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Finite-time stabilization of uncertain delayed-hopfield neural networks with a time-varying leakage delay via non-chattering control

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This article is concerned with the finite-time stabilization (FTSB) of a class of delayed-Hopfield neural networks with a time-varying delay in the leakage term in the presence of parameter uncertainties. To accomplish the target of FTSB, two new finite-time controllers are designed for uncertain delayed-Hopfield neural networks with a time-varying delay in the leakage term. By utilizing the finite-time stability theory and the Lyapunov-Krasovskii functional (LKF) approach, some sufficient conditions for the FTSB of these neural networks are established. These conditions, which can be used for the selection of control parameters, are in the form of linear matrix inequalities (LMIs) and can be numerically checked. Additionally, an upper bound of the settling time was estimated. Finally, our theoretical results are further substantiated by two numerical examples with graphical illustrations to demonstrate the effectiveness of the results.

neural networks, finite-time stabilization, parametric uncertainties, leakage delay, Lyapunov-Krasovskii functional, LMI

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1 Introduction

Neural networks (NNs) have sparked great interest in many researchers due to its successful practical applications in many areas such as computer vision, associative memory, pattern recognition, generalized optimization, and memory design [1–7]. In a realistic NN, a time delay often occurs and creates some oscillations that cause high complexity in the dynamical behavior of the NN.

Many studies have been conducted on the effects of the delays on the stability of NNs [1, 2, 7–12]. One such delay is leakage delay, also known as forgetting delay, whose effect on stability is one of the major topics of research [13–17]. As indicated previously [18], logic circuits and ultra-thin-body geoi devices involve leakage delays [18], which tend to cause instability in NNs; thus, the strategies used for conventional delayed NNs cannot be applied to this problem[19]. There are many studies on the stability of NNs in the presence of forgetting delays [10, 17, 20–25]. Unfortunately, the approaches considered by the abovementioned studies cannot be extended to the finite-time stabilization (FTSB) of delayed-Hopfield NNs with time-varying delays in the leakage terms.

Due to external disturbances and parameter fluctuations, the values of the resistances and capacities of neurons are often uncertain [26]. Therefore, it is of practical interest to consider the uncertain parameters when studying the stability of NNs. Moreover, on the one hand, the parameters of NNs may exhibit some deviations because of modeling errors, external disturbances, and parameter fluctuations [26, 27]; on

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the other hand, in the electronic implementation of NNs, the connection-weight coefficients can be perturbed by the external environment and cause parameter uncertainties [19]. In recent years, there have been many studies on the stability of uncertain NNs [28–30], but none on the FTSB of uncertain delayed-Hopfield NNs (UDHNNs) with a time-varying leakage delay. Hence, there is an urgent need to study UDHNNs with time-varying delays in the leakage terms.

As is well known, the usual stability analyses of UDHNNs with time-varying delays in the leakage terms require asymptotic convergence, which can be rather time-consuming for achieving the desired precision, and hence, they can exceed the scale of human operations they will be useless to the real world applications [31]. However, in practice, we always hope that the physical process is able to achieve convergence within a shorter period of time: a concept known as finite-time stability (FTS), which means that the solutions to a system reach an equilibrium point in finite time [32]. The key issue in FTS is the estimation of the time function, known as the settling time, which indicates when the trajectories have reached the equilibrium point.

Haimo [33] was the first to study the concept of FTS; some interesting results were also obtained by refs. [34–36]. Recently, Moulay et al. developed this theory by extending the subjects of the abovementioned studies to non-autonomous systems in ref. [32] and to time-delay differential equations in ref. [37]. FTS is of major interest to many applications such as secure communications [38] or finite-time attitude tracking for spacecrafts [39]. There are many studies on the FTS of delayed systems [26, 31, 40–52]. However, previously designed controllers such as the pinning controller proposed by refs. [53, 54] cannot ensure the FTS of the delayed NNs considered in our study. More precisely, according to classical control law, controlling the effects of leakage delays based on the inequality $\dot{V}(t) \leqslant -\beta V^{\mu}$ is not difficult due to the requirement $\mu \in [0, 1]$.

Most recent studies on the FTS of delayed systems required the use of the sign function in the designed controllers [47, 55, 56]. However, these functions caused the chattering phenomenon [31, 57, 58] and induced undesirable effects in NNs [58]. Thus, the main difficulty lies in the time-varying leakage delay effect when a classical finite-time controller is used during the investigation of FTSB. Our research aims to solve this problem. The contributions of this article touches upon three aspects.

(1) By using FTS theory and the Lyapunov-Krasovskii functional (LKF) approach, the difficulty of controlling for the effects of leakage delays on FTS is overcome. Our study is the first to consider the FTSB of UDHNNs with time-varying leakage delays.

(2) The control parameters can be directly determined by solving the linear matrix inequalities (LMIs), wherein the connection weights contain parametric uncertainties. These kinds of parameters are of major interest in practice but not common in the study of the FTSB of NNs with time-varying leakage delays.

(3) Unlike classical controllers such as those used by refs. [47, 55, 56], the designed controller in our study does not involve the sign function and can be easily implemented in real applications without inducing the chattering phenomenon. Then, a delay-free controller that is more suited to real physical applications can be used because knowledge regarding the delays is not required.

The remainder of this article is organized as follows. In Sect. 2, some preliminaries useful for the study of a class of uncertain NNs are given. In Sect. 3, the FTSB of a class of UDHNNs with time-varying leakage delays is considered and some criteria are established. Then, Sect. 4 discusses the theoretical results, which are substantiated by two numerical examples. Finally, Sect. 5 contains the concluding remarks.

2 Preliminaries

2.1 Model description

Throughout this article, we use the following notations.

(1) $\mathbf{C}([a,\ b], \mathbb{R}^n)$ stands for the space formed by the continuous functions $\boldsymbol{\phi}\ : [a,\ b] \to \mathbb{R}^n$ equipped with the uniform norm $\|\boldsymbol{\phi}\|_{\mathrm{C}} = \sup\limits_{a\leqslant s\leqslant b} \|\boldsymbol{\phi}(s)\|$.

(2) $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ stand for the n-dimensional real space equipped with the Euclidean norm $\|\ .\ \|$ and the set of $n \times n$ real matrices, respectively.

(3) $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{A}^{-1}$ and $\boldsymbol{A} < 0$ stand for the transpose of $\boldsymbol{A}$, the inverse of a square matrix, and $-\boldsymbol{A}$ is positive definite, respectively.

(4) The function $\nu\ : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to the class $\mathcal{K}$ if it is strictly increasing and $\nu(0) = 0$.

(5) $\lambda_{\max}(\boldsymbol{A})$ and $\boldsymbol{I}_n$ stand for the maximum eigenvalue of $\boldsymbol{A}$ and the n-dimensional identity matrix, respectively.

(6) $(*)$ in a symmetric matrix block stands for the symmetric block.

In this article, we consider the following UDHNN with a time-varying delay in the leakage term:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}(t)x\left(t-\sigma(t)\right) + \boldsymbol{A}(t)\boldsymbol{f}(x(t)) \\ \qquad\quad +\boldsymbol{B}(t)\boldsymbol{f}(x(t-\tau(t))), \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in [-\tau^*,\ 0], \end{cases} \tag{1}$$

where

(1) $\boldsymbol{x}(t) = (x_1(t), \ \ldots, \ x_n(t))^{\mathrm{T}}$, and $\boldsymbol{f} = (f_1, \ldots, f_n)^{\mathrm{T}}$ stand for the neuron-state and the neuron-activation functions, respectively, with $f_j(0) = 0, \ j = 1, \ldots, n$.

(2) $\tau(.)$ and $\sigma(.)$ stand for the time-varying transmission delays and the leakage delay, respectively.

(3) $0 \leqslant \tau(t) \leqslant \bar{\tau}, \ 0 \leqslant \sigma(t) \leqslant \bar{\sigma}$ and $\dot{\sigma}(t) \leqslant \sigma^* < 1$.

According to the results obtained in ref. [26], the parametric uncertainties in system (1) can be written as follows.

(1) $\boldsymbol{C}(t) = \boldsymbol{C} + \Delta C(t), \ \boldsymbol{B}(t) = \boldsymbol{B} + \Delta B(t), \ \boldsymbol{A}(t) = \boldsymbol{A} + \Delta A(t)$, in which $\boldsymbol{C} = \mathrm{diag}\,(c_1, \ldots, c_n)$ is a diagonal matrix with $c_i > 0$ while $\boldsymbol{A} = \left(a_{ij}\right)_{n\times n}$ and $\boldsymbol{B} = \left(b_{ij}\right)_{n\times n}$ are the interconnection weight matrix of the neurons.

(2) $\Delta C(t), \Delta B(t)$, and $\Delta A(t)$ stand for the time-varying parametric uncertainties and satisfy the matched conditions [26]

$$\Delta C(t) = \boldsymbol{E}_C \Delta_1(t) \boldsymbol{E}_C, \ \Delta B(t) = \boldsymbol{E}_B \Delta_2(t) \boldsymbol{E}_B, \tag{2}$$

$$\Delta A(t) = \boldsymbol{E}_A \Delta_3(t) \boldsymbol{E}_A, \tag{3}$$

where $\boldsymbol{E}_A, \ \boldsymbol{E}_B$, and $\boldsymbol{E}_C$ are known constant matrices and $\Delta_i(t), \ i = 1, 2, 3$ is an unknown time-varying matrix that satisfies

$$\Delta_i(t)^{\mathrm{T}} \Delta_i(t) < I_n, \ i = 1, 2, 3. \tag{4}$$

(3) $\boldsymbol{\phi}(.) \in \mathbf{C}([-\tau^*, 0], \ \mathbb{R}^n)$ where $\tau^* = \max\{\bar{\tau}, \bar{\sigma}\}$.

2.2 Definitions and lemmas

Let us introduce the following assumption:

$\mathbf{H_1}$: There exist constants $M_j^-, \ M_j^+$ such that the functions $f_j \ (j = 1, 2 \ldots, n)$ satisfy the following condition:

$$M_j^- \leqslant \frac{f_j(x) - f_j(y)}{x - y} \leqslant M_j^+, \quad j = 1, 2, \ldots, n,$$

for all $x, \ y \in \mathbb{R}$.

Remark 1. Under the assumption ($\mathbf{H_1}$), the existence of solutions of system (1) is ensured, as explained in ref. [59]. In ($\mathbf{H_1}$), the constants M_j^+ and M_j^- can be negative or positive. Consequently, this assumption allows Lurie-type functions and Lipschitz functions if $M_j^+, M_j^- > 0$, and $M_j^- = -M_j^+ < 0$, respectively.

Some useful definitions and lemmas are provided below.

Let Ω be an open subset of $\boldsymbol{C}([-\tau^*, \ 0], \mathbb{R}^n)$ such that $0 \in \Omega$.

Definition 1. The equilibrium point, if it exists, of system (1) is FTS if [37]:

(i) the equilibrium of system (1) is Lyapunov stable;

(ii) for any state $\boldsymbol{\phi}(s) \in \Omega$, there exists $0 \leqslant \mathfrak{T}(\boldsymbol{\phi}) < +\infty$ such that every solution of system (1) satisfies $\boldsymbol{x}(t, \ \boldsymbol{\phi}) = 0$ for all $t \geqslant \mathfrak{T}(\boldsymbol{\phi})$.

The functional

$$T_0(\boldsymbol{\phi}) = \inf\left\{\mathfrak{T}(\phi) \geqslant 0 \ : \ \boldsymbol{x}(t, \ \boldsymbol{\phi}) = 0, \quad \forall t \geqslant \mathfrak{T}(\phi)\right\}.$$

is called the settling time of system (1).

The derivative of V along the trajectories of system (1) is defined in ref. [59] as follows:

$$\dot{V}(\boldsymbol{\phi}) = \dot{V}_{(1)}(\boldsymbol{\phi}) = \limsup_{h\to 0^+} \frac{1}{h}\left[V(\boldsymbol{x}_h(\boldsymbol{\phi})) - V(\boldsymbol{\phi})\right].$$

Lemma 1. If there exist two functions ν and r of class $\mathcal{K}$ and a continuous functional $V : \ \Omega \to \mathbb{R}_+$ such that [37]

(i) $\nu(\boldsymbol{\phi}(0)\|_c) \leqslant V(\boldsymbol{\phi})$;

(ii) $\dot{V}(\boldsymbol{\phi}) \leqslant -r\left(V(\boldsymbol{\phi})\right)$ with $\int_0^{\epsilon} \frac{\mathrm{d}z}{r(z)} < \infty, \quad \forall \epsilon > 0, \ \boldsymbol{\phi} \in \Omega$;

then system (1) is FTS with a settling time satisfying the inequality $T_0(\boldsymbol{\phi}) \leqslant \int_0^{V(\boldsymbol{\phi})} \frac{\mathrm{d}z}{r(z)}$. In particular, if $r(V) = \lambda V^{\rho}$, where $\lambda > 0, \ \rho \in (0, \ 1)$, then the settling time satisfies the inequality

$$T_0(\boldsymbol{\phi}) \leqslant \int_0^{V(\boldsymbol{\phi})} \frac{\mathrm{d}z}{r(z)} = \frac{V^{1-\rho}(0, \ \boldsymbol{\phi})}{\lambda(1-\rho)}. \tag{5}$$

Lemma 2. For a positive definite matrix $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ and any vectors $\boldsymbol{x}, \ \boldsymbol{y} \in \mathbb{R}^n, \ \epsilon > 0$, the following inequality holds [60]:

$$\pm 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \leqslant \epsilon^{-1}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{x} + \epsilon\boldsymbol{y}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{y}.$$

Lemma 3. (Schur Complement [60]): Given three constant matrices $\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2$, and $\boldsymbol{\Omega}_3$, where $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_1^{\mathrm{T}}$ and $0 < \boldsymbol{\Omega}_2 = \boldsymbol{\Omega}_2^{\mathrm{T}}$, then

$$\boldsymbol{\Omega}_1 + \boldsymbol{\Omega}_3^{\mathrm{T}}\boldsymbol{\Omega}_2^{-1}\boldsymbol{\Omega}_3 < 0,$$

if and only if

$$\begin{pmatrix} \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_3^{\mathrm{T}} \\ \boldsymbol{\Omega}_3 & -\boldsymbol{\Omega}_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -\boldsymbol{\Omega}_2 & \boldsymbol{\Omega}_3 \\ \boldsymbol{\Omega}_3^{\mathrm{T}} & \boldsymbol{\Omega}_1 \end{pmatrix} < 0.$$

Lemma 4. For given matrices $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}}, \boldsymbol{H}$, and $\boldsymbol{E}$ of appropriate dimensions [50],

$$\boldsymbol{Q} + \boldsymbol{H}\boldsymbol{F}(t)\boldsymbol{E} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}} < 0$$

holds for all $\boldsymbol{F}(t)$ satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leqslant \boldsymbol{I}_n$ if and only if there exists $\epsilon > 0$ such that

$$\boldsymbol{Q} + \epsilon^{-1}\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}} + \epsilon\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} < 0.$$

In the following, we consider the controlled system:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}(t)x\left(t - \sigma(t)\right) + \boldsymbol{A}(t)\boldsymbol{f}(x(t)) \\ \qquad\quad +\boldsymbol{B}(t)\boldsymbol{f}(x(t - \tau(t))) + \boldsymbol{u}, \quad t > 0, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in [-\tau^*, \ 0], \end{cases} \tag{6}$$

where $\boldsymbol{u}(x) = (u_1(x), \ldots, u_n(x))^{\mathrm{T}}$ stands for the control input.

Remark 2. If system (6) is FTS, the uniqueness of solutions in forward time at the origin cannot be ensured [37]. However, Theorem 3.4 (page 24) shows that in ref. [61], the continuity of the right-hand sides of system (6) with the uniqueness of the solutions in forward time ensure the continuity of the solutions.

3 Main results

In this section, we design two different kinds of controllers that are able to finite-time stabilize the class of UDHNNs with time-varying delays in the leakage term.

Before we investigate the abovementioned problem, we introduce the following notations:

$$M_j = \max\left\{|M_j^-|,\ |M_j^+|\right\},\quad \boldsymbol{L}_f = \mathrm{diag}(M_1,\dots,M_n).$$

The state feedback control inspired by refs. [41, 51, 62] is designed as follows:

$$\begin{aligned}\boldsymbol{u}\,(x(t)) = &-\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma}\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu}\\ &-k_1\,\boldsymbol{x}(t) - \Big[k_2\,\boldsymbol{x}\,(t-\tau(t))^{\mathrm{T}}\,\boldsymbol{x}\,(t-\tau(t))\\ &+k_3\,\boldsymbol{x}\,(t-\sigma(t))^{\mathrm{T}}\,\boldsymbol{x}\,(t-\sigma(t))\Big]\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu},\end{aligned}\tag{7}$$

with $\lambda,\ \nu,\ k_i > 0,\ i = 1,2,3$ as the gain coefficients to be determined and $0<\gamma<1$.

Remark 3. It is well known that the sign functions cause chattering in NNs state [57, 58] and induce undesirable effects. Despite this, a majority of interesting (à supprimer ce mot) FTS results were obtained using the sign functions [47, 55, 56]. To eliminate chattering when taking the FTS approach, we designed a non-chattering controller (7) based on continuous functions under integration. Moreover, under a time-varying leakage delay, the pinning control presented in refs. [53, 54] cannot be utilized to realize the FTS of system (6), which is solved here by establishing some sufficient conditions for the FTS of system (6).

In the following theorem, sufficient conditions in terms of LMIs are established to ensure the FTSB of system (6).

Theorem 1. If there exist positive constants $s_i,\ k_i\ (i = 1,2,3)$ such that the following conditions hold

$$\begin{aligned}&\boldsymbol{I}_n + s_1\,\boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_2\,\boldsymbol{B}(t)\boldsymbol{B}(t)^{\mathrm{T}} + s_3\,\boldsymbol{C}(t)\boldsymbol{C}(t)^{\mathrm{T}}\\ &\quad + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\,\boldsymbol{I}_n < 0,\end{aligned}\tag{8}$$

$$s_2^{-1}\boldsymbol{L}_f^2 - 2k_2\,\boldsymbol{I}_n < 0,\tag{9}$$

$$-(1-\sigma^*) + s_3^{-1} - 2k_3 < 0,\tag{10}$$

then system (6) is FTSB via the controller (7) and the settling time satisfies $\mathfrak{T}\leqslant\mathfrak{T}_1$ where $\mathfrak{T}_1 = \frac{V^{1-\gamma}(0,\phi)}{\lambda(1-\gamma)}$.

Proof. Consider the following LKF

$$V(\boldsymbol{x}(t)) = \boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s.\tag{11}$$

The derivative of eq. (11) along the trajectories of system (6) is estimated as follows:

$$\begin{aligned}\dot V(\boldsymbol{x}(t)) =&\boldsymbol{x}(t)^{\mathrm{T}}\dot{\boldsymbol{x}}(t) + \dot{\boldsymbol{x}}(t)^{\mathrm{T}}\boldsymbol{x}(t)\\ =&\boldsymbol{x}(t)^{\mathrm{T}}\Big[-\boldsymbol{C}(t)\,\boldsymbol{x}(t-\sigma(t)) + \boldsymbol{A}(t)\,\boldsymbol{f}\,(x(t))\\ &+\boldsymbol{B}(t)\,\boldsymbol{f}\,(x(t-\tau(t))) + \boldsymbol{u}\Big]\\ &+\Big[\boldsymbol{A}(t)\,\boldsymbol{f}\,(x(t)) - \boldsymbol{C}(t)\,\boldsymbol{x}(t-\sigma(t))\\ &+B(t)\,f\,(x(t-\tau(t))) + u\Big]^{\mathrm{T}}x(t).\end{aligned}\tag{12}$$

It is easy to see that when $|x(t)|\neq 0$,

$$\begin{aligned}&\boldsymbol{x}^{\mathrm{T}}(t)\left[-\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma}\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2}\right]\\ &\quad= -\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma}.\end{aligned}\tag{13}$$

Then, from ref. [58] and eq. (13), we have

$$\begin{aligned}\dot V(x(t)) \leqslant&2\boldsymbol{x}(t)^{\mathrm{T}}\Big[-\boldsymbol{C}(t)\,\boldsymbol{x}(t-\sigma(t)) + \boldsymbol{A}(t)\,\boldsymbol{f}\,(x(t))\\ &+\boldsymbol{B}(t)\,\boldsymbol{f}\,(x(t-\tau(t)))\Big] - k_1\,\boldsymbol{x}(t)\\ &-\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma}\\ &-k_2\,\boldsymbol{x}\,(t-\tau(t))^{\mathrm{T}}\,\boldsymbol{x}\,(t-\tau(t))\\ &-k_3\,\boldsymbol{x}\,(t-\sigma(t))^{\mathrm{T}}\,\boldsymbol{x}\,(t-\sigma(t))\\ &-(1-\dot\sigma(t))\,\boldsymbol{x}(t-\sigma(t))^{\mathrm{T}}\boldsymbol{x}(t-\sigma(t)).\end{aligned}\tag{14}$$

Furthermore, from Lemma 2, we easily obtain

$$\begin{aligned}2\boldsymbol{x}(t)^{\mathrm{T}}\,\boldsymbol{A}(t)\,\boldsymbol{f}(\boldsymbol{x}(t)) \leqslant& s_1\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}}\boldsymbol{x}(t)\\ &+s_1^{-1}\boldsymbol{f}^{\mathrm{T}}\,(x(t))\,\boldsymbol{f}\,(x(t));\end{aligned}\tag{15}$$

$$\begin{aligned}&2\boldsymbol{x}(t)^{\mathrm{T}}\,\boldsymbol{B}(t)\,\boldsymbol{f}(x(t-\tau(t)))\\ &\leqslant s_2\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{B}(t)\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{x}(t) + s_2^{-1}\boldsymbol{f}^{\mathrm{T}}\,(x(t-\tau(t)))\,\boldsymbol{f}\,(x(t-\tau(t)));\end{aligned}\tag{16}$$

$$\begin{aligned}2\boldsymbol{x}(t)^{\mathrm{T}}\,\boldsymbol{C}(t)\,\boldsymbol{x}(t-\sigma(t))) \leqslant& s_3\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{C}(t)\boldsymbol{C}^{\mathrm{T}}(t)\boldsymbol{x}(t)\\ &+s_3^{-1}\boldsymbol{x}^{\mathrm{T}}(t-\sigma(t))\boldsymbol{x}(t-\sigma(t)).\end{aligned}\tag{17}$$

Thus, combining eqs. (15) and (17), we derive

$$\dot V(x_t) \leqslant\boldsymbol{x}(t)^{\mathrm{T}}\Big[\boldsymbol{I}_n + s_1\,\boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_2\,\boldsymbol{B}(t)\boldsymbol{B}(t)^{\mathrm{T}}$$

$$+ s_3\, \boldsymbol{C}(t)\boldsymbol{C}(t)^{\mathrm{T}} + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n\Big]x(t)$$
$$+ \boldsymbol{x}^{\mathrm{T}}(t-\sigma(t))\Big[-(1-\sigma^*) + s_3^{-1} - 2k_3\Big]\boldsymbol{x}(t-\sigma(t))$$
$$+ \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\Big[s_2^{-1}\boldsymbol{L}_f^2 - 2k_2\boldsymbol{I}_n\Big]\boldsymbol{x}(t-\tau(t))$$
$$- 2\lambda V^{\gamma}(t), \forall\, t > 0. \tag{18}$$

Furthermore, from (8)–(10), we obtain

$$\dot{V}(t) < -2\lambda V^{\gamma}(t), \quad \forall\, t > 0. \tag{19}$$

Thus, since condition (i) in Lemma 1 is ensured by $(\mathbf{H_1})$, system (6) is FTSB via controller (7) and $\mathfrak{T} \leqslant \frac{V^{1-\gamma}(0,\boldsymbol{\phi})}{\lambda(1-\gamma)}$, which completes the proof.

Remark 4. The leakage delay tends to destabilize the NNs [22] and makes the dynamical behavior of the systems more complex. In refs. [17, 26, 47–49], the FTSB of delayed NNs was investigated. However, the approach used in the above-mentioned work fails when $\sigma(.) \neq 0$. The routine employed for conventional delayed NNs cannot be applied to study this kind of delay [19]. In Theorem 1, we present another FTSB result based on a new LKF, which renders our results more general than those of the existing studies.

When there is no parametric uncertainty, system (6) reduces to

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\,\boldsymbol{x}(t-\sigma(t)) + \boldsymbol{A}\,\boldsymbol{f}(x(t)) \\ \qquad\quad +\boldsymbol{B}\,\boldsymbol{f}(x(t-\tau(t))) + \boldsymbol{u}, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in [-\tau^*,\ 0]. \end{cases} \tag{20}$$

If we take $\boldsymbol{E}_A = \boldsymbol{E}_B = \boldsymbol{E}_C = 0$ in Theorem 1, we obtain the following corollary.

Corollary 1. If there exist positive constants s_i, k_i $(i = 1, 2, 3)$ such that the following conditions hold

$$\boldsymbol{I}_n + s_1\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}} + s_2\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}} + s_3\boldsymbol{C}\boldsymbol{C}^{\mathrm{T}} + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n < 0, \tag{21}$$
$$s_2^{-1}\boldsymbol{L}_f^2 - 2k_2\boldsymbol{I}_n < 0, \tag{22}$$
$$-(1-\sigma^*) + s_3^{-1} - 2k_3 < 0, \tag{23}$$

then system (20) is FTSB via the controller (7) and the settling time satisfies $\mathfrak{T} \leqslant \mathfrak{T}_1$.

Remark 5. The established conditions in refs. [63–66] are based on the analytical approach and are invalid when the L_2-norm is used [65]. We would like to mention here that $L_2 \subset L_1$, so the settling time established in our work may be smaller than that obtained in the abovementioned papers and proves the advantage of our results.

Obviously, the conditions of Corollary 1 are not easy to check since they are nonlinear. One approach to tackling such a problem is to make the sufficient conditions in terms of LMIs equivalent to the abovementioned conditions of Corollary 1.

Corollary 2. If there exist positive constants s_i, k_i $(i = 1, 2, 3)$ such that the following LMIs hold:

$$\boldsymbol{\Omega}_0 = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \boldsymbol{L}_f & s_1\boldsymbol{A} & s_2\boldsymbol{B} & s_3\boldsymbol{C} \\ * & -s_1\boldsymbol{I}_n & 0 & 0 & 0 \\ * & * & -s_1\boldsymbol{I}_n & 0 & 0 \\ * & * & * & -s_2\boldsymbol{I}_n & 0 \\ * & * & * & * & -s_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -2k_2\boldsymbol{I}_n & \boldsymbol{L}_f \\ * & -s_2\boldsymbol{I}_n \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} -(1-\sigma^*)\boldsymbol{I}_n - 2k_3\boldsymbol{I}_n & \boldsymbol{I}_n \\ * & -s_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{26}$$

then system (20) is FTSB via the controller (7) and the settling time satisfies $\mathfrak{T} \leqslant \mathfrak{T}_1$.

A simple use of Lemma 3 and Corollary 1 imply the results of Corollary 2. The details of the proof are left to the reader.

Remark 6. The FTS conditions established in refs. [57, 63, 64] are based on the analytical method and cannot be expressed in LMIs, because the inequalities are based on the L_1-norm. In this work, some sufficient conditions in terms of LMIs are established. The LMI approach leads to less conservative conditions than do the non-LMI methods. These kinds of conditions can be easily checked by using the MATLAB LMI toolbox. This (the above-mentioned discussion) proves the advantage of our proposed method.

Remark 7. Unlike the sliding mode approach, the following controller

$$\boldsymbol{u}^*(x(t)) = \begin{cases} -\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \displaystyle\int_{t-\sigma(t)}^{t} \boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)ds\right)^{\gamma} \dfrac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2} \\ -k_1\,\boldsymbol{x}(t) - \Big[k_2\,\boldsymbol{x}(t-\tau(t))^{\mathrm{T}}\,\boldsymbol{x}(t-\tau(t)) \\ +k_3\,\boldsymbol{x}(t-\sigma(t))^{\mathrm{T}}\,\boldsymbol{x}(t-\sigma(t))\Big]\dfrac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2}, \\ \qquad \text{if } \|\boldsymbol{x}(t)\|_2 \neq 0, \\ 0, \quad \text{if } \|\boldsymbol{x}(t)\|_2 = 0 \end{cases} \tag{27}$$

is only discontinuous at the origin and does not contain the sign function, which could lead to the chattering phenomenon. If $\|\boldsymbol{x}(t)\|_2 = 0$ for $t \leqslant \mathfrak{T}$, then the FTS is obtained without appearance of chattering.

Now, when there is no leakage delay, system (6) reduces to

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}(t)\,\boldsymbol{x}(t) + \boldsymbol{A}(t)\,\boldsymbol{f}(x(t)) \\ \qquad\quad +\boldsymbol{B}(t)\,\boldsymbol{f}(x(t-\tau(t))) + \boldsymbol{u}, \quad t > 0, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in [-\tau^*,\ 0]. \end{cases} \tag{28}$$

Then, we have the following result:

Corollary 3. If there exist positive constants s_1, s_2, k_i ($i = 1, 2, 3$) such that the following conditions hold:

$$\boldsymbol{I}_n + s_1\, \boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_2\, \boldsymbol{B}(t)\boldsymbol{B}(t)^{\mathrm{T}} + \boldsymbol{C}(t) + \boldsymbol{C}(t)^{\mathrm{T}} + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n < 0, \tag{29}$$

$$s_2^{-1}\boldsymbol{L}_f^2 - 2k_2\boldsymbol{I}_n < 0, \tag{30}$$

then system (28) is FTSB via the following controller:

$$\boldsymbol{u}\,(x(t)) = -\,k_1\,\boldsymbol{x}(t) - k_2\boldsymbol{x}^{\mathrm{T}}\,(t-\tau(t))\,\boldsymbol{x}\,(t-\tau(t))\,\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu} - \lambda\|\boldsymbol{x}(t)\|^{\gamma}, \tag{31}$$

with $\lambda > 0$, $0 < \gamma < 1$, and $k_i > 0$ ($i = 1, 2, 3$) the control strength. Moreover, the settling time satisfies $\mathfrak{T} \leqslant \frac{\|\phi\|_c^{1-\gamma}}{\lambda(1-\gamma)}$.

Proof. By taking $\sigma(.) = 0$ in the LKF (11) and calculating the derivative of the functional (11) along the trajectories of system (28), we easily obtain a result with arguments similar to the ones of Theorem 1.

Remark 8. Although the stability of NNs with leakage delays has been studied (see for instance refs. [10, 21, 22] and the references therein), all the previous works have been for asymptotic or exponential behaviors rather than the FTSB of UDHNNs with time-varying leakage delays.

When dealing with parametric uncertainties, the conditions of Theorem 1 are not standard LMIs. Consequently, Theorem 1 cannot be used to check the FTSB of system (6). Hence, we formulate a new theorem in which the obtained conditions are LMIs that can be numerically checked.

Theorem 2. If there exist positive constants s_i, k_i, λ_i ($i = 1, 2, 3$) such that the following LMIs hold:

$$\Omega = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \boldsymbol{L}_f & s_1\boldsymbol{A} & s_2\boldsymbol{B} & s_3\boldsymbol{C} & s_1\boldsymbol{E}_A & s_2\boldsymbol{E}_B & s_3\boldsymbol{E}_c \\ * & -s_1\boldsymbol{I}_n & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \boldsymbol{\Omega}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Omega}_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \boldsymbol{\Omega}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -\lambda_1\boldsymbol{I}_n & 0 & 0 \\ * & * & * & * & * & * & -\lambda_2\boldsymbol{I}_n & 0 \\ * & * & * & * & * & * & * & -\lambda_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} -2k_2\boldsymbol{I}_n & \boldsymbol{L}_f \\ * & -s_2\boldsymbol{I}_n \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} -(1-\sigma^*)\boldsymbol{I}_n - 2k_3\boldsymbol{I}_n & \boldsymbol{I}_n \\ * & -s_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{34}$$

where $\boldsymbol{\Omega}_{33} = -s_1\boldsymbol{I}_n + \lambda_1\boldsymbol{E}_A\boldsymbol{E}_A^{\mathrm{T}}$, $\boldsymbol{\Omega}_{44} = -s_2\boldsymbol{I}_n + \lambda_2\boldsymbol{E}_B\boldsymbol{E}_B^{\mathrm{T}}$, $\boldsymbol{\Omega}_{55} = -s_3\boldsymbol{I}_n + \lambda_3\boldsymbol{E}_C\boldsymbol{E}_C^{\mathrm{T}}$, then system (6) is FTSB via the controller (7) and the settling time satisfies $\mathfrak{T} \leqslant \mathfrak{T}_1$.

The proof of Theorem 2 is inspired by the proof of Theorem 3 in ref. [26].

Proof. We prove the theorem in two steps.

Step 1: we prove that the inequality (8) is equivalent to

$$\bar{\boldsymbol{\Omega}} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_1 < 0,$$

where

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} 0 & 0 & s_1\boldsymbol{E}_A\Delta(t)\boldsymbol{E}_A & s_2\boldsymbol{E}_B\Delta(t)\boldsymbol{E}_B & s_3\boldsymbol{E}_C\Delta(t)\boldsymbol{E}_C \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}.$$

According to the matched conditions (2), $\bar{\boldsymbol{\Omega}}$ can be written as follows:

$$\bar{\boldsymbol{\Omega}} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_1 = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \boldsymbol{L}_f^{\mathrm{T}} & s_1\boldsymbol{A}(t) & s_2\boldsymbol{B}(t) & s_3\boldsymbol{C}(t) \\ * & -s_1\boldsymbol{I}_n & 0 & 0 & 0 \\ * & * & -s_1\boldsymbol{I}_n & 0 & 0 \\ * & * & * & -s_2\boldsymbol{I}_n & 0 \\ * & * & * & * & -s_3\boldsymbol{I}_n \end{bmatrix}. \tag{35}$$

Thus, pre-and post-multiplying (35) by the block-diagonal matrix

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{I}_n & 0 & 0 & 0 & 0 \\ 0 & \sqrt{s_1}^{-1}\boldsymbol{I}_n & 0 & 0 & 0 \\ 0 & 0 & \sqrt{s_1}^{-1}\boldsymbol{I}_n & 0 & 0 \\ 0 & 0 & 0 & \sqrt{s_2}^{-1}\boldsymbol{I}_n & 0 \\ 0 & 0 & 0 & 0 & \sqrt{s_3}^{-1}\boldsymbol{I}_n \end{bmatrix},$$

we obtain

$$\boldsymbol{R}\bar{\boldsymbol{\Omega}}\boldsymbol{R} = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \sqrt{s_1}^{-1}L_f & s_1\boldsymbol{A}\,\sqrt{s_1}^{-1} & s_2\boldsymbol{B}\,\sqrt{s_2}^{-1} & s_3\boldsymbol{C}\,\sqrt{s_n}^{-1} \\ * & -\boldsymbol{I}_n & 0 & 0 & 0 \\ * & 0 & -\boldsymbol{I}_n & 0 & 0 \\ * & 0 & 0 & -\boldsymbol{I}_n & 0 \\ * & 0 & 0 & 0 & -\boldsymbol{I}_n \end{bmatrix}.$$

Then, by letting

$$\Omega_2 = \begin{pmatrix} \boldsymbol{I}_n & 0 & 0 & 0 \\ 0 & \boldsymbol{I}_n & 0 & 0 \\ 0 & 0 & \boldsymbol{I}_n & 0 \\ 0 & 0 & 0 & \boldsymbol{I}_n \end{pmatrix},$$

$$\mathbf{\Omega}_3^{\mathrm{T}} = \left[\boldsymbol{I}_n - 2k_1\boldsymbol{I}_n \ \sqrt{s_1}^{-1}\boldsymbol{L}_f \ s_1\boldsymbol{A} \ \sqrt{s_1}^{-1} \ s_2\boldsymbol{B} \ \sqrt{s_2}^{-1} \ s_3\boldsymbol{C} \ \sqrt{s_n}^{-1}\right]$$

and $\mathbf{\Omega}_1 = \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n$, we obtain

$$\bar{\mathbf{\Omega}} < 0 \Leftrightarrow \boldsymbol{R}\bar{\mathbf{\Omega}}\boldsymbol{R} = \begin{pmatrix} \mathbf{\Omega}_1 & \mathbf{\Omega}_3^{\mathrm{T}} \\ \mathbf{\Omega}_3 & -\mathbf{\Omega}_2 \end{pmatrix} < 0, \tag{36}$$

which is equivalent, by a simple use of Lemma 3, to inequality (8).

Step 2: we prove that the inequality $\bar{\mathbf{\Omega}} = \mathbf{\Omega}_0 + \mathbf{\Omega}_1 < 0$ is equivalent to the existence of $\lambda_1, \lambda_2, \lambda_3$ such that

$$\begin{aligned} \mathbf{\Omega}_0 &+ \begin{bmatrix} \lambda_1^{-1}s_1^2\boldsymbol{E}_A\boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \lambda_1\boldsymbol{E}_A\boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \lambda_2^{-1}s_2^2\boldsymbol{E}_B\boldsymbol{E}_B^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & \lambda_2\boldsymbol{E}_B\boldsymbol{E}_B^{\mathrm{T}} & 0 \\ * & * & * & * & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \lambda_3^{-1}s_3^2\boldsymbol{E}_C\boldsymbol{E}_C^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & \lambda_3\boldsymbol{E}_C\boldsymbol{E}_C^{\mathrm{T}} \end{bmatrix} < 0. \end{aligned} \tag{37}$$

Firstly, $\mathbf{\Omega}_1$ can be written as follows:

$$\mathbf{\Omega}_1 = \mathbf{\Omega}_{1_1} + \mathbf{\Omega}_{1_2} + \mathbf{\Omega}_{1_3} \tag{38}$$

with

$$\begin{aligned} \mathbf{\Omega}_{1_1} &= \begin{bmatrix} s_1\boldsymbol{E}_A \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_1(t) \begin{bmatrix} 0 & 0 & \boldsymbol{E}_A & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{E}_A^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} \Delta_1(t)^{\mathrm{T}} \begin{bmatrix} s_1\boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} \mathbf{\Omega}_{1_2} &= \begin{bmatrix} s_2\boldsymbol{E}_B \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_2(t) \begin{bmatrix} 0 & 0 & 0 & \boldsymbol{E}_B & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ \boldsymbol{E}_B^{\mathrm{T}} \\ 0 \end{bmatrix} \Delta_2(t)^{\mathrm{T}} \begin{bmatrix} s_2\boldsymbol{E}_B^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} \mathbf{\Omega}_{1_3} &= \begin{bmatrix} s_3\boldsymbol{E}_C \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_3(t) \begin{bmatrix} 0 & 0 & 0 & 0 & \boldsymbol{E}_C \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \boldsymbol{E}_C^{\mathrm{T}} \end{bmatrix} \Delta_3(t)^{\mathrm{T}} \begin{bmatrix} s_3\boldsymbol{E}_C^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Thus, by applying Lemma 4 to $\mathbf{\Omega}_{1_1}$ with

$$\boldsymbol{Q} = \mathbf{\Omega}_0 + \mathbf{\Omega}_{1_2} + \mathbf{\Omega}_{1_3};$$

$$\boldsymbol{H} = \begin{bmatrix} s_1\boldsymbol{E}_A \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{E}^{\mathrm{T}} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{E}_A^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}, F(t) = \Delta_1(t).$$

It follows from Lemma 4 and eq. (4) that

$$\bar{\mathbf{\Omega}} = \boldsymbol{Q} + \boldsymbol{H}\Delta_1(t)\boldsymbol{E} + \boldsymbol{E}^{\mathrm{T}}\Delta_1(t)^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}} < 0$$

implies that there exists $\lambda_1 > 0$ such that

$$\boldsymbol{Q} + \lambda_1^{-1}\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}} + \lambda_1\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} < 0,$$

i.e.

$$\mathbf{\Omega}_0 + \mathbf{\Omega}_{1_2} + \mathbf{\Omega}_{1_3} + \begin{bmatrix} \lambda_1^{-1}s_1^2\boldsymbol{E}_A\boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \lambda_1\boldsymbol{E}_A\boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} < 0. \tag{39}$$

Then, we applied Lemma 4 to $\mathbf{\Omega}_{1_2}$ by choosing

$$\boldsymbol{Q} = \mathbf{\Omega}_0 + \mathbf{\Omega}_{1_3} + \begin{bmatrix} \lambda_1^{-1} s_1^2 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \lambda_1 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}, \quad (40)$$

$$\boldsymbol{H} = \begin{bmatrix} s_2 \boldsymbol{E}_B \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{E}^{\mathrm{T}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \boldsymbol{E}_B^{\mathrm{T}} \\ 0 \end{bmatrix}, F(t) = \Delta_2(t)$$

and we obtain that eq. (39) is equivalent to the existence of $\lambda_2 > 0$ such that

$$\mathbf{\Omega}_0 + \mathbf{\Omega}_{1_3} + \begin{bmatrix} \lambda_1^{-1} s_1^2 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \lambda_1 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} + \begin{bmatrix} \lambda_2^{-1} s_2^2 \boldsymbol{E}_B \boldsymbol{E}_B^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & \lambda_2 \boldsymbol{E}_B \boldsymbol{E}_B^{\mathrm{T}} & 0 \\ * & * & * & * & 0 \end{bmatrix} < 0. \quad (41)$$

Finally, we applied Lemma 4 to $\mathbf{\Omega}_{1_3}$ by choosing

$$\boldsymbol{Q} = \mathbf{\Omega}_0 + \begin{bmatrix} \lambda_1^{-1} s_1^2 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \lambda_1 \boldsymbol{E}_A \boldsymbol{E}_A^{\mathrm{T}} & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} + \begin{bmatrix} \lambda_2^{-1} s_2^2 \boldsymbol{E}_B \boldsymbol{E}_B^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & \lambda_2 \boldsymbol{E}_B \boldsymbol{E}_B^{\mathrm{T}} & 0 \\ * & * & * & * & 0 \end{bmatrix}, \quad (42)$$

$$\boldsymbol{H} = \begin{bmatrix} s_3 E_C \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{E}^{\mathrm{T}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \boldsymbol{E}_C^{\mathrm{T}} \end{bmatrix}, F(t) = \Delta_3(t).$$

Eq. (41) is equivalent to $\lambda_3 > 0$ such that eq. (37) is satisfied and completes the proof of Step 2.

Now, by pre- and post-multiplying eq. (32) by a block-diagonal matrix $\mathrm{diag}\left(\boldsymbol{I}_n, \boldsymbol{I}_n, \boldsymbol{I}_n, \boldsymbol{I}_n, \boldsymbol{I}_n, \frac{1}{\sqrt{\lambda_1}}\boldsymbol{I}_n, \frac{1}{\sqrt{\lambda_2}}\boldsymbol{I}_n, \frac{1}{\sqrt{\lambda_3}}\boldsymbol{I}_n\right)$ and a simple use of Lemma 3 to obtain the inequality show that eq. (37) is equivalent to eq. (32). Therefore, the conditions of Theorem 1 are satisfied.

Remark 9. It is difficult to obtain an exact NN when modeling a dynamical system, which requires consideration of the parameter uncertainties. The feedback controls given in refs. [47, 48] cannot stabilize the finite-time system (6), unlike the controller (7), which improves and extends the abovementioned works.

Let us introduce the following assumption:

($\mathbf{H_3}$) there exist constants G_i such that

$$|f_i(x)| \leqslant G_i \quad i = 1, \dots, n.$$

Now, through the LMI approach, a new kind of finite-time controller is built for UDHNNs with time-varying leakage delays.

Theorem 3. If there exist positive constants $s_i,\ k_i$ $(i = 1, 3)$ such that the following conditions hold

$$\boldsymbol{I}_n + s_1\ \boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_3\ \boldsymbol{C}(t)\boldsymbol{C}(t)^{\mathrm{T}} + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n < 0, \quad (43)$$

$$-(1-\sigma^*) + s_3^{-1} - 2k_3 < 0, \quad (44)$$

then system (6) is FTSB via the following controller:

$$\begin{aligned} \boldsymbol{u}(x(t)) = &- \lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\sigma(t)}^{t} \boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma} \frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2 + \nu} \\ &- k_3\boldsymbol{x}(t-\sigma(t))^{\mathrm{T}}\boldsymbol{x}(t-\sigma(t))\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2 + \nu} \\ &- k_1\ \boldsymbol{x}(t) - (B + E_B^2)\ G\ \mathrm{sign}(x(t)), \end{aligned} \quad (45)$$

with $\lambda > 0,\ k_i > 0$ $(i = 1, 3)$ as the control strength to be determined. The real number γ satisfies $0 < \gamma < 1$, $G = \max_{1\leqslant i\leqslant n} G_i$ and the settling time satisfies $\mathfrak{T} \leqslant \mathfrak{T}_1$.

Proof. By replacing eq. (45) in eq. (14) and applying eqs. (15), (17), and ($\mathbf{H_3}$), we obtain

$$\begin{aligned} &\dot{V}(x_t) \\ &\leqslant \boldsymbol{x}(t)^{\mathrm{T}}\left[\boldsymbol{I}_n + s_1\ \boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_3\ \boldsymbol{C}(t)\boldsymbol{C}(t)^{\mathrm{T}} + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n\right]\boldsymbol{x}(t)^{\mathrm{T}} \\ &\quad + \boldsymbol{x}^{\mathrm{T}}(t-\sigma(t))\left[-(1-\sigma^*) + s_3^{-1} - 2k_3\right]\boldsymbol{x}(t-\sigma(t)) \\ &\quad - 2\lambda V^{\gamma}(t), \quad \forall\ t > 0. \end{aligned} \quad (46)$$

Thus, from (43) and (44), we obtain

$$\dot{V}(t) < -2\lambda V^{\gamma}(t), \quad \forall\ t > 0.$$

The rest of the proof is straightforward, and so, is omitted.

In the following corollary, we present some new LMI conditions that could be easily checked, and via controller (45), ensure the FTSB of UDHNNs with time-varying leakage delays.

Corollary 4. If there exist positive constants s_i, λ_i, k_i $(i = 1, 3)$ such that the following LMIs hold:

$$\Omega = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \boldsymbol{L}_f & s_1\boldsymbol{A} & s_3\boldsymbol{C} & s_1\boldsymbol{E}_A & s_3\boldsymbol{E}_c \\ * & -s_1\boldsymbol{I}_n & 0 & 0 & 0 & 0 \\ * & * & \boldsymbol{\Omega}_{33} & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Omega}_{55} & 0 & 0 \\ * & * & * & * & -\lambda_1\boldsymbol{I}_n & 0 \\ * & * & * & * & -\lambda_2\boldsymbol{I}_n & 0 \\ * & * & * & * & * & -\lambda_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{47}$$

$$\begin{bmatrix} -(1-\sigma^*)\boldsymbol{I}_n - 2k_3\boldsymbol{I}_n & \boldsymbol{I}_n \\ * & -s_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{48}$$

then system (6) is FTSB via the controller (45).

The proof of Corollary 4 is similar to that of Theorem 2, so it is omitted.

In practice, the exact values of the delay are often poorly known [31]. Also, it is difficult to assess the delays, and most of the time, only approximate values are available [67]. Even a real-time operating system can guarantee only the maximum values for the time-varying delays [68]. A new finite-time controller was designed with the following corollary, in which the knowledge of only the upper bounds $\bar{\sigma}$ of the time-varying delays σ is necessary, rendering the controller more suitable for real applications.

Corollary 5. If there exist positive constants s_i, k_i $(i = 1, 2, 3)$ such that the following LMIs hold:

$$\Omega = \begin{bmatrix} \boldsymbol{I}_n - 2k_1\boldsymbol{I}_n & \boldsymbol{L}_f & s_1\boldsymbol{A} & s_3\boldsymbol{C} & s_1\boldsymbol{E}_A & s_3\boldsymbol{E}_c \\ * & -s_1\boldsymbol{I}_n & 0 & 0 & 0 & 0 \\ * & * & \boldsymbol{\Omega}_{33} & 0 & 0 & 0 \\ * & * & * & \boldsymbol{\Omega}_{55} & 0 & 0 \\ * & * & * & * & -\lambda_1\boldsymbol{I}_n & 0 \\ * & * & * & * & -\lambda_2\boldsymbol{I}_n & 0 \\ * & * & * & * & * & -\lambda_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{49}$$

$$\begin{bmatrix} -(1-\sigma^*)\boldsymbol{I}_n & \boldsymbol{I}_n \\ * & -s_3\boldsymbol{I}_n \end{bmatrix} < 0, \tag{50}$$

then system (6) is FTSB via the continuous controller as follows:

$$\boldsymbol{u}\,(x(t)) = -\,k_1\,\boldsymbol{x}(t) - (B + E_B^2)G\,\mathrm{sign}(x(t))$$

$$-\,\lambda\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-\bar{\sigma}}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\gamma}\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2 + \nu} \tag{51}$$

and the settling time satisfies $\mathfrak{T} \leqslant \mathfrak{T}_1$.

Proof. By replacing eq. (51) in eq. (14), as with Corollary 4, we have

$$\begin{aligned} \dot{V}\,(x_t) \leqslant\ & \boldsymbol{x}(t)^{\mathrm{T}}\Big[\boldsymbol{I}_n + s_1\,\boldsymbol{A}(t)\boldsymbol{A}(t)^{\mathrm{T}} + s_3\,\boldsymbol{C}(t)\boldsymbol{C}(t)^{\mathrm{T}} \\ & + s_1^{-1}\boldsymbol{L}_f^2 - 2k_1\boldsymbol{I}_n\Big]x(t)^{\mathrm{T}} \\ & + \boldsymbol{x}^{\mathrm{T}}\,(t-\sigma(t))\Big[-(1-\sigma^*) + s_3^{-1}\Big]\boldsymbol{x}\,(t-\sigma(t)) \\ & - 2\lambda V^{\gamma}(t), \quad \forall\, t > 0. \end{aligned} \tag{52}$$

Thus, from eqs. (49) and (50), we immediately obtain the result that completes the proof.

4 Applications

In this section, two numerical examples are given to demonstrate the effectiveness of our main theoretical results.

4.1 Example 1

Consider the following delayed-Hopfield NNs with time-varying delays in the leakage terms:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}\,(t-\sigma(t)) + \boldsymbol{A}\,\boldsymbol{f}(x(t)) + \boldsymbol{B}\,\boldsymbol{f}(x(t-\tau)) \\ \qquad +\boldsymbol{u}, \quad t > 0, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s \in [-\tau^*,\ 0], \end{cases} \tag{53}$$

where $n = 2$ and

$$f_1(s) = f_2(s) = \tanh(s), \quad \tau = 1, \quad \sigma(t) = 0.2|\sin t|.$$

The initial conditions are defined as follows:

$$x_1(s) = \phi_1(s) = -1.6,\ x_2(s) = \phi_2(s) = 1.2,\ \forall s \in [-1, 0]$$

and the parameters $\boldsymbol{C}$, $\boldsymbol{A}$, and $\boldsymbol{B}$ are given as follows:

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\quad \boldsymbol{A} = \begin{pmatrix} 2 & -0.1 \\ -5 & -4.5 \end{pmatrix},\quad \boldsymbol{B} = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{pmatrix}.$$

Noting that $\boldsymbol{L}_f = \mathrm{diag}(1, 1)$, $\sigma^* = 0.2$. By solving eqs. (24), (25), (26) with the Matlab LMI toolbox leads to the following solution

$$k_1 = 50,\ k_2 = 3,\ k_3 = 3,\ s_i = 1,\ i = 1, 2, 3.$$

Therefore, according to Corollary 2, the following controller

$$\boldsymbol{u}\,(x(t)) = -\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-0.2|\sin t|}^{t}\boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)^{0.9}\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2 + \nu}$$

$$- 50\, \boldsymbol{x}(t) - \left(3\boldsymbol{x}\,(t-\tau(t))^{\mathrm{T}}\, \boldsymbol{x}\,(t-\tau(t)) + 3\boldsymbol{x}\,(t-\sigma(t))^{\mathrm{T}}\, \boldsymbol{x}\,(t-\sigma(t))\right) \frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu}. \tag{54}$$

Setting $\gamma = 0.9$ and $\nu = 10^{-3}$ stabilizes the finite-time system (53). The trajectories of the state variables are shown in Figure 1, which confirms the effectiveness of our main theoretical results.

Remark 10. In Example 4, nine variables involved in the LMIs are to be solved. It should be pointed out that when LMI becomes larger, the complexity increases because the number of variables to be solved in the LMIs depends on the number of neurons n. Such a complexity is caused by the use of LKF (11).

4.2 Example 2

Consider the following UDHNN with a time-varying delay in the leakage term:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\boldsymbol{C}(t)\boldsymbol{x}\,(t-\sigma(t)) + \boldsymbol{A}(t)\boldsymbol{f}(x(t)) \\ \qquad +\boldsymbol{B}(t)\boldsymbol{f}(x(t-\tau(t))) + \boldsymbol{u}, \quad t>0, \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s\in[-\tau^*,\ 0], \end{cases} \tag{55}$$

$x_1(s) = \phi_1(s) = -1.6$, $x_2(s) = \phi_2(s) = 1.2$, and $s \in [-1, 0]$. The uncertain parameters are $\boldsymbol{E}_A = \boldsymbol{E}_B = \boldsymbol{E}_C = \mathrm{diag}(1,1)$,

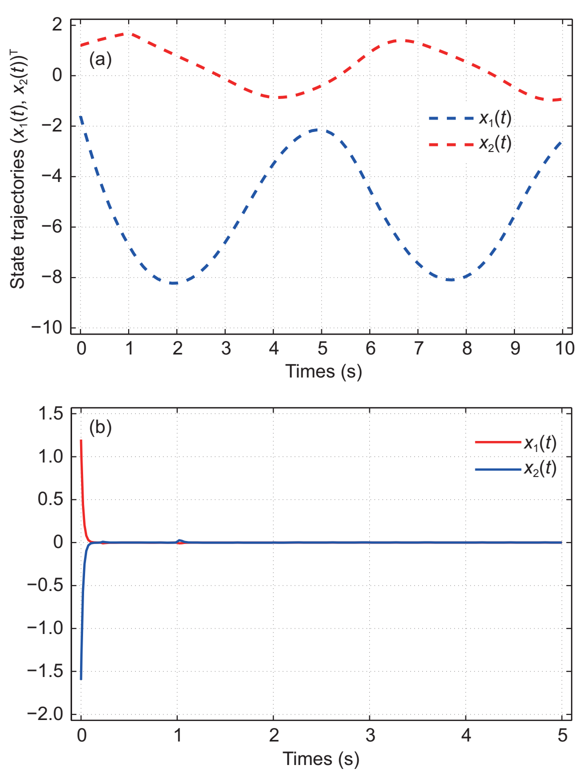

Figure 1 (Color online) State trajectories of system (53) with initial condition $(-1.6, 1.2)^{\mathrm{T}}$. (a) Open-loop evolution of system (53); (b) closed-loop system (53) under controller (54).

$\boldsymbol{\Delta}_i(t) = \mathrm{diag}(\sin(t), \text{and } \sin(t))$, $i = 1,2,3$. Other parameters are similar to those in Sect. 4.1. Using Matlab LMI Toolbox for solving eqs. (32)–(34) and fixing $\gamma = 0.9$, $\nu = 10^{-3}$, we obtain the following solution:

$$s_1 = 0.5522,\ s_2 = 1.1595,\ s_3 = 7.0379;$$
$$k_1 = 50,\ k_2 = 3,\ k_3 = 3;$$
$$\lambda_1 = 0.1314,\ \lambda_2 = 0.3441,\ \lambda_3 = 3.6611.$$

From Theorem 2, we deduce that system (55) is FTSB via the controller

$$\boldsymbol{u}\,(x(t)) = -\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-0.2|\sin t|}^{t} \boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{x}(s)\mathrm{d}s\right)\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu} - 50\, \boldsymbol{x}(t) - \left(3\boldsymbol{x}\,(t-\tau(t))^{\mathrm{T}}\, \boldsymbol{x}\,(t-\tau(t)) + 3\boldsymbol{x}\,(t-\sigma(t))^{\mathrm{T}}\, \boldsymbol{x}\,(t-\sigma(t))\right)\frac{x(t)}{\|\boldsymbol{x}(t)\|^2+\nu}. \tag{56}$$

Furthermore, according to Corollary 5, the following controller

$$\boldsymbol{u}\,(x(t)) = -\left(\boldsymbol{x}(t)^{\mathrm{T}}\boldsymbol{x}(t) + \int_{t-0.2}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{x}(s)\mathrm{d}s\right)^{0.9}\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|^2+\nu} - 50\boldsymbol{x}(t) - (B+I_2)\,\mathrm{sign}(x(t)) \tag{57}$$

also ensures the FTSB of system (4.2). In eq. (57), we do not need to know the time-varying delay σ, and the controller is well suited to real applications. Simulation results of system (55) are depicted in Figure 2.

5 Conclusion

This article deals with the problem of the finite-time stabilization of uncertain delayed-Hopfield NNs with time-varying delays in the leakage term. By using the LKF method and the FTS theory, some sufficient conditions in terms of LMIs are established to ensure the FTSB. Firstly, our proposed results extend the previously known results established in refs. [47–49], where the leakage delay is not taken into consideration. Secondly, our proposed method improves and completes the results of refs. [21, 22], of which only the asymptotic behavior is investigated. Finally, some numerical examples are given to demonstrate the effectiveness of our proposed results. Despite the contribution that provides the FTS, the settling time depends on the initial conditions of the NNs. On the one hand, the variation of the initial values has a strong effect on the estimation of the settling time. On the other hand, in practice, advance knowledge of the initial conditions is very difficult to acquire [69]. For future research, we intend to design a fixed-time controller for NNs with time-varying

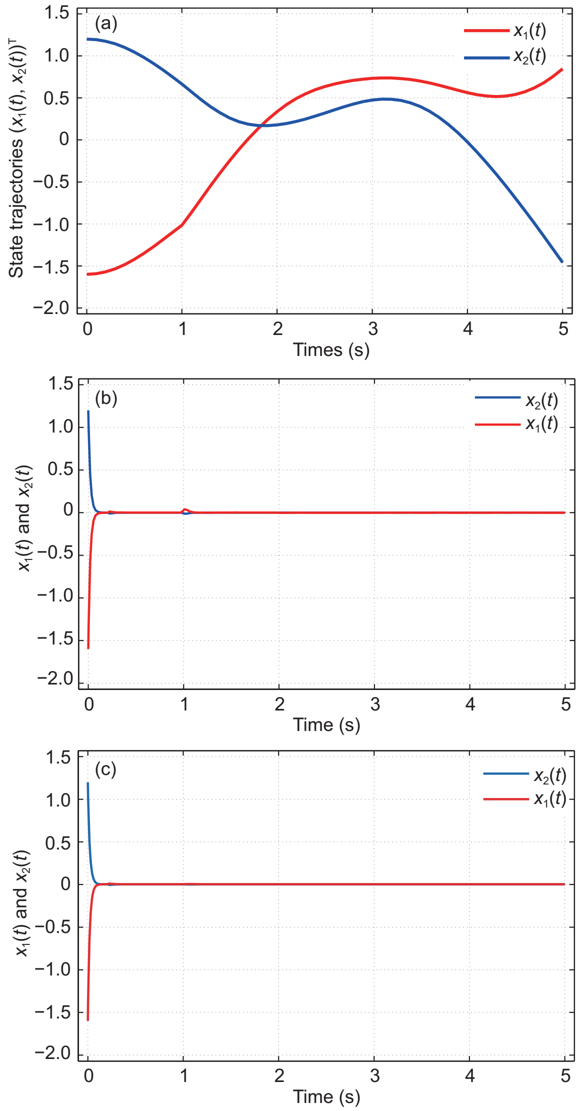

Figure 2 (Color online) FTS of system (55) with initial condition $(-1.6, 1.2)^T$ under different kinds of control protocols. (a) Open-loop evolution of system (55); (b) closed-loop system (55) under controller (56); (c) closed-loop system (55) under controller (57).

leakage delays in which the settling time is independent of the initial conditions.

1 Mathiyalagan K, Hongye Su K, Peng Shi K, et al. Exponential H filtering for discrete-time switched neural networks with random delays. IEEE Trans Cybern, 2015, 45: 676–687
2 Mathiyalagan K, Anbuvithya R, Sakthivel R, et al. Non-fragile H synchronization of memristor-based neural networks using passivity theory. Neural Networks, 2016, 74: 85–100
3 Nitta T. Orthogonality of decision boundaries in complex-valued neural networks. Neural Computation, 2004, 16: 73–97
4 Aouiti C, Alimi A M, Karray F, et al. The design of beta basis function neural network and beta fuzzy systems by a hierarchical genetic algorithm. Fuzzy Sets Syst, 2005, 154: 251–274
5 Aouiti C, Alimi A M, Maalej A. A genetic-designed beta basis function neural network for multi-variable functions approximation. Syst Anal Model Simul, 2002, 42: 975–1009
6 Forti M, Nistri P, Quincampoix M. Generalized neural network for non-smooth nonlinear programming problems. IEEE Trans Circuits Syst I, 2004, 51: 1741–1754
7 Li X D, Song S J. Impulsive control for existence, uniqueness, and global stability of periodic solutions of recurrent neural networks with discrete and continuously distributed delays. IEEE Trans Neural Netw Learning Syst, 2013, 24: 868–877
8 Aouiti C. Neutral impulsive shunting inhibitory cellular neural networks with time-varying coefficients and leakage delays. Cogn Neurodyn, 2016, 10: 573–591
9 Aouiti C, Mhamdi M S, Touati A. Pseudo almost automorphic solutions of recurrent neural networks with time-varying coefficients and mixed delays. Neural Process Lett, 2017, 45: 121–140
10 Aouiti C, Coirault P, Miaadi F, et al. Finite time boundedness of neutral high-order Hopfield neural networks with time delay in the leakage term and mixed time delays. Neurocomputing, 2017, 260: 378–392
11 Li X, Cao J. An impulsive delay inequality involving unbounded time-varying delay and applications. IEEE Trans Automat Contr, 2017, 62: 3618–3625
12 Li X, Ho D, Cao J. Finite-time stability and settling-time estimation of nonlinear impulsive systems. Automatica, 2019, 99: 361–368
13 Li X, Bohner M, Wang C K. Impulsive differential equations: Periodic solutions and applications. Automatica, 2015, 52: 173–178
14 Li X, Zhang X, Song S. Effect of delayed impulses on input-to-state stability of nonlinear systems. Automatica, 2017, 76: 378–382
15 Li X, Fu X. Effect of leakage time-varying delay on stability of nonlinear differential systems. J Franklin Institute, 2013, 350: 1335–1344
16 Stamova I, Stamov T, Li X. Global exponential stability of a class of impulsive cellular neural networks with supremums. Int J Adapt Control Signal Process, 2014, 28: 1227–1239
17 Aouiti C, Miaadi F. Pullback attractor for neutral Hopfield neural networks with time delay in the leakage term and mixed time delays. Neural Comp Appl, 2018, doi: 10.1007/s00521-017-3314-z
18 Hu V P H, Fan M L, Su P, et al. Leakage-delay analysis of ultra-thin-body GeOI devices and logic circuits. In: Proceedings of the 2011 International Symposium on VLSI Technology, Systems and Applications. Taiwan: IEEE, 2011. 1–2
19 Zhang H, Wang Z, Liu D. A comprehensive review of stability analysis of continuous-time recurrent neural networks. IEEE Trans Neural Netw Learning Syst, 2014, 25: 1229–1262
20 Li X, Song S. Stabilization of delay systems: delay-dependent impulsive control. IEEE Trans Automat Contr, 2017, 62: 406–411
21 Li Y, Zeng Z, Wen S. Asymptotic stability analysis on nonlinear systems with leakage delay. J Franklin Institute, 2016, 353: 757–779
22 Li X, Wu J. Sufficient stability conditions of nonlinear differential systems under impulsive control with state-dependent delay. IEEE Trans Automat Contr, 2018, 63: 306–311
23 Aouiti C. Oscillation of impulsive neutral delay generalized high-order Hopfield neural networks. Neural Compu Appl, 2018, 29: 477–495
24 Aouiti C, Mhamdi M S, Cao J, et al. Piecewise pseudo almost periodic solution for impulsive generalised high-order hopfield neural networks with leakage delays. Neural Process Lett, 2017, 45: 615–648
25 Li X, Wu J. Stability of nonlinear differential systems with state-dependent delayed impulses. Automatica, 2016, 64: 63–69
26 Liu X, Jiang N, Cao J, et al. Finite-time stochastic stabilization for BAM neural networks with uncertainties. J Franklin Institute, 2013, 350: 2109–2123
27 Wu Y, Cao J, Alofi A, et al. Finite-time boundedness and stabilization of uncertain switched neural networks with time-varying delay. Neural Networks, 2015, 69: 135–143
28 Cao J, Huang D S, Qu Y. Global robust stability of delayed recurrent neural networks. Chaos Solitons Fractals, 2005, 23: 221–229
29 Ji C, Zhang H G, Wei Y. LMI approach for global robust stability of

Cohen-Grossberg neural networks with multiple delays. Neurocomputing, 2008, 71: 475–485
30 Zhang H G, Wang Y C. Stability analysis of Markovian jumping stochastic Cohen-Grossberg neural networks with mixed time delays. IEEE Trans Neural Netw, 2008, 19: 366–370
31 Aouiti C, Miaadi F. Finite-time stabilization of neutral hopfield neural networks with mixed delays. Neural Proces Lett, 2018, 48: 1645–1669
32 Moulay E, Perruquetti W. Finite time stability and stabilization of a class of continuous systems. J Math Anal Appl, 2006, 323: 1430–1443
33 Haimo V T. Finite time controllers. SIAM J Control Optim, 1986, 24: 760–770
34 Bhat S P, Bernstein D S. Lyapunov analysis of finite-time differential equations. In: Proceedings of the 1995 American Control Conference. IEEE, 1995. 831–1832
35 Bhat S P, Bernstein D S. Finite-time stability of homogeneous systems. In: Proceedings of the 1997 American Control Conference. IEEE, 1997. 2513–2514
36 Bhat S P, Bernstein D S. Finite-time stability of continuous autonomous systems. SIAM J Control Optim, 2000, 38: 751–766
37 Moulay E, Dambrine M, Yeganefar N, et al. Finite-time stability and stabilization of time-delay systems. Syst Control Lett, 2008, 57: 561–566
38 Perruquetti W, Floquet T, Moulay E. Finite-time observers: Application to secure communication. IEEE Trans Automat Contr, 2008, 53: 356–360
39 Du H, Li S, Qian C. Finite-time attitude tracking control of spacecraft with application to attitude synchronization. IEEE Trans Automat Contr, 2011, 56: 2711–2717
40 Mathiyalagan K, Balachandran K. Finite-time stability of fractional-order stochastic singular systems with time delay and white noise. Complexity, 2016, 21: 370–379
41 Huang J, Li C, Huang T, et al. Finite-time lag synchronization of delayed neural networks. Neurocomputing, 2014, 139: 145–149
42 Mathiyalagan K, Park J H, Sakthivel R. Finite-time boundedness and dissipativity analysis of networked cascade control systems. Nonlinear Dyn, 2016, 84: 2149–2160
43 Liu X, Ho D W C, Yu W, et al. A new switching design to finite-time stabilization of nonlinear systems with applications to neural networks. Neural Networks, 2014, 57: 94–102
44 Liu X, Park J H, Jiang N, et al. Nonsmooth finite-time stabilization of neural networks with discontinuous activations. Neural Networks, 2014, 52: 25–32
45 Shen H, Park J H, Wu Z G. Finite-time synchronization control for uncertain Markov jump neural networks with input constraints. Nonlinear Dyn, 2014, 77: 1709–1720
46 Shen J, Cao J. Finite-time synchronization of coupled neural networks via discontinuous controllers. Cogn Neurodyn, 2011, 5: 373–385
47 Wang L, Shen Y. Finite-time stabilizability and instabilizability of delayed memristive neural networks with nonlinear discontinuous controller. IEEE Trans Neural Netw Learning Syst, 2015, 26: 2914–2924
48 Wang L, Shen Y, Ding Z. Finite time stabilization of delayed neural networks. Neural Networks, 2015, 70: 74–80
49 Wang L, Shen Y, Sheng Y. Finite-time robust stabilization of uncertain delayed neural networks with discontinuous activations via delayed feedback control. Neural Networks, 2016, 76: 46–54
50 Wu R, Lu Y, Chen L. Finite-time stability of fractional delayed neural networks. Neurocomputing, 2015, 149: 700–707
51 Yang S, Li C, Huang T. Finite-time stabilization of uncertain neural networks with distributed time-varying delays. Neural Comput Applic, 2017, 28: 1155–1163
52 Lv X, Li X. Finite time stability and controller design for nonlinear impulsive sampled-data systems with applications. ISA Trans, 2017, 70: 30–36
53 Zhou J, Lu J, Lü J. Pinning adaptive synchronization of a general complex dynamical network. Automatica, 2008, 44: 996–1003
54 Liu T, Hill D J, Zhao J. Synchronization of dynamical networks by network control. IEEE Trans Automat Contr, 2012, 57: 1574–1580
55 Zuo Z, Tie L. Distributed robust finite-time nonlinear consensus protocols for multi-agent systems. Int J Syst Sci, 2016, 47: 1366–1375
56 Wan Y, Cao J, Wen G, et al. Robust fixed-time synchronization of delayed Cohen-Grossberg neural networks. Neural Networks, 2016, 73: 86–94
57 Yang X, Ho D W C, Lu J, et al. Finite-time cluster synchronization of T-S fuzzy complex networks with discontinuous subsystems and random coupling delays. IEEE Trans Fuzzy Syst, 2015, 23: 2302–2316
58 Hamayun M T, Edwards C, Alwi H. Fault Tolerant Control Schemes Using Integral Sliding Modes. Cham: Springer International Publishing, 2016. 17–37
59 Hale J K. Theory of Functional Differential Equations. New York: Springer, 1977. 36–56
60 Boyd S, El Ghaoui L, Feron, et al. Linear Matrix Inequalities in System and Control Theory. Philadelphia: Society for Industrial and Applied Mathematics (SIAM), 1994. 20–78
61 Hale J K. Ordinary differential equations. Pure Appl Math, 1980, 20: 36–56
62 Cai Z W, Huang L H. Finite-time synchronization by switching state-feedback control for discontinuous CohenGrossberg neural networks with mixed delays. Int J Machine Learn and Cybernet, 2018, 9: 1683–1695
63 Yang X. Can neural networks with arbitrary delays be finite-timely synchronized? Neurocomputing, 2014, 143: 275–281
64 Zhou C, Zhang W, Yang X, et al. Finite-time synchronization of complex-valued neural networks with mixed delays and uncertain perturbations. Neural Process Lett, 2017, 46: 271–291
65 Li Y, Yang X, Shi L. Finite-time synchronization for competitive neural networks with mixed delays and non-identical perturbations. Neurocomputing, 2016, 185: 242–253
66 Shi L, Yang X, Li Y, et al. Finite-time synchronization of nonidentical chaotic systems with multiple time-varying delays and bounded perturbations. Nonlinear Dyn, 2016, 83: 75–87
67 Léchappé V, Moulay E, Plestan F, et al. New predictive scheme for the control of LTI systems with input delay and unknown disturbances. Automatica, 2015, 52: 179–184
68 Lechappe V, Rouquet S, Gonzalez A, et al. Delay estimation and predictive control of uncertain systems with input delay: Application to a DC motor. IEEE Trans Ind Electron, 2016, 63: 5849–5857
69 Hu C, Yu J, Chen Z, et al. Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks. Neural Networks, 2017, 89: 74–83