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Transfer matrix method for multibody systems (Rui method) and its applications†

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The transfer matrix method for multibody systems, namely the "Rui method", is a new method for studying multibody system dynamics, which avoids the global dynamics equations of the system, keeps high computational speed, and allows highly formalized programming. It has been widely applied to scientific research and key engineering of lots of complex mechanical systems in 52 research directions. The following aspects regarding the transfer matrix method for multibody systems are reviewed systematically in this paper: history, basic principles, formulas, algorithm, automatic deduction theorem of overall transfer equation, visualized simulation and design software, highlights, tendency, and applications in 52 research directions in over 100 key engineering products.

multibody system dynamics, transfer matrix method for multibody systems, Rui method, automatic deduction theorem of overall transfer equation, theory, computation, software, applications

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1 Introduction

Multibody system dynamics (MSD), a research hotspot of today's mechanics, offers an important foundation for the design of dynamics performance and tests of engineering productions in many fields including aeronautics, astronautics, weapons, vehicles, ships, robots, trains, general machinery and human, etc. Diverse multibody system dynamics methods (MSDM) developed rapidly in the past 50 years have significantly promoted the developments of modern engineering technology. At the same time, the following problems have been faced by ordinary MSDM [1]: (1) The global dynamics equations of the system are necessary and their deduction processes, generally speaking, are rather complicated. (2) The order of the system matrix of complex multibody systems (MS) is quite high which causes the computational speed to decrease significantly. Now, the fastest computational speed of the computer in the world, IBM's Summit, is 2×10^{17} times per second, nearly 7×10^{24} times per year. For example, the required computational times for the dynamics optimal design of firing sequence to realize the best firing precision for a 40 tubes launch rocket system are far more than 40! (about 8×10^{47}), thus it takes a very long time! It can not satisfy the requirement of dynamics design of lots of complex mechanical systems if using ordinary MSDM. (3) The very high order of system matrix and the dynamics coupling among rigid and flexible bodies will cause the ill-conditioning when computing the eigenvalue problem of a complex linear multi-rigid-flexible-body system (MRFS), and will cause the non-self-conjugate of eigenvalue problem of a MRFS, eigenvectors not orthogonal in ordinary meaning, and the difficulty to analyze precisely the dynamics response of a linear MRFS with

†The predecessor of this paper was reported and was awarded the best paper as ref. [1] in the 14th International Conference on Multibody Systems, Nonlinear Dynamics, and Control, August 26–29, 2018, Quebec City, Canada. The title of this paper just was the topic of an International Symposium in the International Conference.
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modal methods.

The classical transfer matrix method (TMM) was established to study the vibration problem of one dimensional systems comprised of elastic elements in 1920s. However, TMM can neither be used for the vibration characteristics of MRFS, nor for the dynamics of time-variant and nonlinear MS with large-motion.

In order to solve above difficulties faced by ordinary MSDM and achieve the rapid computation of dynamics of complex mechanical systems, Rui [2] presented the transfer matrix method for multibody systems (MSTMM) in 1993 for the first time, which then was called "Rui method" by many professors, such as Jens Wittenburg evaluated "Rui method represents a totally new approach for simulating the dynamics of multibody system and is very promising" in the preface of ref. [3], John Herbst evaluated "It is very valuable to popularize Rui method in the study fields of multibody system dynamics and complicated mechanical engineering" in the preface of ref. [4]. Many scholars and experts made great contributions to the development of the method. This method has become a new efficient formalized MSDM and has been widely applied to scientific research and engineering fields by continuous theoretical research and engineering practices as well as extensive international academic exchanges during the past 25 years. It has been proved that the method is effective for solving practical engineering problems. Lots of difficulties appearing in many national key engineering projects have been overcome, and many important technological achievements have been realized by using this method. All of these show the powerful function and broad application prospect of the method.

According to incomplete statistics up to now, engineering problems of about 100 products in 52 fields including weapons, ships, aeronautics, astronautics, machinery, etc. have been successfully solved. 6 monographs [3–8] concerned with Rui method and its applications have been published. More than 300 papers in over 100 journals by more than 200 researchers from nearly 60 institutions studying the method have been published, more than 10 software copyrights and 100 patents have been authorized, moreover, 4 national technology progress and invention awards of China have been realized regarding the method [1].

This paper introduces systematically the development, history, theory, software, highlights, tendency and 52 applications in important engineering of Rui method.

2 History of Rui method

The development of Rui method mainly goes through three stages: transfer matrix method for linear multibody systems (linear MSTMM) [3,5,9–11] (1993–), discrete time transfer matrix method for multibody systems (MSDTTMM) [3,5,12–14] (1998–), and the new version of transfer matrix method for multibody systems (NV-MSTMM) [5,15] (2013–).

Rui (1993) [2] presented MSTMM firstly in his dissertation for dcotoral degree, which then was published in 1995 [7]. It solved basically the eigenvalue problem of a complex linear MS, avoided the ill-conditioning appeared in computation, and improved significantly the computational speed. Rui et al. (1997) [9] presented the concept of "augmented eigenvector" and "augmented operator" as well as the orthogonality of augmented eigenvectors for linear MS and MRFS, and the exact analysis of dynamics response of a linear MRFS using a modal method was firstly achieved. Yun et al. (2006) [16] presented the TMM for a two-dimensional system, and the dynamics analysis of a two-dimensional system was achieved by using pure TMM. Lu, Bestle and Wang et al. (2006) [17–19] presented the TMM for linear controlled multibody system (CMS), the active vibration control of CMS was achieved.

Rui et al. (1998) [12] presented the discrete time transfer matrix method for multi-rigid-body systems (MSDTTMM) and gradually improved this method. In MSDTTMM, the high efficiency of MSTMM and wide application range of time integration procedures are combined. The constant transfer matrices and state vectors described in modal coordinates in linear MSTMM were extended into time-dependent transfer matrices and state vectors described in physical coordinates respectively. Rui et al. (1999) [13] presented the MSDTTMM for MRFS and gradually improved it. He et al. (2007) [14] presented the Riccati MSDTTMM by introducing Riccati transformation to solve MSD consisting of numerous elements. Rong et al. (2010) [20] presented MSDTTMM for CMS. The dynamics of MRS, MRFS, and CMS, especially complex launching systems, were firstly studied using Rui method without global dynamics equations and with high computational speed.

Nevertheless, it is necessary to keep the time step small so as to ensure the computational precision and stability owing to linearization of nonlinear functions in MSDTTMM. Rui et al. (2013) [15] presented the NV-MSTMM without linearization, in which accelerations, angular accelerations, internal forces and torques are chosen as state variables of the state vector.

Rui et al. (2011) [21] presented the concept of topology figure of system dynamics model and the automatic deduction theorem of overall transfer equation. Rui et al. (2016) [22] improved the automatic deduction theorem. Zhou et al. (2016) [23,24] presented the deduction method of overall transfer equations for linear CMS. He et al. (2006) [25] presented the mixed methods of Rui method with other

MSDM and the finite element method to take advantages of each of these methods. Rui et al. (2014) [26] developed the visualized simulation and design software MSTMMSim for MSD with Rui method executing as its core for further engineering applications.

"Transfer Matrix Method for Multibody Systems and Its Applications" was taken as the title of the international symposium in the 14th International Conference on Multibody Systems, Nonlinear Dynamics, and Control (MSNDC), held in August 26-29, 2018, Quebec City, Canada. And "Rui method" was mentioned in 15 papers presented in the conference.

3 Basic theory of Rui method

3.1 Modular modeling principle and topology figure of dynamics model

The basic idea of Rui method is: firstly, divide a MS into the elements containing bodies (including rigid bodies, flexible bodies, lumped masses, etc.) and hinges (including ball-and-sockets, pins, springs, rotary springs, dampers and rotary dampers, etc.), all of which share parallel status; then establish the transfer equations and transfer matrices of these elements, the library of transfer matrices of hundreds of elements can be found in refs. [3,5]; finally, deduce the overall transfer equation and overall transfer matrix of the system automatically according to the topology figure of system dynamics model. Due to the features of without the global dynamics equations of system and always keeping very low order of system matrix, the computational speed of the method is always high comparing with ordinary MSDM.

The topology figure of dynamics model of system [21], as shown in Figure 1, is a new graphical representation method in Rui method, it can intuitively describe the relationship among the state vectors of elements. In the topology figure of dynamics model of a MS, a circle (◯) represents a body element, and an arrow (→) represents a hinge element and transfer direction. Numbers represent the sequence numbers of elements, while "0" denotes the boundary point. Because the transfer direction is specified from tips to the root of a system, for a tree system, all elements can be treated as ones with multiple input ends and single output end (the element with single input and single output end is its special case). The first and second subscripts i and j in a state vector $\boldsymbol{z}_{i,j}$ or $\boldsymbol{Z}_{i,j}$ represent sequence number of the body and hinge connected with the point $P_{i,j}$ respectively. If the second subscript j=0, $\boldsymbol{z}_{i,0}$ or $\boldsymbol{Z}_{i,0}$ represents the state vector of a boundary point.

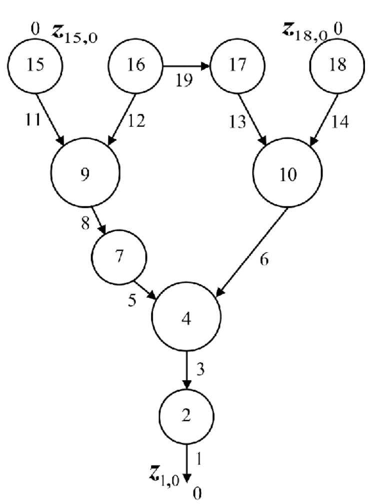

Figure 1 Topology figure of dynamics model of MS.

3.2 New version of transfer matrix method for multibody systems

In NV-MSTMM [5,15], the state vector of an element moving in space is defined as

$$\boldsymbol{z} = [\ddot{x}, \ddot{y}, \ddot{z}, \dot{\Omega}_x, \dot{\Omega}_y, \dot{\Omega}_z, m_x, m_y, m_z, q_x, q_y, q_z, 1]^{\mathrm{T}}. \quad (1)$$

The transfer equation of an element j can be obtained by re-writing its dynamics equations:

$$\boldsymbol{z}_{j,j+1} = \boldsymbol{U}_j(t_i)\boldsymbol{z}_{j,j-1}, \quad (2)$$

where $\boldsymbol{U}_j(t_i)$ is the transfer matrix of the element j and is a known function of kinematic parameters of system at current time t_i, whose order is 13×13 for an element with two ends moving in space.

The overall transfer equation of a chain MS is

$$\boldsymbol{z}_{n,0} = \boldsymbol{U}_{1-n}\boldsymbol{z}_{1,0}, \quad (3)$$

with the overall transfer matrix

$$\boldsymbol{U}_{1-n} = \boldsymbol{U}_n \cdots \boldsymbol{U}_2\boldsymbol{U}_1, \quad (4)$$

where $\boldsymbol{z}_{n,0}$ and $\boldsymbol{z}_{1,0}$ are state vectors of output and input ends of the system, respectively; $\boldsymbol{U}_i (i = 1, 2, \cdots, n)$ are the transfer matrices of elements.

The highest order of overall transfer matrix $\boldsymbol{U}_{1-n}$ is 13×13 no matter how large the DOF of a chain MS is, which is much smaller compared with ordinary MSDM, leading to very high computational speed. The unknown state variables of the state vectors at boundary ends can be obtained by applying the boundary conditions and solving the overall transfer equation. Next, state vectors of all joint points of the system can be got at time t_i by solving transfer equations of elements. The generalized coordinates and velocities of the system at the next time can be obtained by using any integration scheme. The above procedures can be repeatedly executed until required time.

From principle, this method belongs to an exact approach of dynamics analysis, because the transfer equations of elements as well as overall transfer equations of system are exact and the boundary conditions are strictly satisfied.

For a MRFS, except for eq. (3), the generalized coordinates equations describing the deformation need to be supplied:

$$\dot{\boldsymbol{Y}}_i = \boldsymbol{M}_i^{-1}\boldsymbol{\Psi}_{i,I}\boldsymbol{Q}_{i,I} + \boldsymbol{M}_i^{-1}\boldsymbol{\Psi}_{i,O}\boldsymbol{Q}_{i,O} + \boldsymbol{M}_i^{-1}\boldsymbol{Q}_i^0, \tag{5}$$

where $\boldsymbol{Y}_i$ is the generalized velocity column matrix of flexible body i, composed of general velocities of the floating frame and the first time derivatives of the generalized deformation coordinates. The deduction and the detailed expression of this equation can be found in ref. [27].

3.3 Discrete time transfer matrix method for multibody systems

In MSDTTMM [3,5,12], the state vectors of rigid body and elastic body with spatial motion are defined respectively as

$$\boldsymbol{z} = \left[x, y, z, \theta_x, \theta_y, \theta_z, m_x, m_y, m_z, q_x, q_y, q_z, 1\right]^{\mathrm{T}}, \tag{6}$$

and

$$\boldsymbol{z} = \left[x, y, z, \theta_x, \theta_y, \theta_z, m_x, m_y, m_z, q_x, q_y, q_z, q^1, q^2, \cdots, q^n, 1\right]^{\mathrm{T}}. \tag{7}$$

The dynamics equations of element can be assembled into a transfer equation similar to eq. (2) by using linearization and numerical integration procedures. n is the highest mode order describing the deformation. The overall transfer equation and overall transfer matrix of the system have the same forms as eqs. (3) and (4) for a chain MS.

3.4 Transfer matrix method for linear multibody systems

For linear MRFS [3,5,10], the state vector of element vibrating in space in modal coordinates is defined as

$$\boldsymbol{Z} = \left[X, Y, Z, \Theta_x, \Theta_y, \Theta_z, M_x, M_y, M_z, Q_x, Q_y, Q_z\right]^{\mathrm{T}}. \tag{8}$$

The transfer equation of element j can be got by modal transformation of its dynamics equations:

$$\boldsymbol{Z}_{j,j+1} = \boldsymbol{U}_j\boldsymbol{Z}_{j,j-1}. \tag{9}$$

For a linear chain MRFS, the overall transfer equation and overall transfer matrix of the system have the same forms as eqs. (3) and (4).

The eigenfrequency equation of the MS can be obtained by using boundary conditions of the system in the overall transfer equation:

$$\det\left(\overline{\boldsymbol{U}}_{1-n}\right) = 0. \tag{10}$$

The eigenvalue problems of MS can be solved by solving eqs. (10), (3) and (9).

The new concept of body dynamics equations of body elements and system are presented as

$$\boldsymbol{M}_i\boldsymbol{v}_{i,tt} + \boldsymbol{C}_i\boldsymbol{v}_{i,t} + \boldsymbol{K}_i\boldsymbol{v}_i = \boldsymbol{f}_i, \tag{11}$$

and

$$\boldsymbol{M}\boldsymbol{v}_{tt} + \boldsymbol{C}\boldsymbol{v}_t + \boldsymbol{K}\boldsymbol{v} = \boldsymbol{f}, \tag{12}$$

where

$$\begin{aligned}
&\boldsymbol{M} = \mathrm{diag}\left(\boldsymbol{M}_{B_1}, \boldsymbol{M}_{B_2}, \ldots, \boldsymbol{M}_{B_n}\right),\\
&\boldsymbol{K} = \mathrm{diag}\left(\boldsymbol{K}_{B_1}, \boldsymbol{K}_{B_2}, \ldots, \boldsymbol{K}_{B_n}\right),\\
&\boldsymbol{f} = \left[\boldsymbol{f}_{B_1}^{\mathrm{T}} \quad \boldsymbol{f}_{B_2}^{\mathrm{T}} \quad \cdots \quad \boldsymbol{f}_{B_n}^{\mathrm{T}}\right]^{\mathrm{T}},\\
&\boldsymbol{v} = \left[\boldsymbol{v}_{B_1}^{\mathrm{T}} \quad \boldsymbol{v}_{B_2}^{\mathrm{T}} \quad \cdots \quad \boldsymbol{v}_{B_n}^{\mathrm{T}}\right]^{\mathrm{T}}.
\end{aligned} \tag{13}$$

The meaning of symbols in above equations can be found in ref. [10].

Applying modal technology, let

$$\boldsymbol{v} = \sum_{k=1}^{\infty} \boldsymbol{V}^k q^k(t), \tag{14}$$

where

$$\boldsymbol{V}^k = \left[\boldsymbol{V}_{B_1}^k, \boldsymbol{V}_{B_2}^k, \cdots, \boldsymbol{V}_{B_n}^k\right]^{\mathrm{T}}, \ (k = 1, 2, \cdots) \tag{15}$$

is a new concept named augmented eigenvector, and its components include translational and angular displacements in modal coordinates of joint points in the kth order mode.

The orthogonality of augmented eigenvectors can be found as

$$\begin{aligned}
&\left\langle \boldsymbol{M}\boldsymbol{V}^k, \boldsymbol{V}^p \right\rangle = \delta_{k,p}M_p,\\
&\left\langle \boldsymbol{K}\boldsymbol{V}^k, \boldsymbol{V}^p \right\rangle = \delta_{k,p}K_p.
\end{aligned} \tag{16}$$

For proportional damping $\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}$, the differential equations eq. (12) can be decoupled by using the orthogonality of augmented eigenvectors:

$$\ddot{q}^k(t) + (\alpha + \beta\omega_k^2)\dot{q}^k(t) + \omega_k^2 q^k(t) = p^k(t), \tag{17}$$

where

$$p^k(t) = \sum_j \left\langle \boldsymbol{f}_j(x_1, t), \boldsymbol{V}_j^k(x_1) \right\rangle / M_k. \tag{18}$$

The system's augmented eigenvectors can be obtained according to the state vector of each element. The generalized coordinates and the dynamics response of the system can be obtained by solving the eqs. (17) and (14) respectively.

3.5 Riccati transfer matrix method for multibody systems

The Riccati transformation was introduced to further reduce the order of the system matrix and improve the numerical stability of MSTMM, yielding the Riccati transfer matrix method for multibody systems (RMSTMM). The two-point boundary value problem is converted to an initial value problem by using Riccati transformation, and the advantages of MSTMM are retained. The Riccati transformation and recursive transfer equation have the same forms for all Rui methods including NV-MSTMM [5], MSDTTMM [3,5,14] and linear MSTMM [28]. RMSTMM with NV-MSTMM for a chain MRS moving in space is introduced hereinafter.

Modifying the state vector in eq. (1) as

$$\boldsymbol{z} = \left[\ddot{x}, \ddot{y}, \ddot{z}, \dot{\Omega}_x, \dot{\Omega}_y, \dot{\Omega}_z, m_x, m_y, m_z, q_x, q_y, q_z\right]^{\mathrm{T}}. \tag{19}$$

So, the form of the transfer equation needs to be modified as

$$\boldsymbol{z}_{j,O} = \boldsymbol{U}_j \boldsymbol{z}_{j,I} + \boldsymbol{f}_j, \tag{20}$$

that is

$$\begin{bmatrix} \boldsymbol{z}_a \\ \boldsymbol{z}_b \end{bmatrix}_{j,O} = \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{21} & \boldsymbol{T}_{22} \end{bmatrix}_j \begin{bmatrix} \boldsymbol{z}_a \\ \boldsymbol{z}_b \end{bmatrix}_{j,I} + \begin{bmatrix} \boldsymbol{f}_a \\ \boldsymbol{f}_b \end{bmatrix}_j, \tag{21}$$

where $\boldsymbol{z}_a$ denotes the six known state variables being zeros at the input end of the system, and $\boldsymbol{z}_b$ contains the six remaining unknowns.

Introducing the Riccati transformation of state vector of a connecting point P

$$\boldsymbol{z}_{a,P} = \boldsymbol{S}_P \boldsymbol{z}_{b,P} + \boldsymbol{e}_P, \tag{22}$$

where $\boldsymbol{S}$ is the Riccati transfer matrix of point P and is a 6×6 matrix; $\boldsymbol{e}$ is a column matrix corresponding to the non-homogeneous term. At the input end of the system $\boldsymbol{z}_{a,1,I} = \boldsymbol{0}$, there are

$$\begin{aligned} &\boldsymbol{S}_{1,I} = \boldsymbol{0}_{6\times 6}, \\ &\boldsymbol{e}_{1,I} = \boldsymbol{0}. \end{aligned} \tag{23}$$

The Riccati transformation of state vector for any element j can be written as

$$\boldsymbol{z}_{a,j,I} = \boldsymbol{S}_{j,I} \boldsymbol{z}_{b,j,I} + \boldsymbol{e}_{j,I}, \tag{24}$$

$$\boldsymbol{z}_{a,j,O} = \boldsymbol{S}_{j,O} \boldsymbol{z}_{b,j,O} + \boldsymbol{e}_{j,O}. \tag{25}$$

From eqs. (21) and (24), one can obtain

$$\begin{aligned} \boldsymbol{z}_{b,j,I} = &\left(\boldsymbol{T}_{21,j}\boldsymbol{S}_{j,I} + \boldsymbol{T}_{22,j}\right)^{-1} \boldsymbol{z}_{b,j,O} \\ &-\left(\boldsymbol{T}_{21,j}\boldsymbol{S}_{j,I} + \boldsymbol{T}_{22,j}\right)^{-1}\left(\boldsymbol{f}_{b,j} + \boldsymbol{T}_{21,j}\boldsymbol{e}_{j,I}\right). \end{aligned} \tag{26}$$

Recursive formulations of $\boldsymbol{S}$ and $\boldsymbol{e}$ can be obtained by substituting eq. (26) into the first equation of eq. (21) and comparing with eq. (25):

$$\boldsymbol{S}_{j,O} = \left(\boldsymbol{T}_{11,j}\boldsymbol{S}_{j,I} + \boldsymbol{T}_{12,j}\right)\left(\boldsymbol{T}_{21,j}\boldsymbol{S}_{j,I} + \boldsymbol{T}_{22,j}\right)^{-1}, \tag{27}$$

$$\boldsymbol{e}_{j,O} = \left(\boldsymbol{f}_{a,j} + \boldsymbol{T}_{11,j}\boldsymbol{e}_{j,I}\right) - \boldsymbol{S}_{j,O}\left(\boldsymbol{f}_{b,j} + \boldsymbol{T}_{21,j}\boldsymbol{e}_{j,I}\right). \tag{28}$$

The Riccati transformation of state vector of the output end of the system can be obtained according to eq. (25):

$$\boldsymbol{z}_{a,n,O} = \boldsymbol{S}_{n,O} \boldsymbol{z}_{b,n,O} + \boldsymbol{e}_{n,O}. \tag{29}$$

The total state vectors of the output end of the system can be obtained according to its boundary condition and the above equations. Next, the state vectors of each element can be obtained according to eqs. (24)–(26). By using RMSTMM to solve dynamics of a chain MRS moving in space, the highest order of the matrices involved is 6.

3.6 Transfer matrix method for controlled multibody systems

(1) Discrete time transfer matrix method for controlled multibody systems.

The control subsystem [20] converts the feedback output state vector $\tilde{z}_f$ of the feedback element j obtained from the sensor to the corresponding control input $\boldsymbol{u}_C$ to realize the control of the controlled element i.

The transfer equation of element m can be obtained by linearizing its dynamics equations as

$$\boldsymbol{z}_{m,m+1}(t_i) = \boldsymbol{U}_m(t_i)\boldsymbol{z}_{m,m-1}(t_i) + \boldsymbol{G}_m(t_i)\boldsymbol{u}_{C,m}(t_i), \tag{30}$$

where $\boldsymbol{U}_m(t_i)$ is a fully known function at time instant t_i, $\boldsymbol{G}_m(t_i)$ is relevant to the control input $\boldsymbol{u}_{C,m}$, and they are the transfer matrices of the element m.

The overall transfer equation of CMS is

$$\boldsymbol{z}_{n,0} = \boldsymbol{U}_{1-n,1}\boldsymbol{z}_{1,0} + \boldsymbol{U}_{1-n,2}\boldsymbol{u}_C, \tag{31}$$

where $\boldsymbol{U}_{1-n,1} = \boldsymbol{U}_n \cdots \boldsymbol{U}_2\boldsymbol{U}_1$, $\boldsymbol{U}_{1-n,2} = \boldsymbol{U}_n \cdots \boldsymbol{U}_{i+1}\boldsymbol{G}_i$. The control input $\boldsymbol{u}_C$ is usually a function with respect to the corresponding reference input state vector $\tilde{z}_R$ and feedback output state vector $\tilde{z}_f$, and can be obtained according to the designed control law.

(2) Transfer matrix method for linear controlled multibody systems.

The body dynamics equation of a linear CMS [19] can be written as

$$\boldsymbol{M}\boldsymbol{v}_{tt} + \boldsymbol{C}\boldsymbol{v}_t + \boldsymbol{K}\boldsymbol{v} = \boldsymbol{f} + \boldsymbol{Q}_C\boldsymbol{u}_C. \tag{32}$$

The differential equation of generalized coordinates of a CMS in the kth order mode can be got by using the orthogonality of augmented eigenvectors:

$$\ddot{q}^k(t) + (\alpha + \beta\omega_k^2)\dot{q}^k(t) + \omega_k^2 q^k(t) = p_f^k(t) + p_C^k(t). \tag{33}$$

The dynamics response analysis of the linear CMS can be achieved by using eqs. (14) and (33) once the control law of the CMS is known. It can be carried out as follows to design the control law of a CMS.

Defining the state vector of system in the kth order mode as

$$\boldsymbol{\eta}_k = [q^k, \dot{q}^k]^{\mathrm{T}}. \tag{34}$$

The state-space equation for the kth order mode can be obtained as

$$\dot{\boldsymbol{\eta}}_k(t) = \boldsymbol{A}_k\boldsymbol{\eta}_k(t) + \boldsymbol{B}_k p_C^k(t) + \boldsymbol{D}_k p_f^k(t), \tag{35}$$

where

$$\boldsymbol{A}_k = \begin{bmatrix} 0 & 1 \\ -\omega_k^2 & -(\alpha+\beta\omega_k^2) \end{bmatrix}, \quad \boldsymbol{B}_k = \boldsymbol{D}_k = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{36}$$

The independent modal space optimal control is taken as an example, and the kth order modal performance index has the following form:

$$J_k = \int_0^\infty (\boldsymbol{\eta}_k^{\mathrm{T}}\boldsymbol{Q}_k\boldsymbol{\eta}_k + p_C^k R_k p_C^k)\mathrm{d}t. \tag{37}$$

The kth order modal control force p_C^k can be written as

$$p_C^k = -R_k^{-1}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_k\boldsymbol{\eta}_k, \tag{38}$$

where $\boldsymbol{P}_k$ is a 2×2 matrix and satisfies the following Riccati equation:

$$-\boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{P}_k - \boldsymbol{P}_k\boldsymbol{A}_k - \boldsymbol{Q}_k + \boldsymbol{P}_k\boldsymbol{B}_k R_k^{-1}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{P}_k = \boldsymbol{0}. \tag{39}$$

The control force in physical coordinates can be obtained after obtaining various order modal control forces:

$$\boldsymbol{u}_C = \sum_{k=1}^{N} M^k p_C^k \left(\boldsymbol{V}^{k,\mathrm{T}}\boldsymbol{Q}_C\right)^+, \tag{40}$$

where $(*)^+$ is generalized inverse of $(*)$; N is the highest order of mode considered.

3.7 Transfer matrix method for two-dimensional system

The transfer matrix method for two-dimensional system [16] is introduced briefly by a rectangular thin plate in x-y plane vibrating in the z-direction. The thin plate can be divided into $n\times m$ grid elements by dividing it into n pieces with equal width in the x-direction and dividing each piece into m pieces in the y-direction. Every grid element is treated as a lumped mass at its center, and the elastic effect between two adjacent grid elements is equivalent to massless elastic beam connecting them. The plate is equivalent to a plane net structure vibrating in the z-direction composed of $n\times m$ lumped masses, $(n+1)\times m$ beams in the x-direction, and $n\times(m+1)$ beams in the y-direction. The x-direction and the y-direction are the main and auxiliary transfer direction respectively.

The state vector of each connecting point is defined as

$$\boldsymbol{Z} = \left[Z,\ \Theta_x,\ \Theta_y,\ M_x,\ M_y,\ Q_z\right]^{\mathrm{T}}. \tag{41}$$

In the main transfer direction, the state vectors of the input end of hinge elements in the $(i+1)$th column and the output end of hinge elements in the $(i-1)$th column are respectively defined as

$$\boldsymbol{Z}_{i,i+1} = \left[\boldsymbol{Z}_{(i,i+1),2}^{\mathrm{T}},\quad \boldsymbol{Z}_{(i,i+1),4}^{\mathrm{T}},\quad \cdots,\quad \boldsymbol{Z}_{(i,i+1),2m}^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{42}$$

and

$$\boldsymbol{Z}_{i,i-1} = \left[\boldsymbol{Z}_{(i,i-1),2}^{\mathrm{T}},\quad \boldsymbol{Z}_{(i,i-1),4}^{\mathrm{T}},\quad \cdots,\quad \boldsymbol{Z}_{(i,i-1),2m}^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{43}$$

The transfer equation of element in the ith column and the overall transfer equation can be written as

$$\boldsymbol{Z}_{i+1,i} = \boldsymbol{U}_i\boldsymbol{Z}_{i-1,i}, \tag{44}$$

and

$$\boldsymbol{Z}_{2n+2,2n+1} = \boldsymbol{U}_{2n+1}\boldsymbol{U}_{2n}\cdots\boldsymbol{U}_2\boldsymbol{U}_1\boldsymbol{Z}_{0,1}. \tag{45}$$

3.8 Automatic deduction theorem of overall transfer equation

The overall transfer equations of MS with any topological structure can be deduced automatically by handwriting or computer [21,22]. The overall transfer matrix of a chain MS can be deduced automatically by successive premultiplication of the transfer matrices of all elements in the transfer path from the tip to the root of the system. The overall transfer equation of a closed-loop MS can be deduced automatically by regarding the original system as a chain MS after "cutting" at the junction of any two adjacent elements of the system and considering the couple of "cutting point" as "boundary ends" with the same state vectors. The overall transfer equation of a tree MS can be deduced automatically after considering the geometric relationships and forces between the first input end and other input ends of each body element with multiple input ends. The overall transfer equation of a general MS including closed-loop subsystems can be deduced automatically by regarding the original system as a tree MS after "cutting" at the junction of any two adjacent elements in the closed-loop subsystems and considering the couple of "cutting point" as "boundary ends" with the same state vectors.

4 Visualized simulation and design software of Rui method

Taking advantages of without global dynamics equations of system and low system matrix order of Rui method, and using open source software such as OCC and Qt, etc., a new software MSTMMSim [26] was developed for dynamics simulation and design of MS.

MSTMMSim mainly includes three modules as following: pre-processor, Rui method solver, and post-processor. By using C++ language and Qt framework, the graphical user interface is developed. The OCC geometric modeling kernel library is integrated. Various kinds of CAD solid models produced by other software can be imported by users. The basic geometric modeling can be realized by calling the application program interface of OCC. Complex geometric models can be built rapidly by Boolean operations, such as, difference, intersection and union, etc. According to the connecting relations and material properties defined by the users, the parameters and topology structure of MS can be generated automatically. The pre-processor is mainly used to set up the models of the mechanical system including functional model, 3D solid model, and dynamics model. The Rui method solver reads the defined parameters, computes the dynamics of system, and stores the computational results. The computational results are returned to the users in any form including numerical values, curves, animation, pictures, and tables, etc., in post-processor. The visualized simulation interface of a tank dynamics is shown in Figure 2.

MSTMMSim achieves the following functions: direct calling of various 3D model file, visualized modeling, automatic generation of the topology structure and dynamics parameters of MS, fast computation of MSD, data processing, animation displaying, etc.

5 Highlights and development tendency of Rui method

5.1 Highlights of Rui method

(1) It changes the study mode and simplifies the study procedure of MSD: the complicated global dynamics equations of system are not required for the first time.

(2) Transfer matrices of elements and overall transfer equation of various MS can be obtained directly by calling library of transfer matrices and using automatic deduction theorem.

(3) The order of system matrix is much lower and the computational speed of MSD is much higher compared with ordinary MSDM, the ill-conditioning eigenvalue computation of a linear complex MS is avoided, and exact analysis of dynamics response of a linear complex MS is realized with the modal method by constructing augmented eigenvectors and its orthogonality.

(4) The dynamics optimization designs of various key engineering products are really realized because of its very high computational speed, as being evaluated in ref. [29] that “has been shown to be more computationally efficient than many other modeling approaches”. The computational time ratio between Rui method and Lagrange equation is shown in Figure 3.

5.2 Development tendency of Rui method

(1) Based on the theories, software, standards, patents as well as a great deal of engineering practices, the large commercial software based on Rui method, such as MSTMMSim et al., will be improved, to satisfy the strong requirement for fast simulation, design of dynamics performance and experiments of the mechanical systems in lots of industries.

(2) The dynamic performance design and test for complex CMS should satisfy the requirement for fast design of control law and high-efficiency control of the dynamics.

(3) The library of transfer matrices of various kinds of mechanical elements, control subsystems and controlled elements will be extended continuously.

(4) The database of the software based on Rui method will be extended continuously.

(5) Applications of Rui method in engineering fields will be further popularized.

6 52 practical applications of Rui method in scientific research and key engineering

The complex mechanical systems are modeled as MS, and the quantitative relationships between the dynamics performance and system global parameters are established by using Rui method and its software. The exact prediction and op-

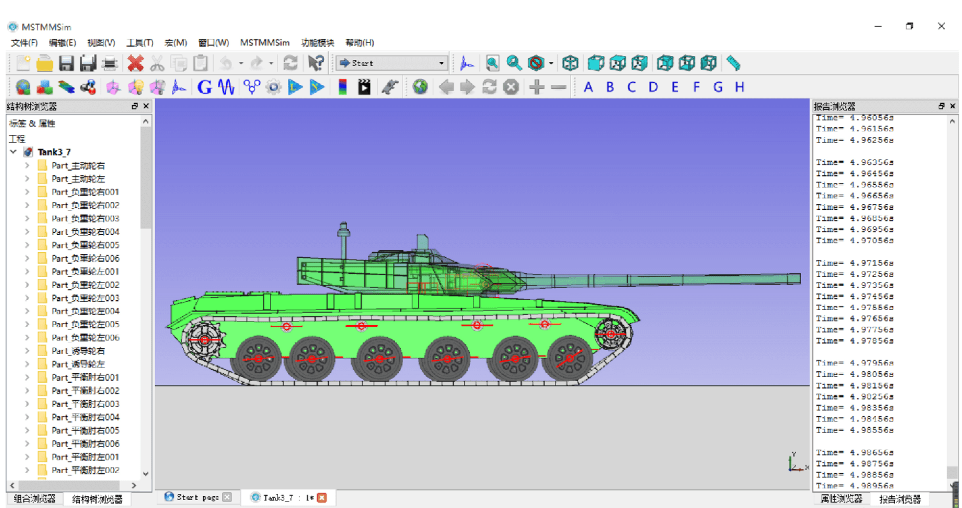

Figure 2 Animation interface of the tank dynamics.

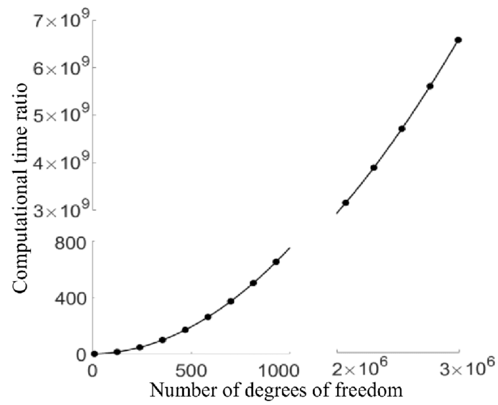

Figure 3 Computational time ratio between Rui method and Lagrange equation.

timal design of mechanical system dynamics are realized with high computational speed, the blindness caused by inability to carry out fast computation is overcome and the dynamics performance of lots of complex mechanical systems is dramatically improved. For example, the theory and technology system of launch dynamics of various weapons has been established based on Rui method and its software, so that the design level of dynamics performance and launch safety of weapons have been improved greatly, the cost of weapons has been reduced greatly and the development cycle of weapons has been shortened greatly, and won 3 national technology progress and invention awards of China. In fact, the dynamics performance and the level of dynamics design and test of more than 100 mechanical products in 52 practical application directions have been improved substantially: tank, self-propelled artillery, "metal storm", shipborne gun, cannon, spin tube gun, antiaircraft gun, vehicular MLRS, shipborne MLRS, airborne MLRS, vehicular missile system, missile, launch vehicle, rocket, feeding platform, parachute-submissile, aerospace aircraft, helicopter, inertial measurement unit system, submarine, underwater towed system, ship, ship's anti-vibration system, vehicle suspension, truck cranes, heavy duty machine tool, fly-cutting machine tool, five-axis CNC machine tool, machine tool spindle, large-scale rotary machine, piezoelectric actuator, intelligent flexible linkage devices, controlled flexible manipulators, immersed tunnel, earthquake resistant civil structures, super long stay cable, robots, mobile concrete truck, road roller, vibration compaction, vibration screen, wind turbine, wind turbine tower, gas turbine, high pressure compressor, low pressure rotor of gas turbine, roots blower, diesel engine, high pressure gas well, floating bridge, wing, servo turret, etc. [1,4,6,30–33].

7 Conclusions

The Rui method reveals the strict transfer relationship among state vectors of MS and provides totally new theory and platform for dynamics simulation and design with high computational speed for various complex mechanical systems. The paper systematically reviews the developments of Rui method and its applications, including: history, basic principles, visualized simulation and design software, highlights and tendency. The paper also illustrates the applications in the very wide fields of 52 scientific research and key practical engineering in over 100 kinds of products. We hope that Rui method will play more and more important role for the progress of technology and human being.

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