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Energy dependence on modes of electric activities of neuron driven by different external mixed signals under electromagnetic induction

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Energy supply and release play an important role in individual neuron and neural network. In this paper, the electrical activities and Hamilton energy of neuron are investigated when external mixed signals (i.e., the periodic stimulus current and the periodic electromagnetic field) are imposed on the neuron under the electromagnetic induction. As a result, the Hamilton energy is much dependent on the mode transition, the multiple electric activity modes and the numerical analysis of Hamilton energy are more complicated under various parameters. When the periodic high-low frequency electromagnetic radiation is imposed in neuron, it is found that the electrical activities are more complex, and the changing of energy is obvious. In addition, the response of electrical activity and Hamilton energy is much dependent on the changing of amplitude A, B when the external high-low frequency signal is imposed on the neuron, meanwhile, the energy of bursting state is lower than the one of spiking state. It can be used for investigation about the energy coding in the neuron even the neuron networks.

Hamilton energy, electromagnetic induction, electric activity, multiple growing modes

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1 Introduction

The neuron is the basic unit of nervous system and its the one directly associated with diseases [1,2], such as senile dementia, epilepsy, etc. Hence the studies on neuron network and dynamic analysis are done by many researchers using neuron models [3–6] so as to observe electrophysiological features and information encoding of neurons in reference to previously established work. The neuron network has been studied in refs. [7–10] by the methods of electrical coupling, phase synchronization, coherence resonance, spiral wave, and the like. In addition, the external signals [11–13] are always considered in an individual neuron or the neuron network to investigate the mode transformation which approximates to the real neuron. The refs. [14,15] found that the neuron model under various parameters have different responses by applying the periodic high-low frequency current and the periodic high-low frequency electromagnetic radiation under the Gaussian white noise.

The state of neuron electrical activities should be introduced, in order to illustrate the richness and complexity of spiking behavior [16,17] of an individual neuron in response to simple pluses of external stimulus signal. The different states [18] are classified as phasic bursting, tonic bursting, mixed model (bursting then spiking), subthreshold oscillations, bistability of resting spiking state and so on. This rich dynamical behavior of a single neuron have been proven in many biological experiments [19–22] and theoretical neuronal models. The theoretical neuronal models are applied widely in the study of diseases in real life. For example, epilepsy is a chronic brain dysfunction disease because of sudden abnormal discharge of neurons in the brain, the stu-

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dies of ref. [23] suggested that calcium wave propagation in astrocytes determines the propagation of seizure-like discharges in the connected neuron. The ref. [24] investigated effect of the coordinated rest stimulations on controlling absence seizure, and found coordinated rest stimulation is effective on controlling absence seizures in proper ranges of stimulation parameters. In addition, the seizures dynamic [25] has been built in a neural field model of cortical-thalamic circuitry, it is worthy to point out seizure duration is significantly affected by a time-varying delay. Based on recent experiments on gamma-aminobutyric acid (GABA) astrocytes, a modified GABAergic astrocyte model [26] has been established to observe calcium dynamics and involvement in seizure activity. Meanwhile, ref. [27] of stochastic fluctuations of permittivity coupling regulate seizure dynamics in partial epilepsy observed that with the help of permittivity noise our stochastic Epileptor model can trigger the seizure dynamics.

The investigation about energy consumption in the nervous system has been conducted in experiments rather than quantitative theoretical analyses due to the complexity of the brain. The mode transformation of electrical activities and emergence of action potential in neuron are associated with the energy encoding and energy metabolism [28]. There have some difficulties to accurately detect the energy supply and consumption, thus the Hamilton energy [29] is used to estimate the dependence of state on energy in neuron and oscillator models by using Helmholtz theorem. Energy is consumed diffusely in the metabolic process of the biological system, and the electrical activities can be maintained in the neuron, therefore, a type of Hamilton statistical function [30] is defined to describe the energy degree in some oscillator by using Helmholtz's theorem in Hindmarsh-Rose (HR) neuron model. Helmholtz's theorem [31] guarantees that the vector field f is uniquely determined by the divergence, curl and boundary conditions, therefore we can decompose f into the sum of one divergence-free vector f_c that accounts for the whole rotational tensor of f plus one gradient vector field f_d that carries its whole divergence ($f(x)=f_c(x)+f_d(x)$).

In this paper, the electrical activities and the Hamilton energy in the neuron are investigated by considering the periodic high-low frequency stimulus current or the periodic high-low frequency electromagnetic field. According to the Helmholtz's theorem, the Hamilton energy function H and its change with time are approached under different external stimuli. Under the different parameters (e.g., N, ω, B, A), the state transformation of the electric activities and the energy are investigated simultaneously.

2 Models and methods

It is well known that the fluctuation in membrane potential could induce the change of electromagnetic field or the distribution of ion concentration in cells. Therefore, when the electromagnetic induction is considered in the neuron, the improved HR neuron model [32] is established according to the Maxwell electromagnetic induction theorem [33]. Here the improved HR neuron model is driven by both external current and external electromagnetic radiation, which are considered as a periodic high-low frequency signal, respectively.

2.1 External stimulation current with periodic high-low frequency signal

The dynamic equations of the improved HR neuron model driven by external stimulation current are described as follow:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = y - ax^3 + bx^2 - z - k_1\rho(\varphi)x + I_{\mathrm{ext}}, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = c - dx^2 - y, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = r[s(x+1.56) - z], \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}t} = kx - k_2\varphi, \end{cases} \tag{1}$$

where x, y, z, φ describe the membrane potential, the slow current associated with recovery variable, the adaption current, and the magnetic flux across the membrane of neuron, respectively. kx and $k_2\varphi$ describe the membrane potential-induced changes on magnet flux and the leakage of magnet flux, respectively. The term $k_1\rho(\varphi)x$ is the feedback current on the membrane potential induced by electromagnetic induction, where k_1 is the feedback gain. The physical unit is verified as follows:

$$i' = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}t} = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi}\frac{\mathrm{d}\kappa}{\mathrm{d}t} = \rho(\varphi)V = k_1\rho(\varphi)x, \tag{2}$$

where V is the induced voltage and has the same physical units as variable x, and the dependence of electric charge on magnet flux defined by the memory-conductance is

$$\rho(\varphi) = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}t} = \alpha + 3\beta\varphi^2, \tag{3}$$

where α, β are parameters, q is the charge across the memistor. The $\rho(\varphi)$ is the memory conductance of a magnetic flux-controlled memistor [34,35], which is used for describing the coupling between magnetic flux and membrane potential of neurons.

$$I_{\mathrm{ext}} = I + A\cos(\omega\ t) + B\cos(N\omega\ t), \tag{4}$$

where the constant current I and periodic forcing $A\cos(\omega t)+B\cos(N\omega t)$ with frequency diversity are imposed on the external stimulation synchronously and simultaneously.

The information transition and energy coding [36–38] play a significant role in the signal processing of the neuron electrical activities. According to the Helmholtz theorem, any space vector field can be decomposed into the superposition of vortex field and gradient field. Therefore, Hamilton energy can be estimated by Helmholtz theorem [39], and the general vector function of position can be written as a gradient plus a curl as follows:

$$F(\mathbf{r})=F_d(\mathbf{r})+F_c(\mathbf{r})=-\nabla U+\nabla\times\mathbf{W}, \tag{5}$$

where W is a vector function and U is a scalar function, the minus sign in front of ∇U is a convention. So the dynamical system described by eq. (1) can be written by

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\phi}\end{pmatrix}=f_c(x,y,z,\phi)+f_d(x,y,z,\phi), \tag{6}$$

where f_c and f_d can be read as follows respectively:

$$f_c(x,y,z,\varphi)=J(x,y,z,\varphi)\cdot\nabla H =\begin{pmatrix}y-z+I_{\text{ext}}-\varphi\\ c-dx^2\\ rs(x+1.56)\\ kx\end{pmatrix}, \tag{7}$$

$$f_d(x,y,z,\varphi)=R(x,y,z,\varphi)\cdot\nabla H =\begin{pmatrix}-ax^3+bx^2-k_1\rho(\varphi)x+\varphi\\ -y\\ -rz\\ -k_2\varphi\end{pmatrix}, \tag{8}$$

where H is the Hamilton energy function, and the change of energy results from the work done in the force field, H could be defined as follows:

$$\begin{cases}\nabla H^{\mathrm{T}}f_c(x,y,z,\varphi)=0,\\ \nabla H^{\mathrm{T}}f_d(x,y,z,\varphi)=\dfrac{\mathrm{d}H}{\mathrm{d}t}=\dot{H}.\end{cases} \tag{9}$$

Therefore, the Hamilton energy can be estimated according to the criterion in eq. (9), and we can get

$$(y-z+I_{\text{ext}}-\varphi)\frac{\partial H}{\partial x}+(c-dx^2)\frac{\partial H}{\partial y} +rs(x+1.56)\frac{\partial H}{\partial z}+kx\frac{\partial H}{\partial \varphi}=0. \tag{10}$$

According to eq. (10), the Hamilton energy function H is calculated by

$$H=\frac{2}{3}dx^3-2cx+rs(x+1.56)^2 +(y-z+I_{\text{ext}}-\varphi)^2+kx^2. \tag{11}$$

Furthermore, the change of Hamilton energy versus time is approached by

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t}&=2dx^2\dot{x}-2c\dot{x}+2rs(x+1.56)\dot{x}\\ &\quad+2(y-z+I_{\text{ext}}-\phi)\times(\dot{y}-\dot{z}-\dot{\phi})+2kx\dot{x}\\ &=2(y-z+I_{\text{ext}}-\phi)(-y)\\ &\quad-2(y-z+I_{\text{ext}}-\phi)(-rz)\\ &\quad-2(y-z+I_{\text{ext}}-\phi)(-k_2\phi)\\ &\quad+[2dx^2-2c+2rs(x+1.56)+2kx]\\ &\quad\times[-ax^3+bx^2-k_1\rho(\varphi)x+\varphi]\\ &=\nabla H^{\mathrm{T}}f_d.\end{aligned} \tag{12}$$

In addition, the external stimulus of constant current and high-low frequency current $I_{\text{ext}}=I+A\cos(\omega t)+B\cos(N\omega t)$ are imposed into the eqs. (11) and (12), the results are:

$$\begin{aligned}H&=\frac{2}{3}dx^3-2cx+rs(x+1.56)^2\\ &\quad+[y-z+I+A\cos(\omega t)+B\cos(N\omega t)-\phi]^2\\ &\quad+kx^2,\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t}&=(-2y+2rz+2k_2\varphi)\\ &\quad\times[y-z+I+A\cos(\omega t)+B\cos(N\omega t)-\varphi]\\ &\quad+[2dx^2-2c+2rs(x+1.56)+2kx]\\ &\quad\times[-ax^3+bx^2-k_1\rho(\varphi)x+\varphi].\end{aligned} \tag{14}$$

2.2 External electromagnetic field with periodic high-low frequency signal

The dynamic equations of the improved HR neuron model driven by the external electromagnetic radiation are described as follows:

$$\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t}=y-ax^3+bx^2-z-k_1\rho(\phi)x+I,\\ \dfrac{\mathrm{d}y}{\mathrm{d}t}=c-dx^2-y,\\ \dfrac{\mathrm{d}z}{\mathrm{d}t}=r[s(x+1.56)-z],\\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}t}=kx-k_2\varphi+\varphi_{\text{ext}},\end{cases} \tag{15}$$

$$\phi_{\text{ext}}=A\cos(\omega t)+B\cos(N\omega t), \tag{16}$$

where I is the constant current, φ_{ext} is the external electromagnetic radiation. Hamilton energy can be estimated by Helmholtz theorem, and in this case of the improved HR neuron model, the two sub-vector fields (i.e., f_c and f_d) according to the criterion in eq. (6) are described as follows:

$$f_c(x,y,z,\varphi)=J(x,y,z,\varphi)\cdot\nabla H =\begin{pmatrix}y-z+I-\varphi\\ c-dx^2\\ rs(x+1.56)\\ kx+\varphi_{\text{ext}}\end{pmatrix}, \tag{17}$$

$$f_d(x,y,z,\varphi) = R(x,y,z,\varphi)\cdot\nabla H = \begin{pmatrix} -ax^3+bx^2-k_1\rho(\varphi)x+\varphi \\ -y \\ -rz \\ -k_2\varphi \end{pmatrix}. \tag{18}$$

Therefore, the Hamilton energy can be estimated according to the criterion in eq. (9), and we can get

$$(y-z+I-\varphi)\frac{\partial H}{\partial x}+(c-dx^2)\frac{\partial H}{\partial y} +rs(x+1.56)\frac{\partial H}{\partial z}+(kx+\varphi_{\text{ext}})\frac{\partial H}{\partial \varphi}=0. \tag{19}$$

An appropriate solution can be approached in eq. (19), and the Hamilton energy is calculated as follows:

$$H=\frac{2}{3}dx^3-2cx+rs(x+1.56)^2 +(y-z+I-\varphi)^2+kx^2+2\varphi_{\text{ext}}x. \tag{20}$$

The change of Hamilton energy versus time is approached by

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t} &= 2dx^2\dot{x}-2c\dot{x}+2rs(x+1.56)\dot{x} \\ &\quad +2(y-z+I-\phi)\times(\dot{y}-\dot{z}-\dot{\varphi}) \\ &\quad +2kx\dot{x}+2\varphi_{\text{ext}}\dot{x} \\ &= 2(y-z+I-\varphi)(-y) \\ &\quad -2(y-z+I-\varphi)(-rz) \\ &\quad -2(y-z+I-\varphi)(-k_2\varphi) \\ &\quad +[2dx^2-2c+2rs(x+1.56)+2kx+2\varphi_{\text{ext}}] \\ &\quad \times[-ax^3+bx^2-k_1\rho(\varphi)x+\varphi] \\ &= \nabla H^{\mathrm{T}} f_d. \end{aligned} \tag{21}$$

Considering the external electromagnetic radiation of high-low frequency signal $\varphi_{\text{ext}=}A\cos(\omega t)+B\cos(N\omega t)$, then eqs. (20) and (21) become

$$H=\frac{2}{3}dx^3-2cx+rs(x+1.56)^2+(y-z+I-\phi)^2 +kx^2+2[A\cos(\omega\, t)+B\cos(N\omega\, t)]x, \tag{22}$$

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t} &= (-2y+2rz+2k_2\varphi)(y-z+I-\varphi) \\ &\quad +[2dx^2-2c+2rs(x+1.56) \\ &\quad +2kx+2A\cos(\omega\, t)+2B\cos(N\omega\, t)] \\ &\quad \times(-ax^3+bx^2-k_1\rho(\varphi)x+\varphi). \end{aligned} \tag{23}$$

In the following section, the energy transformation will be calculated under different parameters to investigate the response and mode transition of neuronal activities.

3 Numerical results and discussion

In the numerical studies, the parameters are selected as the same values in most of the previous works, such as a=1, b=3, c=1, d=5, s=4, r=0.006, k_1=0.4, α=0.4, β=0.02, k_2=0.5, k=0.9, the fourth-order Runge-Kutta algorithm is used, the time step is set as 0.01, and the initial values for (x, y, z, φ) are set as (0.2, 0.3, 0.1, 0). The transient period is 5000 time units, the external mixed signal is added after the 1000 times units (marking with a black arrow in figure), the transition of the electrical activities and energy are investigated when the external mixed signal is imposed in the neuron at t=1000 time units.

3.1 Numerical results of external stimulation current with periodic high-low frequency signal

In this section, we will investigate the electrical activities and Hamilton energy of the neuron by adding the periodic forcing in the external stimulation current in the model 2.1.

3.1.1 The effects on electrical activities and Hamilton energy with changing the constant current

From the physical view, the occurrence of action potential and transition of the neuronal electrical activities depend on the energy supply and release, therefore, it's significant to detect the changing of Hamilton energy under the different external mixed signals (i.e., constant I and high-low frequency current $A\cos(\omega t)+B\cos(N\omega t)$) in HR neuron model. The Hamilton energy is dependent on all the variables (i.e., x, I, k) in eqs. (15) and (22). With the changing of external constant current I and the variables k, extensive numerical results of the distribution of the different states are shown in Figure 1.

With the increasing of external constant current I and the parameter k, as can be seen in Figure 1, the electrical state is much affected by this parameters. If the variable k is fixed, for small values I, the electrical activity of the neuron is found in a quiescent state, after increasing I beyond some threshold value, the mode in electrical activities of the neuron undergoes a transition from the quiescent state to the spiking one. However, if we further increase I, the electrical activity of the neuron will become the bursting state. That is the mode in electrical activities of neuron undergoes a succession of two transitions (quiescent state→spiking state→bursting state).

Similarly, if I is fixed, with the changing of parameter k, there has a succession in the electrical activity. For small value I, the electrical activity state undergoes a transition from the spiking state to quiescent state or from the bursting state to spiking state with the increasing of k; however, for big value I, the mode retains the bursting state in neuronic electrical activities. In addition, if the constant current I is greater than 4.6, the electrical activities become the spiking state. According to the previous work and the results in Figure 1, we picked a value 0.9 for the parameter k that produced the best, which is adopted in the following in-

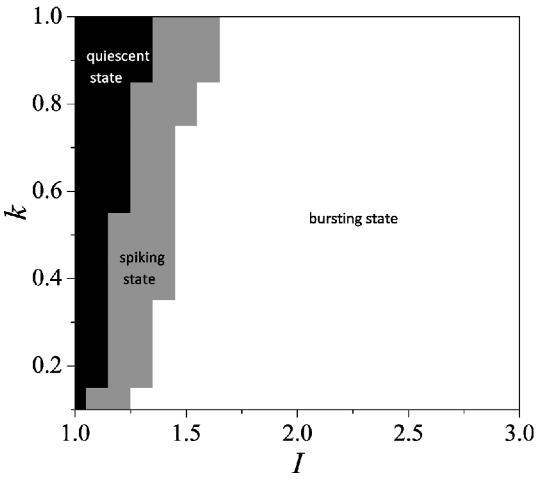

Figure 1 Distribution of the different states in the two-parameter phase space *k*-*I* under different external stimulus currents $I_{\text{ext}}=I+A\cos(\omega t)+B\cos(N\omega t)$. A=0.15, B=0.1, ω=0.01, N=10.

vestigation. Furthermore, the evolution of action potential and energy function with time are calculated under the different external constant currents I=1.4, 1.9 in Figure 2.

Figure 2 shows that the energy function is association with the discharge mode under the external stimulation current, it is obviously that energy function fluctuates with the lower amplitude under the bursting state. The external high-low frequency periodic signals are added at t=1000 time units, it is found that the electrical activities are suppressed into quiescent state when I=1.4, the potential mechanism could be that the energy which is stored at the electromagnetic field is used for resisting the changing of external stimulation current. Furthermore, it shows that the energy will become smaller when the electrical activities change from the bursting state into the complex chaotic state at t=1000 time units, so the Hamilton energy of the regular electrical activities is smaller than the chaotic one. The potential mechanism might be that the transformation of the neuronal electrical activities will release the more energy.

According to the count of average Hamilton energy, when I=1.4, the electrical activity is in spiking state, the average Hamilton energy is 28.660 before the 1000 times units, and the Hamilton energy is 24.667 when the time is ranging from 1000 to 5000 time units. Above results show that energy is lower under bursting states while spiking states make neuron hold higher energy. When I=1.9, the electrical activity is in bursting state, the Hamilton energy is 23.059 before the 1000 times units. The external current I=1.9 is bigger than I=1.4, however, the average Hamilton energy under I=1.9 is lower than the one under I=1.4. In addition, when I=1.9, the Hamilton energy is 27.394 when time is ranging from 1000 to 5000 time units, it is still lower than the average Hamilton energy (the value is 28.660) under the spiking state. These results also show that the neuron under spiking state holds higher energy.

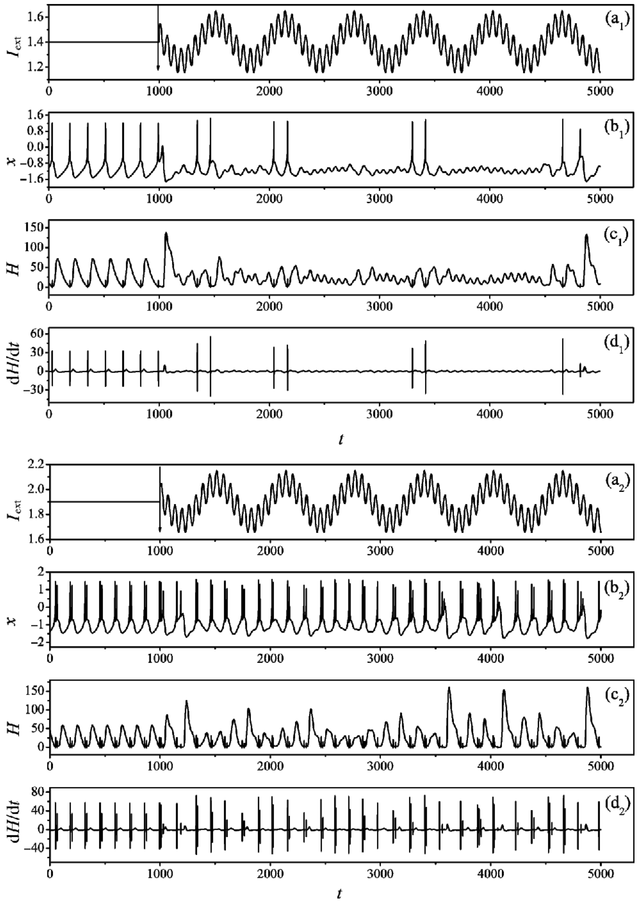

Figure 2 Evolution of action potential and energy function with time are calculated by changing the external mixed signal at A=0.15, B=0.1, ω=0.01, N=10. (a_1)–(d_1) I=1.4; (a_2)–(d_2) I=1.9. The period current is imposed on the neuron from t=1000 time units.

3.1.2 The effects on electrical activities and Hamilton energy with the different parameters

The diversity of the external mixed current also can alter the model of the neuronal electrical activities, which can arose the alteration of the Hamilton energy, hence the different parameters (i.e., A, B, N, ω) should be considered in the improved HR model.

Power spectrum indicates the change in signal power with frequency, that is, the distribution of signal power in frequency domain. The spectra of the time series are obtained by the last Fourier transformation. Last Fourier analysis converts a signal from its original domain (sampled time series) to the power spectrum in the frequency domain, and the data are taken in the time period t=5000 time units, each plot is provided by the average of 500 runs. Power spectrum is often used in the representation and analysis of power signals, the definition of power spectrum is described as follows:

$$S(f)=\lim_{T\to\infty}\frac{|F_T(f)|^2}{2\pi T}, \tag{24}$$

where $F_T(f)$=F[$f_T(t)$], F[.] express the last Fourier transformation, $f_T(t)$ is power signals.

In order to observe the change of the electrical activities under different external stimulus, power spectrum of the time series for four different parameters (N=0.1, 1.0, 10, 100) are depicted in Figure 3. From Figure 3, it is found that when N is 0.1, there only has a distinct peak, the spectrum peak is very high and sharp, when the parameter N is 1.0, there have four obvious spectrum peaks, it indicates the electrical activities become more chaotic than before. When N is 10, 100, the power spectrum become only a distinct spectrum peak, thus this state is similar to the state when N=0.1. Meanwhile, the frequency of the spectrum peak is not considerably shifted.

So as to investigate the changing of the energy in neuron, evolution of action potential and energy function with time are calculated by changing the external mixed signal in Figure 4, it shows that the different ratios of the high-low frequency current could make neuron present different electrical activities modes. Comparing with the state under the different parameter, the Hamilton energy changes with the electrical activity varies. The energy is dependent on the electrical activities model rather than the external stimulus signal, and the more complex electrical activities are, the higher the energy when the different ratios of the high-low frequency stimulus current are imposed at t=1000 time units.

Furthermore, the different anger frequencies will affect the model selection of electrical activities, thus the different anger frequencies value ω should be considered in the improved HR neuron model. The evolution of action potential and energy function with time are calculated by changing the parameter ω under the external mixed signal in Figure 5.

As it can be seen in Figure 5 the mode of the electrical activities can be affected by the anger frequency ω, and multiple modes of bursting state and spiking state can be induced under the external stimulus current. It is found that the discharge with the high rhythm makes each action potential hold low energy unit, the underlying potential might be that the discharge fluctuation with the high rhythm will release more energy when the electrical activities transmit the signal.

In addition, the influence of the high-low frequency current amplitude is calculated under the external stimulus current in Figures 6 and 7. With the increasing of the external stimulus current amplitude, the model of electrical activities can convert in spiking state and bursting state mutually, and the Hamilton energy changed with the different modes of the electrical activities. The energy and electrical activities suffer a rapid shift when the external high-low frequency current is added at t=1000 time units.

It is obviously in Figure 6 that the shift between the bursting state and spiking state is obvious and the Hamilton energy has higher amplitude with the increasing of the HF current amplitude B. The results in Figure 7 confirm that the electrical activities state changes from the mixed state (bursting state and spiking state) into the period double bursting state, meanwhile, the Hamilton energy has a period transformation and a lower value with the increasing of the LF current amplitude A. The potential mechanism might be that the generation of action potential is associated with energy release. The superimposition of different external

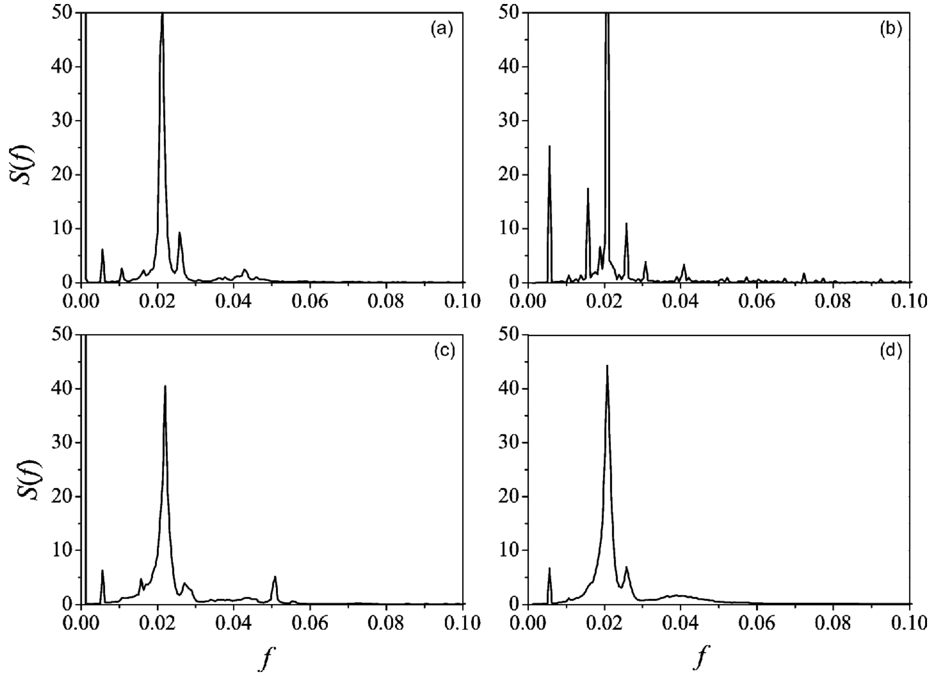

Figure 3 Power spectrum of the time series for four different parameters. (a) N=0.1; (b) N=1.0; (c) N=10; (d) N=100. The external stimulus currents is $I_{ext}=I+A\cos(\omega t)+B\cos(N\omega t)$, A=0.15, B=0.1, ω=0.01, I=1.6.

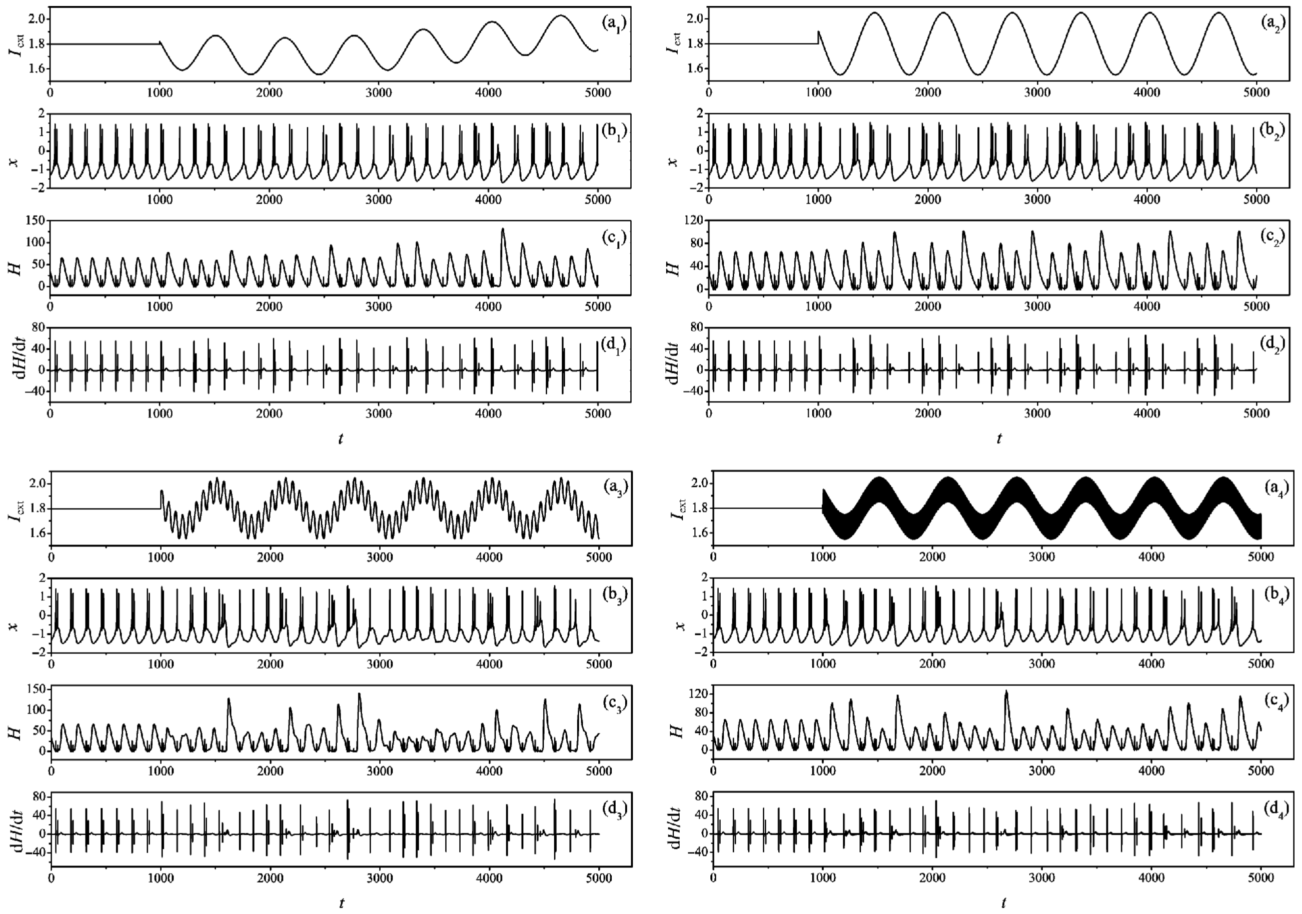

Figure 4 Evolution of action potential and energy function with time are calculated by changing the external mixed signal at A=0.15, B=0.1, ω=0.01, I=1.8. (a$_1$)–(d$_1$) N=0.1; (a$_2$)–(d$_2$) N=1; (a$_3$)–(d$_3$) N=10; (a$_4$)–(d$_4$) N=100.

period high-low frequencies current are imposed according to the multiple channels, thus the electrical activities could be adjusted synchronously.

3.2 Numerical results of external electromagnetic field with periodic high-low frequency signal

In the last section, the electrical activities and Hamilton energy of the neuron are discussed by adding the external stimulus current. In the following section, the electrical activities and Hamilton energy of the neuron will be investigated by adding the periodic forcing in the electromagnetic field in the model 2.2. Only are the external stimulus current imposed in the electrical activities of the neuron before t=1000 time units, the external period high-low frequency electromagnetic radiation is imposed in the neuron at t=1000 time units in circuit simulation.

3.2.1 The effects on electrical activities and Hamilton energy with changing the constant current

The distribution of quiescent state, spiking state and bursting state in the two-parameter phase space k-I under the different periodic electromagnetic radiation $\varphi_{\text{ext}}=A\cos(\omega t)+B\cos(N\omega t)$ are calculated in Figure 8. According to Figure 8, with the increasing of external constant current I and the parameter k, it is found that the model shift of the neuronal electrical activities are distinct by adding the periodic forcing in the electromagnetic field.

If the variable k is fixed, for small values I, the electrical activity of the neuron is found in a quiescent state, after increasing I beyond some threshold value, the mode in electrical activities would convert from the quiescent state to spiking state. If we further increase I, the electrical activity of the neuron will become the bursting state. Similarly, if I is fixed, with the changing of parameter k, the electrical activity undergoes a succession of two transitions (quiescent state→spiking state→bursting state). However, for big value I, electrical activity is much depended on I, the mode retains the bursting state in neuronic electrical activities. In addition, if the constant current I is greater than 4.6, the electrical activities become the spiking state. There we also picked a value 0.9 for the parameter k that produced the best, which is adopted in the following investigation.

Meanwhile, we select the evolution of action potential and energy function with time to investigate the mode transition with the changing of the periodic electromagnetic radiation

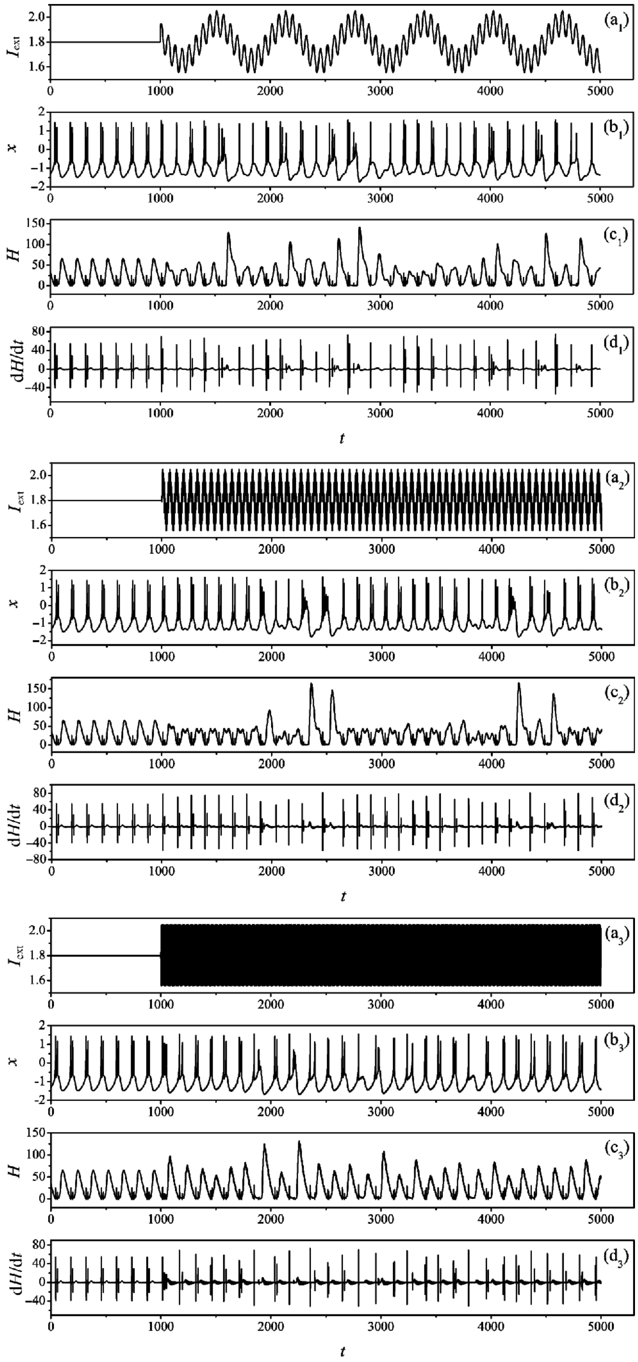

Figure 5 Evolution of action potential and energy function with time are calculated by changing the external mixed signal at A=0.15, B=0.1, N=10, I=1.8. (a$_1$)–(d$_1$) ω=0.01; (a$_2$)–(d$_2$) ω=0.1; (a$_3$)–(d$_3$) ω=1.0.

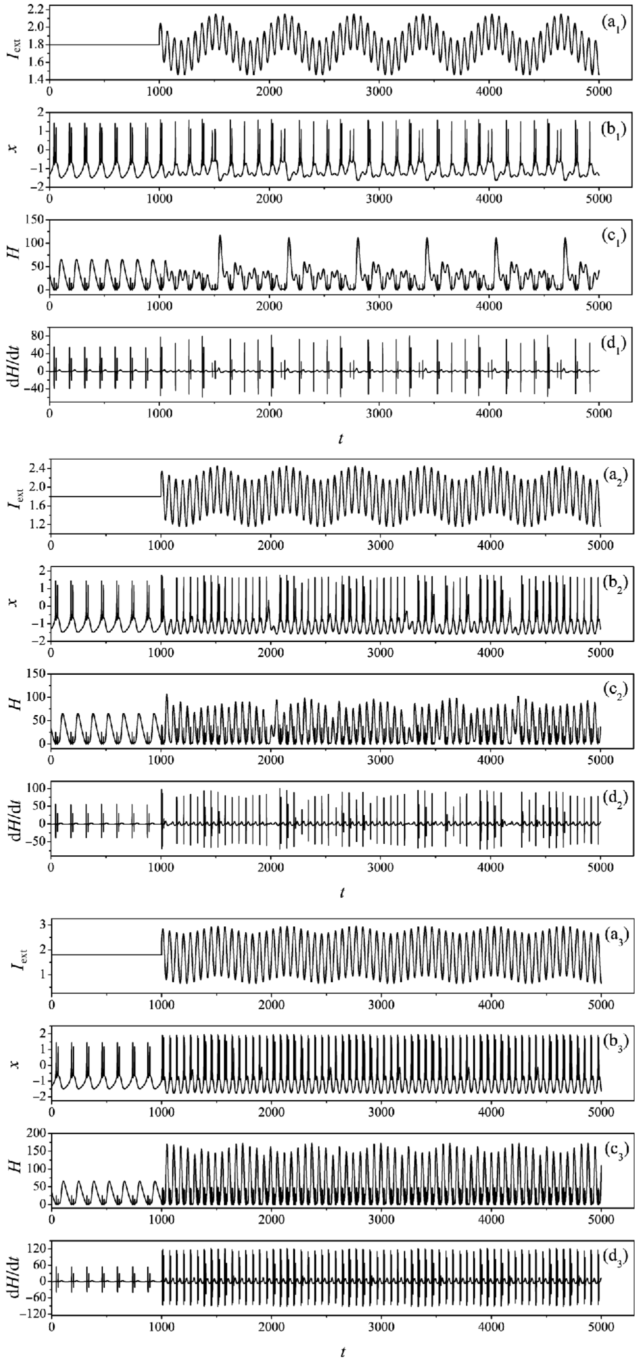

Figure 6 Evolution of action potential and energy function with time are calculated by changing the external mixed signal at A=0.15, N=10, ω=0.01, I=1.8. (a$_1$)–(d$_1$) B=0.2; (a$_2$)–(d$_2$) B=0.5; (a$_3$)–(d$_3$) B=1.0.

when the constant current I=1.4, 1.8 in Figure 9.

The results in Figure 9 confirm that the electrical activity of the neuron is in the spiking state without the periodic electromagnetic radiation at fist, then it is changing between the quiescent state and spiking state at I=1.4 when the periodic electromagnetic radiation is imposed at t=1000 time units. When the constant current I=1.8, the electrical activity is in bursting state in all of the time. It is distinct that the Hamilton energy of the period regular bursting state is lower than the one of spiking state. According to the calculation, when the constant I=1.4, the average Hamilton energy is 28.660 before 1000 time units, and the average Hamilton energy is 27.748 when the time is ranging from 1000 to 5000 in Figure 9(c$_1$). Simultaneously, when I=1.8, the average Hamilton energy is 26.381 before 1000 time units, and the average Hamilton energy is 26.195 when the time is ranging from 1000 to 5000 in Figure 9(c$_2$). The potential mechanism might be that the periodic electromagnetic radiation make

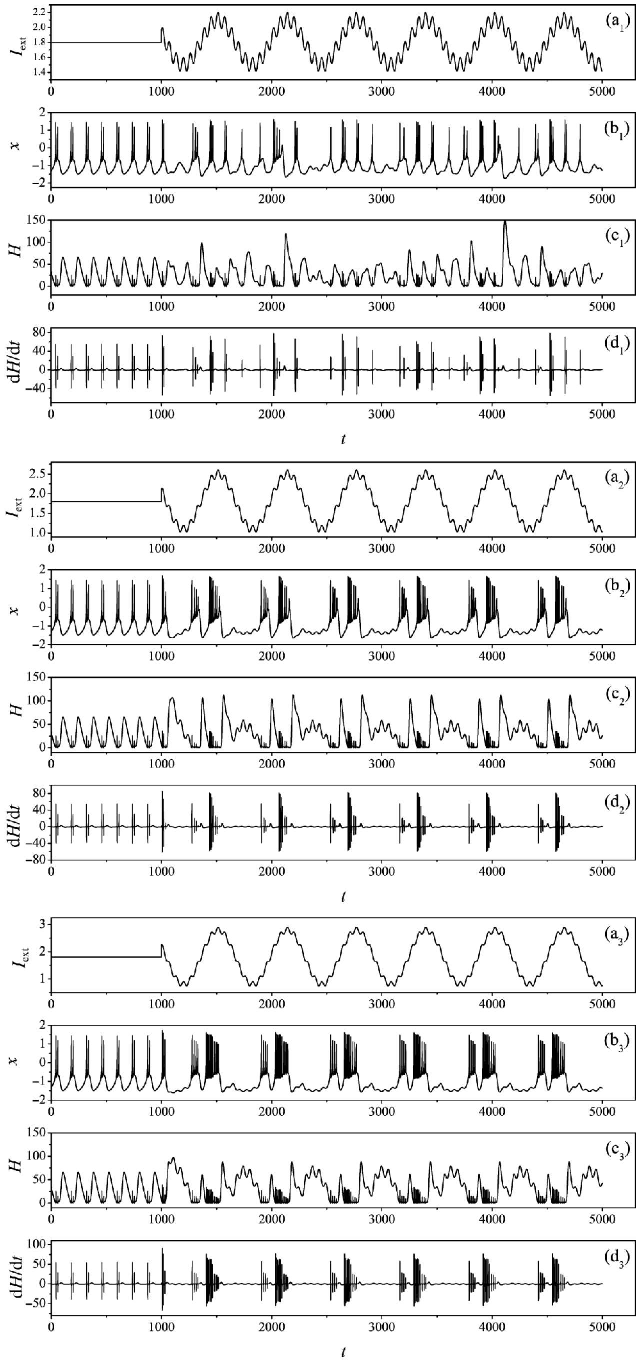

Figure 7 Evolution of action potential and energy function with time are calculated by changing the external mixed signal at B=0.1, ω=0.01, N=10, I=1.8. (a$_1$)–(d$_1$) A=0.3; (a$_2$)–(d$_2$) A=0.7; (a$_3$)–(d$_3$) A=1.0.

the neuron absorb external energy flow, thus the electrical activities become into the bursting state. Bursting state is helpful to release the energy, so the Hamilton energy of the bursting state is lower than the spiking state.

3.2.2 *The effect on electrical activities and Hamilton energy with the different parameters*

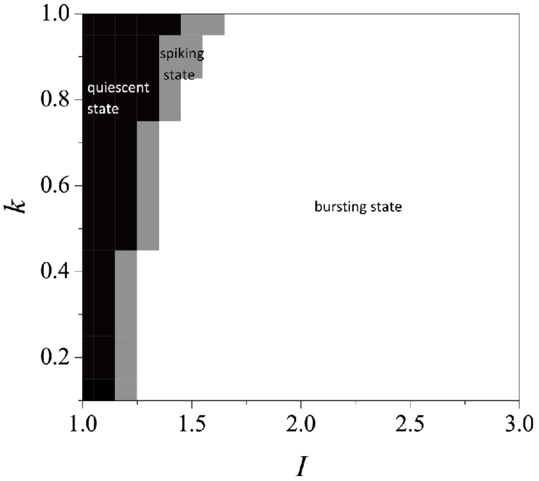

Figure 8 Distribution of the different states in the two-parameter phase space k-I under the different periodic electromagnetic radiation $\varphi_{ext}=A\cos(\omega t)+B\cos(N\omega t)$. A=0.1, B=0.2, ω=0.01, N=10.

Further, the parameters (i.e., A, B, N, ω) of the periodic electromagnetic radiation should be considered in the neuronal electrical activities. In order to observe the change of electrical activities under the periodic electromagnetic radiation $\varphi_{ext}=A\cos(\omega t)+B\cos(N\omega t)$, the power spectrums of time series for four different parameters (N=0.1, 1.0, 10, 100) are depicted in Figure 10.

From Figure 10, it is found that when N is 0.1, there have two distinct spectrum peaks, the spectrum peak is high and sharp, and it means that the electrical activities are irregular. According to calculation, we know that electrical activities are sudden spiking intermittently. When the parameter N is 1.0, 10, there have three spectrum peaks, the electrical activities are sudden periodic spiking intermittently. When N is around 1000, there has a high spectrum peak, it means that the electrical activities are regular and the degree of ordering is big. According to calculation, we know that the electrical activities are in spiking states, these results are consistent with the fluctuation shown by Figure 11.

In addition, the Hamilton energy of the neuron electrical activities under the different frequency ratios is shown in Figure 11. Comparing Figure 4 with Figure 11, it is found that the transition of electrical activities and energy under the periodic electromagnetic radiation in Figure 11 are obvious than the phenomena under the periodic external current in Figure 4. From these results, we know that the Hamilton energy of the multiple mode in Figure 11(a$_1$)–(d$_2$) is lower than the energy of the spiking state in Figure 11(a$_3$)–(d$_4$) when the period high-low frequency electromagnetic radiation is imposed in the neuron. The electrical activities and Hamilton energy have a large transition under the periodic electromagnetic radiation.

The average Hamilton energies of electrical activities are calculated under the different frequency ratios of electromagnetic radiations. The average Hamilton energy is 28.737 before the 1000 time units, when the frequency ratio N=0.1, 1.0, 10, 100, the average Hamilton energy are 27.050,

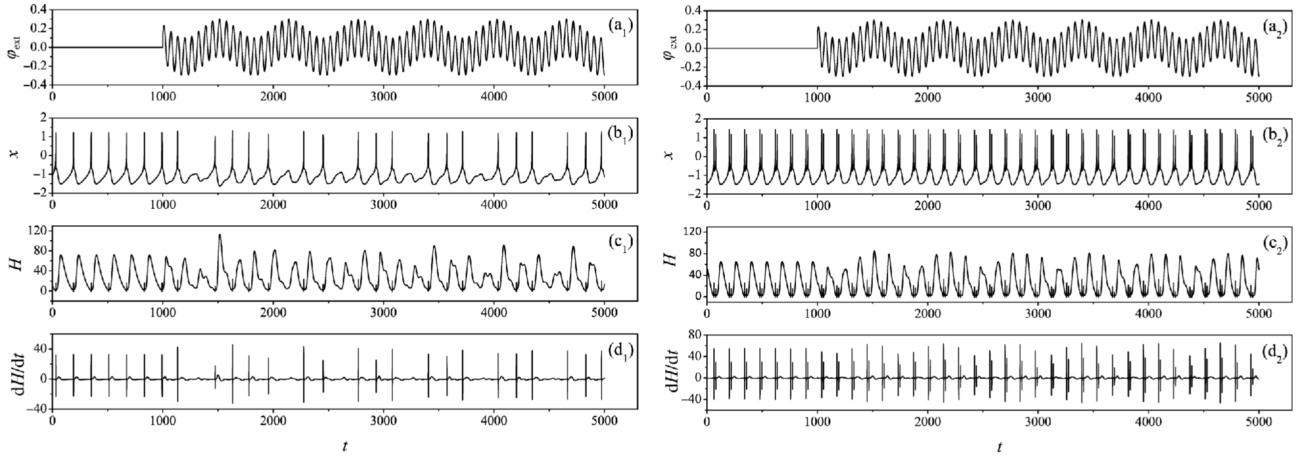

Figure 9 Evolution of action potential and energy function with time are calculated by changing the periodic electromagnetic radiation at A=0.1, B=0.2, ω=0.01, N=10. (a$_1$)–(d$_1$) I=1.4; (a$_2$)–(d$_2$) I=1.8. The period electromagnetic radiation is imposed on the neuron from t=1000 time units.

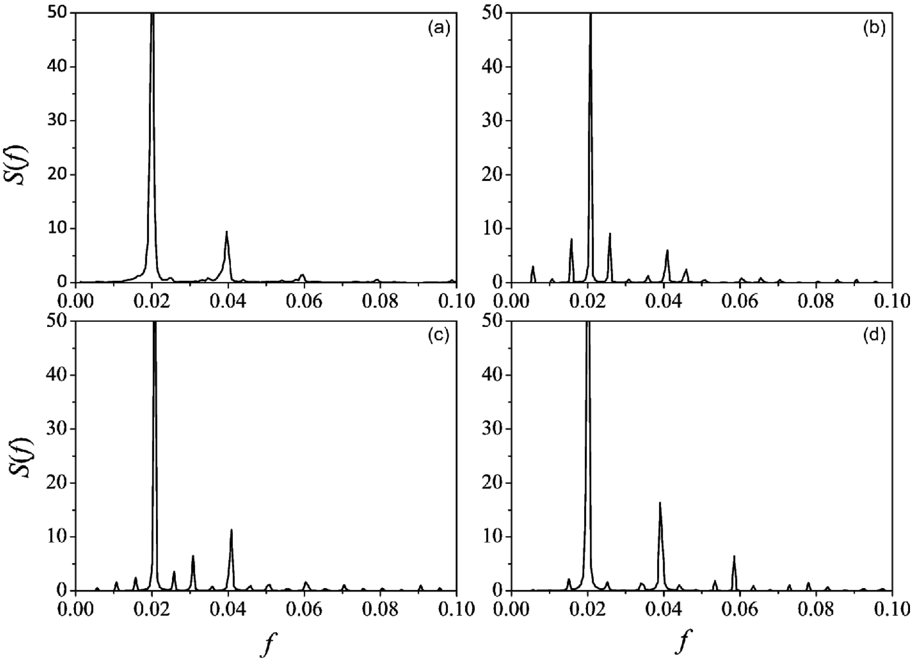

Figure 10 Power spectrum of the time series for four different parameters. (a) N=0.1; (b) N=1.0; (c) N=10; (d) N=100. The periodic electromagnetic radiation is $\varphi_{ext}=A\cos(\omega t)+B\cos(N\omega t)$, A=0.1, B=0.2, ω=0.01, I=1.4.

25.018, 27.808, 30.127 respectively in the neuron. This state that the Hamilton energy of the spiking state is higher than the one in sudden periodic spiking intermittently, this is consistent with the results in Figure 11.

The different anger frequencies of the periodic high-low frequency electromagnetic radiation could affect the mode selection of electrical activities. Therefore, the different anger frequency values ω should be considered in the improved HR neuron model eq. (15). The evolution of action potential and energy function with time are calculated by changing the anger frequency (i.e., ω=0.01, 0.1) of the periodic electromagnetic radiation in Figure 12.

As can be seen in Figure 12 the electrical activity is in the spiking state without the periodic forcing electromagnetic radiation at first, and the average Hamilton energy is 28.737. When the periodic forcing electromagnetic radiation is imposed in the neuron, as the frequency ω increases, the electrical activity has a transition from the multiple mode (quiescent state and spiking state) into the spiking state, the Hamilton energy also increases simultaneously. The average energy are calculated when the frequency ω=0.01, 0.1 here, the values are 27.808 and 29.974 respectively, the trend of the data is consistent with Figure 12.

Consider the effects under the different high frequency signal amplitudes B in Figure 13, where the value of B is selected as 0.3, 0.5, 0.9. Firstly, the average Hamilton energy

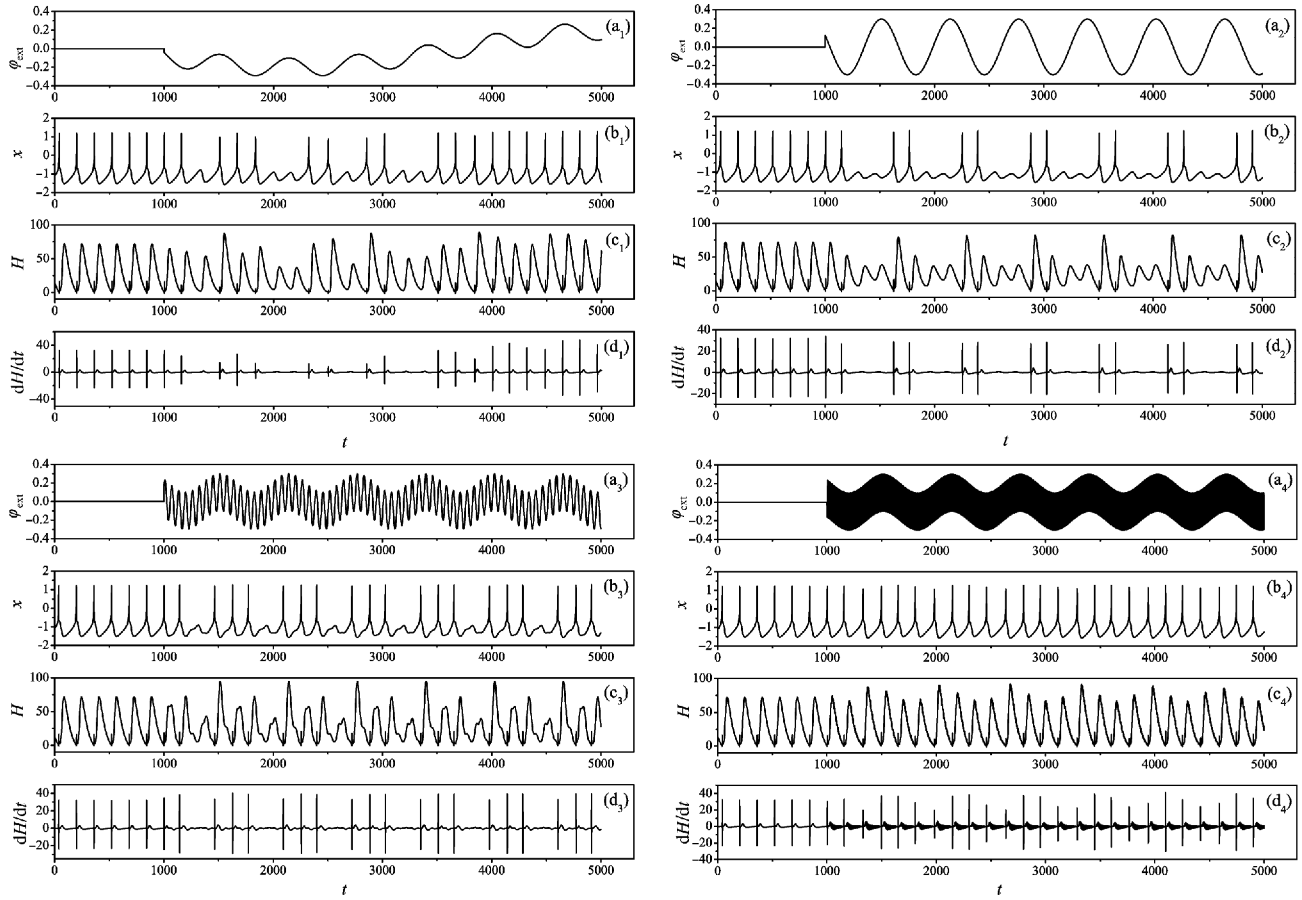

Figure 11 Evolution of action potential and energy function with time are calculated by changing the periodic electromagnetic radiation at A=0.1, B=0.2, ω=0.01, I=1.4. (a_1)–(d_1) N=0.1; (a_2)–(d_2) N=1; (a_3)–(d_3) N=10; (a_4)–(d_4) N=100.

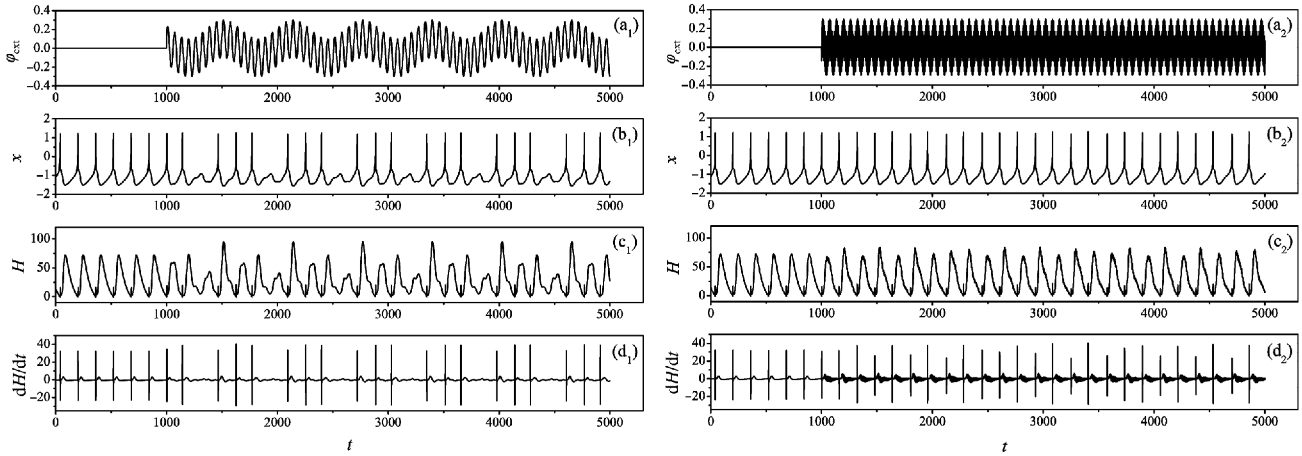

Figure 12 Evolution of action potential and energy function with time are calculated by changing the periodic electromagnetic radiation at A=0.1, B=0.2, N=10, I=1.4. (a_1)–(d_1) ω=0.01; (a_2)–(d_2) ω=0.1.

are calculated, the average energy is 28.737 before 1000 time units without the period forcing electromagnetic radiation. After adding the periodic electromagnetic radiation, when the amplitude B is 0.3, the average Hamilton energy is 29.485; when the amplitude B is 0.5, the average Hamilton energy is 28.941; when the amplitude B is 0.9, the average Hamilton energy is 43.259. As it can been seen, the electrical activity is in the spiking state without the periodic forcing electromagnetic radiation at first. When the periodic forcing electromagnetic radiation is imposed in the neuron, the electric activity has hardly altered under the amplitude B is 0.3, the electrical activities appear the bursting state when

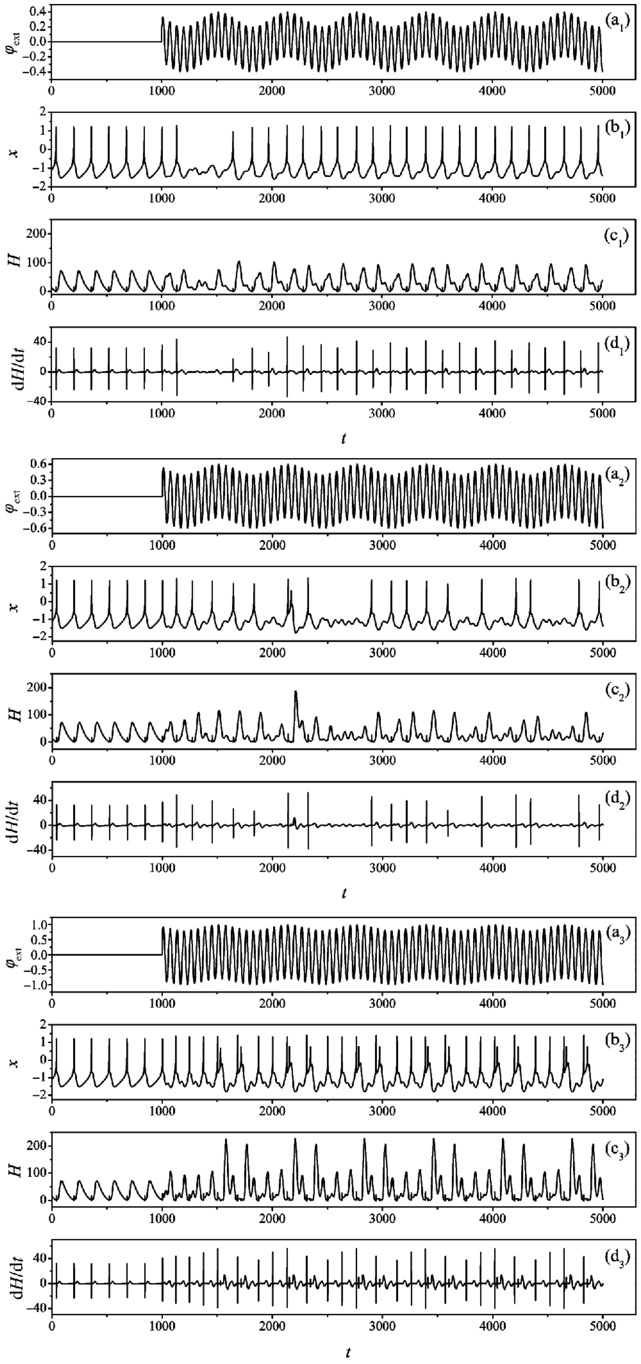

Figure 13 Evolution of action potential and energy function with time are calculated by changing the periodic electromagnetic radiation at A=0.1, N=10, ω=0.01, I=1.4. (a_1)–(d_1) B=0.3; (a_2)–(d_2) B=0.5; (a_3)–(d_3) B=0.9.

B=0.5. As the amplitude increases (B=0.9), the period multiple modes of the bursting state and spiking state are observed. It is interesting that the average energy (the value is 43.259) under B=0.9 is higher than the one under B=0.5 (the value is 28.941) when the periodic forcing are imposed in the electromagnetic field. In addition, it is found that neuron gives response sensitively to amplitude than angular frequency completely, and the potential mechanism is that external stimulus signals can input enough energy to induce mode transition while angular frequency can cause slight modulation on firing rhythm at fixed intensity.

Furthermore, the different amplitudes A of the low frequency electromagnetic radiation are investigated in the neuron in Figure 14. The results show that the electrical activities appear the multiple modes with the increasing of the amplitude. Meanwhile, the period energy transformation can be observed with the time series before the t=1000 time

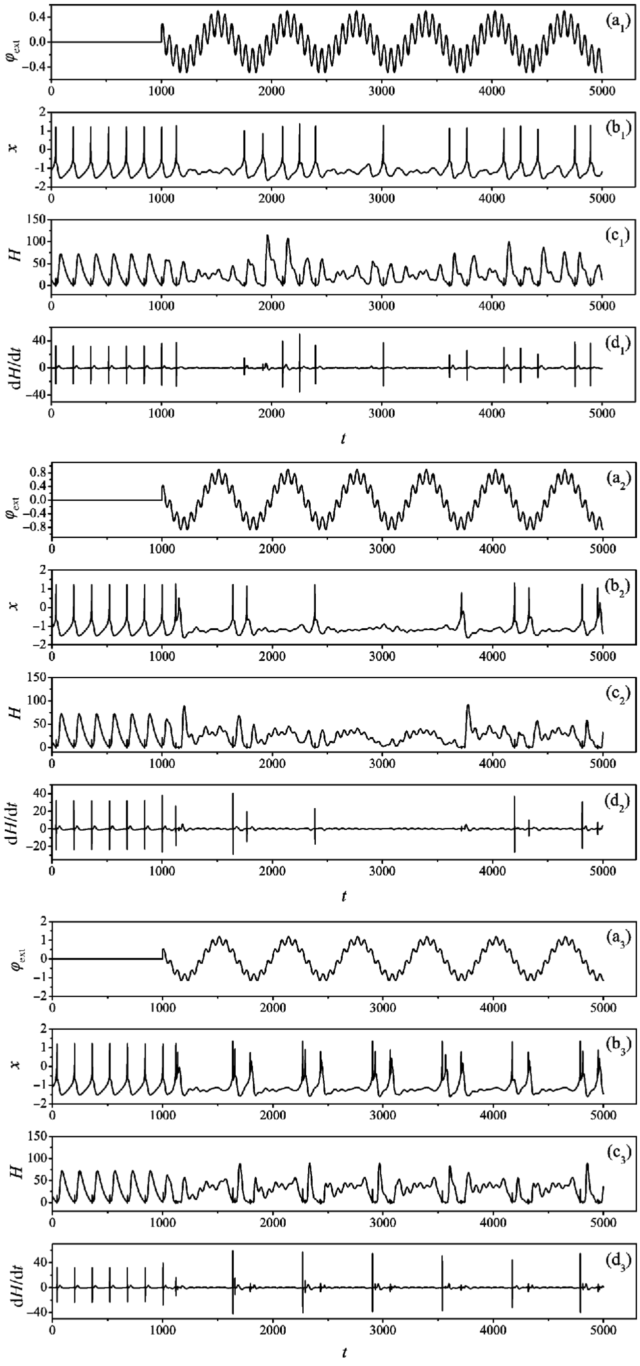

Figure 14 Evolution of action potential and energy function with time are calculated by changing the periodic electromagnetic radiation at B=0.2, ω=0.01, N=10, I=1.4. (a_1)–(d_1) A=0.3; (a_2)–(d_2) A=0.7; (a_3)–(d_3) A=1.0.

units without the period electromagnetic radiation. When the high-low frequency electromagnetic radiation is imposed in the neuron, the amplitude of energy is lower than before. According to the calculation, the average Hamilton energy value is 28.737 without the high-low frequency electromagnetic radiation. When the different amplitudes A are imposed in the neuron, the transition of energy has been observed obviously. The average Hamilton energy value is 26.177 under the amplitude A=0.3, the value is 23.884 under A=0.7, and the average Hamilton energy is 28.568 under A=1.0. It shows that spiking state make neuron hold higher energy. The potential mechanism could be that the multiple modes (i.e., bursting state, chaotic state) could be helpful to release the energy in the neuron fleetly. In addition, the bigger amplitude values have been surveyed in the improved HR model. When the amplitude A is smaller than some threshold value (around 6.0), the transition from periodical neuronal spiking into mixed mode is one-way conversion. After increasing the amplitude A beyond some threshold value, the neuron electrical activities is suppressed and the Hamilton energy is power, this is consistent with the previous results.

4 Conclusions

In a summary, the electrical activities and Hamilton energy are investigated when the mixed stimulus current (i.e., constant current I, periodic HF current $B\cos(N\omega t)$, periodic LF current $A\cos(\omega t)$) or the periodic high-low frequency electromagnetic radiation are imposed in the improved HR neuron model. It is found that the mode of electrical activities under the periodic electromagnetic radiation is more complicated than the one under the mixed stimulus current, furthermore, so is the change of energy. In addition, as it can be seen the changing of energy is much dependent on the modes transition of electrical activities rather than the external forcing current directly in the neuron. In addition, the electrical activity and Hamilton energy have a distinct variety with the changing of amplitude A, B, meanwhile, during the transition from spiking state to the multiple modes, the Hamiltonian energy is fast diminished. These phenomena of the electrical activities and Hamilton energy are associated with the energy absorption and energy release in the presence of complex electromagnetic condition. These results are instructive for further revealing the rich nonlinear dynamical behavior and mechanisms of electrical activities, and may be helpful for us to comprehensively understand the information processing in biological neuronal systems.

In these previous investigations, the transition of the electrical and Hamilton energy are discussed under the different external stimulation without additive phase diversity or the Gaussian white noise. Therefore, it is interesting to discuss the open problems: the effects of phase diversity or the Gaussian white noise could be investigated under the different external signals in the further work, meanwhile, the transition of the electrical activities and Hamilton energy could be surveyed in the neuron network.

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