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Dynamic multi-objective differential evolution algorithm based on the information of evolution progress

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The multi-objective differential evolution (MODE) algorithm is an effective method to solve multi-objective optimization problems. However, in the absence of any information of evolution progress, the optimization strategy of the MODE algorithm still appears as an open problem. In this paper, a dynamic multi-objective differential evolution algorithm, based on the information of evolution progress (DMODE-IEP), is developed to improve the optimization performance. The main contributions of DMODE-IEP are as follows. First, the information of evolution progress, using the fitness values, is proposed to describe the evolution progress of MODE. Second, the dynamic adjustment mechanisms of evolution parameter values, mutation strategies and selection parameter value based on the information of evolution progress, are designed to balance the global exploration ability and the local exploitation ability. Third, the convergence of DMODE-IEP is proved using the probability theory. Finally, the testing results on the standard multi-objective optimization problem and the wastewater treatment process verify that the optimization effect of DMODE-IEP algorithm is superior to the other compared state-of-the-art multi-objective optimization algorithms, including the quality of the solutions, and the optimization speed of the algorithm.

information of evolution progress, multi-objective differential evolution algorithm, optimization effect, optimization speed, convergence

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1 Introduction

Most daily production and living activities, such as industrial production, transportation, commerce, and healthcare, are commonly described as multi-objective optimization problems (MOPs). These MOPs need to optimize multiple objectives interrelated or even mutually constrained at the same time. It is remarkable that the optimal solution of the multi-objective optimization problems is a Pareto solution set, which is composed of some optimal solutions obtained simultaneously, rather than a single solution [1–5]. Multi-objective evolutionary algorithms have been widely studied to solve MOPs in the engineering and scientific applications during the last three decades, such as the nosiheptide fermentation process, blast furnace gas system, and wastewater treatment processes. Among these algorithms, the multi-objective differential evolution (MODE) algorithm, which is a parallel search algorithm with few parameters and easy implementation [6,7], has attracted widespread attention from researchers. However, the solution quality reduces as the increase of the complexity between the objectives, and the optimization time increases when there are more and more optimization goals. The optimization effect will restrict its application in practice. In order to keep the advantage of MODE algorithm, it is a significant challenge to improve the optimization effect, including the quality of solutions, the uniformity of solutions, and the optimization speed of the algorithm [8,9].

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To address the above problem, cooperation with other methods has been considered as a promising approach for differential evolution algorithm [10,11]. For example, Santana-Quintero et al. [12] presented a hybrid algorithm which combined MODE with a local search engine for constrained MOPs. In this algorithm, MODE was utilized to generate an initial approximation of the Pareto front, and rough set theory was used to improve the spread and quality of the initial approximation. The results showed that the proposed algorithm could find better results, and was able to solve standard bi-objective constrained test problems and real-world problems at a moderate computational cost. Cheng et al. [13] proposed a grid-based adaptive multi-objective differential evolution algorithm. In this algorithm, the objective space was divided into grids according to the non-dominated solutions in the population. Then, the parent selection, parameter control, and population update were implemented based on grid index values exploiting the feedback information. Experimental results showed that the proposed algorithm was superior to the other multi-objective evolutionary algorithms on four test suites by using three performance metrics. Jamali et al. [14] proposed a multi-objective differential evolution algorithm with fuzzy inference-based dynamic adaptive mutation factor (MODE-FM) for MOPs. In the proposed work, the mutation factor was adjusted by a nine-rule fuzzy logic inference system, which took the number of generation and population diversity as inputs. MODE-FM was successfully used in bi-objective optimization processes and a five degree of freedom vehicle vibration model, and it was superior to the other methods in the literature. Although these algorithms improve the optimization effect by combining the advantages of other algorithms, they destroy the simple but effective search framework of MODE algorithm. And they consume a long time for calculating parameters and adjusting structures in algorithms. As a result, the improvement of optimization effect is limited when dealing with complex problems [15,16].

In order to maintain the structure of MODE algorithm and make better use of its advantages, researchers carry out a lot of exploratory work after analyzing the characteristics of MODE algorithm deeply [17–19]. Zheng and Zhang [20] presented a jumping gene multi-objective differential evolution (JGDE) algorithm. In JGDE, the jumping gene operator was employed to promote the population diversity in the first component, and an elitism leading mechanism was designed to accelerate the convergence in the second component. Experimental studies showed the exploitation ability and the exploration ability were improved, and JGDE algorithm had the superiority over the other competitive algorithms in both convergence and diversity. Wang et al. [21] proposed a self-adaptive differential evolution (APDDE) algorithm to deal with real-time high-dimensional optimization problems. APDDE algorithm introduced the corresponding values for individual iteration during the differential evolution, integrated the detecting values into two mutation strategies to produce offspring population, and then implemented a new mutation strategy based on the best vector of each predefined group to keep balance of the exploitation and exploration capabilities. The experimental results showed that the proposed APDDE algorithm had good performance when dealing with high dimensional and multimodal problems. Fan et al. [22] designed a self-adaptive weight vector adjustment strategy for decomposition-based multi-objective differential evolution (AWDMODE) algorithm. In AWDMODE algorithm, an adaptive adjustment strategy, which distinguished the shapes and adjusted weight vectors dynamically, was introduced to ensure the accurate and effective of guidance. The experimental results showed that AWDMODE outperformed the compared algorithms and had the potential to deal with the MOPs. The above algorithms can improve the optimization effect of MODE. But they lack theoretical proof of convergence, which hinders the development and application of the algorithm in engineering fields [23].

Based on the analysis above, in this study, a dynamic multi-objective differential evolution based on the information of evolution progress (DMODE-IEP) algorithm is developed for MOPs. In DMODE-IEP algorithm, the information of evolution progress, defined by the fitness values of individual, is used to describe the degree of evolution. The dynamic adjustment mechanisms for evolutionary parameter values, mutation strategies and selection parameter value based on the information of evolution progress are designed in DMODE-IEP, in order to balance the global exploration ability and the local exploitation ability. To this end, DMODE-IEP algorithm is effective to improve the quality of solutions and the speed of optimization.

The remainder of this paper is organized as follows. In Section 2, the traditional MODE algorithm and the features of evolutionary process in MODE algorithm are introduced as preliminaries. In Section 3, the MODE-IEP algorithm is proposed in detail, including the definition of the information of evolution progress, the dynamic adjustment mechanisms of the evolutionary parameter values, mutation strategies and selection parameter value. Section 4 is devoted to proof the convergence of MODE-IEP algorithm. Section 5 reports some experimental results of MODE-IEP, which demonstrate some merits in optimization accuracy and optimization speed against other existing methods. Finally, the conclusion is given in Section 6.

Remark 1: MOPs include maximizing objective function and minimizing objective function. The maximizing objective function and the minimizing objective function can be transformed by mathematical transformation. Therefore, only the problem of minimizing multi-objective optimization is discussed in this study.

2 MODE algorithm

2.1 Optimal solutions of MOPs

In solving MOPs, x^* Pareto-dominated x means that for all functions to be optimized, the function value of x^* is less than or equal to the function value of x on the corresponding function, and the function value of x^* is strictly less than the function value of x on at least one function.

In the entire domain of feasible solution set, x^* is a Pareto solution if x^* is not dominated by any other x. Pareto solution set is constituted by all Pareto solutions obtained.

2.2 Basic flows of MODE algorithm

MODE algorithm is a parallel direct search method for MOPs. The multiple solution spaces are searched in parallel drawing on biological evolutionary ideas for the global optimal solutions. The basic flows of MODE algorithm are similar to the other general multi-objective evolutionary algorithms.

(1) Initialization

Generate initial populations by a random method.

(2) Mutation

Target vector is the parent vector of current generation. Donor vector is obtained from the parent vector after differential mutation operation.

(3) Crossover

Trial vector is formed by exchanging some elements of donor vector and target vector. At present, the common modes of crossover are binomial crossover and exponential crossover.

(4) Selection

The next generation populations are obtained by non-dominant sorting based on Pareto selection mechanism.

(5) Iteration

Algorithm returns to the second step and continues evolutionary operation if termination condition is not satisfied. Otherwise, the optimal solutions constitute a Pareto optimal solution set. Algorithm outputs the optimal solutions and ends the evolution process.

2.3 Characteristics of evolution progress in MODE algorithm

The basic flows of MODE algorithm show that the algorithm generates an initial population by random. The individuals of the initial population are distributed randomly. The fitness values of individuals in the initial population depend on the stochastic distribution of initial individuals. In the process of MODE, the algorithm performs mutation operations and crossover operations on the individuals, and then selects the next population by a greedy selection mechanism. The goal of selection mechanism is to minimize the value of objective functions.

With the advancement of the optimization process, all individuals move towards the optimal solutions and distribute uniformly on the Pareto front. The distances between the individual and the optimal solution become smaller, and the fitness differences between individuals gradually decreased accordingly. Therefore, the variation on fitness difference reflects the degree of evolution progress. And it can be used as a standard for improving parameters.

3 DMODE-IEP algorithm

The scheme of proposed DMODE-IEP (as shown in Algorithm 1) consists of four innovative works. First, the information of evolution progress (IEP), defined by the fitness values, is designed to characterize the evolution process and standardize the information about the evolution progress. Second, the dynamic adjustment mechanism of evolution parameter values, based on IEP, is proposed to adjust the mutation rate and crossover rate dynamically. Third, the IEP-based dynamic adjustment mechanism of mutation strategy, is designed to balance the global exploration capability and the local exploitation capability. Finally, adjustable selection parameter value, based on IEP, is proposed for improving the optimization speed. In the following parts, these four works are described in detail.

3.1 Information of evolution progress

The variation of fitness difference reflects the evolutionary state of the population, which can be used as the embodiment of the information of evolution progress. According to the

Algorithm 1 DMODE-IEP algorithm

Input: the population dimension D, the maximum population generation $T_{\max}$, the initial population generation as t=0, the initial populationsize as NP^0, the initial mutation rate as F^0, and the initial crossover rate as Cr^0

Output: the non-dominated solutions $\mathbf{x}$

```
x^0 ← Generate the initial vector;
while t < T_max do
    θ^t ← Calculate the information of evolution progress;
    F^t ← Calculate the mutation rate;
    v^t ← Generate the donor vector;
    Cr^t ← Calculate the crossover rate;
    u^t ← Generate the trial vector;
    NP^{t+1} ← Calculate the population size of next generation;
Obtain the non-dominated solutions of this population generation;
Update the population generation t=t+1;
end while
Output the non-dominated solutions.
```

characteristics of evolution progress in MODE algorithm, as explained in Section 2.3, IEP in the tth generation population in DMODE-IEP algorithm is given as

$$\theta^{t} = (f_{\psi}^{t} - f_{\sigma}^{t}) / (f_{\psi}^{t} + f_{\sigma}^{t}) , \tag{1}$$

where f_{ψ}^{t} and f_{σ}^{t} are the maximum value and minimum value of individual fitness for all objective functions in the tth generation population.

The value of IEP reflects the characteristics of evolution progress, realizes a quantitative description of the evolutionary state, and shows evolutionary degree of the algorithm to a certain extent. Therefore, it can be used as an important basis for adjusting the evolutionary parameter values and evolutionary strategies.

3.2 Dynamic adjustment mechanisms of evolutionary parameter values

It is generally known that the main evolution processes of MODE algorithm are mutation operation and crossover operation. The main evolutionary parameters are mutation rate F and crossover rate Cr.

3.2.1 Dynamic adjustment mechanisms of mutation rate

Mutation is an operation that zooms in or zooms out the target vector $\mathbf{x}^t$ by the differential vector and the donor vector $\mathbf{v}^t$. It is an important operation in the process of MODE algorithm.

Mutation rate is an evolutionary parameter that determines the scaling degree of the differential vector, which affects the search range of algorithm directly. The smaller mutation rate means the smaller perturbation to the differential vector, which is helpful to find the optimal solutions quickly. But it is prone to fall into the local optimum and increase the convergence time. A larger mutation rate means a larger perturbation to the differential vector, which is helpful to increase the diversity of population. But the randomness of the algorithm increases, and the efficiency of the algorithm decreases.

According to the evolutionary law in MODE algorithm, different mutation rates are required at different stages of the evolution progress. In DMODE-IEP algorithm, a larger mutation rate is used to ensure the diversity of the population at the beginning of the algorithm (the first fifth of the maximum generation), and a smaller mutation rate is used to preserve the optimal solutions at the end of the algorithm (the last fifth of the maximum generation).

The definition of the mutation rate in the tth generation population is given as

$$F_p^t = F_p^{t-1} \cdot [\mu_{p,L}^{t} + (\mu_{p,H}^{t} - \mu_{p,L}^{t})\theta^{t}] , \tag{2}$$

where F_p^t is the mutation rate of the pth individual in the tth generation, F_p^{t-1} is the mutation rate of the pth individual in the $(t-1)$th generation, $\mu_{p,H}^{t}$ and $\mu_{p,L}^{t}$ are the upper and lower limits of the mutation rate of the pth individual in the tth generation, respectively. And $\mu_{p,H}^{t}$ is greater than 1, $\mu_{p,L}^{t}$ is less than 1.

3.2.2 Dynamic adjustment mechanisms of crossover rate

Crossover is an operation that exchanges some elements between the target vector and the donor vector. The individual obtained after the crossover operation is the trial vector $\mathbf{u}^t$.

Crossover rate is an evolutionary parameter that determines the probability of individual mutations, which affects the probability of producing new individuals directly. It has a great impact on population diversity. The smaller crossover rate means the smaller probability of mutation for individuals, which makes it easy to find optimal solutions. But it is not conducive to maintain population diversity. The larger crossover rate means the larger probability of mutation for individuals, which increases population diversity. But it is not conducive to find optimal solutions quickly.

In DMODE-IEP algorithm, the setting values of crossover rate are adjusted dynamically by comparing the individual fitness values with the average fitness value to improve population diversity.

The definition of the crossover rate in the tth generation population is given as

$$Cr_p^t = \begin{cases} Cr_p^{t-1}, & f_p^t < f_m^t, \\ Cr_p^{t-1}[\rho_{p,L}^{t} + (\rho_{p,H}^{t} - \rho_{p,L}^{t}) / \theta^{t}], & f_p^t \geq f_m^t, \end{cases} \tag{3}$$

where Cr_p^t is the crossover rate of the pth individual in the tth generation, Cr_p^{t-1} is the crossover rate of the pth individual in the $(t-1)$th generation, f_p^t is the fitness value of the pth individual in the tth generation, f_m^t is the average fitness value of the pth individual in the tth generation, $\rho_{p,H}^{t}$ and $\rho_{p,L}^{t}$ are the upper and lower limits of the crossover rate for the pth individual in the tth generation, respectively. $\mu_{p,H}^{t}$ is greater than 1, $\mu_{p,L}^{t}$ is less than 1.

3.3 Dynamic adjustment mechanisms of mutation strategies

Mutation is a key step of differential evolutionary algorithm. There are several mutation strategies can be chosen depending on different mutation mechanisms. The mode of mutation strategies is distinguished by the type of target vector and the number of difference vectors used. The common mutation strategies include as follows.

(1) DE/rand/1

$$\mathbf{v}_p^t = \mathbf{x}_{r1}^t + F_p^t \cdot (\mathbf{x}_{r2}^t - \mathbf{x}_{r3}^t); \tag{4}$$

(2) DE/best/1

$$\mathbf{v}_p^t = \mathbf{x}_{\text{best}}^t + F_p^t \cdot (\mathbf{x}_{r1}^t - \mathbf{x}_{r2}^t), \tag{5}$$

where $\mathbf{v}_p^{\ t}$ is the trial vector of the pth individual in the tth generation, $F_p^{\ t}$ is the mutation rate, $\mathbf{x}_{r1}^{\ t}$, $\mathbf{x}_{r2}^{\ t}$ and $\mathbf{x}_{r3}^{\ t}$ are three different random individuals in the tth generation, $\mathbf{x}_{\text{best}}^t$ is the best individual in the tth generation, r1, r2, and r3 are random real numbers different from p within the range of population size.

DE/rand/1 is a mutation strategy based on the random individual in the population. It is helpful for maintaining population diversity, but it increases the search scope. DE/best/1 is a mutation strategy based on the best individual in the population. It has a fast convergence speed, but it is easy to fall into the local optimum.

In DMODE-IEP algorithm, DE/rand/1 and DE/best/1 are used simultaneously to take advantage of their respective advantages. DE/rand/1 is used at the early stage of the algorithm, which focuses on enhancing the global exploration capability. DE/best/1 is used at the end stage of DMODE-IEP algorithm, which focuses on enhancing the local exploitation capability.

The mutation strategy in the tth generation population is described as

$$\mathbf{v}_p^t = \begin{cases} \mathbf{x}_{r1}^t + F_p^t \cdot (\mathbf{x}_{r2}^t - \mathbf{x}_{r3}^t), & \theta^t > \frac{2}{3}\theta^0, \\ \mathbf{x}_{\text{best}}^t + F_p^t \cdot (\mathbf{x}_{r1}^t - \mathbf{x}_{r2}^t), & \theta^t \le \frac{2}{3}\theta^0, \end{cases} \tag{6}$$

where θ^0 is the IEP in initial population.

3.4 Dynamic adjustment mechanisms of selection parameter value

Selection is an operation that sorts all individuals sequentially to get optimal solutions by a non-dominant sorting strategy. The smaller population size means fewer comparisons and shorter time. However, it reduces the diversity of population and weakens the explore ability. The larger population size means wider distribution and more individuals. However, it increases the calculation time and reduces the search ability.

In DMODE-IEP algorithm, the individuals are pre-sorted according to their fitness values. The population size is adjusted based on the IEP dynamically. The population size increases to provide more alternative solutions at the beginning of algorithm, and the population size decreases to improve the speed at the end of algorithm.

This proposed DMODE-IEP adopts a Pareto-based selection mechanism to sort the target vector and the donor vector in the tth generation, and selects NP^{t+1} individuals for the next generation finally.

$$NP^{t+1} = (1 - \theta^t) \cdot NP^t. \tag{7}$$

It can be seen that the population size decreases gradually as the algorithm progresses, which can reduce the number of comparison operations and decrease the optimization time significantly. The lower limit of population size is half of the initial population in order to ensure the optimization performance of the algorithm.

3.5 Procedure of DMODE-IEP algorithm

The procedure of the dynamic multi-objective differential evolutionary algorithm based on the information of evolution progress is as follows.

(1) Step 1: Initialization

The population dimension, the initial population size, the initial mutation rate and the initial crossover rate are defined in this step. A D-dimensional initial population with a population size of NP^0 is generated randomly according to eq. (8).

$$\begin{aligned} &\mathbf{x}_p^0 = \left[x_{p,1}^0, x_{p,2}^0, \dots, x_{p,q}^0\right]^{\mathrm{T}}, \\ &q = [1, 2, \cdots, D],\ p = [1, 2, \cdots, NP^0]. \end{aligned} \tag{8}$$

The initial population generation of DMODE-IEP algorithm is set as t=0, the maximum population evolutionary generation is set as $T_{\max}$, the initial mutation rate is set as F^0, and the initial crossover rate is set as Cr^0.

(2) Step 2: Mutation

IEP is calculated according to eq. (1). The mutation rate of the population is calculated according to eq. (2). The donor vector is produced by the mutation strategy obtained from eq. (6).

(3) Step 3: Crossover

The crossover rate of the population is calculated according to eq. (3). The trial vector is produced by the crossover operation obtained from eq. (9) using binomial crossover strategy.

$$u_{p,q}^t = \begin{cases} v_{p,q}^t, & \text{rand}[0,1] \le Cr_p^t, \quad q = q_{\text{rand}}, \\ x_{p,q}^t, & \text{others}, \end{cases} \tag{9}$$

where $u_{p,q}^t$ is the trial vector obtained by crossover in the tth generation, rand[0, 1] is the random number between 0 and 1, and q_{rand}=1, 2, …, D is a flag bit chosen at random to ensure at least one element in the target vector can be inherited during evolution.

(4) Step 4: Selection

Population size of the next generation is calculated according to eq. (7). Individuals of the next generation are obtained by a non-dominance sorting strategy.

(5) Step 5: Iteration

Return to Step 2. Repeat mutation, crossover, selection, and generate new individuals if generation is less than $T_{\max}$. Otherwise, the Pareto solution set is formed by the best individuals, and the evolutionary process ends.

4 Proof of convergence

4.1 Related definitions and lemmas

Definition 1 $\mathbf{x}_p'$ and $\mathbf{x}_p$ are reachable if $\mathrm{P}\left\{\lim_{t\to\infty}\left\|\mathbf{x}_p-\mathbf{x}_p'\right\|_\infty=0\right\}>0$.

The occurrence probability of event $\{\cdot\}$ is set as $\mathrm{P}\{\cdot\}$, the norm in feasible region is set as $\|\cdot\|_\infty$, an arbitrary solution is set as $\mathbf{x}_p$, and $\mathbf{x}_p$ is marked as $\mathbf{x}_p'$ when $\mathbf{x}_p$ experiences mutation operation and crossover operation.

Lemma 1 [24] The algorithm can converge to the global optimal solution set if the multi-objective optimization problem has global optimal solutions and satisfies the following conditions.

(1) $\mathbf{x}_p'$ is reachable from $\mathbf{x}_p$ by mutation and crossover, as any two solutions in the feasible domain.

(2) The evolution of population is monotonic. $\mathbf{x}_p^{t+1}$ is better than $\mathbf{x}_p^{t}$, or at least not worse than $\mathbf{x}_p^{t}$.

4.2 Proof

Theorem 1 DMODE-IEP converges to the global optimal solutions with probability one if the multi-objective optimization problem has a global optimal solution set and $f(x)$ is continuous in its search space.

$$\mathrm{P}\left\{\lim_{t\to\infty}\|\mathbf{x}-\mathbf{x}^*\|_\infty=0\right\}=1\ , \tag{10}$$

where $\mathbf{x}$ is the optimal solution set, and $\mathbf{x}^*$ is the standard Pareto optimal solution set.

Proof

(1) $\mathbf{v}_p$ is obtained from arbitrary solution $\mathbf{x}_p$ by mutation in DMODE-IEP algorithm. An arbitrary vector is selected as the difference vector in the evolutionary process of DMODE-IEP, and the mutation rate is an arbitrary value in the feasible domain. Therefore, the probability that $\mathbf{v}_p$ is an arbitrary point is greater than zero.

(2) $\mathbf{u}_p$ is obtained from $\mathbf{v}_p$ by crossover in DMODE-IEP algorithm. The probability of reaching from $\mathbf{v}_p$ to $\mathbf{u}_p$ is $P_{cr}=\left[\frac{1}{D}\cdot\frac{1}{Cr_p^{t+1}}\right]^{NP}$. $Cr_p^{t+1}>0$ and $D>0$, so $P_{cr}>0$, $\mathbf{u}_p$ is reachable from $\mathbf{v}_p$.

Hence, $\mathbf{x}_p$ and $\mathbf{x}_p'$ are two arbitrary solutions in the feasible domain. $\mathbf{x}_p'$ is reachable from $\mathbf{x}_p$ by mutation and crossover.

(3) A non-dominant sorting strategy is used during selection in DMODE-IEP algorithm, which guarantees the solution set in the $(t+1)$th generation is better than that in the tth generation, at least not inferior to the solution set in the tth generation.

Hence, the evolution is monotonic. $\mathbf{x}_p^{t+1}$ is better than $\mathbf{x}_p^{t}$, at least not inferior to $\mathbf{x}_p^{t}$.

(4) Eq. (10) is proved by Lemma 1. DMODE-IEP algorithm converges to the global optimal solutions with probability one.

5 Experiments and result analysis

To verify the optimization effect of DMODE-IEP algorithm, experiments are carried out on the standard test functions, CEC2018 many objectives benchmark problems and wastewater treatment examples. All the experiments are programmed with MATLAB version 2014, and are run on a PC with a clock speed 2.6 GHz and 4 GB RAM, under a Microsoft Windows 8.0 environment.

5.1 Experiments of the standard test functions

Standard test functions ZDT1 and DTLZ2 are selected for experiments.

(1) Standard test function ZDT1

$$\mathrm{Min}\begin{cases} f_1(x)=x_1,\\ f_2(x)=g(x)\left(1-\sqrt{(x_1/g(x))}\right),\\ g(x)=1+9\sum_{i=2}^{m}x_i/(m-1),\end{cases} \tag{11}$$

s.t. m=30, $0\le x_i\le 1,\ i=1,2,\dots,m.$

(2) Standard test function DTLZ2

$$\mathrm{Min}\begin{cases} f_1(x)=\cos\left(\frac{\pi}{2}x_1\right)\cos\left(\frac{\pi}{2}x_2\right)(1+g(x)),\\ f_2(x)=\cos\left(\frac{\pi}{2}x_1\right)\sin\left(\frac{\pi}{2}x_2\right)(1+g(x)),\\ f_3(x)=\sin\left(\frac{\pi}{2}x_1\right)(1+g(x)),\\ g(x)=\sum_{i=3}^{m}(x_i-0.5)^2,\end{cases} \tag{12}$$

s.t. m=12, $0\le x_i\le 1,\ i=1,2,\dots,m.$

5.1.1 *Experimental design*

Based on empirical value, the initial crossover rate is 0.5, the initial mutation rate is 0.2, and the initial population size is 200. The values of these parameters will be adjusted adaptively as the evolution progresses. And the maximum generation is 300.

The quality of optimal solutions is described quantitatively by inversed generational distance (IGD) and spacing (SP).

$$\mathrm{IGD}(\mathbf{P}^*,\mathbf{P})=\sum_{\mathbf{x}\in\mathbf{P}^*}\mathrm{mindis}(\mathbf{x},\mathbf{P})/|\mathbf{P}^*|\ , \tag{13}$$

$$SP=\sqrt{(K-1)^{-1}\sum_{i=1}^{K}(\bar{d}-d_i)}\ , \tag{14}$$

where $\mathbf{P}^*$ is the standard Pareto optimal solution set, $\mathbf{P}$ is the Pareto optimal solution set obtained by DMODE-IEP, and mindis($\mathbf{x}$, $\mathbf{P}$) is the minimum Euclidean distance between $\mathbf{x}$ and $\mathbf{P}$. K is the number of non-dominated solutions, $\bar{d}$ is the average Euclidean distance of all solutions, and d_i is the

Euclidean distance between the *i*th solution and its nearest solution.

IGD is a performance index that describes the distance between Pareto optimal solutions and standard Pareto optimal solutions. Pareto optimal solutions are closer to standard Pareto optimal solutions when IGD is smaller. SP is a performance index that characterizes the distribution uniformity of the optimal solutions. The distribution uniformity is better when SP is smaller.

5.1.2 Experimental results

The standard test functions ZDT1 and DTLZ2 are experimented by DMODE-IEP algorithm adopting the above experimental design. Figure 1 shows the comparisons of optimal solutions and standard Pareto optimal solutions on ZDT1 and DTLZ2.

As shown in Figure 1(a), the optimal solutions of the two-objective optimization problem can be obtained by DMODE-IEP algorithm effectively. The obtained optimal solutions, which are basically consistent with the standard Pareto optimal solutions, have high quality. They also have uniform distribution and a wide range. As shown in Figure 1(b), the three-objective optimization problem can be solved by DMODE-IEP algorithm effectively. The optimal solutions obtained are approximated to the standard Pareto optimal solutions, and spread over Pareto front uniformly. These optimal solutions have good convergence, uniformity, and diversity.

5.1.3 Result analysis

In order to test the effect of improvement mechanisms, different versions of DMODE-IEP algorithm are defined as follows. DMODE-IEP-1 algorithm is DMODE-IEP algorithm without information of evolution progress. DMODE-IEP-2 algorithm is DMODE-IEP algorithm without proposed dynamic adjustment mechanism of evolution parameter values. DMODE-IEP-3 algorithm is DMODE-IEP algorithm without proposed dynamic adjustment mechanism of mutation strategies. DMODE-IEP-4 algorithm is DMODE-IEP algorithm without proposed adjustable selection parameter value.

All results were averaged on 30 independent runs. The optimization effects of DMODE-IEP algorithm, α-DEMO algorithm [25], MODE-RMO algorithm [26], non-dominated ordering genetic algorithm (NSGA-II) [27], and multi-objective particle swarm algorithm (MOPSO) [28] on ZDT1 and DTLZ2 are shown in Tables 1 and 2. The optimization times are shown in Table 3.

The optimization effects of DMODE-IEP on ZDT1 are shown in Table 2. The mean and variance of IGD are 1.14×10^{-3} and 1.21×10^{-4}, respectively. The mean of IGD is better than other DE-based algorithms, and it is only one-fifth of NSGA-II algorithm (5.63×10^{-3}). The variance of IGD is suboptimal, and it is just higher than MODE-RMO algorithm (1.13×10^{-4}). The mean and variance of SP are 2.44×10^{-3} and 5.31×10^{-4}, respectively. The mean and variance of SP have the smallest value in all DE-based algorithms, and even in all algorithms compared. Obviously, DMODE-IEP algorithm can get high quality optimal solutions. The improved strategies of DMODE-IEP algorithm have significant advantages in solving two-objective optimization problems.

Table 3 presents the optimization effects of DMODE-IEP on DTLZ2. The mean and variance of IGD are 3.02×10^{-3} and 4.33×10^{-5}, respectively. The mean of IGD is suboptimal, which is just higher than MODE-RMO (1.03×10^{-3}). The variance of IGD is better than other DE-based algorithms, and even it is the best one in all algorithms. The mean and variance of SP are 6.43×10^{-2} and 4.53×10^{-3}, respectively. The mean of SP is better than other algorithms. The variance of SP is the smallest in all DE-based algorithms, and just higher than MOPSO (4.23×10^{-3}). From the above analysis, it can be seen that DMODE-IEP has good convergence and diversity for three-objective optimization problems.

As shown in Table 4, the optimization time of DMODE-IEP on ZDT1 is 92 s. It is better than the other algorithms. The optimization time of DMODE-IEP on DTLZ2 is 213 s,

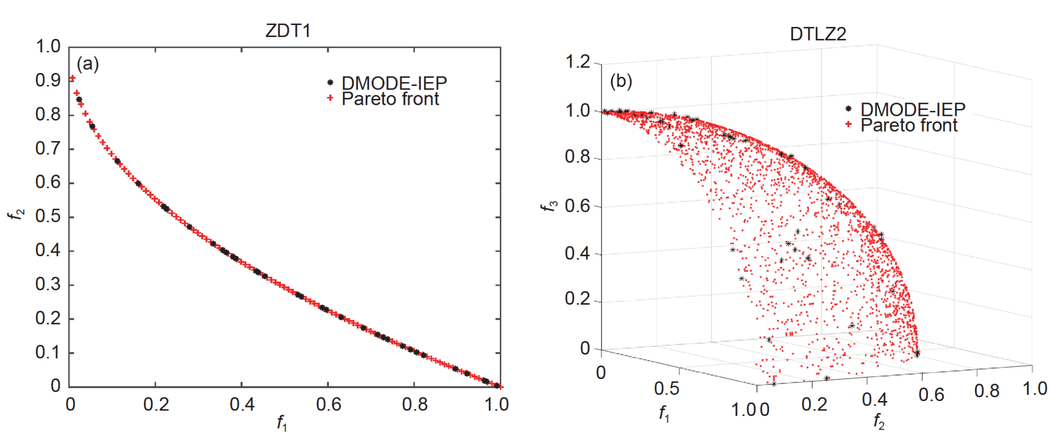

Figure 1 (Color online) Comparisons of optimal solutions and standard Pareto optimal solutions. (a) ZDT1; (b) DTLZ2.

Table 1 The optimization effects of different algorithms on ZDT1

Algorithm	IGD		SP	
	Mean	Variance	Mean	Variance
DMODE-IEP-1	2.12×10^{-3}	1.82×10^{-4}	4.13×10^{-2}	3.16×10^{-3}
DMODE−IEP-2	1.63×10^{-3}	1.15×10^{-4}	3.56×10^{-3}	7.11×10^{-4}
DMODE-IEP-3	1.72×10^{-3}	1.43×10^{-4}	3.71×10^{-3}	7.86×10^{-4}
DMODE-IEP-4	2.14×10^{-3}	1.87×10^{-4}	4.12×10^{-2}	3.22×10^{-3}
DMODE-IEP	1.14×10^{-3}	1.21×10^{-4}	2.44×10^{-3}	5.31×10^{-4}
α-DEMO	1.65×10^{-3}	1.77×10^{-4}	2.67×10^{-2}	6.81×10^{-4}
MODE-RMO	1.61×10^{-3}	1.13×10^{-4}	3.77×10^{-2}	1.15×10^{-3}
NSGA-II	5.63×10^{-3}	3.37×10^{-4}	5.29×10^{-2}	9.32×10^{-3}
MOPSO	2.11×10^{-3}	1.58×10^{-4}	3.75×10^{-2}	2.65×10^{-3}

Table 2 The optimization effects of different algorithms on DTLZ2

Algorithm	IGD		SP	
	Mean	Variance	Mean	Variance
MODE-IEP-1	1.14×10^{-2}	6.53×10^{-5}	7.91×10^{-1}	4.86×10^{-3}
MODE-IEP-2	5.96×10^{-3}	5.11×10^{-5}	8.43×10^{-2}	4.63×10^{-3}
MODE-IEP-3	5.67×10^{-3}	5.32×10^{-5}	8.66×10^{-2}	4.69×10^{-3}
MODE-IEP-4	1.17×10^{-2}	6.55×10^{-5}	7.97×10^{-1}	4.92×10^{-3}
MODE-IEP	3.02×10^{-3}	4.33×10^{-5}	6.43×10^{-2}	4.53×10^{-3}
α-DEMO	5.10×10^{-3}	7.24×10^{-5}	5.65×10^{-1}	5.67×10^{-3}
MODE-RMO	1.03×10^{-3}	3.27×10^{-4}	5.26×10^{-1}	4.84×10^{-3}
NSGA-II	2.13×10^{-1}	5.71×10^{-3}	5.03×10^{-1}	5.71×10^{-3}
MOPSO	1.07×10^{-2}	4.54×10^{-5}	3.55×10^{-1}	4.23×10^{-3}

Table 3 The optimization times of different algorithms (unit: s)

Algorithm	ZDT1	DTLZ2
DMODE-IEP-1	117	223
DMODE-IEP-2	111	217
DMODE-IEP-3	112	219
DMODE-IEP-4	114	220
DMODE-IEP	92	213
α-DEMO	101	209
MODE-RMO	98	211
NSGA-II	121	227
MOPSO	110	218

which is slower than other DE-based algorithms. Overall, the search speed of DMODE-IEP has advantages in solving two-objective and three-objective optimization problems.

5.2 Experiments of CEC2018 many objectives benchmark problems

Based on empirical value, the initial crossover rate is 0.5, the initial mutation rate is 0.2, and the initial population size is 300. The values of these parameters will be adjusted adaptively as the evolution progresses.

Table 4 The description of the many objectives benchmark problems

Group	Properties	Problem
1	Linear	MaF1
2	Concave	MaF2
		MaF6
3	Nonseparable, Convex	MaF11
	Nonseparable, Concave	MaF12
4	Multimodal	MaF7
	Unimodal	MaF13

5.2.1 *Experimental design*

The MaF series benchmark problems [5] as CEC2018 many objectives benchmark problems are selected for experiments. Four groups of many objectives benchmark problems are shown in Table 4, and run on PlatEMO.

5.2.2 *Experimental results*

The many objectives benchmark problems are experimented by DMODE-IEP algorithm adopting the above experimental design. Figures 2–5 show the optimal solutions of the benchmark problems with three and ten objectives.

5.2.3 Result analysis

The characteristic of problem in the first group is linear, and there is no single optimal solution in any subset of objectives. As shown in Figure 2, DMODE-IEP algorithm is effective for this kind of three objective optimization problem, but it needs to be improved in the process of solving ten objectives. The characteristic of problem in the second group is concave. As shown in Figure 3, DMODE-IEP algorithm is valid for

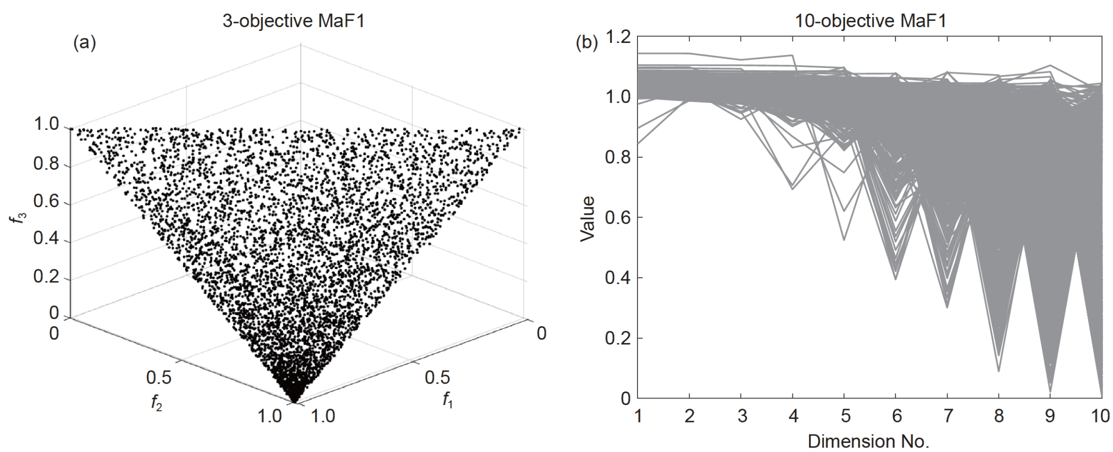

Figure 2 The optimal solutions of benchmark problems with three and ten objectives are shown by Cartesian coordinates and parallel coordinates, respectively (Group 1). (a) The optimal solutions of 3-objective MaF1; (b) the optimal solutions of 10-objective MaF1.

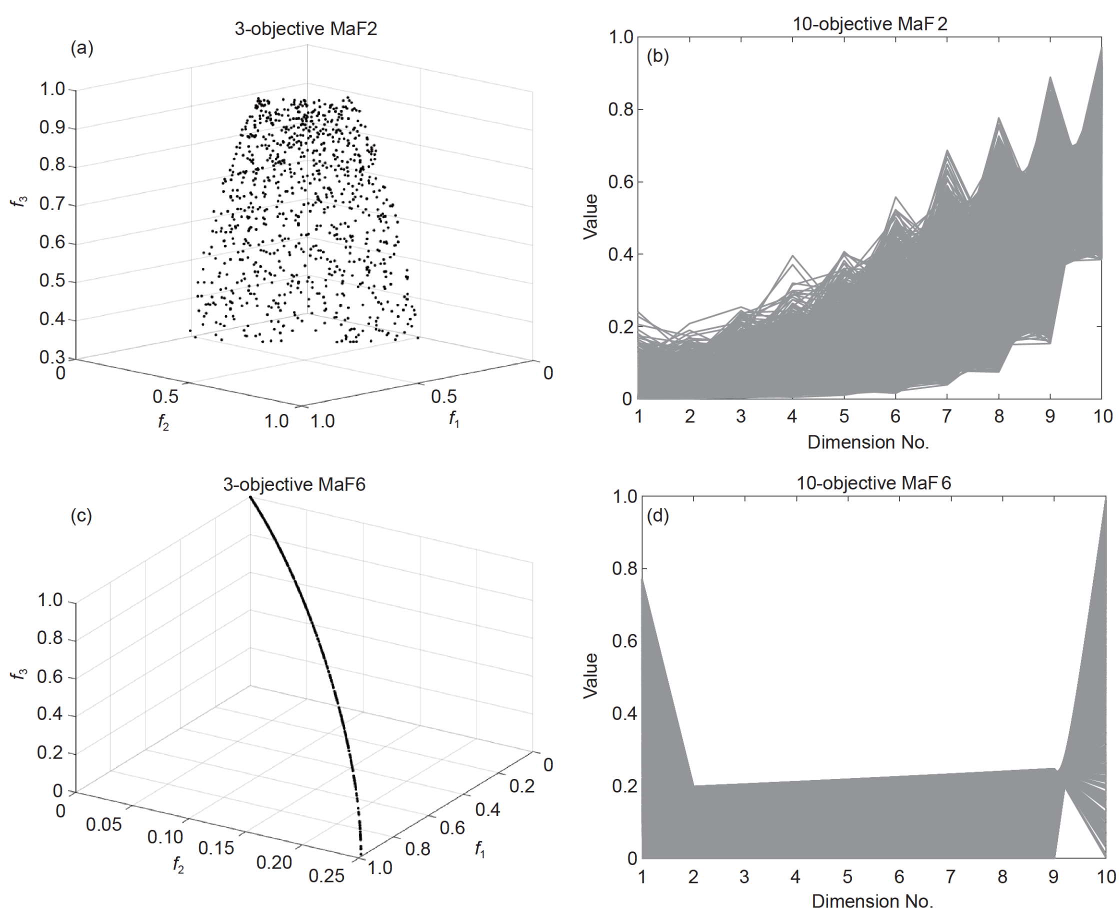

Figure 3 The optimal solutions of benchmark problems with three and ten objectives are shown by Cartesian coordinates and parallel coordinates, respectively (Group 2). (a) The optimal solutions of 3-objective MaF2; (b) the optimal solutions of 10-objective MaF2; (c) the optimal solutions of 3-objective MaF6; (d) the optimal solutions of 10-objective MaF6.

dealing with degenerate fronts problems, and it performs concurrent convergence on different objectives. The characteristic of problem in the third group is complicated fitness landscapes. As shown in Figure 4, DMODE-IEP algorithm can handle scaled disconnected fronts effectively. The multimodal and unimodal problems are visible in the fourth group. As shown in Figure 5, the disconnected and degenerate fronts can be obtained by DMODE-IEP algorithm.

5.3 Experiments of wastewater treatment process

The optimal control of wastewater treatment process is a multi-objective optimization control problem, including ensuring water quality, improving wastewater treatment efficiency, decreasing wastewater treatment costs, and reducing energy consumption.

5.3.1 Experimental design

The activated sludge process is a common method for treating wastewater in China, which is a biochemical treatment technology based on the activated sludge. The main treatment process consists of a reaction tank, secondary sedimentation tank, sludge return system, and excess sludge removal system.

(1) Experimental design

The optimized values of the dissolved oxygen concentration in the fifth tank ($S_{O,5}$) and the nitrate nitrogen concentration in the second tank ($S_{NO,2}$) are calculated by DMODE-IEP for energy conservation and emission reduction based on the benchmark simulation platform of wastewater treatment. A proportional integral controller is used to track the optimal values by operating oxygen transfer coefficient in the fifth zone and internal return flow.

(2) Optimization control objective

The operation cost of wastewater treatment plant includes aeration energy consumption of the blower, pumping energy consumption of the reflux pump, and transportation cost of the sludge. Aeration energy and pumping energy are the main components of energy consumption, the sum of which accounts for more than 80% of the operation cost. At the same time, there are strict regulations on the quality of water discharged. Wastewater treatment plants are required to pay corresponding sewage charges according to the discharge of pollutants.

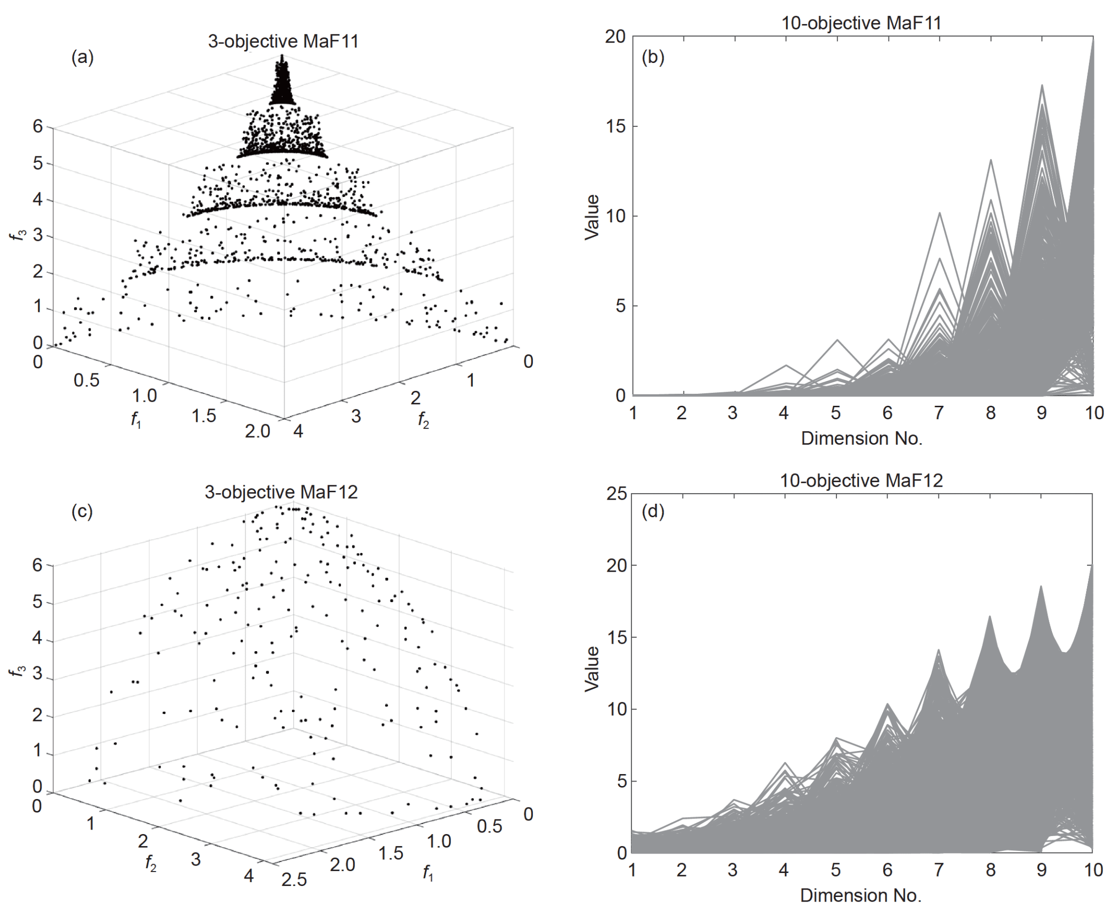

Figure 4 The optimal solutions of benchmark problems with three and ten objectives are shown by Cartesian coordinates and parallel coordinates, respectively (Group 3). (a) The optimal solutions of 3-objective MaF11; (b) the optimal solutions of 10-objective MaF11; (c) the optimal solutions of 3-objective MaF12; (d) the optimal solutions of 10-objective MaF12.

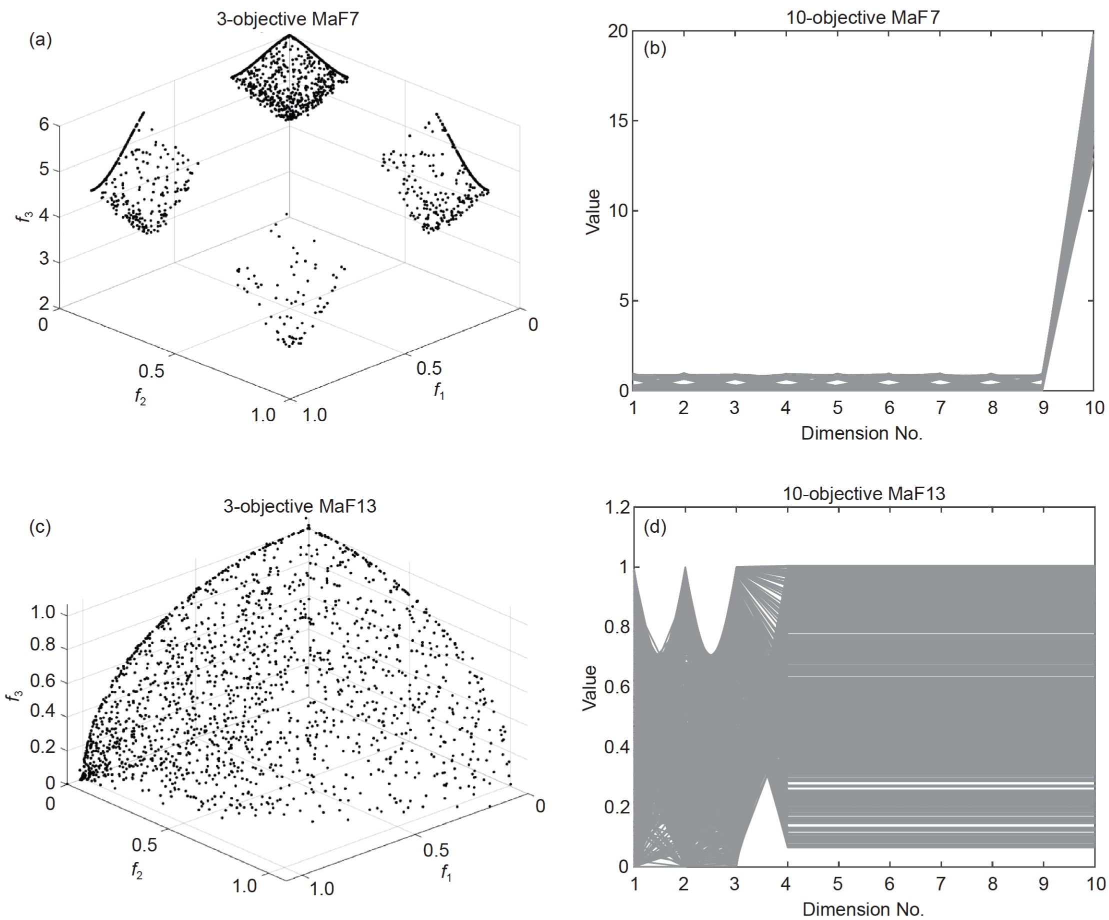

Figure 5 The optimal solutions of benchmark problems with three and ten objectives are shown by Cartesian coordinates and parallel coordinates, respectively (Group 4). (a) The optimal solutions of 3-objective MaF7; (b) the optimal solutions of 10-objective MaF7; (c) the optimal solutions of 3-objective MaF13; (d) the optimal solutions of 10-objective MaF13.

Therefore, the minimization of aeration energy (AE), pumping energy (PE), and effluent quality (EQ) is defined as the optimization control target. The optimization control objective is given as

$$
\mathrm{Min}\begin{cases}
\mathrm{AE} = \dfrac{24}{7}\displaystyle\int_{t=7}^{t=14}\sum_{l=3}^{5}\left[0.0007\times K_L a_l(t)^2\left(\dfrac{V_l}{1333}\right)+0.3267\times K_L a_l(t)\left(\dfrac{V_l}{1333}\right)\right]\mathrm{d}t,\\
\mathrm{PE} = \dfrac{1}{T}\displaystyle\int_{t=7\text{ days}}^{t=14\text{ days}}\left(0.004\cdot Q_a(t)+0.008\cdot Q_r(t)+0.05\cdot Q_w(t)\right)\cdot\mathrm{d}t,\\
\mathrm{EQ} = \dfrac{1}{T\cdot 1000}\displaystyle\int_{t=7\text{ days}}^{t=14\text{ days}}\left[\begin{array}{c}2\cdot SS_e(t)+COD_e(t)+30\cdot S_{Nkj,e}(t)\\ +10\cdot S_{\mathrm{NO},e}(t)+2\cdot BOD_e(t)\end{array}\right]\cdot Q_e(t)\mathrm{d}t,
\end{cases}
\tag{15}
$$

$$\text{s.t. } S_{\mathrm{NH}}\le 4,\ COD\le 100,\ TSS\le 30,\ N_{\mathrm{tot}}\le 18,\ BOD_5\le 10,$$

where $K_La_l(t)$ is the oxygen transfer coefficient of tank l, V_l is the volume of tank l, $K_La_l(t)$=0–10 h^{-1}, l=3, 4, 5, cycle T=7, $Q_a(t)$ is the internal return flow rate, $Q_r(t)$ is the sludge flow rate, $Q_w(t)$ is the pollutant flow rate, $SS_e(t)$ is the concentration of suspended solids, $COD_e(t)$ is the chemical oxygen demand, $S_{Nkj,e}(t)$ is the total nitrogen concentration, $S_{\mathrm{NO},e}(t)$ is the ammonia concentration, $BOD_e(t)$ is the biochemical oxygen demand, $Q_e(t)$ is the effluent flow rate, S_{NH} is the ammonia nitrogen concentration, COD is the chemical oxygen demand, TSS is the total suspended solids, N_{tot} is the total nitrogen concentration and BOD_5 is the five-day biochemical oxygen demand.

It can be seen that the oxygen transfer coefficients in three aerobic tanks are operable variables of AE. The concentration of dissolved oxygen at the end of aerobic tank not only affects the nitrification reaction but also affects the denitrification reaction through internal return flow. Thus, the controllable variable of AE is $S_{O,5}$, and the operable variable of AE is the oxygen transfer coefficient in the fifth tank (K_La_5). It can be seen that the internal return flow, external return flow, and sludge discharge are operable variables for operating PE. The nitrate nitrogen concentration at the end of anoxic tank affects the rate of denitrification. Thus, the controllable variable of PE is $S_{NO,2}$, and the operable variable of PE is internal return flow (Q_a).

Therefore, $S_{O,5}$ and $S_{NO,2}$ are controllable variables, while K_La_5 and Q_a are operating variables in the wastewater treatment process by the analyzing influencing factors of AE and PE.

(3) Parameter setting

The initial crossover rate of DMODE-IEP is set as 0.4, the initial mutation rate is set as 0.2, the initial population size is set as 500, and the maximum evolutionary generation is set as 500. The controller coefficients for $S_{O,5}$ are K_p=200 and K_i=50, the controller coefficients for $S_{NO,2}$ are K_p=8000 and K_i=5000.

5.3.2 *Experimental results*

(1) Sunny days

The wastewater treatment system operates smoothly without influence from the external environment on sunny days. The influent flow and pollutant concentration express cyclic changes during day and night, weekdays and weekends. The multi-objective optimization of wastewater treatment on sunny days is carried out by DMODE-IEP. The optimal setting values of $S_{O,5}$ and $S_{NO,2}$ are shown in Figure 6.

As shown in Figure 6, the optimal setting values are adjusted dynamically by DMODE-IEP according to the changes of influent flow and water quality with the goal of energy saving and emission reduction. The changing trends of optimal setting values of $S_{O,5}$ and $S_{NO,2}$ are consistent with the changes of influent water. The values are high during the day and weekdays, but low during at night and on weekends.

(2) Rainy days

The wastewater treatment system is affected by rainfall. The water inflow increases suddenly, and the pollutant concentration decreases significantly. The multi-objective optimization of wastewater treatment on rainy days is carried out by DMODE-IEP. The optimal setting values of $S_{O,5}$ and $S_{NO,2}$ are shown in Figure 7.

As shown in Figure 7, the optimal setting values of $S_{O,5}$ and

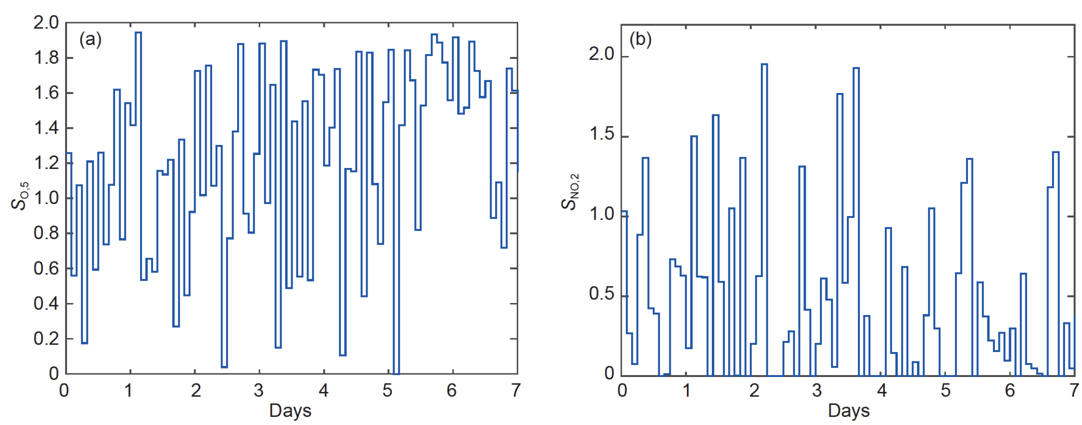

Figure 6 (Color online) The optimal setting values (on sunny days). (a) The value of $S_{O,5}$; (b) the value of $S_{NO,2}$.

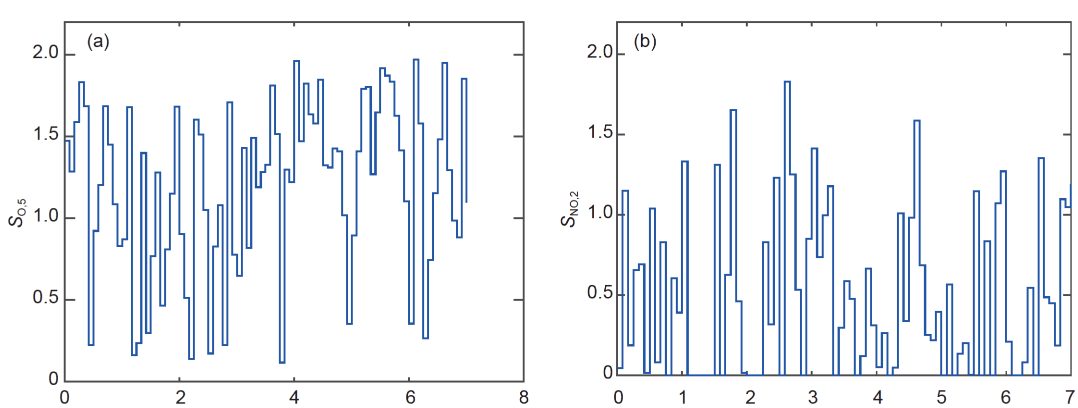

Figure 7 (Color online) The optimal setting values (on rainy days). (a) The value of $S_{O,5}$; (b) the value of $S_{NO,2}$.

Table 5 The details of emission pollutants (on sunny days)

Emission pollutants	Limit value (mg/L)	Max value (mg/L)	Average value (mg/L)	Mini value (mg/L)
S_{NH}	4.0	4.0	3.9	3.6
COD	100.0	53.4	49.8	44.3
TSS	30.0	18.7	16.8	9.9
N_{tot}	18.0	18.0	17.4	13.9
BOD_5	10.0	10.0	8.8	1.7

Table 6 The details of emission pollutants (on rainy days)

Emission pollutants	Limit value (mg/L)	Max value (mg/L)	Average value (mg/L)	Mini value (mg/L)
S_{NH}	4.0	4.0	3.9	3.8
COD	100.0	58.3	52.4	45.7
TSS	30.0	19.8	17.9	10.3
N_{tot}	18.0	18.0	17.6	14.3
BOD_5	10.0	10.0	8.9	1.8

$S_{\mathrm{NO},2}$ are adjusted by DMODE-IEP according to the influent water condition on rainy days, with the increase of the influent water and the decrease of the pollutant concentration.

5.3.3 Result analysis

The optimal setting values of $S_{\mathrm{O},5}$ and $S_{\mathrm{NO},2}$ are tracked by the proportional integral controller, manipulating oxygen transfer coefficient in the fifth tank and internal recirculation flow. The details of emission pollutants on sunny days and rainy days are shown in Tables 5 and 6.

As shown in Tables 5 and 6, the indexes of emission pollutants are qualified, and the water quality meets the national wastewater discharge standards. Most of the values are close to the limit values, which can reflect the good effect of optimal control and low energy consumption of the system.

6 Conclusions

In this paper, a dynamic multi-objective differential evolution algorithm based on the information of evolution progress is proposed to improve the optimization effect of the multi-objective differential evolution algorithm. The proposed DMODE-IEP algorithm adjusts evolution parameter values, mutation strategies, and selection parameter values dynamically to improve the quality of optimal solutions and reduce the search time. It is proved that DMODE-IEP algorithm can converge to the global optimum theoretically. The conclusions are obtained by comparing with other multi-objective optimization algorithms.

(1) The information of evolution progress, defined by fitness, reflects the characteristics of the evolution process and realizes the quantitative description of the evolutionary degree.

(2) The dynamic adjustment mechanisms of evolutionary parameter values, mutation strategies, and selection parameter values are designed based on the information of evolution progress, in order to balance the global exploration ability and the local exploitation ability.

(3) The experimental results show that the quality of the solutions and the optimization speed of DMODE-IEP algorithms are better than other algorithms when solving standard test functions. Meanwhile, DMODE-IEP algorithm can be applied to the wastewater treatment process to obtain the dynamic optimal setting values of dissolved oxygen and nitrate nitrogen, providing an effective optimization method for the complex system.

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