SCIENCE CHINA
Technological Sciences

• Article •

April 2021 Vol. 64 No. 4: 879–889
https://doi.org/10.1007/s11431-020-1686-5

Event-triggered fault-tolerant consensus control with control allocation in leader-following multi-agent systems

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Received May 8, 2020; accepted July 8, 2020; published online January 11, 2021

Event-triggered consensus in leader-following multi-agent systems with actuator fault is considered in this paper, in which the fault investigated can be multiplicative fault and outage fault. An event-triggered mechanism is utilized to relieve the communication burden of the interconnected system. Then, control allocation is proposed to solve actuator fault in the multi-agent systems for the first time. Compared with the existing fault-tolerant methods, the proposed method can guarantee that the consensus errors converge to zero asymptotically without the traditional rank assumption. Meanwhile, the *Zeno* behavior of the event-triggered system is proved to be avoided. Simulation results are also provided to verify the effectiveness of the proposed method.

multi-agent systems, fault-tolerant control, consensus, event-triggered mechanism, control allocation, optimal

Citation: Wang X X, Liu Z X, Chen Z Q. Event-triggered fault-tolerant consensus control with control allocation in leader-following multi-agent systems. Sci China Tech Sci, 2021, 64: 879–889, https://doi.org/10.1007/s11431-020-1686-5

1 Introduction

Enlightened by the group behavior in nature, more and more researchers begin to investigate the multi-agent systems (MASs), which is composed of multiple individual agents. Through cooperation among agents, MASs can complete tasks that are difficult or impossible to be achieved by single systems. Moreover, MASs can be applied to plenty of fields, such as formation control [1–4], flocking [5–7], coverage control [8, 9], collective behavior [10, 11], containment control [12, 13], etc. Among the investigation of MASs, the most important one is the consensus problem, which requires the states of all the agents to be consistent with the evolution of time. The detailed investigation on consensus can be found in ref. [14]. Moreover, due to the limitation of communication cost in reality, event-triggered method is proposed to relieve the communication burden. Furthermore, it can extend the lifespan of the actuator by effectively reducing the number of action updates. Compared with single systems, the communication cost among agents in MASs is larger, which is more urgent for being reduced. The research of event-triggered method in MASs has been widely investigated over the past years, and for more detailed event-triggered research in MASs, readers can refer to refs. [15–22].

Despite the research history of consensus in MASs is relatively long, few researchers pay attention to the situation when faults appear during the running process of MASs. Once faults occur in the interconnected system, they can cause fatal impact on the stability of the system. Since constrained by actual circumstance, such as complex environmental, extraordinary complexity of the tasks, faults are inevitable in reality. Therefore, a fault-tolerant control (FTC) is crucial for engineering application of MASs. Based on the controller design, the existing fault-tolerant consensus research can be divided into two kinds: time-scheduled FTC and event-triggered FTC.

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For the time-scheduled FTC, literatures [23–32] mainly conclude the classic and recent research. In ref. [23], team consensus for multi-agent systems in presence of actuator's partial loss of effectiveness (PLOE) fault is analyzed. In ref. [24], a cooperative actuator fault accommodation strategy for linear time-invariant MASs with a directed and switching topology is proposed. In ref. [25], a robust adaptive FTC of MASs with undetectable actuator fault is proposed. In ref. [26], the fault-tolerant tracking control for linear and Lipschitz nonlinear MASs subject to actuator fault and the leader's bounded unknown input is considered. In ref. [27], the fault-tolerant control for both leaderless and leader-following MASs with time-varying actuator fault is investigated. In ref. [28], an optimal control approach with an Off-policy Reinforcement Learning solving the Hamilton-Jacobi-Bellman equation is proposed for FTC in leaderless MASs. In ref. [29], a distributed continuous adaptive FTC consensus protocol for leaderless MASs is investigated.

Moreover, there exists the extreme situation that the actuator is totally loss of effectiveness, which can also be regarded as the outage fault. And the related research is relatively less. In ref. [30], consensus problem with actuators' PLOE and outage faults is considered, while the consensus errors of the faulty system cannot converge to zero. In ref. [31], a performance fault recovery control problem for MASs with PLOE and outage faults is considered, which also cannot guarantee the consensus errors converge to zero. In ref. [32], the cooperative output regulation problem for linear MASs with both PLOE and outage faults is considered. Although the algorithm in ref. [32] solves the outage fault, its algorithm is obtained based on the relatively strict rank assumption. It is decided by the nature of the assumption that it can only handle some special kinds of outage fault. Moreover, the above assumption exists broadly in literatures dealing with outage fault in multi-agent systems. That lack of an efficient fault-tolerant controller dealing with outage fault without the strict rank assumption motivates us to investigate a more suitable controller.

As to the event-triggered mechanism, few literatures focus on the FTC in event-triggered multi-agent systems. In ref. [33], the event-triggered fault-tolerant control for a class of networked control systems with actuator's additive fault and external disturbance is studied. In ref. [34], the distributed adaptive event-triggered fault-tolerant consensus of general linear MASs is considered to deal with the multiple fault of the actuator. In ref. [35], the adaptive double event-triggered consensus control problem for linear MASs subject to multiplicative and additive actuator fault is considered. In ref. [36], a fault-tolerant event-triggered control protocol is developed to obtain the leader-following consensus of the multi-agent systems with switching topology and actuator fault. In ref. [37], a sliding mode FTC algorithm based on event-triggered technology for second-order leader-follower MASs is proposed to deal with the actuator's partial failure. In ref. [38], the fully distributed observer-based adaptive fault-tolerant synchronization problem of multi-agent systems with event-triggered control mechanisms is investigated. In ref. [39], the problem of consensus tracking control for event-triggered multi-agent systems with stochastic actuator fault is investigated. To the best of the authors' knowledge, ref. [39] is the only literature that can be used to deal with the outage fault in an event-triggered mechanism, and its controller is obtained by the fixed-gain method. Compared with the time-scheduled method, the fault-tolerant algorithm in event-triggered mechanism is rare and the fault considered is somewhat simple. And the event-triggered mechanism can reduce the occurrence of actuator fault by reducing actuator action updates. Based on the advantage of event-triggered mechanism and the related research state, the authors try to find a more valid algorithm on dealing with various faults with event-triggered mechanism.

Moreover, it is noted that control allocation (CA) is applied to solve different kinds of faults in single systems. Compared with other traditional FTC methods, control allocation can deal with faults without reconfiguring the controller, and it can deal with outage fault. The work in ref. [40] can be seen as the most pioneering work, in which an on-line sliding mode control allocation scheme for FTC in a single systems is proposed to deal with PLOE and outage faults simultaneously. For more basic knowledge about control allocation, readers can refer to ref. [41]. Unfortunately, few researchers consider the application of control allocation in FTC of MASs. In ref. [42], the fault-tolerant attitude tracking control for an over-actuated spacecraft subject to actuator fault and external disturbances is investigated. In ref. [43], an active FTC for spacecraft attitude maneuvers with actuator saturation and faults is proposed. Further, in ref. [44], the relationship between optimal controller design and CA is investigated. It is shown that for a particular condition, the two controller designs can have the exactly same performance. Nevertheless, none of them provides a comprehensive analysis on control allocation dealing with FTC in MASs. That absence of more efficient controllers in dealing with actuator fault in multi-agent systems and the superiority of control allocation in solving various faults motivate us to investigate a novel fault-tolerant consensus control in leader-following multi-agent systems.

Based on the above discussion, a novel distributed event-triggered fault-tolerant consensus control in leader-following multi-agent systems is proposed in this paper. The main contributions of this paper can be summarized as follows.

• The dynamics of event-triggered leader-following multi-agent systems with control allocation dealing with actuator fault is proposed for the first time, while the existing works, such as refs. [42] and [43], only consider some particular application of multi-agent systems with control allocation.

• The obtained controller is event-triggered, which can efficiently reduce the communication burden of the system. It is proved that the *Zeno* behavior of the event-triggered system can be avoided. Moreover, the proposed fault-tolerant consensus controller is distributed, and it can deal with faults without reconfiguring the controller.

• The controller obtained in this paper can deal with actuator fault, especially the outage fault, and the faults can simultaneously appear in multiple agents. Moreover, the consensus errors of the multi-agent systems with outage fault can asymptotically converge to zero without the rank assumption, whereas part of the current related works can only obtain bounded consensus errors, such as refs. [30, 31]. And the work in ref. [39] cannot guarantee the optimal performance of the faulty system under outage fault. The rest of the existing literatures that can guarantee the consensus errors converge to zero under outage fault are based on the strict rank assumption in ref. [32], which can only handle some special outage fault.

The rest of the paper is organized as some preliminary knowledge is shown in sect. 2. Problem formulation is expressed in sect. 3. The design and analysis of the proposed method are given in sect. 4. Simulations are demonstrated to show the effectiveness of the proposed method in sect. 5. Finally, a conclusion is summarized in sect. 6.

Notation: R^n denotes the set of $n \times 1$ real vector. $R^{n\times m}$ is the set of $n \times m$ real matrices. I denotes the identity matrix with appropriate dimensions. $\|x\|$ denotes the norm of x. $W > 0$ implys that the matrix is positive definite. $diag(A_1, A_2, \cdots, A_n)$ is the block-diagnoal matrix with $A_1, A_2, \cdots, A_n$ on its principal diagonal. $rank(A)$ denotes the rank of matrix $\boldsymbol{A}$.

2 Preliminary

2.1 Basic graph theory

This paper considers the multi-agent systems with one leader agent and N follower agents. A directed graph is applied to model the communication topology of the multi-agent systems, which is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. $\mathcal{V} = \{\upsilon_1, \upsilon_2, \cdots, \upsilon_N\}$ is the set of all follower agents, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edge between follower agents, $\mathcal{A} = [a_{ij}]_{N\times N}$ stands for the corresponding adjacency matrix. The element of $\mathcal{E}$ is $e_{ij}, i, j = 1, 2, \cdots, N$, when there exists communication from agent i to agent j, $e_{ij} = (\upsilon_i, \upsilon_j) \in \mathcal{E}$. Otherwise, $e_{ij} \notin \mathcal{E}$. And $e_{ii} \notin \mathcal{E}, i = 1, 2, \cdots, N$. If $e_{ij} = (\upsilon_i, \upsilon_j) \in \mathcal{E}$, then a_{ij} is defined as 1. Otherwise, $a_{ij} = 0$. The definition of neighbor of agent i is defined as $\mathcal{N}_i = \{\upsilon_j \in \mathcal{V} | e_{ji} = (\upsilon_j, \upsilon_i) \in \mathcal{E}\}$. The node υ_0 denotes the leader agent, and if there exists an edge between the leader agent and the follower agent i, $d_{0i} = 1, i \in \{1, 2, \cdots, N\}$. Otherwise, $d_{0i} = 0$. Define matrix $\mathcal{D} = diag(d_{01}, d_{02}, \cdots, d_{0N}) \in R^{N\times N}$. The definition of Laplacian matrix $\mathcal{L} \in R^{N\times N}$ is

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j. \end{cases}$$

Therefore, the whole information of the leader-following MAS can be described as $\boldsymbol{G} = \mathcal{L} + \mathcal{D}$, and the element of matrix $\boldsymbol{G}$ can be $g_{ij}, i, j = 1, 2, \cdots, N$. Moreover, if there exists a directed path from every agent to every other agent, the graph is said to be strongly connected.

For the stability analysis of the interconnected system, we suppose the following assumption holds.

Assumption 2.1 The communication topology among the follower agents is strongly connected. And there exists at least one follower agent that is connected to the leader agent, i.e., there exists at least one $d_{0i} > 0, i \in \{1, 2, \cdots, N\}$.

For more detailed property that a strongly connected graph meets, readers can refer to ref. [45].

2.2 Control allocation

Control allocation (CA) is one effective method to handle actuator redundancy for different control strategies handing actuator fault [46].

The following Figure 1 demonstrates the main structure of control allocation. Generally, the controller design of CA consists of three parts. Firstly, an algorithm is designed to calculate an virtual control input v that can maintain the stability of the system in a fault-free condition. Then, a control allocation algorithm is proposed to map the virtual control input into individual actuator dimensions to guarantee that the total control input u generated in each dimension of the actuator amounts to the desired virtual control input. Finally, the control input in each dimension will be computed to achieve its desired value.

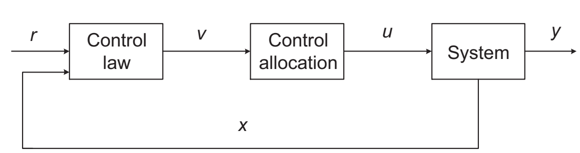

Figure 1 Control allocation.

Remark 2.1 Control allocation is common in handling various kinds of actuator fault in single systems. Compared with the traditional fault-tolerant control methods, it can deal with faults without reconfiguring the controller. Furthermore, it is able to handle the outage fault without controller accommodation and can guarantee the consensus errors converge to zero asymptotically.

3 Problem formulation

In this section, the dynamics of leader-following multi-agent systems with control allocation and the event-triggered mechanism in multi-agent systems are demonstrated.

3.1 System dynamics

A class of linear leader-following multi-agent systems with one leader agent and N follower agents is considered in this paper. Suppose that the state of agents is denoted by $x_i \in R^n$, $i = 0, 1, 2, \cdots, N$, the controller is described by $u_i \in R^m$, $i = 1, 2, \cdots, N$. It is assumed that $m > n$ throughout this paper.

The dynamic of the leader-following fault-free MASs can be formulated as

$$\dot{\boldsymbol{x}}_0 = \boldsymbol{A}\boldsymbol{x}_0, \tag{1}$$

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{B}\boldsymbol{u}_i, i = 1, 2, \cdots, N, \tag{2}$$

where 0 denotes the leader agent, and $i = 1, 2, \cdots, N$ denotes the follower agents, $\boldsymbol{A} \in R^{n\times n}$, $\boldsymbol{B} \in R^{n\times m}$ are the corresponding matrices.

Moreover, the following assumptions are supposed to hold throughout this paper.

Assumption 3.1 The matrix pair $(\boldsymbol{A}, \boldsymbol{B})$ is controllable.

Assumption 3.2 The matrix $\boldsymbol{B}$ in eq. (2) is not full rank, i.e., $rank(\boldsymbol{B}) < \min(m, n)$.

Remark 3.1 According to Assumption 3.2, the matrix $\boldsymbol{B}$ can be factorized as $\boldsymbol{B} = \boldsymbol{B}_v\boldsymbol{C}$, where $\boldsymbol{B}_v \in R^{n\times k}$, $\boldsymbol{C} \in R^{k\times m}$, and $rank(\boldsymbol{B}) = rank(\boldsymbol{B}_v) = rank(\boldsymbol{C}) = k$.

When faults occur, the real control input of agent i on the hth level can be u^F_{ih}, which can be formulated as

$$u^F_{ih} = (1 - \rho_{ih}(t))u_{ih}, i = 1, 2, \cdots, N, h = 1, 2, \cdots, m, \tag{3}$$

where $0 \leq \rho_{ih}(t) \leq 1$ represents the extent of failure. $\rho_{ih}(t) = 0$ indicates a fault-free condition; $0 < \rho_{ih}(t) < 1$ denotes the partial loss of effectiveness (PLOE) fault; $\rho_{ih}(t) = 1$ implies the outage fault. When all the $\rho_{ih}(t)$, $i = 1, 2, \cdots, N$, $h = 1, 2, \cdots, m$ is zero, the system is fault-free. Moreover, it is supposed in this work that the leader agent remains healthy during the operation of the system.

Remark 3.2 The partial loss of effectiveness, which can also be called the multiplicative fault, exists broadly in reality and has been investigated over the past years. And lots of controllers have been proposed to guarantee the stability of the faulty systems. However, few literatures focus on dealing with the outage fault. And the outage fault is common in reality and desperates for being handled, while the results on analyzing outage fault is not satisfactory for real applications, such as refs. [30–32, 39].

Based on eqs. (1) and (2), the dynamics of the leader-following MASs with control allocation can be expressed as

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{B}_v\boldsymbol{v}_i, \\ \boldsymbol{v}_i = \boldsymbol{C}\boldsymbol{u}_{iv}, i = 1, 2, \cdots, N, \end{cases} \tag{4}$$

where $\boldsymbol{v}_i \in R^k$ denotes the virtual control input calculated in the control allocation manner.

For further investigation, we have the following assumption.

Assumption 3.3 In the presence of up to any $(m - k)$ actuators undergo outage fault, the remaining actuators can still be used to implement control signals to achieve a desired control objective [47].

Remark 3.3 The Assumption 3.3 guarantees the existence of fesible solutions of the faulty system. Compared with refs. [32] and [47], the algorithm proposed in this paper does not need the traditional rank assumption: $rank(\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{\mathcal{P}}_i(\boldsymbol{t}))) = rank(\boldsymbol{B})$, where $\boldsymbol{\mathcal{P}}_i(\boldsymbol{t}) = diag(\rho_{i1}(t), \rho_{i2}(t), \cdots, \rho_{im}(t)), i = 1, 2, \cdots, N$. Theoretically speaking, the traditional rank assumption can only deal with some special outage fault since the rank assumption is too strict.

The object of this paper is to derive all the follower agents to converge to the leader, which is the classic consensus problem. The definition of consensus in math can be expressed as

$$\lim_{t\to\infty} \|\boldsymbol{x}_i(\boldsymbol{t}) - \boldsymbol{x}_0(\boldsymbol{t})\| = 0, i = 1, 2, \cdots, N,$$

where $\|\cdot\|$ usually denotes the 2-norm.

3.2 Event-triggered mechanism

In event-triggered mechanism, the controller updates only at some discrete instants, which can efficiently save the communication cost among agents and extend the lifespan of the actuator. Especially for the faulty system, it can reduce the occurrence of faults by reducing actuator action updates.

For agent i, the time sequence $\{t^i_k\}^\infty_{k=0}$ with monotonically increased property is regarded as the set of triggerd instants. The element of this sequence satisfies that: $0 \leq t^i_k < t^i_{k+1}, k =$

$1, 2, \cdots$, where t_k^i is the kth triggered instant of agent i. Moreover, the state and controller of agent i at t_k^i can be denoted as $\hat{x}_{i,k}, \hat{u}_{i,k}$, where $\hat{x}_{i,k} = x_i(t_k^i), \hat{u}_{i,k} = u_i(t_k^i)$ for all $k \in N^+$. And the obtained event-triggered controller for agent i is denoted by $\hat{u}_i$.

4 Proposed fault-tolerant consensus controller

The main result of the research is proposed in this section. A control allocation method is utilized to guarantee the consensus of the faulty multi-agent systems with event-triggered mechanism. And the *Zeno* behavior of the event-triggered mechanism is proved to be avoided.

Firstly, to describe the difference in the neighbor of agent i, we define the consensus error of agent i as

$$\boldsymbol{e_i(t)} = \sum_{j \in N_i} a_{ij}(\boldsymbol{x_i} - \boldsymbol{x_j}) + d_{0i}(\boldsymbol{x_i} - \boldsymbol{x_0}), i = 1, 2, \cdots, N. \quad (5)$$

Remark 4.1 The following work is devoted to minimize $\boldsymbol{e_i(t)}$. Based on Assumption 2.1 and the transitivity of states, if eq. (5) is minimized for all agents, the consnesus of the system can be achieved.

In the event-triggered mechanism, the consensus error of agent i at t_k^i can be denoted as $\boldsymbol{\hat{e}_{i,k}}$. And we define $\boldsymbol{\delta_i} = \boldsymbol{e_i(t)} - \boldsymbol{\hat{e}_{i,k}}, t \in [t_k^i, t_{k+1}^i)$ for agent i. In order to obtain the optimal event-triggered controller, we have the following assumption.

Assumption 4.1 The event-triggered sampling controller is Lipschitz continuous in a compact set [48]. Then there exists a constant M such that

$$\|\boldsymbol{u_i} - \boldsymbol{\hat{u}_i}\| \leq M\|\boldsymbol{\delta_i}\|. \quad (6)$$

Based on the control allocation and adaptive dynamic programming, the following performance index function of agent i is defined as

$$J_{iv} = \int_0^\infty \left[\boldsymbol{v_i^{\mathrm{T}} R_{iv}(t) v_i} + \boldsymbol{e_i^{\mathrm{T}}(t) Q e_i(t)}\right]\mathrm{d}t,\ i = 1, 2, \cdots, N, \quad (7)$$

where $\boldsymbol{Q} \in R^{n\times n}$ and $\boldsymbol{R_{iv}(t)} \in R^{k\times k}$ are diagonal positive definite matrices.

One can further obtain optimal $\boldsymbol{u_{iv}}$ by solving the following problem:

$$\begin{aligned} &\min \quad \boldsymbol{u_{iv}^{\mathrm{T}} W_i^{-1}(t) u_{iv}}, \\ &\text{subject to} \quad \boldsymbol{C u_{iv}} = \boldsymbol{v_i}, \end{aligned} \quad (8)$$

where $\boldsymbol{W_i^{-1}(t)} = \boldsymbol{(W_i{}^{-1}(t))^{\mathrm{T}}} \in R^{m\times m} > 0$.

Usually, $\boldsymbol{W_i(t)}$ indicates the actuator effectiveness of agent i. In a fault-free condition, $\boldsymbol{W_i(t)}$ is chosen to be an unit matrix $\boldsymbol{I}$. If there exists fault in the actuator of agent i, $W_i(t)$ can be chosen as $\boldsymbol{W_i(t)} = (1+\varepsilon)\boldsymbol{I} - \boldsymbol{\mathcal{P}_i(t)}$, $\varepsilon \to 0^+$ is a constant positive number to guarantee the matrix $\boldsymbol{W_i(t)}$ to be positive definite when outage fault occurs, and $\boldsymbol{\mathcal{P}_i(t)} = diag(\rho_{i1}(t), \rho_{i2}(t), \cdots, \rho_{im}(t))$. Since $rank(\boldsymbol{C}) = k$, $\boldsymbol{C}$ has a nullspace of dimension $m-k$, in which $\boldsymbol{u_{iv}}$ can be perturbed without affecting the system dynamics [44].

For comparision, the performance index function in the traditional optimal manner is defined as

$$J_{iu} = \int_0^\infty \left[\boldsymbol{u_i^{\mathrm{T}}(t) R_{iu}(t) u_i} + \boldsymbol{e_i^{\mathrm{T}}(t) Q e_i(t)}\right]\mathrm{d}t,\ i = 1, 2, \cdots, N, \quad (9)$$

where $\boldsymbol{R_{iu}(t)} \in R^{m\times m}$ is diagonal positive definite.

Further, to distinguish the controller obtained by eqs. (7), (8) from (9), $\boldsymbol{u_i}$ calculated by eq. (9) is replaced with $\boldsymbol{u_{iu}}$, so as to the consensus error $\boldsymbol{e_i(t)}$.

Based on the following three Lemmas that exist in literatures, we can further analyze the stability of the MASs.

Lemma 4.1. [44]

Consider eqs. (7) and (9) and assume that the matrices $\boldsymbol{R_{iu}(t)}$ and $\boldsymbol{R_{iv}(t)}$ are related as

$$\boldsymbol{C R_{iu}^{-1}(t) C^{\mathrm{T}}} = \boldsymbol{R_{iv}^{-1}(t)}.$$

Then the following holds: If $\boldsymbol{u_{iu}^*}$ and $\boldsymbol{v_i^*}$ are the optimal controls associated with eqs. (7) and (9), then

$$\boldsymbol{C u_{iu}^*} = \boldsymbol{v_i^*}.$$

And the corresponding $\boldsymbol{x_i} - trajectories$ are the same.

Lemma 4.2. [44]

The control laws generated by eqs. (7), (8) and (9) are the same in the following case. If, for given $\boldsymbol{R_{iv}(t)}$ and $\boldsymbol{W_i(t)}$, the matrix $\boldsymbol{R_{iu}(t)}$ is chosen as

$$\boldsymbol{R_{iu}(t)} = \boldsymbol{W_i{}^{-1}(t)} + \boldsymbol{C^{\mathrm{T}}[R_{iv}(t) - (C W_i(t) C^{\mathrm{T}})^{-1}]C}.$$

Lemma 4.3. [49]

For a system formulated by

$$\dot{\boldsymbol{x}} = f(t, \boldsymbol{x(t)}, \boldsymbol{u(t)}). \quad (10)$$

Its corresponding performance index function is

$$J(\boldsymbol{x(0)}, \boldsymbol{u}) = \int_0^{\mathrm{T}} g(t, \boldsymbol{x(t)}, \boldsymbol{u(t)})\mathrm{d}t + h(\boldsymbol{x(T)}), \quad (11)$$

where $f(t, \boldsymbol{x(t)}, \boldsymbol{u(t)})$ and $g(t, \boldsymbol{x(t)}, \boldsymbol{u(t)})$ are continuous functions on R^{1+n+m}. $\boldsymbol{x(0)}$ denotes the initial state of the system.

Let $H(t, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = g(t, \boldsymbol{x(t)}, \boldsymbol{u(t)}) + \boldsymbol{\lambda} f(t, \boldsymbol{x(t)}, \boldsymbol{u(t)})$ be the Hamiltonian function. If $\boldsymbol{u^*}$ is the controller that yields a local minimum for the performance index function (11), and

$\boldsymbol{x}^*(\boldsymbol{t})$, $\boldsymbol{\lambda}^*(t)$ are the corresponding state and co-state. Then it is necessary that

$$\begin{cases} \dot{x}^*(t) = f(t, x^*, u^*) = \left(\dfrac{\partial H(t, x^*, u^*, \lambda^*)}{\partial \lambda}\right), x^*(0) = x(0), \\ \dot{\lambda}^*(t) = -\dfrac{\partial H(t, x^*, u^*, \lambda^*)}{\partial x}, \lambda^*(T) = \dfrac{\partial h(x^*(T))}{\partial x}. \end{cases} \tag{12}$$

And for all $t \in [0, T]$,

$$\frac{\partial H(t, \boldsymbol{x}^*, \boldsymbol{u}^*, \boldsymbol{\lambda}^*)}{\partial \boldsymbol{u}} = 0. \tag{13}$$

Further, the trigger function for agent i at $t \in [t_k^i, t_{k+1}^i)$ is defined as

$$f_i = \|\boldsymbol{\delta}_i\| - \frac{\|\boldsymbol{r}_i \hat{\boldsymbol{u}}_i^*\|}{M\|\boldsymbol{r}_i\|}, i = 1, 2, \cdots, N, \tag{14}$$

where $\boldsymbol{R}_{iu}(\boldsymbol{t}) = \boldsymbol{r}_i^{\mathrm{T}}(\boldsymbol{t})\boldsymbol{r}_i(\boldsymbol{t})$. And when $f_i > 0$, the event is triggered, which means the time t_{k+1}^i occurs; when $f_i \leq 0$, the controller of agent i does not update.

Based on the discussion and Lemmas 4.1–4.3, the main theorem proposed in this paper can be expressed as follows.

Theorem 4.1. For the system formulated by eqs. (1), (4) with eqs. (7) and (9) as its corresponding performance index functions. Suppose Assumptions 2.1, 3.1, 3.2, 3.3, 4.1 hold. If for given $\boldsymbol{R}_{iv}(\boldsymbol{t})$ and $\boldsymbol{W}_i(\boldsymbol{t})$, $\boldsymbol{R}_{iu}(\boldsymbol{t})$ is selected as $\boldsymbol{R}_{iu}(\boldsymbol{t}) = \boldsymbol{W}_i^{-1}(\boldsymbol{t}) + \boldsymbol{C}^{\mathrm{T}}[\boldsymbol{R}_{iv}(\boldsymbol{t}) - (\boldsymbol{C}\boldsymbol{W}_i(\boldsymbol{t})\boldsymbol{C}^{\mathrm{T}})^{-1}]\boldsymbol{C}$. The trigger function f_i for agent i at $t \in [t_k^i, t_{k+1}^i)$ satisfies: $f_i \leq 0$. Then the optimal controller $\boldsymbol{u}_{iv}^*$ for agent i, $i = 1, 2, \cdots, N$, at $t \in [t_k^i, t_{k+1}^i)$ in dealing with both fault-free and faulty condition can be

$$\begin{cases} \hat{\boldsymbol{u}}_{iv}^*(\boldsymbol{t}) = -0.5 g_{ii} \boldsymbol{W}_i(\boldsymbol{t})\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{W}_i(\boldsymbol{t})\boldsymbol{C}^{\mathrm{T}})^{-1}\boldsymbol{C}\boldsymbol{R}_{iu}^{-1}(\boldsymbol{t})\boldsymbol{B}^{\mathrm{T}}\hat{\boldsymbol{\lambda}}_i^*(\boldsymbol{t}), \\ \dot{\hat{\boldsymbol{\lambda}}}_i^*(\boldsymbol{t}) = -(2\boldsymbol{Q}\hat{\boldsymbol{e}}_{i,k} + \boldsymbol{A}^{\mathrm{T}}\hat{\boldsymbol{\lambda}}_i^*(\boldsymbol{t})), \end{cases} \tag{15}$$

where $\hat{\boldsymbol{\lambda}}_i^* \in R^n$ denotes the auxiliary variable to be calculated. Moreover, the event-triggered leader-following MASs is asymptotically stable in both fault-free and faulty condition.

Proof.

For the event-triggered leader-following multi-agent systems, the authors define the following two kind Lyapunov functions for agent i:

$$V_{iv}(\boldsymbol{e}_{iv}(\boldsymbol{t})) = \int_t^{\infty} (\boldsymbol{v}_i^{\mathrm{T}}\boldsymbol{R}_{iv}(\boldsymbol{t})\boldsymbol{v}_i + \boldsymbol{e}_{iv}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e}_{iv})\mathrm{d}t, \quad i = 1, 2, \cdots, N, \tag{16}$$

$$V_{iu}(\boldsymbol{e}_{iu}(\boldsymbol{t})) = \int_t^{\infty} (\boldsymbol{u}_{iu}^{\mathrm{T}}\boldsymbol{R}_{iu}(\boldsymbol{t})\boldsymbol{u}_{iu} + \boldsymbol{e}_{iu}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e}_{iu})\mathrm{d}t, \quad i = 1, 2, \cdots, N. \tag{17}$$

Moreover, the time derivative of the consensus error for agent i with event-triggered mechanism by the two forementioned methods can be calculated as

$$\dot{\boldsymbol{e}}_{iv}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{e}_{iv}(\boldsymbol{t}) + g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iv}^* - \sum_{j \in \mathcal{N}_i} \boldsymbol{B}\hat{\boldsymbol{u}}_{jv}^*, i = 1, 2, \cdots, N, \tag{18}$$

$$\dot{\boldsymbol{e}}_{iu}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{e}_{iu}(\boldsymbol{t}) + g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^* - \sum_{j \in \mathcal{N}_i} \boldsymbol{B}\hat{\boldsymbol{u}}_{ju}^*, i = 1, 2, \cdots, N. \tag{19}$$

Due to the relationship between $\boldsymbol{R}_{iv}(\boldsymbol{t})$ and $\boldsymbol{R}_{iu}(\boldsymbol{t})$, $V_{iv}(\boldsymbol{e}_{iv}(\boldsymbol{t}))$ and $V_{iu}(\boldsymbol{e}_{iu}(\boldsymbol{t}))$ can obtain the controller that can have the same property in handling the optimal problem. That is, if $\boldsymbol{u}_{iu}^*$ obtained by minimizing eq. (9) can guarantee the stability of eqs. (1), (2) in a fault-free condition, then $\boldsymbol{u}_{iv}^*$ obtained by minimizing eqs. (7), (8) can guarantee the consensus of eqs. (1), (4) in spite of faults.

Therefore, we will later devote to analyzing whether $\boldsymbol{u}_{iu}^*$ can maintain the stability of eqs. (1), (2) in a fault-free condition.

Based on Lemma 4.3, we can obtain the traditional time-triggered optimal controller as

$$\begin{cases} \boldsymbol{u}_{iu}^*(\boldsymbol{t}) = -0.5 g_{ii}\boldsymbol{R}_{iu}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{\lambda}_i^*, \\ \dot{\boldsymbol{\lambda}}_i^*(\boldsymbol{t}) = -(2\boldsymbol{Q}\boldsymbol{e}_{iu} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{\lambda}_i^*). \end{cases} \tag{20}$$

Furthermore, one can obtain

$$\begin{aligned} &\boldsymbol{e}_{iu}^{\mathrm{T}}(\boldsymbol{t})\boldsymbol{Q}\boldsymbol{e}_{iu}(\boldsymbol{t}) + \boldsymbol{u}_{iu}^{*T}(\boldsymbol{t})\boldsymbol{R}_{iu}\boldsymbol{u}_{iu}^*(\boldsymbol{t}) \\ &\quad = -\boldsymbol{\lambda}_i^{*T}(\boldsymbol{t})\left(\boldsymbol{A}\boldsymbol{e}_{iu}(\boldsymbol{t}) + g_{ii}\boldsymbol{B}\boldsymbol{u}_{iu}^* - \sum_{j \in \mathcal{N}_i} \boldsymbol{B}\boldsymbol{u}_{ju}^*\right). \end{aligned} \tag{21}$$

According to ref. [50], one can get that $\frac{\partial V_{iu}}{\partial \boldsymbol{e}_{iu}} = \boldsymbol{\lambda}_i^*(\boldsymbol{t})$. Especially in the triggered instant t_k^i, we have the following:

$$\frac{\partial V_{iu}^{\mathrm{T}}}{\partial \hat{\boldsymbol{e}}_{iu}}\left[\boldsymbol{A}\hat{\boldsymbol{e}}_{iu} + g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^* - \sum_{j \in \mathcal{N}_i} \boldsymbol{B}\hat{\boldsymbol{u}}_{ju}^*\right] = -[\hat{\boldsymbol{e}}_{iu}^{\mathrm{T}}\boldsymbol{Q}\hat{\boldsymbol{e}}_{iu} + \hat{\boldsymbol{u}}_{iu}^{*T}\boldsymbol{R}_{iu}\hat{\boldsymbol{u}}_{iu}^*]. \tag{22}$$

For agent i in the event-triggered mechanism, we have $\boldsymbol{u}_{ju}^* = \hat{\boldsymbol{u}}_{ju}^*$, $j \in \mathcal{N}_i$, at $t \in [t_k^i, t_{k+1}^i)$. Thus, we can obtain the following equation:

$$-2\boldsymbol{u}_{iu}^{*T}\boldsymbol{R}_{iu} = g_{ii}\frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}\boldsymbol{B}. \tag{23}$$

The time derivative of the corresponding Lyapunov function of agent i can be calculated as

$$\begin{aligned} \dot{V}_{iu}(\boldsymbol{e}_{iu}) &= \frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}\dot{\boldsymbol{e}}_{iu} \\ &= \frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}(\boldsymbol{A}\boldsymbol{e}_{iu}(\boldsymbol{t}) + g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^* - \sum_{j \in \mathcal{N}_i} a_{ij}\boldsymbol{B}\hat{\boldsymbol{u}}_{ju}^*). \end{aligned} \tag{24}$$

Take the above eqs. (22), (23) into eq. (24), one can obtain that

$$\begin{aligned}\dot{V}_{iu}(\boldsymbol{e}_{iu}) =&\frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}\left(\boldsymbol{A}\boldsymbol{e}_{iu}(t)+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_{ju}^{*}\right)\\ =&-\boldsymbol{e}_{iu}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_{iu}(t)-\boldsymbol{u}_{iu}^{*T}(t)\boldsymbol{R}_{iu}\boldsymbol{u}_{iu}^{*}(t)\\ &-\frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}(g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\boldsymbol{u}_{ju}^{*})\\ &+\frac{\partial V_{iu}^{\mathrm{T}}}{\partial \boldsymbol{e}_{iu}}(g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_{ju}^{*}).\end{aligned}\tag{25}$$

Due to that $\boldsymbol{u}_{ju}^{*}=\hat{\boldsymbol{u}}_{ju}^{*}$, $j\in\mathcal{N}_i$ at $t\in[t_k^i,t_{k+1}^i)$, we can then simplify (25) as

$$\begin{aligned}\dot{V}_{iu}(\boldsymbol{e}_{iu}(t)) =&-\boldsymbol{e}_{iu}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_{iu}(t)+\boldsymbol{u}_{iu}^{*T}\boldsymbol{R}_{iu}\boldsymbol{u}_{iu}^{*}(t)-2\boldsymbol{u}_{iu}^{*T}\boldsymbol{R}_{iu}\hat{\boldsymbol{u}}_{iu}^{*}(t)\\ =&-\boldsymbol{e}_{iu}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_{iu}(t)+\|\boldsymbol{r}_i\boldsymbol{u}_{iu}^{*}-\boldsymbol{r}_i\hat{\boldsymbol{u}}_{iu}^{*}\|^2-\|\boldsymbol{r}_i\hat{\boldsymbol{u}}_{iu}^{*}\|^2\\ \le&-\boldsymbol{e}_{iu}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_{iu}(t)+M^2\|\boldsymbol{r}_i\|^2\|\boldsymbol{\delta}_i\|^2-\|\boldsymbol{r}_i\hat{\boldsymbol{u}}_{iu}^{*}\|^2.\end{aligned}\tag{26}$$

Based on the trigger condition, we can know that: $\|\boldsymbol{\delta}_i\|^2\le\frac{\|\boldsymbol{r}_i\hat{\boldsymbol{u}}_{iu}^{*}\|^2}{M^2\|\boldsymbol{r}_i\|^2}$ at $t\in[t_k^i,t_{k+1}^i)$. Thus, the above formula can be

$$\dot{V}_{iu}(\boldsymbol{e}_{iu})\le-\boldsymbol{e}_{iu}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{e}_{iu}(t)\,.\tag{27}$$

Because the matrix $\boldsymbol{Q}$ is positive definite, we can know that the consensus error $\boldsymbol{e}_{iu}, i=1,2,\cdots,N$ can asymptotically converge to zero.

According to the above analysis, we can conclude that the optimal controller $\boldsymbol{u}_{iv}^{*}$ can guarantee the consensus of the event-triggered leader-following MASs with PLOE and outage faults.

The proof is completed.

Remark 4.2 As long as the virtual control input $\boldsymbol{v}_i\in R^k$ can guarantee the consensus of multi-agent system in a fault-free condition, fault-tolerant control in event-triggered leader-following multi-agent systems is addressed automatically by $\boldsymbol{W}_i(t)$ without reconfiguring the controller in spite of faults, which is decided by the nature of control allocation.

Theoretically speaking, the significant difference between the time-scheduled controller and the event-triggerd controller is that the latter can reduce the communication cost. Thus the number of event-time must be finite to guarantee the efficiency of the event-triggered controller. The *Zeno* behavior means that the number of triggers by the event is infinite in a limited time. In other words, if agent i causes *Zeno* behavior, the event-triggerd mechanism can be the same with the traditional time-scheduled mechanism, which cannot reduce the communication cost. In this part, we will analyze the *Zeno* behavior in this proposed event-triggered controller.

Usually in liteatures, if $(t_{k+1}^i-t_k^i)$ has a lower positive bound, then it is divergent. Thus the *Zeno* behavior can be excluded. Therefore, the researchers usually need to give the minimum lower bound on the interval of the trigger time, which can effectively illustrate the feasibility of the algorithm. The following theorem shows the *Zeno* behavior of the proposed controller.

Theorem 4.2. The *Zeno* behavior of the event-triggered leader-following multi-agent systems can be avoided, and the lower bound $T_k^i=t_{k+1}^i-t_k^i$ of agent i, $i=1,2,\cdots,N$, between two triggered instants can be bounded by

$$T_k^i>\frac{\|\boldsymbol{r}_i\hat{\boldsymbol{u}}_i^{*}\|}{M\|\boldsymbol{r}_i\|\times\|\boldsymbol{A}\hat{\boldsymbol{e}}_i+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_i^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}\|}.\tag{28}$$

Proof.

Firstly, one can know that the time derivative of the consensus error for agent i can be

$$\dot{\boldsymbol{e}}_i(t)=\boldsymbol{A}\boldsymbol{e}_i(t)+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_i^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}, i=1,2,\cdots,N.$$

Moreover, according to ref. [37], the following inequation by some mathmatically calculation can be obtained:

$$\begin{aligned}\frac{\mathrm{d}\|\boldsymbol{e}_i-\hat{\boldsymbol{e}}_i\|}{\mathrm{d}t}&\le\|\dot{\boldsymbol{e}}_i-\dot{\hat{\boldsymbol{e}}}_i\|\\ &=\|\dot{\boldsymbol{e}}_i\|\\ &=\|\boldsymbol{A}[\boldsymbol{e}_i(t)-\hat{\boldsymbol{e}}_i]+\boldsymbol{A}\hat{\boldsymbol{e}}_i+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_i^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}\|.\end{aligned}$$

Based on $\|\boldsymbol{x}+\boldsymbol{y}\|\le\|\boldsymbol{x}\|+\|\boldsymbol{y}\|$, the following inequation can be obtained:

$$\begin{aligned}\|\dot{\boldsymbol{e}}_i-\dot{\hat{\boldsymbol{e}}}_i\|\le&\|\boldsymbol{A}[\boldsymbol{e}_i(t)-\hat{\boldsymbol{e}}_i(t)]\|\\ &+\|\boldsymbol{A}\hat{\boldsymbol{e}}_i(t)+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_{iu}^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}\|.\end{aligned}$$

Because the solution of differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}+P(x)y=S(x)$ can be expressed as

$$y=\mathrm{e}^{-\int P(x)\mathrm{d}x}\left[\int S(x)e^{\int P(x)\mathrm{d}x}\mathrm{d}x+C\right],$$

the solution of the above inequation can be calculated as

$$\begin{aligned}&\|\boldsymbol{e}_i-\hat{\boldsymbol{e}}_i\|\\ &\le\mathrm{e}^{\|\boldsymbol{A}\|T_k^i}\left[\int_{t_k^i}^{t_{k+1}^i}\|\boldsymbol{A}\hat{\boldsymbol{e}}_i+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_i^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}\|\mathrm{e}^{-\int_{t_k^i}^{t_{k+1}^i}\|\boldsymbol{A}\|\mathrm{d}t}\mathrm{d}t\right].\end{aligned}$$

To further calculation, the following inequation is obtained as

$$\|\boldsymbol{e}_i-\hat{\boldsymbol{e}}_i\|\le T_k^i\|\boldsymbol{A}\hat{\boldsymbol{e}}_i+g_{ii}\boldsymbol{B}\hat{\boldsymbol{u}}_i^{*}-\sum_{j\in\mathcal{N}_i}\boldsymbol{B}\hat{\boldsymbol{u}}_j^{*}\|.$$

Because when the function is triggered, the δ_i meets the following:

$$\|e_i - \hat{e}_i\| > \frac{\|r_i \hat{u}_i^*\|}{M\|r_i\|}.$$

As for $\|r_i \hat{u}_i^*\|$, it is greater than zero. According to the property of norm, if $\|r_i \hat{u}_i^*\| = 0$, then $r_i \hat{u}_i^* = 0$. Moreover, due to $R_{iu}(t) = r_i^{\mathrm{T}}(t) r_i(t)$, one can get that $\hat{u}_i^{*\mathrm{T}} R_{iu}(t) \hat{u}_i^* = 0$. Further, because $R_{iu}(t)$ is positive definite, it is then obtained that $\hat{u}_i^* = [0, 0, \cdots, 0]^{\mathrm{T}}$. However, the proposed controller is obtained by supposing the Assumption 3.3 hold, which implies that $\hat{u}_i^* \neq [0, 0, \cdots, 0]^{\mathrm{T}}$. Thus, one can obtain that $\|r_i \hat{u}_i^*\| > 0$.

Thus, the lower bound of T_k^i can be obtained as

$$T_k^i > \frac{\|r_i \hat{u}_i^*\|}{M\|r_i\| \times \|A\hat{e}_i + g_{ii} B\hat{u}_i^* - \sum\limits_{j\in\mathcal{N}_i} B\hat{u}_j^*\|}.$$

Because the above inequation holds, we can conclude that the *Zeno* behavior can be excluded in the proposed event-triggered controller.

The proof is completed.

Remark 4.3 At the begining of the operation, $\|A\hat{e}_i + g_{ii} B\hat{u}_i^* - \sum\limits_{j\in\mathcal{N}_i} B\hat{u}_j^*\|$ cannot be zero. However, with the controller involved, the consensus error $e_i(t)$ can be decreased. When $\|A\hat{e}_i + g_{ii} B\hat{u}_i^* - \sum\limits_{j\in\mathcal{N}_i} B\hat{u}_j^*\| \to 0$, we can know that $T_k^i \to \infty$, which means that the event is no longer triggered, and the consensus is achieved.

Moreover, the general control system block diagram of the ith following agent, $i = 1, 2, \cdots, N$, can be expressed by the following Figure 2.

5 Simulation

To verify the effectiveness and superiority of the novel method, the simulation results of event-triggered leader-following multi-agent systems are shown in this section.

The directed communication graph of the system is shown in Figure 3, where vertex 0 denotes the leader agent and vertexes 1–5 denote follower agents.

The corresponding Laplacian matrix $\mathcal{L}$ and G are

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Define the state of the system as $x_i = [x_{i1}, x_{i2}]^{\mathrm{T}} \in R^2, i = 0, 1, 2, 3, 4, 5$. The parameters of the system are set as

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}, B_v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
$$Q = 10 I_{10\times10},\ R_v = 10 I_{5\times5}, M = 500.$$

In order to show the effectiveness of the proposed method, the following simulation results are shown.

In this simulation, the faults occur in the operation of the system, which is common in practice. The overall running time of this simulation is T=25 s. The detailed simulation parameters are as follows.

For $t \in (0, 12.5]$, the system is fault-free and the related matrices can be set as

$$W_1(t) = W_2(t) = W_3(t) = W_4(t) = W_5(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For $t \in (12.5, 25]$, faults occur in different follower agents with various kinds and the faults can be modelled by the following functions:

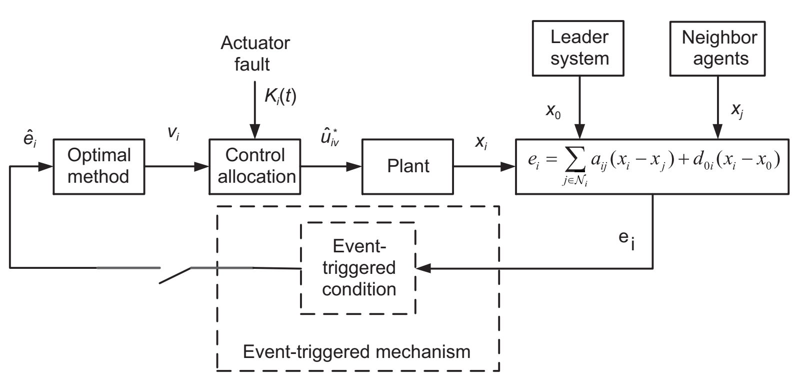

Figure 2 The control block diagram of the ith following agent.

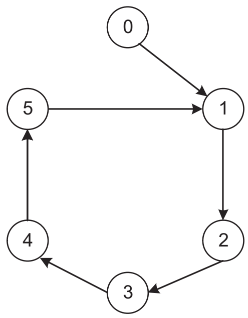

Figure 3 Communication topology.

$$\mathcal{P}_1(t)=\begin{bmatrix}1&0&0\\0&\sin(t\pi/30)&0\\0&0&1\end{bmatrix},\ \mathcal{P}_2(t)=\begin{bmatrix}t/60&0&0\\0&1&0\\0&0&0.8\end{bmatrix},$$
$$\mathcal{P}_3(t)=\begin{bmatrix}e^{-t/30}&0&0\\0&1&0\\0&0&0.1\end{bmatrix},\ \mathcal{P}_4(t)=\begin{bmatrix}1&0&0\\0&(t/30)^2&0\\0&0&1\end{bmatrix},$$
$$\mathcal{P}_5(t)=\begin{bmatrix}1&0&0\\0&0.4&0\\0&0&0.9\end{bmatrix}.$$

And ε is set to be 0.0001.

Remark 5.1 In the simulation, the selection of $\mathcal{P}_i(t), i=1,2,3,4,5$, obeies the following two rules: (1) the diagonal elements of $\mathcal{P}_i(t)$ falls within the range of [0, 1], which indicates the extent of actuator fault; (2) the occurence number of 1 in $\mathcal{P}_i(t)$ is no more than $m-k$, which obeies the Assumption 3.3.

The corresponding simulation results are Figures 4–6. Since faults occur at the time t=12.5 s, Figure 4 indicates that the consensus in both PLOE and outage faults can be reached without reconfiguring the controller. In Figure 5, the subgraph denotes the initial states of all agents, and we can see from Figure 5 that all agents converge to the same state with the evolution of time.

The following Figure 6 shows that the *Zeno* behavior of the system can be avoided. The triggered time can be shown in Figure 6.

According to Remark 4.3, we can know that when the consensus is achieved the event is no longer triggered, which explains why the system is not triggered after some specific time instants.

Remark 5.2 The event-triggered fault-tolerant consensus control for leader-following MASs with control allocation is propsed for the first time. And the exisiting literatures in this filed is relatively rare. Moreover, the advantage between this paper and the existing literatures can be directly obtained by the previous discussion. Thus, a comparision simulation with other methods is not included in this section.

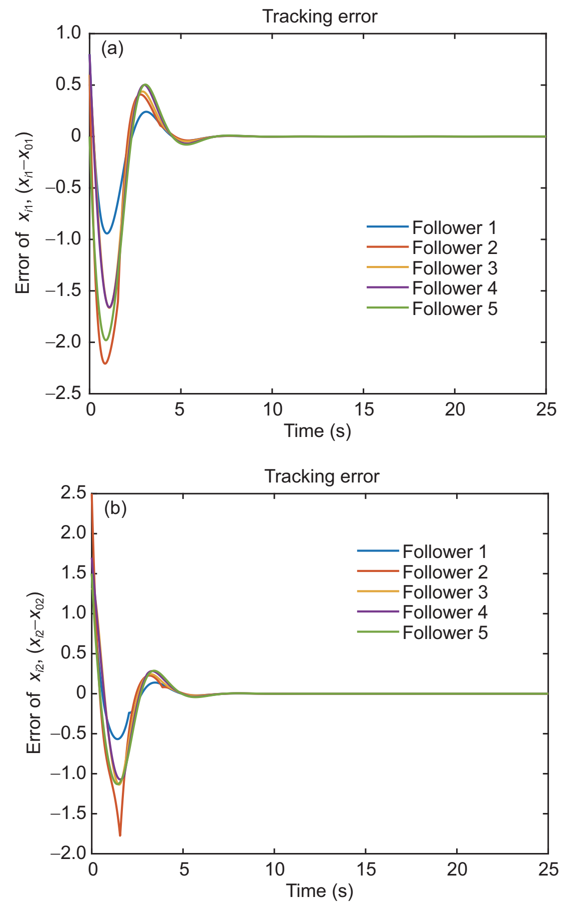

Figure 4 Tracking error in each dimension. (a) Error of $x_{i1}, i=1,2,\cdots,5$. (b) Error of $x_{i2}, i=1,2,\cdots,5$.

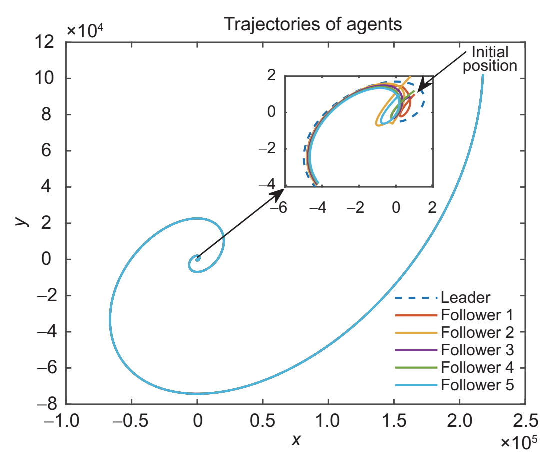

Figure 5 Trajectories of agents.

6 Conclusion

Event-triggered fault-tolerant control in leader-following multi-agent systems with control allocation has been studied for the first time in this paper. The proposed method can deal

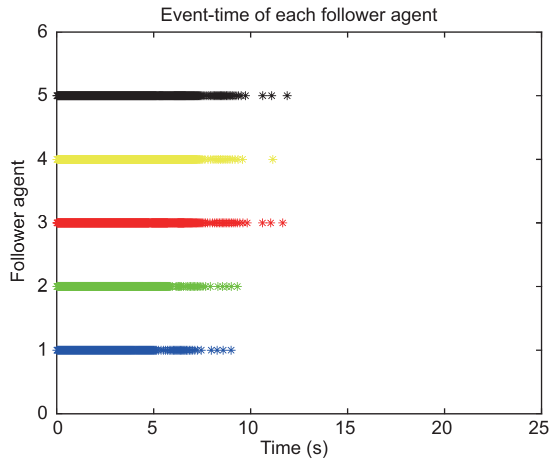

Figure 6 Event-time of each follower agent.

with multiplicative fault and outage fault without reconfiguring the controller. Compared with the existing research on FTC in multi-agent systems, the proposed method can guarantee the consensus errors converge to zero asymptotically without the strict rank assumption and can save communication cost effectively. Moreover, the *Zeno* behavior of the system has been proved to be avoided. And we will focus on investigating the consensus of event-triggered multi-agent systems with more general kinds of faults in the future.

This work was supported by the National Key Research and Development Program of China (Grant No. 2018YFB1307803), and the National Natural Science Foundation of China (Grant No. 61973175).

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