_{\max}(\boldsymbol{P})}V+\frac{2\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}\sqrt{V}. \tag{26}$$

Let $W=\sqrt{V}$, so $\dot{W}=\frac{\dot{V}}{2\sqrt{V}}$ and eq. (26) is equal to

$$\dot{W} \leq -\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}W+\frac{\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}.$$

Considering $W(t)=\int_{t_0}^{t}\dot{W}(\tau)d\tau+W(t_0)$, there is

$$\begin{aligned}\dot{W} \leq &-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}\int_{t_0}^{t}\dot{W}(\tau)\mathrm{d}\tau\\ &+\frac{\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}W(t_0).\end{aligned} \tag{27}$$

Applying Gronwall-Bellman inequality to eq. (27), we can get

$$\begin{aligned}\dot{W} \leq &\left(\frac{\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}W(t_0)\right)\\ &\cdot e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)}.\end{aligned} \tag{28}$$

Integrate the two sides of inequality eq. (28) to get

$$\begin{aligned}\int_{t_0}^{t}\dot{W}(\tau)\mathrm{d}\tau \leq &\int_{t_0}^{t}e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(\tau-t_0)}\mathrm{d}\tau\\ &\cdot\left(\frac{\lambda_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}W(t_0)\right).\end{aligned} \tag{29}$$

Calculating eq. (29) gives

$$\begin{aligned}W \leq &W(t_0)e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)}\\ &+\frac{2\lambda^2_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\sqrt{\lambda_{\min}(\boldsymbol{P})}(1-2\lambda_{\max}(\boldsymbol{P})L)}\left(1-e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)}\right).\end{aligned} \tag{30}$$

Denote $\|\boldsymbol{E}\| \leq \frac{\sqrt{V}}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}=\frac{W}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}$, $W(t_0) \leq \sqrt{\lambda_{\max}(\boldsymbol{P})}\,\|\boldsymbol{E}(t_0)\|$, and eq. (30) is transformed into

$$\begin{aligned}\|\boldsymbol{E}\| \leq &\frac{\sqrt{\lambda_{\max}(\boldsymbol{P})}}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}\boldsymbol{E}(t_0)e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)}\\ &+\frac{2\lambda^2_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\lambda_{\min}(\boldsymbol{P})(1-2\lambda_{\max}(\boldsymbol{P})L)}\left(1-e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)}\right).\end{aligned} \tag{31}$$

Then it concludes that

$$\|\boldsymbol{E}\| \leq \max\left\{\frac{2\lambda^2_{\max}(\boldsymbol{P})(Lr_0+L_0)}{\lambda_{\min}(\boldsymbol{P})(1-2\lambda_{\max}(\boldsymbol{P})L)}, \frac{\sqrt{\lambda_{\max}(\boldsymbol{P})}}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}\boldsymbol{E}(t_0)\right\}.$$

Considering $\lim_{t\to\infty} e^{-\frac{1-2\lambda_{\max}(\boldsymbol{P})L}{2\lambda_{\max}(\boldsymbol{P})}(t-t_0)} = 0$, it is reasonable to obtain that as $t \to \infty$,

$$\|\boldsymbol{E}\| \leq \frac{2\lambda_{\max}^2(\boldsymbol{P})(Lr_0 + L_0)}{\lambda_{\min}(\boldsymbol{P})(1 - 2\lambda_{\max}(\boldsymbol{P})L)}.$$

Therefore, it can be obtained that the system (20) is uniformly ultimately bounded with respect to internal uncertainties and external disturbances, and considering $\boldsymbol{E}_o = \boldsymbol{TE}$, it concludes that

$$\|\boldsymbol{E}_o\| \leq \max\left\{\frac{2\,\|\boldsymbol{T}\|\,\lambda_{\max}^2(\boldsymbol{P})(Lr_0 + L_0)}{\lambda_{\min}(\boldsymbol{P})(1 - 2\lambda_{\max}(\boldsymbol{P})L)}, \frac{\sqrt{\lambda_{\max}(\boldsymbol{P})}}{\sqrt{\lambda_{\min}(\boldsymbol{P})}}\,\|\boldsymbol{T}\|\,\boldsymbol{E}_o(t_0)\right\},$$

and as $t \to \infty$,

$$\|\boldsymbol{E}_o\| \leq \frac{2\,\|\boldsymbol{T}\|\,\lambda_{\max}^2(\boldsymbol{P})(Lr_0 + L_0)}{\lambda_{\min}(\boldsymbol{P})(1 - 2\lambda_{\max}(\boldsymbol{P})L)}.$$

The proof of Theorem 1 is completed.

Remark 4. Therefore, to ensure the convergence of state trajectories, two steps should be considered.

Step 1 First of all, select the feasible observer gains $\boldsymbol{\beta}$ to guarantee that $\boldsymbol{T}_{ww}$ is Hurwitz, where $\boldsymbol{T}_{ww}$ is calculated according to the designed RESO.

Step 2 Under Assumption 2, the constants L, L_0 are determined for the derivative of total disturbance. Then choose appropriate control bandwidth ω_o to satisfy the sufficient condition $1 - 2\lambda_{\max}(\boldsymbol{P})L > 0$.

In addition to the convergence, the observer gains $\boldsymbol{\beta}$ and controller bandwidth ω_o still have an effect on control performance, but this is not the point of the text.

5 Simulation

Two representative examples are considered in the simulation, where example 1 is the single high-order measurable output, example 2 is the multi-output plant, and the effect of measurement noise is discussed.

5.1 Example 1

For the following system:

$$\begin{cases} x^{(3)} = f(x, \dot{x}, \ddot{x}, d) + b_0 u, \\ y = \ddot{x}, \end{cases} \tag{32}$$

where $b_0 = 1, \dot{f} = 0.02\sin(0.125t) + 0.25x_1 + 0.1x_2$. With $x_1 = x, x_2 = \dot{x}, x_3 = \ddot{x}, x_4 = f$ and $\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^{\mathrm{T}}$, there is

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}u + \boldsymbol{E}\dot{f}, \\ y = \boldsymbol{CX}, \end{cases}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ b_0 \\ 0 \end{bmatrix}, \boldsymbol{E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}^{\mathrm{T}}.$$

(1) Design of stable controller

Clearly, this plant is not completely observable because

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{CA} & \boldsymbol{CA}^2 & \boldsymbol{CA}^3 \end{bmatrix}^{\mathrm{T}}, \mathrm{rank}(\boldsymbol{N}) = 2 < 4.$$

So make some changes for matrix $\boldsymbol{A}$ and disturbance f, now $\boldsymbol{A}$ and f are transformed into $\boldsymbol{A}_n, f_n$, and then this plant is observable.

$$\boldsymbol{A}_n = \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&0&0&0 \end{bmatrix}, \dot{f_n} = \dot{f} - x_1.$$

Select

$$\boldsymbol{M} = \begin{bmatrix} 1&0&-\beta_{13}&0 \\ 0&1&-\beta_{23}&0 \\ 0&0&-\beta_{33}&1 \\ 0&0&1&0 \end{bmatrix},$$

which is invertible, and

$$\boldsymbol{M}^{-1} = \begin{bmatrix} 1&0&0&\beta_{13} \\ 0&1&0&\beta_{23} \\ 0&0&0&1 \\ 0&0&1&\beta_{33} \end{bmatrix}.$$

The generalized RESO is

$$\dot{\hat{\boldsymbol{w}}} = \begin{bmatrix} 0&1&-\beta_{13} \\ 0&0&-\beta_{23} \\ 1&0&-\beta_{33} \end{bmatrix}\hat{\boldsymbol{w}} + \begin{bmatrix} \beta_{23} - \beta_{13}\beta_{33} \\ 1 - \beta_{23}\beta_{33} \\ \beta_{13} - \beta_{33}^2 \end{bmatrix}\boldsymbol{y} + \begin{bmatrix} -\beta_{13}b_0 \\ -\beta_{23}b_0 \\ -\beta_{33}b_0 \end{bmatrix}u,$$

and estimated state is

$$\hat{\boldsymbol{X}} = \boldsymbol{M}^{-1}\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \\ y \end{bmatrix} = \begin{bmatrix} \hat{w}_1 + \beta_{13}y \\ \hat{w}_2 + \beta_{23}y \\ y \\ \hat{w}_3 + \beta_{33}y \end{bmatrix}.$$

To guarantee that $\boldsymbol{T}_{ww}$ is Hurwitz, the feasible region of $\beta_{13}, \beta_{23}, \beta_{33}$ is obtained in the shaded area of Figure 3. Clearly, it is relatively easy to find the observer gains for Hurwitz $\boldsymbol{T}_{ww}$.

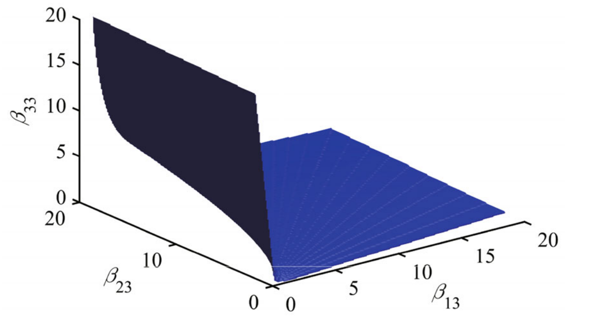


Figure 3 (Color online) Feasible region of observer gains for Hurwitz $\boldsymbol{T}_{ww}$ in example 1.

Then select parameters $\beta_{13} = 8, \beta_{23} = 3, \beta_{33} = 2$ in the shaded area of Figure 3 to make $\boldsymbol{T}_{ww}$ Hurwitz, and design the ADRC control law u for virtual reference r as follows:

$$u = \frac{k_1(r - \hat{x}_1) + k_2(\dot{r} - \hat{x}_2) + k_3(\ddot{r} - \hat{x}_3) - \hat{x}_4 + r^{(3)}}{b_0},$$

where the virtual reference r can be converted from the actual reference, as explained in Sect. 3.

Under parameterized control gain $\omega_c = 5$ in eq. (6), the simulation results are shown below with simulation step $h = 0.001$.

The output of system state and estimated state is shown in Figure 4, which implies that the estimated state does not coincide with system state under this set of parameters, and Figure 5 also illustrates that not only estimation error is bounded, but the output error is also bounded. However, the boundary can be reduced through tuning parameters $\omega_c, \beta_{13}, \beta_{23}, \beta_{33}$, and generally, increasing the parameters can achieve it, but a trade-off between performance and stability should be considered in an integrated manner.

(2) Effect of measurement noise

When white noise exists in system output, the estimation error and output error are shown below, where the mean value is 0.01, and the amplitude is between 0 and 0.02 for random white noise.

In Figure 6 and 7, the estimation error and output error are displayed when random white noise exists, where the noise frequency of Figure 6 is 100 times that of Figure 7.

As depicted by the simulation results, the generalized ADRC controller can achieve bounded stability with respect to the disturbance and measurement noise. Comparing with the condition without noise, the output is unsmooth, which implies the measurement noise has a negative effect on measurable states and estimated states. Therefore, the measurable states with less noise should be firstly considered, and the states with severe noise can be neglected when designing the generalized RESO.

Figure 7 shows better performance than Figure 6, which indicates the ADRC controller has the certain capability to deal with noise, especially for low-frequency noise. However, when facing the high-frequency measurement noise, it is necessary to denoise before designing the RESO, and some

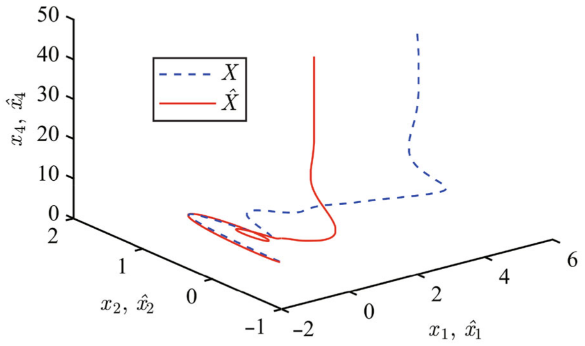


Figure 4 (Color online) Measurable states X of system and estimated states $\hat{X}$ of generalized RESO in example 1.

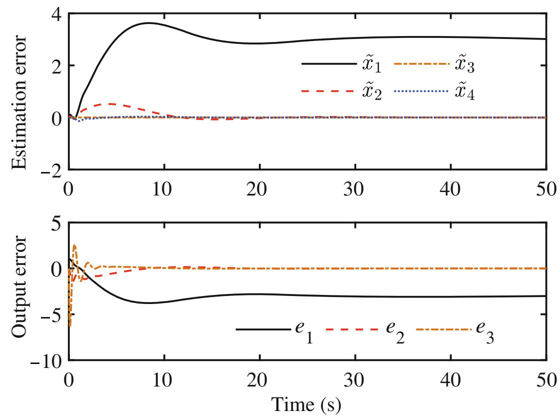


Figure 5 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 1.

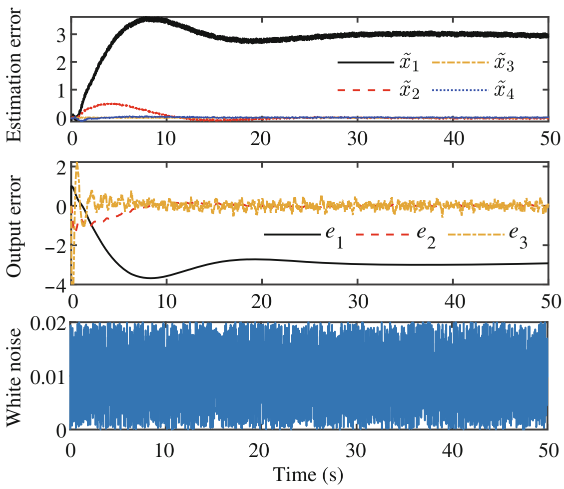


Figure 6 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 1 when high-frequency white noise exists.

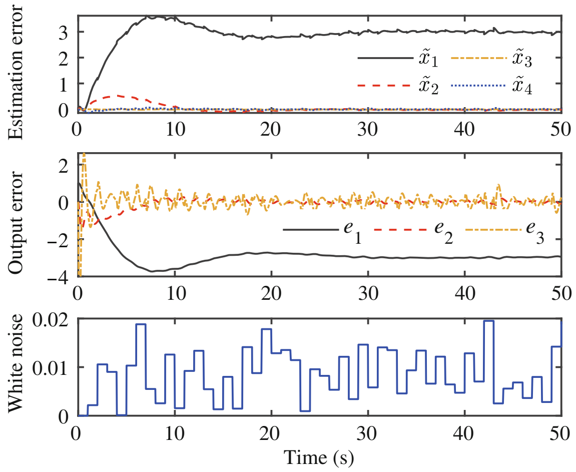


Figure 7 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 1 when low-frequency white noise exists.

denoising methods combining with ADRC have been developed.

5.2 Example 2

For the nonlinear multi-output system

$$\begin{cases} x^{(4)} = f(x, \cdots, x^{(3)}, d) + b_0 u, \\ \boldsymbol{y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \dot{x} & x + \ddot{x} \end{bmatrix}^{\mathrm{T}}, \end{cases} \tag{33}$$

where $b_0 = 1, \dot{f} = 0.25x\dot{x} + 0.098\sin(0.125t + 0.25t) + 0.25x_1 + 0.47$. Let $x_i = x^{(i-1)}(i = 1, \cdots, 4), x_5 = f$, and $\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_5 \end{bmatrix}^{\mathrm{T}}$, there is

$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}u + \boldsymbol{E}\dot{f}, \\ \boldsymbol{y} = \boldsymbol{CX}, \end{cases}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0\\0\\0\\b_0\\0 \end{bmatrix}, \boldsymbol{E} = \begin{bmatrix} 0\\0\\0\\0\\1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 0&1 \\ 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \end{bmatrix}^{\mathrm{T}}.$$

(1) Design of stable controller

Clearly, due to

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{CA} & \cdots & \boldsymbol{CA}^4 \end{bmatrix}^{\mathrm{T}}, \ \mathrm{rank}(\boldsymbol{N}) = 5 = 5,$$

this plant is completely observable.

Then we can design the linear transformation matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & -\beta_{13} & 0 & 0 \\ -\beta_{21} & 0 & 0 & 1 & 0 \\ -1 & -\beta_{32} & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Obviously, $\boldsymbol{M}$ is invertible, and

$$\boldsymbol{M}^{-1} = \begin{bmatrix} \frac{1}{\beta_{13}+1} & 0 & 0 & 0 & \frac{\beta_{13}}{\beta_{13}+1} \\ 0 & 0 & 0 & 1 & 0 \\ \frac{-1}{\beta_{13}+1} & 0 & 0 & 0 & \frac{1}{\beta_{13}+1} \\ \frac{\beta_{21}}{\beta_{13}+1} & 1 & 0 & 0 & \frac{\beta_{13}\beta_{21}}{\beta_{13}+1} \\ \frac{1}{\beta_{13}+1} & 0 & 1 & \beta_{32} & \frac{\beta_{13}}{\beta_{13}+1} \end{bmatrix}.$$

The generalized RESO is

$$\dot{\hat{\boldsymbol{w}}} = \begin{bmatrix} \frac{-\beta_{13}\beta_{21}}{\beta_{13}+1} & -\beta_{13} & 0 \\ \frac{1}{\beta_{13}+1} & 0 & 1 \\ \frac{\beta_{32}}{\beta_{13}+1} & 0 & 0 \end{bmatrix} \hat{\boldsymbol{w}} + \begin{bmatrix} 1 & \frac{-\beta_{13}^2\beta_{21}}{\beta_{13}+1} \\ \beta_{32} - \beta_{21} & \frac{\beta_{13}}{\beta_{13}+1} \\ -1 & \frac{-\beta_{32}}{\beta_{13}+1} \end{bmatrix} \boldsymbol{y} + \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix} u,$$

and estimated state is

$$\hat{\boldsymbol{X}} = \boldsymbol{M}^{-1} \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{\hat{w}_1}{\beta_{13}+1} + \frac{\beta_{13} y_2}{\beta_{13}+1} \\ y_1 \\ \frac{-\hat{w}_1}{\beta_{13}+1} + \frac{y_2}{\beta_{13}+1} \\ \frac{\beta_{21}\hat{w}_1}{\beta_{13}+1} + \hat{w}_2 + \frac{\beta_{13}\beta_{21}y_2}{\beta_{13}+1} \\ \frac{\hat{w}_1}{\beta_{13}+1} + \hat{w}_3 + \beta_{32}y_1 + \frac{\beta_{13}y_2}{\beta_{13}+1} \end{bmatrix}.$$

Similarly, the feasible region of $\beta_{13}, \beta_{21}, \beta_{32}$ is given in Figure 8. To ensure $\boldsymbol{T}_{ww}$ is Hurwitz, select parameters $\beta_{13} = 6, \beta_{21} = 18, \beta_{32} = 2$.

In this example, the estimated state can follow the system state more accurately under this set of parameters, see Figure 9. In Figure 10, the estimation error and output error are bounded, which show the effectiveness of the proposed generalized RESO and the correctness of the analysis.

(2) Effect of measurement noise

For the multi-output plant, simulations are carried out when random white noise exists in both output channels. Similarly, the noise is evenly distributed within 0–0.02, and the results are shown below.

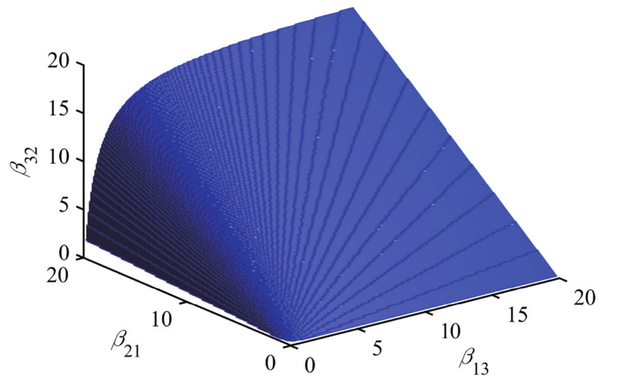


Figure 8 (Color online) Feasible region of observer gains for Hurwitz $\boldsymbol{T}_{ww}$ in example 2.

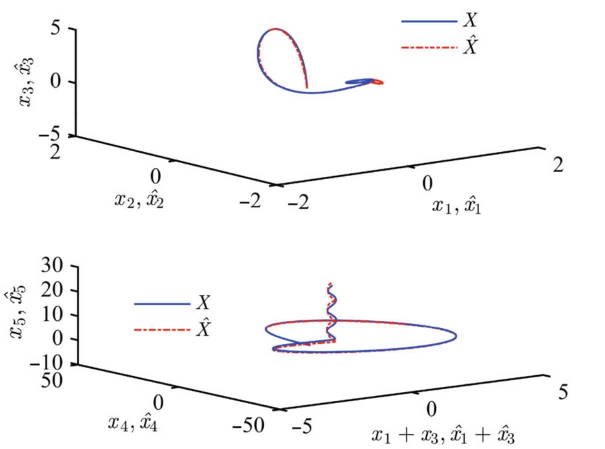


Figure 9 (Color online) Measurable states X of system and estimated states $\hat{X}$ of generalized RESO in example 2.

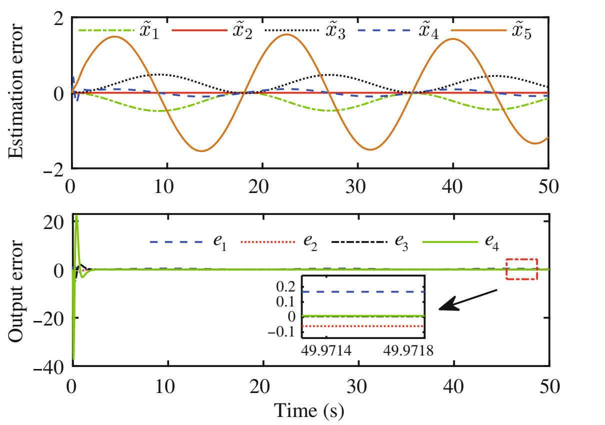


Figure 10 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 2.

Different from example 1, example 2 is a high-order plant with multi-output, and generalized RESO is designed using all the measurable information. As shown in Figures 11 and 12, the measurement noise has the same effect as that of example 1. It implies the measurement noise can affect the estimated states, and the ADRC controller does have the certain capability to resist some noise. Besides, the denoising method is also needed when dealing with severe noise.

6 Conclusion

Focusing on a class of nonlinear uncertain systems with linear output, this paper presents a generalized RESO to make full use of the model information, and saves the derivation process of constructing auxiliary variables. This method includes the existing results, and can be applied to more general cases, such as any order measurable states and linear

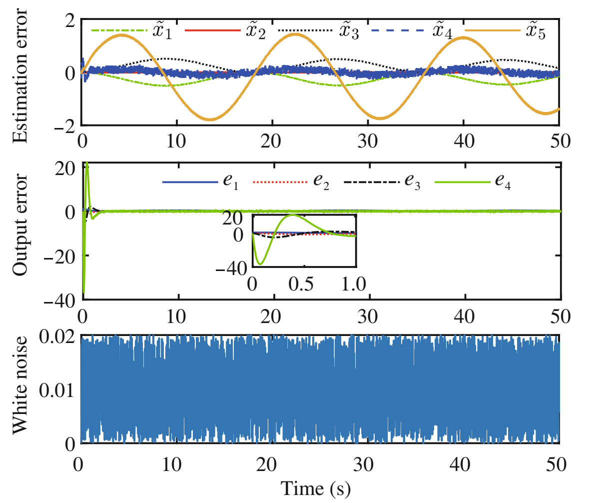


Figure 11 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 2 when high-frequency white noise exists.

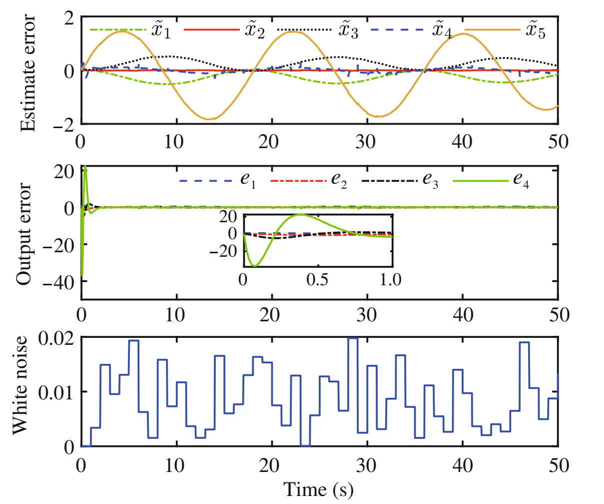


Figure 12 (Color online) Estimation error $\tilde{x}_i$ and output error e_i of generalized ADRC in example 2 when low-frequency white noise exists.

combination states. Also, the stability of generalized RESO is demonstrated through the Lyapunov function, and the closed-loop ADRC system is proven to be uniformly ultimately bounded. Besides, the boundedness in the entire time domain is also obtained, which is decided by both parameters and disturbances. Finally, simulations of the two examples show the effectiveness of the proposed RESO, and imply that the estimation and output errors of ADRC are bounded, even with some measurement noise.

However, the versatility of the generalized RESO means the diversity of the forms, so designing the best RESO will be the next issue to be considered. Furthermore, the application to actual plants will also be taken into account.

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61973175, 61973172 and 62073177), the Key Technologies Research and Development Program of Tianjin (Grant No. 19JCZDJC32800), and Tianjin Research Innovation Project for Postgraduate Students (Grant No. 2020YJSZXB02).

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