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Distributed disturbance-observer-based vibration control for a flexible-link manipulator with output constraints

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The issue of output constraints is studied for a flexible-link manipulator in the presence of unknown spatially distributed disturbances. The manipulator can be taken as an Euler-Bernoulli beam and its dynamic is expressed by partial differential equations. On account of the uncertainty of disturbances, we present a disturbance observer to estimate infinite dimensional disturbances on the beam. The observer is proven exponentially stable. Considering the problem of output constraints in the practical engineering, we propose a novel distributed vibration controller based on the disturbance observer to fulfill the position regulation of the joint angle and suppress elastic deflections on the flexible link, while confining the regulating errors of output in a suitable scope that we can assign. The closed-loop system is demonstrated exponentially stable based on an integral-barrier Lyapunov function. Simulations validate the effectiveness of the design scheme.

flexible manipulator, distributed vibration control, disturbance observer, partial differential equation, output constraint, distributed parameter system

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1 Introduction

Flexible-link manipulators have been widely applied in recent years, because of their favorable properties [1] such as lower energy consumption, faster operational speed and lighter weight. However, the flexible characteristics also make the dynamic model of flexible manipulators be distributed and take on the form of partial differential equations (PDEs), generating more difficulties for vibration control design. In order to suppress the vibration of flexible systems, researchers have proposed many approaches [2–17]. In general case, researchers did not consider the problems of distributed disturbances and output constraints simultaneously, while these problems usually happen in the actual engineering.

First, we consider developing a disturbance observer that acts as a compensator in the control scheme to resist the adverse impact caused by exogenous disturbances in practical engineering. Many methods [18–20] for overcoming disturbances have been used in manipulators. Concerning the observer design, a Luenberger-like observer has been designed for a general class of linear singular time-delay systems in ref. [21]. In ref. [22], the authors developed a disturbance observer to estimate the unknown compounded disturbance for a class of uncertain nonaffine nonlinear systems. In ref. [23], a general systematic approach is proposed to solve the disturbance observer design problem for robotic manipulators. However, the observers used in the

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foregoing studies are finite dimensional observers expressed by ordinary differential equations (ODEs), they are not suitable for the system in our study. Since the flexible link is a distributed parameter system (DPS), the dynamic model is described by PDEs and the distributed disturbances are infinite dimensional. Consequently, it is required to design a distributed infinite dimensional disturbance observer.

Then, the issue of output constraint that usually appeared in practical engineering is concerned. To avoid the instability or even the damage of flexible-link manipulators caused by excessive vibrations, we consider a control scheme that can limit the regulating errors of output (joint angular position, vibratory deflection on the beam) in a suitable range that we can determine. Previous researchers [24,25] have proposed many methods to solve the problem of output constraints. For distributed parameter systems whose dynamic described by PDEs, a boundary control with the boundary output constraint [25] has been proposed so that the vibration of cantilever beam can be suppressed. However, the authors only discussed boundary disturbance of a cantilever beam, and the closed-loop system is only uniform ultimate bounded (UUB). By contrast, we consider the distributed disturbance along the rotatable link, and the controller could validate overall exponential stability of the system. Many works have been done for designing distributed controllers [26–28], but they neglect the output constraints in the actual engineering.

We have recently proposed a state observer [29] and a distributed controller [30], but the problems on distributed disturbances and output constraints did not considered. In this paper, we study a flexible-link manipulator with unknown infinite dimensional distributed disturbances and concern output constraints of system simultaneously. The flexible link can be taken as an Euler-Bernoulli beam and its dynamics takes on the form of PDEs. Considering the unknown disturbances caused by noisy environments in the practical engineering, we introduce an infinite dimensional distributed disturbance observer that can estimate distributed disturbances on the flexible link and is validated exponentially stable. Then, we propose a novel distributed controller based on the disturbance observer to fulfill the position regulation of the joint angle and suppress elastic deflections on the flexible link. In addition, the controller could restrict the regulating errors of output in a prescribed scope to keep the system secure, based on an integral-barrier Lyapunov function. The closed-loop system is validated exponentially stable and the effectiveness of the proposed scheme is demonstrated by numerical simulations. The detailed contributions are summarized as follows:

(1) The unknown exogenous distributed disturbances on flexible manipulators can be estimated by our infinite dimensional disturbance observer, which is the precondition for developing the distributed controller. Furthermore, the adverse influences of boundary disturbance and distributed disturbance on the beam are all resolved.

(2) The issue of output constraints is addressed with the disturbance-observer-based controller. The distributed controller could restrict vibrations of the manipulator in a range that we could assign, increasing security in the practical engineering.

(3) On the basis of solving the problems on distributed disturbances and output constraints, the controller can achieve the position regulation of joint angle and suppress vibrations on the flexible link with exponential rate, instead of UUB.

2 System description

In this paper, a flexible-link manipulator in the presence of unknown spatially distributed disturbances is studied. We discuss a kind of flexible-link manipulator whose surfaces are bonded with two piezoelectric plates [26]. Note that the piezoelectric materials, acting as the distributed actuators and sensors, are distributed along the flexible link. Without loss of generality, we simplify the system into a typical Euler-Bernoulli beam, whose schematic is shown in Figure 1. The distributed force generated by piezoelectric material is directly expressed by $u(x, t)$ for simplification of analysis. Frame $X_0O_0Y_0$ is the fixed global inertial coordinate and frame XOY is the local coordinate. The joint motor is rigidly clamping the hub of the beam and a tip mass is attached to its free end. In Figure 1, EI, ρ, and L represent the bending stiffness, mass per unit length and length of the beam; Let $\theta(t)$ represent the joint angular position, I_h the inertia of the hub, m the mass of the payload, $w(x, t)$, $d(x, t)$ and $r(x, t)$ the vibratory deflection, distributed varying disturbance and position of point P in Frame $X_0O_0Y_0$, respectively; $u_0(t)$, $u_L(t)$, $d_0(t)$, $d_L(t)$ are the torque input generated by joint motor, force input generated by the actuator at the end, boundary disturbance acting on the motor, and boundary disturbance acting on the payload, respectively.

Assumption 1. The flexible beam only moves in the horizontal plane and the effect of gravity is not taken into consideration [8].

Assumption 2. The manipulator is flexible transverse to its length in the plane of motion and rigid in other directions. The link elongations are small enough to be neglected [8].

Remark 1. For simplicity and clarity, we use derivative notations as follows: $\dot{*} = \frac{\partial(*)}{\partial t}$, $\ddot{*} = \frac{\partial^2(*)}{\partial t^2}$, $(*)_x = \frac{\partial(*)}{\partial x}$, $(*)_{xx} = \frac{\partial^2(*)}{\partial x^2}$, $(*)_{xxx} = \frac{\partial^3(*)}{\partial x^3}$, and $(*)_{xxxx} = \frac{\partial^4(*)}{\partial x^4}$, where $x \in [0, L]$ and $t \in [0, \infty)$ respectively denote the position variable and time variable.

As the spillover effect occurred in the controller obtained by using lumped parameter modeling method, we apply

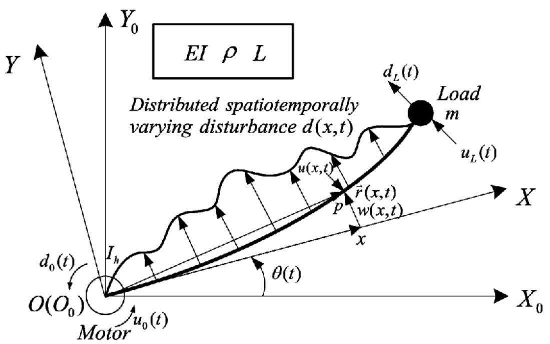

Figure 1 Schematic of the flexible-link manipulator with unknown disturbances.

distributed parameter modeling method whose advantage is that it does not ignore high frequency dynamics. Thus, the dynamic model will take on the form of PDEs. We first define the position of point P in the inertial coordinate as $r(x, t)=x\theta(t)+w(x,t)$. Then we consider the kinetic energy E_k, the potential energy E_P, and the non-conservative work W_{nc} of the system, and employ Hamilton's approach to get the distributed parameter model of the flexible-link manipulator. The readers who are interest in the details could refer to ref. [29]. Combined the characteristics of piezoelectric materials with the beam, we give the simplified dynamic model of the system as the following equations:

$$I_h\ddot{\theta}(t) - EIw_{xx}(0,t) = u_0(t) + d_0(t), \tag{1}$$

$$\rho\ddot{r}(x,t) + EIw_{xxxx}(x,t) = u(x,t) + d(x,t), \tag{2}$$

$$m\ddot{r}(L,t) - EIw_{xxx}(L,t) = u_L(t) + d_L(t), \tag{3}$$

$$w(0,t) = 0,\ w_x(0,t) = 0,\ w_{xx}(L,t) = 0, \tag{4}$$

where eq. (2) is the governing equation and eqs. (3) and (4) represent boundary conditions. Noting that the boundary disturbances $d_0(t)$, $d_L(t)$, and distributed infinite dimensional disturbance $d(x,t)$, $x \in (0,L)$ are unknown and time-varying, we could develop a disturbance observer to estimate them. Therefore, before we develop the control law, an effective disturbance observer should be first studied.

3 Design of infinite dimensional disturbance observer

In this section, an infinite dimensional disturbance observer is studied to estimate the boundary disturbances $d_0(t)$ and $d_L(t)$, and the distributed infinite dimensional disturbance $d(x,t)$ on the manipulator. Since we next design a controller based on the observer, the estimate errors should be rapidly regulated to zero. For simplicity and clarity in the following study, the time symbol t is omitted. For example, the distributed infinite dimensional disturbance $d(x,t)$ is rewritten as $d(x)$.We define variables $\hat{d}_0$, $\hat{d}_L$, and $\hat{d}(x)$ as estimations of d_0, d_L, and $d(x)$, respectively, and introduce auxiliary variables φ_0, φ_L, and $\varphi(x)$ for the observer development. First, we design

$$\dot{\hat{d}}_0 = k_1\left(d_0 - \hat{d}_0\right), \tag{5}$$

$$\dot{\hat{d}}_L = k_2\left(d_L - \hat{d}_L\right), \tag{6}$$

$$\dot{\hat{d}}(x) = k_3[d(x) - \hat{d}(x)]. \tag{7}$$

Then, we give

$$\varphi_0 = \hat{d}_0 - k_1 I_h\dot{\theta}, \tag{8}$$

$$\varphi_L = \hat{d}_L - k_2 m\dot{r}(L), \tag{9}$$

$$\varphi(x) = \hat{d}(x) - k_3\rho\dot{r}(x). \tag{10}$$

Taking the time derivative of eqs. (8)–(10), and substituting eqs. (1)–(3) and (5)–(7), we obtain

$$\begin{aligned}\dot{\varphi}_0 &= k_1\left(d_0 - \hat{d}_0\right) - k_1 I_h\ddot{\theta}\\ &= k_1\left[I_h\ddot{\theta} - EIw_{xx}(0) - u_0 - \hat{d}_0\right] - k_1 I_h\ddot{\theta}\\ &= -k_1[EIw_{xx}(0) + u_0] - k_1\hat{d}_0,\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{\varphi}_L &= k_2\left(d_L - \hat{d}_L\right) - k_2 m\ddot{r}(L)\\ &= k_2\left[m\ddot{r}(L) - EIw_{xxx}(L) - u_L - \hat{d}_L\right] - k_2 m\ddot{r}(L)\\ &= -k_2[EIw_{xxx}(L) + u_L] - k_2\hat{d}_L,\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{\varphi}(x) &= k_3[d(x) - \hat{d}(x)] - k_3\rho\ddot{r}(x)\\ &= k_3\left[\rho\ddot{r}(x) + EIw_{xxxx}(x) - u(x) - \hat{d}(x)\right] - k_3\rho\ddot{r}(x)\\ &= -k_3[-EIw_{xxxx}(x) + u(x)] - k_3\hat{d}(x).\end{aligned} \tag{13}$$

Based on eqs. (8)–(13), we propose the infinite dimensional disturbance observer as follows:

$$\begin{aligned}&\dot{\varphi}_0 = -k_1[EIw_{xx}(0) + u_0] - k_1\hat{d}_0,\\ &\hat{d}_0 = \varphi_0 + k_1 I_h\dot{\theta},\\ &\dot{\varphi}_L = -k_2[EIw_{xxx}(L) + u_L] - k_2\hat{d}_L,\\ &\hat{d}_L = \varphi_L + k_2 m\dot{r}(L),\\ &\dot{\varphi}(x) = -k_3[-EIw_{xxxx}(x) + u(x)] - k_3\hat{d}(x),\\ &\hat{d}(x) = \varphi(x) + k_3\rho\dot{r}(x).\end{aligned} \tag{14}$$

where k_1, k_2, and k_3 are the design positive constants. The true values θ, $w(x)$, and $w_{xxx}(x)$ could be directly measured by rotary encoders and piezoelectric sensors, which have been explained in ref. [26]. The first time derivative and first spatial derivative of them can be calculated by a backwards difference algorithm [25].

Assumption 3. The change rate of the disturbances can be assumed negligible in comparison with the disturbance estimation error dynamics i.e., $\dot{d}_0 = \dot{d}_L = \dot{d}(x) \approx 0$. There are positive constants $\bar{D}_0$,$\bar{D}_L$,and $\bar{D}$ such that $|d_0| \leq \bar{D}_0$, $|d_L| \leq \bar{D}_L$, and $|d(x)| \le \bar{D}$, $x \in [0, L]$. This is a reasonable assumption in most of practical engineering.

Theorem 1. All the signals of the infinite dimensional disturbance observer in eq. (14) are bounded and the disturbance observer is exponentially stable.

Proof. First, we define the estimate errors as $\tilde{d}_0 = d_0 - \hat{d}_0$, $\tilde{d}_L = d_L - \hat{d}_L$, and $\tilde{d}(x) = d(x) - \hat{d}(x)$. Then, a Lyapunov function candidate is developed as follows:

$$V_{ob} = \frac{1}{2}\tilde{d}_0^{\,2} + \frac{1}{2}\tilde{d}_L^{\,2} + \frac{1}{2}\int_0^L \tilde{d}^2(x)\mathrm{d}x. \tag{15}$$

Differentiating eq. (15) with respect to time, substituting eq. (14), we obtain

$$\begin{aligned}
\dot{V}_{ob} &= \tilde{d}_0\dot{\tilde{d}}_0 + \tilde{d}_L\dot{\tilde{d}}_L + \int_0^L \tilde{d}(x)\dot{\tilde{d}}(x)\mathrm{d}x \\
&= \tilde{d}_0(\dot{d}_0 - \dot{\hat{d}}_0) + \tilde{d}_L(\dot{d}_L - \dot{\hat{d}}_L) \\
&\quad + \int_0^L \tilde{d}(x)\left[\dot{d}(x) - \dot{\hat{d}}(x)\right]\mathrm{d}x \\
&= -\tilde{d}_0(\dot{\varphi}_0 + k_1 I_h\ddot{\theta}) - \tilde{d}_L[\dot{\varphi}_L + k_2 m\ddot{r}(L)] \\
&\quad - \int_0^L \tilde{d}(x)[\dot{\varphi}(x) + k_3\rho\ddot{r}(x)]\mathrm{d}x \\
&= -\tilde{d}_0\left[-k_1(EIw_{xx}(0) + u_0) - k_1\hat{d}_0 + k_1 I_h\ddot{\theta}\right] \\
&\quad -\tilde{d}_L\left[-k_2(EIw_{xxx}(L) + u_L) - k_2\hat{d}_L + k_2 m\ddot{r}(L)\right] \\
&\quad - \int_0^L \tilde{d}(x)\big[-k_3(-EIw_{xxxx}(x) + u(x)) \\
&\quad - k_3\hat{d}(x) + k_3\rho\ddot{r}(x)\big]\mathrm{d}x \\
&= -\tilde{d}_0(k_1 d_0 - k_1\hat{d}_0) - \tilde{d}_L(k_2 d_L - k_2\hat{d}_L) \\
&\quad - \int_0^L \tilde{d}(x)(k_3 d(x) - k_3\hat{d}(x))\mathrm{d}x \\
&= -k_1\tilde{d}_0^{\,2} - k_2\tilde{d}_L^{\,2} - k_3\int_0^L \tilde{d}^2(x)\mathrm{d}x \\
&\le -\lambda_{ob}V_{ob},
\end{aligned} \tag{16}$$

where $0 < \lambda_{ob} \le \min\{k_1, k_2, k_3\}$. Then, the above inequality gives

$$V_{ob}(t) \le e^{-\lambda_{ob}}V_{ob}(0). \tag{17}$$

From eq. (17), we conclude that the proposed infinite dimensional disturbance observer is bounded and the estimate errors $\tilde{d}_0$, $\tilde{d}_L$, and $\tilde{d}(x)$ exponentially tend to zero, illustrating that the disturbance observer is exponentially stable such that $\hat{d}_0 \to d_0$, $\hat{d}_L \to d_L$, and $\hat{d}(x) \to d(x)$ for $x \in [0, L]$, $t \to \infty$.

4 Design of distributed disturbance-observer-based controller with output constraints

Considering output constraints in the practical engineering, we discuss a distributed vibration controller which could make the output errors be limited in a prescribed range. The output errors are defined as follows:

$$e_1 = \theta - \theta_d, \tag{18}$$

$$e_2 = w(L), \tag{19}$$

$$e_3(x) = w(x),\ x \in (0, L), \tag{20}$$

where the desired joint angular position θ_d is a constant. The corresponding prescribed upper limits are B_1, B_2, and $B_3(x)$, $x \in (0, L)$. Thus, the control scheme should be developed to meet the conditions $|e_1| < B_1$, $|e_2| < B_2$, and $|e_3(x)| < B_3(x)$, $x \in (0, L)$. Note that the initial values of output error must also satisfy $|e_1(0)| < B_1$, $|e_2(0)| < B_2$, and $|e_3(x, 0)| < B_3(x)$, $x \in (0, L)$. Since we should also concern the unknown disturbances, we will develop a novel disturbance-observer-based controller with output constraints for the infinite dimensional distributed parameter system. Moreover, The controller should make the joint angular position θ regulates to the desired position θ_d, while regulating displacement $w(x)$ and its speed $\dot{w}(x)$ to zero.

With these objects, we first introduce the following auxiliary error functions:

$$S_1 = e_1 + \dot{e}_1, \tag{21}$$

$$S_2 = e_2 + \dot{e}_2, \tag{22}$$

$$S_3(x) = e_3(x) + \dot{e}_3(x),\ x \in (0, L), \tag{23}$$

and subsequently design the distributed disturbance-observer-based vibration controller as follows:

$$\begin{aligned}
u_0 &= -EIw_{xx}(0) - \hat{d}_0 - I_h\dot{e}_1 - e_1 - \mu_1 S_1 - \mu_2 S_1\ln\frac{2B_1^2}{B_1^2 - e_1^2} \\
&\quad - \left(e_1 + \frac{S_1}{2}\frac{e_1^2}{B_1^2 - e_1^2} + I_h S_1\frac{e_1\dot{e}_1}{B_1^2 - e_1^2}\right)\Big/\ln\frac{2B_1^2}{B_1^2 - e_1^2},
\end{aligned} \tag{24}$$

$$\begin{aligned}
u_L &= mL\ddot{\theta} - EIw_{xxx}(L) - \hat{d}_L - m\dot{e}_2 - e_2 - \mu_3 S_2 \\
&\quad - \mu_4 S_2\ln\frac{2B_2^2}{B_2^2 - e_2^2} - \left(e_2 + \frac{S_2}{2}\frac{e_2^2}{B_2^2 - e_2^2} + mS_2\frac{e_2\dot{e}_2}{B_2^2 - e_2^2}\right) \\
&\quad \Big/\ln\frac{2B_2^2}{B_2^2 - e_2^2},
\end{aligned} \tag{25}$$

$$\begin{aligned}
u(x) &= \rho x\ddot{\theta} + EIw_{xxxx}(x) - \hat{d}(x) - \rho\dot{e}_3(x) - e_3(x) \\
&\quad - \mu_5 S_3(x) - \mu_6 S_3(x)\ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \\
&\quad - \Bigg[e_3(x) + \frac{S_3(x)}{2}\frac{e_3^2(x)}{B_3^2(x) - e_3^2(x)} \\
&\quad + \rho S_3(x)\frac{e_3(x)\dot{e}_3(x)}{B_3^2(x) - e_3^2(x)}\Bigg] \\
&\quad \Big/\ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)},
\end{aligned} \tag{26}$$

where μ_i, i=1,2,...,6 are design positive constants. All the true values in the controller can be obtained either by measurement using rotary encoders and piezoelectric sensors, or by first time or spatial derivative calculation using a backwards difference algorithm [26]. Then we give the stability analysis of the distributed controller.

Theorem 2. For the system described by eqs. (1)–(4), on condition that the initial states of the system are bounded and satisfying $|e_1(0)| < B_1$, $|e_2(0)| < B_2$, $|e_3(x,0)| < B_3(x)$, $x \in (0,L)$, with the disturbance observer in eq. (14) and the distributed vibration controller in eqs. (24)–(26), the following properties hold:

(1) The proposed disturbance observer in eq. (14) is exponentially stable in the following sense: $\hat{d}_0 \to d_0$, $\hat{d}_L \to d_L$, and $\hat{d}(x) \to d(x)$ for $x \in (0,L)$, $t \to \infty$.

(2) The output errors e_1, e_2, and $e_3(x)$ will be limited in the prescribed range $|e_1| < B_1$, $|e_2| < B_2$, and $|e_3(x)| < B_3(x)$, for $x \in (0,L), \forall t \in (0,\infty)$.

(3) The closed-loop system is bounded and the system errors are exponentially convergent, i.e., $\theta \to \theta_d$, $\dot\theta \to \dot\theta_d$, $w(x) \to 0$, and $\dot w(x) \to 0$ as $t \to \infty$, $\forall x \in [0,L]$.

Proof. First, a Lyapunov function candidate is designed as follows.

$$V = V_1 + V_2 + V_3 + V_{ob}, \tag{27}$$

where

$$V_1 = \frac{1}{2} I_h S_1^2 \ln\frac{2B_1^2}{B_1^2 - e_1^2} + \frac{1}{2} e_1^2 \ln\frac{2B_1^2}{B_1^2 - e_1^2} + \frac{1}{2} e_1^2, \tag{28}$$

$$V_2 = \frac{1}{2} m S_2^2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} + \frac{1}{2} e_2^2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} + \frac{1}{2} e_2^2, \tag{29}$$

$$\begin{aligned} V_3 = {} & \frac{1}{2}\int_0^L \rho S_3^2(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \frac{1}{2}\int_0^L e_3^2(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x + \frac{1}{2}\int_0^L e_3^2(x)\mathrm{d}x, \end{aligned} \tag{30}$$

and integral-barrier Lyapunov terms are $\ln\frac{2B_1^2}{B_1^2 - e_1^2}$, $\ln\frac{2B_2^2}{B_2^2 - e_2^2}$, and $\int_0^L \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)}\mathrm{d}x$. Differentiating eq. (27) along the closed-loop system with respect to time, and substituting eqs. (1)–(3), we obtain

$$\dot V = \dot V_1 + \dot V_2 + \dot V_3 + \dot V_{ob}, \tag{31}$$

where

$$\begin{aligned} \dot V_1 = {} & (I_h S_1 \dot e_1 + S_1 I_h \ddot\theta) \ln\frac{2B_1^2}{B_1^2 - e_1^2} + I_h S_1^2 \frac{e_1 \dot e_1}{B_1^2 - e_1^2} \\ & + e_1 \dot e_1 \ln\frac{2B_1^2}{B_1^2 - e_1^2} + e_1^2 \frac{e_1 \dot e_1}{B_1^2 - e_1^2} + e_1 \dot e_1 \\ = {} & \left\{ I_h S_1 \dot e_1 + S_1 [u_0 + d_0 + EIw_{xx}(0)] \right\} \ln\frac{2B_1^2}{B_1^2 - e_1^2} \\ & + I_h S_1^2 \frac{e_1 \dot e_1}{B_1^2 - e_1^2} + e_1 \dot e_1 \ln\frac{2B_1^2}{B_1^2 - e_1^2} + e_1^2 \frac{e_1 \dot e_1}{B_1^2 - e_1^2} + e_1 \dot e_1, \end{aligned} \tag{32}$$

$$\begin{aligned} \dot V_2 = {} & (m S_2 \dot e_2 + S_2 m \ddot w(L)) \ln\frac{2B_2^2}{B_2^2 - e_2^2} + m S_2^2 \frac{e_2 \dot e_2}{B_2^2 - e_2^2} \\ & + e_2 \dot e_2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} + e_2^2 \frac{e_2 \dot e_2}{B_2^2 - e_2^2} + e_2 \dot e_2 \\ = {} & \left\{ m S_2 \dot e_2 + S_2 \left[u_L + d_L + EIw_{xxx}(L) - mL\ddot\theta \right] \right\} \\ & \times \ln\frac{2B_2^2}{B_2^2 - e_2^2} + m S_2^2 \frac{e_2 \dot e_2}{B_2^2 - e_2^2} + e_2 \dot e_2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} \\ & + e_2^2 \frac{e_2 \dot e_2}{B_2^2 - e_2^2} + e_2 \dot e_2, \end{aligned} \tag{33}$$

$$\begin{aligned} \dot V_3 = {} & \int_0^L [\rho S_3(x) \dot e_3(x) + S_3(x) \rho \ddot w(x)] \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L \rho S_3^2(x) \frac{e_3(x) \dot e_3(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L e_3(x) \dot e_3(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L e_3^2(x) \frac{e_3(x) \dot e_3(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x + \int_0^L e_3(x) \dot e_3(x) \mathrm{d}x \\ = {} & \int_0^L \left\{ \rho S_3(x) \dot e_3(x) + S_3(x) \left[-\rho x \ddot\theta - EIw_{xxxx}(x) \right.\right. \\ & \left.\left. + u(x) + d(x) \right] \right\} \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L \rho S_3^2(x) \frac{e_3(x) \dot e_3(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L e_3(x) \dot e_3(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L e_3^2(x) \frac{e_3(x) \dot e_3(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & + \int_0^L e_3(x) \dot e_3(x) \mathrm{d}x, \end{aligned} \tag{34}$$

$$\dot V_{ob} = -k_1 \tilde d_0^2 - k_2 \tilde d_L^2 - k_3 \int_0^L \tilde d^2(x) \mathrm{d}x. \tag{35}$$

Substituting the controller eqs. (24)–(26) into eqs. (32)–(34), we arrive at (Supporting Information)

$$\begin{aligned} \dot V \le {} & -\mu_1 S_1^2 \ln\frac{2B_1^2}{B_1^2 - e_1^2} - e_1^2 \ln\frac{2B_1^2}{B_1^2 - e_1^2} - e_1^2 \\ & -\mu_3 S_2^2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} - e_2^2 \ln\frac{2B_2^2}{B_2^2 - e_2^2} - e_2^2 \\ & -\mu_5 \int_0^L S_3^2(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x \\ & - \int_0^L e_3^2(x) \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)} \mathrm{d}x - \int_0^L e_3^2(x) \mathrm{d}x \\ & - \left(k_1 - \frac{1}{\gamma_1} \right) \tilde d_0^2 - \left(k_2 - \frac{1}{\gamma_2} \right) \tilde d_L^2 \\ & - \left(k_3 - \frac{1}{\gamma_3} \right) \int_0^L \tilde d^2(x) \mathrm{d}x \\ \le {} & -\lambda V, \end{aligned} \tag{36}$$

where

$$0 < \lambda_{ob} \le \min\{k_1, k_2, k_3\}, \tag{37}$$

$$0 < \lambda \le \min\left\{\frac{2\mu_1}{I_h}, \frac{2\mu_3}{m}, \frac{2\mu_5}{\rho}, 2, 2k_1 - \frac{2}{\gamma_1}, 2k_2 - \frac{2}{\gamma_2}, 2k_3 - \frac{2}{\gamma_3}\right\}, \tag{38}$$

$$k_1 - \frac{1}{\gamma_1} > 0, \tag{39}$$

$$k_2 - \frac{1}{\gamma_2} > 0, \tag{40}$$

$$k_3 - \frac{1}{\gamma_3} > 0. \tag{41}$$

From eq. (36), it yields

$$V(t) \le \mathrm{e}^{-\lambda t} V(0). \tag{42}$$

According to eq. (42), we conclude that the proposed disturbance observer in eq. (14) and all the signals of the closed-loop system are bounded, and the errors e_1, e_2, $e_3(x)$, $\tilde{d}_0$, $\tilde{d}_L$, and $\tilde{d}(x)$, $x \in (0, L)$ exponentially tend to zero. Thus, the proposed disturbance observer in eq. (14) is exponentially stable in the following sense: $\hat{d}_0 \to d_0$, $\hat{d}_L \to d_L$, and $\hat{d}(x) \to d(x)$ for $x \in (0, L)$, $t \to \infty$, and the system errors are exponentially convergent, i.e., $\theta \to \theta_d$, $\dot{\theta} \to \dot{\theta}_d$, $w(x) \to 0$, $\dot{w}(x) \to 0$ as $t \to \infty$, $\forall x \in [0, L]$. As $V(t)$ is bounded, the integral-barrier Lyapunov function terms $\ln\frac{2B_1^2}{B_1^2 - e_1^2}$, $\ln\frac{2B_2^2}{B_2^2 - e_2^2}$, and $\int_0^L \ln\frac{2B_3^2(x)}{B_3^2(x) - e_3^2(x)}\mathrm{d}x$ are bounded for $\forall t \in (0, \infty)$, then the regulating errors e_1, e_2, and $e_3(x)$ will remain in the prescribed set $|e_1| < B_1$, $|e_2| < B_2$, and $|e_3(x)| < B_3(x)$, for $x \in (0, L)$, $\forall t \in (0, \infty)$, which results in Theorem 2 and achieves our control goal.

5 Simulations

In this section, MATLAB simulation is applied to demonstrate the favorable performance of the design scheme. The system parameters are selected as ρ=0.2 kg m^{-1}, L=1 m, EI=2 N m^2, I_h=0.5 kg m^2, m=0.6 kg. The space step and time step are given by Δx=0.1 m and Δt=1×10^{-4} s, respectively. The initial joint angular position and desired position are chosen as $\theta(0)$=0.2 rad and θ_d=0.5 rad, respectively.

We select the disturbances as d_0=d_L=0.2+0.01sin(0.1πt)+0.02sin(0.2πt), $d(x)$=0.02sin(0.5πxt), and set the initial values of observer as $\hat{d}_0(0) = \hat{d}_L(0) = \hat{d}(x, 0) = 0$. The gains in the observer are given by k_1=k_2=k_3=15.

First, the simulations for illustrating the effectiveness of disturbance observer are shown in Figures 2–5. In Figure 2,

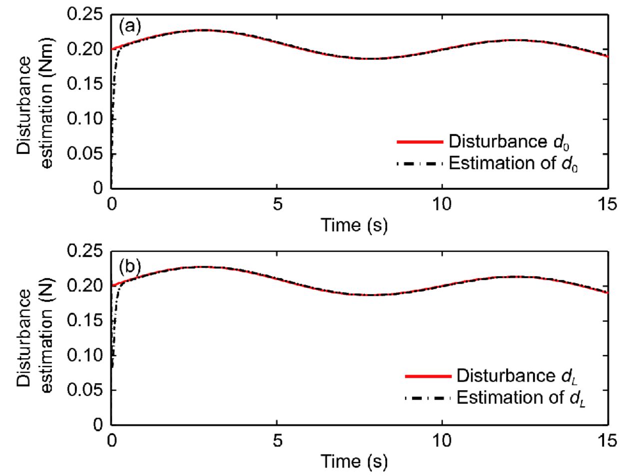

Figure 2 (Color online) Estimation of disturbance. (a) Estimation of d_0; (b) estimation of d_L.

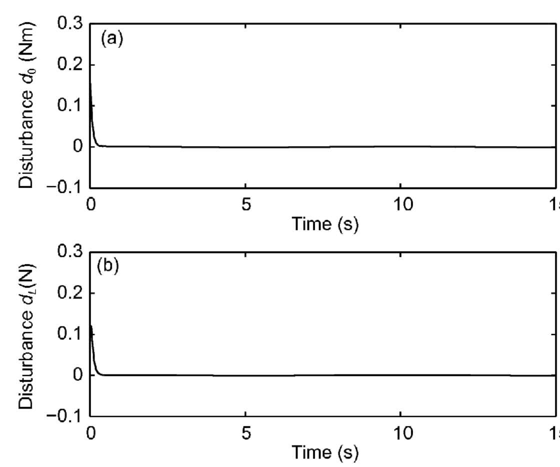

Figure 3 Estimation error. (a) Estimation error of d_0; (b) estimation error of d_L.

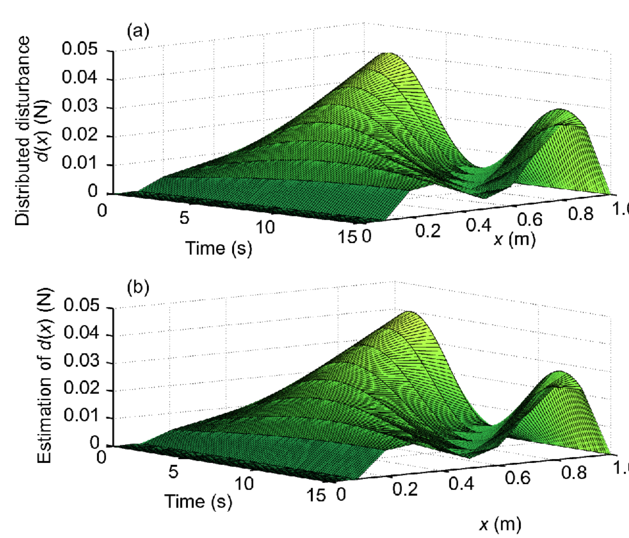

Figure 4 (Color online) Estimation of disturbance. (a) Disturbance $d(x)$; (b) estimation of $d(x)$.

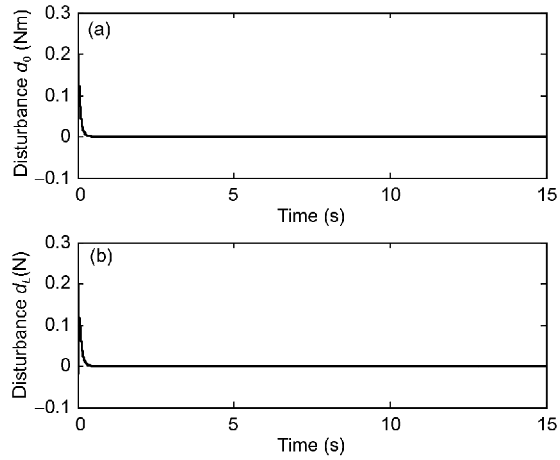

Figure 5 Estimation error of $d(x)$.

the solid and dashed lines show the practical and estimated values of disturbances d_0 and d_L, respectively. Figure 4 displays the practical and estimated value of $d(x)$. In Figures 3 and 5, the errors between the corresponding true and estimated values are described. Figures 2–5 demonstrate that the disturbance observer could approach distributed disturbances rapidly and accurately.

Then, we set the upper bounds of regulating errors as B_1=0.301, B_2=0.02, and $B_3(x) = 0.1[\exp(0.2x) - 1]$, $x \in (0, L)$, satisfying the initial conditions $B_1 > |e_1(0)| = 0.3$, $B_2 > |e_2(0)| = 0$, and $B_3(x) > |e_3(x, 0)| = 0$. In order to satisfy inequalities (29)-(32), $\mu_2 \geq \gamma_1$, $\mu_4 \geq \gamma_2$, and $\mu_6 \geq \gamma_3$, we set the control parameters as μ_1=μ_2=μ_3=μ_4=5, μ_5=μ_6=1.7, and γ_i=1, i=1, 2, 3. The simulation results shown in Figures 6–9 illustrate the effectiveness of proposed control scheme. Figure 6 shows that the control law can make the joint regulate to the desired joint angular position θ_d=0.5 rad and its speed $\dot{\theta}_d = 0$. In Figure 7 , vibrations $w(x)$ and the speed $\dot{w}(x)$ on the beam are effectively and fast suppressed. The control

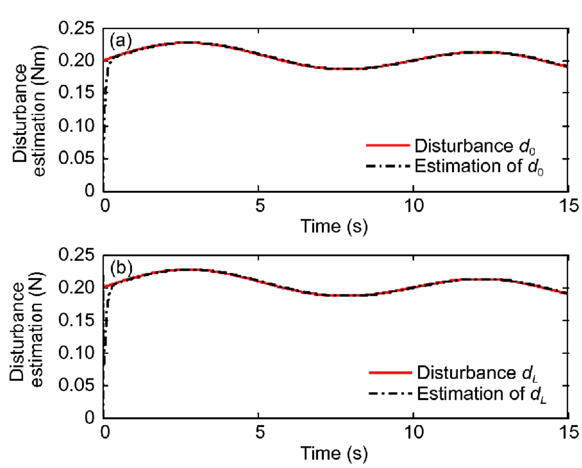

Figure 6 (Color online) Angle and angular speed regulation. (a) Joint angle regulation; (b) joint angular speed regulation.

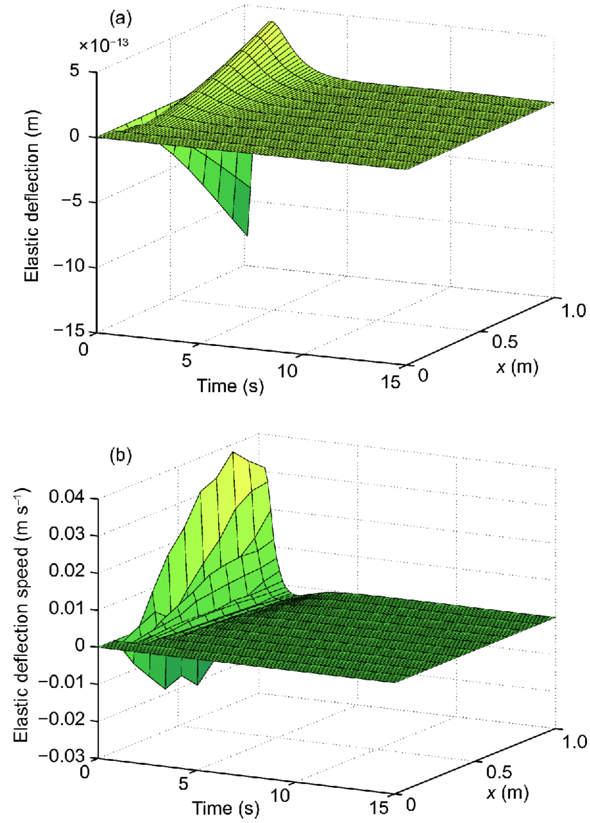

Figure 7 (Color online) Suppression of Elastic deflection and its speed on the whole beam. (a) Elastic deflection $w(x, t)$; (b) elastic deflection speed $w(x, t)$.

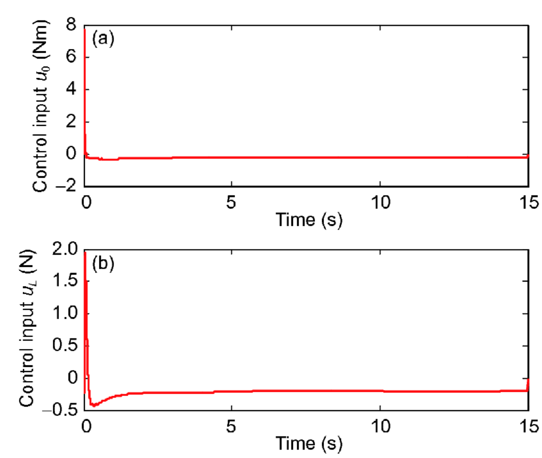

Figure 8 (Color online) Control input u_0 and u_L. (a) Control input u_0; (b) control input u_L.

inputs u_0 and u_L are displayed in Figure 8, and distributed control $u(x)$ in Figure 9. In order to ensure if the regulating errors of output are limited in the upper bounds, we plot the differences between prescribed limits and regulating errors in Figures 10 and 11. We can apparently conclude $|e_1| < B_1$, $|e_2| < B_2$, $|e_3(x)| < B_3(x)$, for $x \in (0, L), \forall t \in (0, \infty)$, which demonstrate the regulating errors are successfully limited in

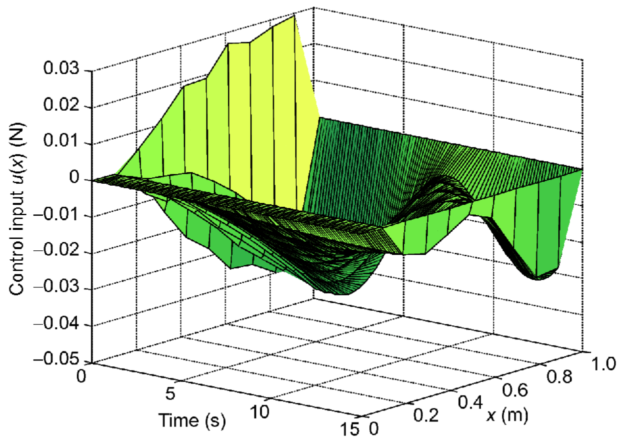

Figure 9 (Color online) Distributed control $u(x)$.

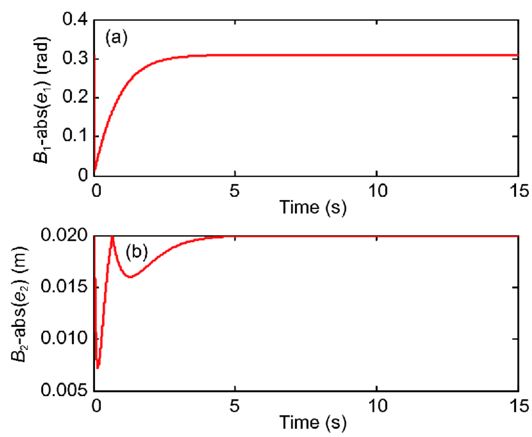

Figure 10 (Color online) Difference between the prescribed limit and regulating error. (a) $B_1-|e_1|$; (b) $B_2-|e_2|$.

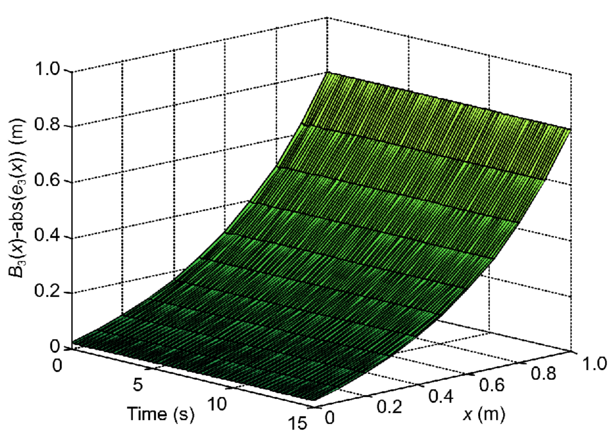

Figure 11 (Color online) Distributed error $B_3(x)-|e_3(x)|$.

our prescribed bounds.

From Figures 6–11, we arrive at the results that the distributed disturbance-observer-based controller can implement the position regulation of joint angle and suppress elastic deflections on the link, and limit the regulating errors in prescribed bounds.

6 Coclusion

In this paper, we develop a novel distributed disturbance-observer-based vibration controller with output constraints for a flexible-link manipulator with unknown spatially distributed disturbances. In the presence of distributed disturbances on the beam and regulating error constrains of output, the distributed controller can effectively regulate angular position and suppress elastic vibration simultaneously, which achieves our control goal and implies the practical significance in the engineering. However, this control scheme cannot deal with the important issue of regulating speed error constraints of output. In the future work, we could consider more advanced technology to cope with this difficulty.

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Supporting Information

The supporting information is available online at tech.scichina.com and www.springerlink.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

1 Zhang L, Liu J. Adaptive boundary control for flexible two-link manipulator based on partial differential equation dynamic model. IET Control Theor Appl, 2013, 7: 43–51
2 Lee H H, Prevost J. A coupled sliding-surface approach for the trajectory control of a flexible-link robot based on a distributed dynamic model. Int J Control, 2005, 78: 629–637
3 Sohn J W, Han Y M, Choi S B, et al. Vibration and position tracking control of a flexible beam using SMA wire actuators. J Vib Control, 2009, 15: 263–281
4 He W, Meng T, Huang D, et al. Adaptive boundary iterative learning control for an Euler-Bernoulli beam system with input constraint. IEEE Trans Neural Netw Learning Syst, 2018, 29: 1539–1549
5 Han Z J, Xu G Q. Exponential stabilisation of a simple tree-shaped network of Timoshenko beams system. Int J Control, 2010, 83: 1485–1503
6 Han Z J, Xu G Q. Exponential stability of Timoshenko beam system with delay terms in boundary feedbacks. ESAIM-COCV, 2011, 17: 552–574
7 He W, He X, Zou M, et al. PDE model-based boundary control design for a flexible robotic manipulator with input backlash. IEEE Trans Contr Syst Technol, 2018, 1–8
8 Ge S S, Zhang S, He W. Vibration control of an Euler-Bernoulli beam under unknown spatiotemporally varying disturbance. Int J Control, 2011, 84: 947–960
9 Khot S, Yelve N P, Tomar R, et al. Active vibration control of cantilever beam by using PID based output feedback controller. J Vib Control, 2012, 18: 366–372
10 Guo B Z, Jin F F. The active disturbance rejection and sliding mode control approach to the stabilization of the Euler-Bernoulli beam equation with boundary input disturbance. Automatica, 2013, 49: 2911–2918
11 He W, Meng T, Zhang S, et al. Trajectory tracking control for the flexible wings of a micro aerial vehicle. IEEE Trans Syst Man Cybern Syst, 2018, 99: 1–11
12 Paranjape A A, Guan J, Chung S J, et al. PDE boundary control for flexible articulated wings on a robotic aircraft. IEEE Trans Robot,

2013, 29: 625–640
13 Jiang T, Liu J, He W. Boundary control for a flexible manipulator with a robust state observer. J Vib Control, 2016, 24: 260–271
14 Zhang S, He W, Huang D. Active vibration control for a flexible string system with input backlash. IET Control Theor Appl, 2016, 10: 800–805
15 Liu Z, Liu J, He W. Modeling and vibration control of a flexible aerial refueling hose with variable lengths and input constraint. Automatica, 2017, 77: 302–310
16 He W, Ouyang Y, Hong J. Vibration control of a flexible robotic manipulator in the presence of input deadzone. IEEE Trans Ind Inf, 2017, 13: 48–59
17 He W, Zhang S. Control design for nonlinear flexible wings of a robotic aircraft. IEEE Trans Contr Syst Technol, 2017, 25: 351–357
18 Park S K, Lee S H. Disturbance observer based robust control for industrial robots with flexible joints. In: 2007 International Conference on Control, Automation and Systems. Seoul, 2007. 584–589
19 Nikoobin A, Haghighi R. Lyapunov-based nonlinear disturbance observer for serial n-link robot manipulators. J Intell Robot Syst, 2009, 55: 135–153
20 Yun J N, Su J B. Design of a disturbance observer for a two-link manipulator with flexible joints. IEEE Trans Contr Syst Technol, 2014, 22: 809–815
21 Zheng G, Efimov D, Bejarano F J, et al. Interval observer for a class of uncertain nonlinear singular systems. Automatica, 2016, 71: 159–168
22 Chen M, Ge S S. Direct adaptive neural control for a class of uncertain nonaffine nonlinear systems based on disturbance observer. IEEE Trans Cybern, 2013, 43: 1213–1225
23 Mohammadi A, Tavakoli M, Marquez H J, et al. Nonlinear disturbance observer design for robotic manipulators. Control Eng Practice, 2013, 21: 253–267
24 Tee K P, Ren B, Ge S S. Control of nonlinear systems with time-varying output constraints. Automatica, 2011, 47: 2511–2516
25 He W, Ge S S. Vibration control of a flexible beam with output constraint. IEEE Trans Ind Electron, 2015, 62: 5023–5030
26 Ge S S, Lee T H, Gong J Q. A robust distributed controller of a single-link SCARA/Cartesian smart materials robot. Mechatronics, 1999, 9: 65–93
27 D'Andrea R, Dullerud G E. Distributed control design for spatially interconnected systems. IEEE Trans Automat Contr, 2003, 48: 1478–1495
28 Wu F. Distributed control for interconnected linear parameter-dependent systems. IEE Proc-Control Theor Appl, 2003, 150: 518–527
29 Yang H, Liu J, Lan X. Observer design for a flexible-link manipulator with PDE model. J Sound Vib, 2015, 341: 237–245
30 Yang H, Liu J. Distributed piezoelectric vibration control for a flexible-link manipulator based on an observer in the form of partial differential equations. J Sound Vib, 2016, 363: 77–96