SCIENCE CHINA
Technological Sciences

• Article •

December 2017 Vol.60 No.12:1831–1840
doi: 10.1007/s11431-017-9053-7

Study on mixed H_2/H_∞ robust control strategy of four wheel steering system

ZHAO WanZhong[1,2*] & QIN XiaoXi[1]

[1] *School of Energy & Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China;*
[2] *The State Key Laboratory of Fluid Power and Mechatronic Systems, Zhejiang University, Hangzhou 310027, China*

Received January 13, 2017; accepted April 12, 2017; published online July 4, 2017

Four wheel steering (4WS) technology can effectively improve the vehicle handling stability and driving safety. In order to fully consider the influence of the rear wheel steering, the vehicle dynamics model of 4WS vehicle, including the rear wheel steering by wire and two degrees of freedom vehicle model of 4WS vehicle, is established in this paper. The desired yaw rate is obtained according to the variable transmission ratio strategy. The yaw rate tracking strategy is applied to 4WS vehicle and rear wheel steering resistance moment is taken into account. Based on the robust control theory, H_2/H_∞ mixed robust controller design is carried to research the stability control of 4WS vehicle. Finally, the closed-loop simulation added driver model based on preview theory is carried out. The simulation results indicate that the designed H_2/H_∞ mixed robust controller can achieve the stability control.

4WS vehicle, handling stability, stability control, H_2/H_∞ control, weight function

Citation: Zhao W Z, Qin X X. Study on mixed H_2/H_∞ robust control strategy of four wheel steering system. Sci China Tech Sci, 2017, 60: 1831–1840, doi: 10.1007/s11431-017-9053-7

1 Introduction

The steering system, one of the four major automobile chassis systems, greatly influences the driving safety and handling stability of vehicle. With the development of technology, four wheel steering (4WS) technology has been more and more paid attention by domestic and foreign automobile enterprises and scholars and has become the main development direction of the automobile power steering technology in the future. 4WS technology could be traced back to the Japanese Society of automotive engineers and technology conference in the 1960s. In order to improve the handling stability of the vehicle, an engineer proposed active rear steering technology and then the major car companies had started the research of 4WS technology. Due to the limitations of electronic technology, automation technology and control technology, 4WS technology research was shelved. Since the 4WS vehicle has a relatively large improvement on handling stability, the 4WS technology has returned to the people's vision. In 2008, the new BMW 7 Series equipped with 4WS system came into market. At high speed, the steering angles of front wheel and rear wheel are in the same phase. While at low speed, they are in the opposite phase. The maximum steering angle of rear wheel is up to 3°. Subsequently, infiniti also launched M37S equipped with 4WS system and it had a great improvement on steering actuator. In April 2015, Cadillac debuted CT6 using 4WS technology in the United States show. It added a similar independent steering mechanism driven by a 12 V servo motor, with deluxe five link suspension system to achieve precise control for rear wheel steering.

At present, the research on the 4WS vehicle is mainly focused on the control strategy. The control strategies about

*Corresponding author (email: zwz@nuaa.edu.cn)

4WS system can be divided into two types. One is designed to make the sideslip angle zero, corresponding to trajectory preserving problem. The other one is designed to track desired yaw rate. 4WS controller was designed by Chen et al. [1] to make the vehicle have a good path tracking capability in both longitudinal and lateral direction using preview theory, robust theory and adaptive path following. Through the study on four wheel steering and four wheel driving, sliding mode controller based on the model tracking was designed by Nonaka and Oda [2]. The simulation results showed that the vehicle had a good performance on path tracking. Variable gain concept was introduced into the four wheel steering and independent driving integration research by von Vietinghoff and Kiencke [3]. The nonlinear model and control were verified in the virtual business software. Considering the velocity changing in the study, Li et al. [4] got 4WS vehicle time-varying model. LPV control was applied to 4WS system and the simulation results showed that the handling characteristics, safety and comfort of the vehicle driving were improved significantly with a large velocity-varying range. And decoupling control with H_∞ performance for 4WS vehicles under varying longitudinal velocity was studied. The results showed that this control scheme could improve handling characteristics, safety and comfort proved from theory to practice in this paper [5]. Yin et al. [6] applied the μ synthesis theory to the study of the 4WS vehicle. Considering the uncertainty of the model, a μ synthesis robust controller was designed with optimized weight functions to attenuate the external disturbances. The numerical simulation showed that the designed μ synthesis robust controller could improve the performance of a 4WS vehicle. Song [7] proposed a dynamic model of the 4WS car with two degrees of freedom, and its theoretical analysis was obtained from the kinetic equations and the transient response. Tracking the dynamics of a general nonlinear planar reference model was proposed by Russell and Gerdes [8]. The modified closed-loop system was demonstrated to be stable and had robust tracking performance to reasonable levels of model uncertainty. Experimental results demonstrated successful emulation of the low friction reference model's planar dynamics for a variety of driving maneuvers and reference friction coefficients. Mashadi et al. [9] designed an integrated 4WS+DYC control system to guide a vehicle on a desired path. Simulation results showed that the proposed controller had the ability to make the vehicle track the desired path. The autodriver algorithm is originally developed on the dynamics of 4WS vehicles by Marzbani et al. [10]. The uncertainty of the four wheel steering was analyzed on the basis of the singular boundary theory and Lyapunov stability theory by Huang et al. [11].

In summary, most of the existing researches focus on the path tracking of the 4WS vehicle, while the researches about the handling stability are few. Moreover, based on the vehicle model, the existing studies on stability control start the design ignoring the influence of rear wheel steering actuator. So in this paper, the vehicle dynamics model of 4WS vehicle, including the rear wheel steering by wire and two degrees of freedom vehicle model of four wheel steering, is established. The desired yaw rate is obtained according to the variable transmission ratio strategy. The yaw rate tracking strategy is applied to 4WS vehicle and rear wheel steering resistance moment is taken into account. Based on the robust control theory, H_2/H_∞ mixed robust controller design is carried to study the stability control of 4WS system. Finally, the closed-loop simulation added driver model based on preview theory is carried out. The simulation results indicate that the designed H_2/H_∞ mixed robust controller can achieve the stability control.

2 4WS vehicle dynamics model

2.1 The rear wheel steering by wire system model

Differential equation of motion of steering motor [12]:

$$\begin{cases} J_\mathrm{m}\ddot{\theta}_\mathrm{m} + B_\mathrm{m}\dot{\theta}_\mathrm{m} + T_\mathrm{a} = T_\mathrm{m}, \\ T_\mathrm{m} = K_\mathrm{t} i, \end{cases} \tag{1}$$

where J_m is the moment of inertia of steering motor; B_m is damping coefficient of steering motor; T_m is electromagnetic torque of steering motor; i is electric current of steering motor; K_t is electromagnetic torque constant; θ_m is steering angle of steering motor; T_a is output torque of steering motor.

Electrical equation:

$$\begin{cases} L\dot{i} + Ri + E = U, \\ E = K_\mathrm{b}\dot{\theta}_\mathrm{m}, \end{cases} \tag{2}$$

where L is the steering motor inductance; R is steering motor resistance; E is steering motor back EMF; U is the voltage of the motor; K_b is the steering motor back EMF constant.

Differential equations of motion of rack:

$$\begin{cases} M_\mathrm{r}\ddot{x}_\mathrm{r} + B_\mathrm{r}\dot{x}_\mathrm{r} + F_\mathrm{R} = NT_\mathrm{a} / r_\mathrm{p}, \\ \theta_\mathrm{sg} = x_\mathrm{r} / r_\mathrm{p}, \end{cases} \tag{3}$$

where M_r is the mass of rack; B_r is damping coefficient of rack; F_R is steering resistance of rack; N is reduction ratio of the steering motor; x_r is displacement of the rack; r_p is radius of the gear; θ_sg is steering angle of the gear.

The rack force can be equivalent to the steering column:

$$M_\mathrm{r} r_\mathrm{p}{}^2\ddot{\theta}_\mathrm{sg} + B_\mathrm{r} r_\mathrm{p}{}^2\dot{\theta}_\mathrm{sg} + T_\mathrm{R} = NT_\mathrm{a}, \tag{4}$$

where T_R is back moment of tire equivalent to the steering column.

2.2 Tire model

Tire is an important part of automobile. The force of automobile is mostly from the contact between tire and road surface, so the tire model plays an important role in the simulation of vehicle handling stability. In this paper, mainly considering the lateral dynamics of the tire and taking into account the follow-up control strategies, the linear tire model is established [13]. The work range of the tire is in the linear region of the tire. At the moment the lateral acceleration is less than 0.4 g and the slip angle is less than 5°.

The slip angle:

$$\alpha_{\mathrm{f}}=\beta+\frac{a}{u}r-\delta_{\mathrm{f}}, \tag{5}$$

$$\alpha_{\mathrm{r}}=\beta-\frac{b}{u}r-\delta_{\mathrm{r}}, \tag{6}$$

where α_{f} and α_{r} are the slip angles of front and rear tire; δ_{f} and δ_{r} are the steering angles of front and rear wheel; β is sideslip angle of 4WS vehicle; r is yaw rate of 4WS vehicle; a and b are distances between the front and rear axle to the vehicle mass center; u is the speed of vehicle.

Tire lateral force:

$$F_{Y\mathrm{f}}=k_1\alpha_{\mathrm{f}}, \tag{7}$$

$$F_{Y\mathrm{r}}=k_2\alpha_{\mathrm{r}}, \tag{8}$$

where k_1 and k_2 are the cornering stiffness of front and rear wheel; $F_{Y\mathrm{f}}$ and $F_{Y\mathrm{r}}$ are the lateral forces of front and rear wheel.

The self-aligning torque of tire:

$$T_{\mathrm{f}}=F_{Y\mathrm{f}}d, \tag{9}$$

$$T_{\mathrm{r}}=F_{Y\mathrm{r}}d, \tag{10}$$

where d is the pneumatic trail; T_{f} and T_{r} are the self-aligning torques of front and rear wheel.

2.3 4WS vehicle model

For the 4WS vehicle, the main research is the vehicle lateral dynamics [14]. Aiming at the research on lateral dynamics of the 4WS vehicle, a 4WS vehicle model is established neglecting the vehicle rolling and only considering the lateral and yaw degrees of freedom. The two degree of freedom vehicle model is shown in Figure 1.

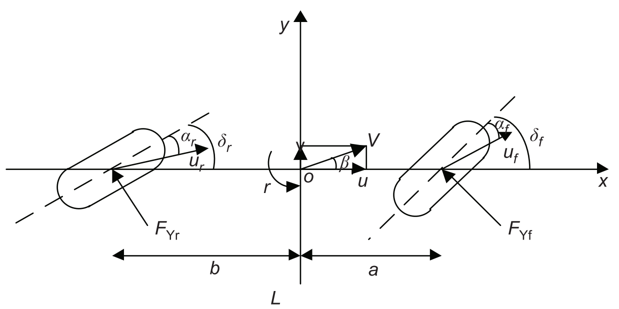

Figure 1 Two degree of freedom vehicle dynamics model.

As is shown in Figure 1, the car on vertical axle center is symmetrical. Ignoring the role of the car suspension, the car only moves parallel to the ground, that is, the car displacements along the z axis, the pitch around y axis and the roll motion around x axis are not considered. The change of tire characteristics caused by the load change is neglected. The slip angles of left and right wheel are the same. The dynamic equations of the 4WS vehicle in the lateral and yaw can be obtained from Figure 1.

$$\begin{cases} mu(\dot{\beta}+r)=F_{Y\mathrm{f}}\cos\delta_{\mathrm{f}}+F_{Y\mathrm{r}}\cos\delta_{\mathrm{r}},\\ I_z\dot{r}=F_{Y\mathrm{f}}a\cos\delta_{\mathrm{f}}-F_{Y\mathrm{r}}b\cos\delta_{\mathrm{r}}, \end{cases} \tag{11}$$

where m is the mass of vehicle; I_z is the yaw moment of inertia.

According to the linear tire model, eq. (11) can be converted to eq. (12):

$$\begin{cases} mu(\dot{\beta}+r)=-k_1\left(\beta+\dfrac{a}{u}r-\delta_{\mathrm{f}}\right)\cos\delta_{\mathrm{f}}-k_2\left(\beta-\dfrac{b}{u}r-\delta_{\mathrm{r}}\right)\cos\delta_{\mathrm{r}},\\ I_z\dot{r}=-k_1\left(\beta+\dfrac{a}{u}r-\delta_{\mathrm{f}}\right)a\cos\delta_{\mathrm{f}}+k_2\left(\beta-\dfrac{b}{u}r-\delta_{\mathrm{r}}\right)b\cos\delta_{\mathrm{r}}. \end{cases} \tag{12}$$

Due to the smaller wheel steering angle, $\cos\delta_{\mathrm{f}}$ and $\cos\delta_{\mathrm{r}}$ can be regarded as 1 to finish 4WS vehicle linear two degrees of freedom model.

$$\begin{cases} mu(\dot{\beta}+r)=-(k_1+k_2)\beta-\dfrac{k_1a-k_2b}{u}r+k_1\delta_{\mathrm{f}}+k_2\delta_{\mathrm{r}},\\ I_z\dot{r}=-(k_1a-k_2b)\beta-\dfrac{k_1a^2+k_2b^2}{u}r+k_1a\delta_{\mathrm{f}}-k_2b\delta_{\mathrm{r}}. \end{cases} \tag{13}$$

Further state space model can be obtained:

$$\begin{cases} \dot{\boldsymbol{x}}=\boldsymbol{Ax}+\boldsymbol{Bu},\\ \boldsymbol{y}=\boldsymbol{Cx}+\boldsymbol{Du}, \end{cases} \tag{14}$$

where

$$\boldsymbol{x}=\begin{bmatrix}\beta & r\end{bmatrix}^{\mathrm{T}},\ u=\begin{bmatrix}\delta_{\mathrm{f}} & \delta_{\mathrm{r}}\end{bmatrix}^{\mathrm{T}},\ \boldsymbol{y}=\begin{bmatrix}\beta & r\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{A}=\begin{bmatrix} -\dfrac{k_1+k_2}{mv} & -\dfrac{k_1a-k_2b}{mv^2}-1\\ -\dfrac{k_1a-k_2b}{I_z} & -\dfrac{k_1a^2+k_2b^2}{I_zv} \end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix} \dfrac{k_1}{mv} & \dfrac{k_2}{mv}\\ \dfrac{k_1a}{I_z} & -\dfrac{k_2b}{I_z} \end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \boldsymbol{D}=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

2.4 Driver model

In order to study the influence of the 4WS vehicle on dynamic characteristics, a closed-loop system model including driver model must be established. A closed loop of the human vehicle to complete tracking for any path is formed. In this paper, the first order preview model is used as shown in Figure 2.

Under the geodetic coordinate system XOY, the position of the vehicle mass center is (X, Y) and the included angle between vehicle longitudinal axis and the X axis is Φ (yaw angle of the vehicle). Then X, Y and Φ can be obtained as following.

$$\begin{cases} X = X_0 + u\int_0^t \cos(\beta+\Phi)\mathrm{d}t, \\ Y = Y_0 + u\int_0^t \sin(\beta+\varphi)\mathrm{d}t, \\ \Phi = \Phi_0 + \int_0^t r\mathrm{d}t, \end{cases} \tag{15}$$

where X_0, Y_0 and Φ_0 are the position when t is zero.

In the process of driving, the driver decides the size of the steering wheel angle according to the displacement deviation of the preview point and the steering angle deviation of the current position of the vehicle. The displacement deviation ε_y is determined by the lateral displacement of the desired path Y_d and the lateral displacement of the current Y when preview time is T_p:

$$\varepsilon_y = Y_d(t+T_p) - (Y(t) + T_p\dot{Y}(t)), \tag{16}$$

where Y_d is the desired lateral displacement; Y is the lateral displacement of the current; T_p is the preview time.

The error of the driving angle of current position and the road direction angle can be expressed as:

$$\varepsilon_\Phi = \Phi - \int_0^t \kappa u \mathrm{d}t, \tag{17}$$

where κ is the curvature of road.

And then, the steering wheel angle can be expressed as the product of the weighted sum of the vehicle travel displacement error and the direction error and the driver's operation delay.

$$\theta_{sw} = \left(K_1\varepsilon_y + K_2\varepsilon_\Phi\right)e^{\tau_d s}, \tag{18}$$

where θ_{sw} is the steering wheel angle; K_1 and K_2 are the compensation gain of driver's position and orientation error; τ_d is the delay time.

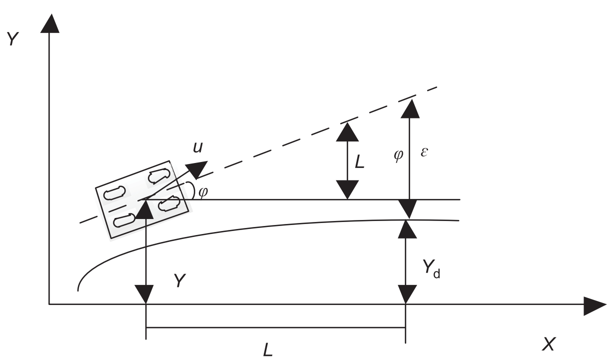

Figure 2 First order preview model.

3 Stability control strategy

The purpose of vehicle stability control is to improve the steady state and transient response, and to improve the vehicle handling stability and the ability to resist external interference [15]. In this paper, we first make the 4WS vehicle with ideal steering characteristics by the law of variable transmission ratio, and thus get the ideal yaw rate. According to the robust control theory, H_2/H_∞ mixed robust controller is designed to change output torque of the rear wheel steering motor to change the steering angle of the rear wheel and then 4WS vehicle stability control is achieved by tracking ideal yaw rate.

3.1 Variable transmission ratio

When the vehicle turns, the ratio of the steering wheel angle to the front wheel angle of the vehicle is defined as the steering gear ratio:

$$i_s = \frac{\theta_{sw}}{\delta_f}. \tag{19}$$

For the traditional front wheel steering vehicle, the steering ratio is determined by the structural parameters of the automobile and it is a fixed value so the steering characteristic varies greatly with the speed. The changing transmission ratio makes the steering characteristics present a fixed proportion relation unconcerned with speed of the vehicle, namely to meet the same steering characteristics at low speed and high speed which can relieve the burden of the driver. The transmission ratio is called the ideal transmission ratio. Combined with the research of the subsequent control strategy, the law of variable transmission ratio is used with constant yaw rate gain.

According to the law of variable transmission ratio derived from constant yaw rate gain, the relationship between steering wheel angle and front wheel steering angle is:

$$i_s = \frac{\theta_{sw}}{\delta_f} = \frac{u/L}{G_{sw}(1+Ku^2)}, \tag{20}$$

where L is wheelbase of the vehicle; G_{sw} is the constant yaw rate gain; K is stability factor, $K = \frac{m}{L^2}\left(\frac{a}{k_2} - \frac{b}{k_1}\right)$.

Given driver command inputs (steering wheel angle), a planar bicycle model of the reference dynamics is simulated, which is computed using a linear tire model (δ_r=0).

$$\begin{cases} mu(\dot{\beta} + \mathrm{r}) = -(k_1+k_2)\beta - \dfrac{k_1a - k_2b}{u}r + k_1\delta_f, \\ I_z\dot{r} = -(k_1a - k_2b)\beta - \dfrac{k_1a^2 + k_2b^2}{u}r + k_1a\delta_f. \end{cases} \tag{21}$$

The steady-state yaw rate r^* is a constant and now $\dot{\beta}$ and $\dot{r}$ are both zero. The steady-state yaw rete r^* is the desired yaw rate. The relationship between desired yaw rate and front wheel steeering angle is:

$$\frac{r^*}{\delta_{\mathrm{f}}}=\frac{u/L}{1+Ku^2}. \tag{22}$$

3.2 Control system model

Compared with the front wheel steering vehicle, 4WS vehicle is easier to improve the handling stability of vehicle by rear wheel steering system. In this paper, a robust controller is designed to track the ideal yaw rate so as to obtain the desired steering characteristics. In 4WS vehicle, the front wheel steering system is the traditional mechanical steering and the rear wheel steering system is the steer-by-wire system, so the performance of the rear wheel steering system directly determines whether the four wheel steering vehicle can achieve the desired performance, In other words, if the performance of the rear wheel steering system is poor, at this point, the 4WS vehicle performance is even worse than the front wheel steering vehicle. Therefore, the rear wheel steering system is taken into account in four wheel steering vehicle.

In order to realize the stability control of the yaw rate, the controller must satisfy the following requirements: the yaw rate tracking, the path tracking and the suppression of the disturbance.

Using the eqs. (4) and (12), the state space model of the stability control for the 4WS vehicle can be derived as:

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}_1\boldsymbol{w}+\boldsymbol{B}_2\boldsymbol{u},\\ \boldsymbol{y}=\boldsymbol{C}\boldsymbol{x}+\boldsymbol{D}_1\boldsymbol{w}+\boldsymbol{D}_2\boldsymbol{u},\end{cases} \tag{23}$$

where

$$\boldsymbol{x}=\begin{bmatrix}\beta & r & \theta_{\mathrm{sg}} & \dot{\theta}_{\mathrm{sg}}\end{bmatrix}^{\mathrm{T}},\ u=T_{\mathrm{a}},\ \boldsymbol{w}=\begin{bmatrix}\delta_{\mathrm{f}} & d_{\mathrm{r}} & f_{\mathrm{y}}\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{A}=\begin{bmatrix}-\dfrac{k_1+\Delta k_1+k_2+\Delta k_2}{mv} & -\dfrac{(k_1+\Delta k_1)a+(k_2+\Delta k_2)b}{mv^2}-1 & \dfrac{k_2+\Delta k_2}{Gmv} & 0\\ -\dfrac{(k_1+\Delta k_1)a-k_2b}{I_z} & -\dfrac{\left(k_1+\Delta k_1\right)a^2+\left(k_2+\Delta k_2\right)b^2}{I_zv} & -\dfrac{(k_2+\Delta k_2)b}{GI_z} & 0\\ 0 & 0 & 0 & 1\\ \dfrac{d(k_2+\Delta k_2)}{GM_{\mathrm{r}}r_{\mathrm{p}}^2} & -\dfrac{d(k_2+\Delta k_2)b}{GvM_{\mathrm{r}}r_{\mathrm{p}}^2} & \dfrac{-d(k_2+\Delta k_2)}{G^2M_{\mathrm{r}}r_{\mathrm{p}}^2} & -\dfrac{B_{\mathrm{r}}}{M_{\mathrm{r}}}\end{bmatrix},$$

$$\boldsymbol{B}_1=\begin{bmatrix}\dfrac{k_1+\Delta k_1}{mu} & 0 & \dfrac{1}{mu}\\ \dfrac{(k_1+\Delta k_1)a}{I_z} & 0 & \dfrac{l_{\mathrm{w}}}{I_z}\\ 0 & 0 & 0\\ 0 & -\dfrac{1}{M_{\mathrm{r}}r_{\mathrm{p}}^2} & 0\end{bmatrix},\ \boldsymbol{B}_2=\begin{bmatrix}0 & 0 & 0 & \dfrac{N}{M_{\mathrm{r}}r_{\mathrm{p}}^2}\end{bmatrix}^{\mathrm{T}},\ \boldsymbol{C}=[0\ \ 1\ \ 0\ \ 0],\ \boldsymbol{D}_1=[0\ \ 0\ \ 0],\ \boldsymbol{D}_2=[0],$$

where β is sideslip angle of 4WS vehicle; r is yaw rate of 4WS vehicle; θ_{sg} is steering angle of the gear; T_{a} is output torque of steering motor; δ_{f} are the steering angle of front wheel; d_{r} is the road surface random disturbance; f_{y} is the lateral wind disturbance; l_{w} is the distance between center of pressure and vehicle centroid, Δk_1 and Δk_2 are the parameter uncertainty of k_1 and k_2.

3.3 H$_2$/H$_\infty$ mixed robust control strategy

In the stability control of 4WS vehicle, the car is interfered by the road surface, the lateral wind and the sensor noise when tracking the ideal yaw rate. H$_\infty$ control theory is a kind of modern control theory through optimizing the infinite norm of performance in the space. As a generalization of H$_\infty$ control problem, the H$_2$/H$_\infty$ mixed robust problem is one of the important robust performance problems. H$_2$/H$_\infty$ mixed control combines optimal performance and robustness of the system to enable the system to get the two most concerned system characteristics for designers by solving an optimal controller.

H$_\infty$ standard design problem is shown in the Figure 3, where u is the input signal; w is the interference signal; z is the controlled output; y is the output signal; $G(s)$ is the controlled object; $K(s)$ is the controller.

The state space expression of transfer function matrix $\boldsymbol{G}(s)$ is

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}_1\boldsymbol{w}+\boldsymbol{B}_2\boldsymbol{u},\\ \boldsymbol{z}=\boldsymbol{C}_{\mathrm{r}}\boldsymbol{x}+\boldsymbol{D}_{11}\boldsymbol{w}+\boldsymbol{D}_{12}\boldsymbol{u},\\ \boldsymbol{y}=\boldsymbol{C}_2x+\boldsymbol{D}_{21}\boldsymbol{w}+\boldsymbol{D}_{22}\boldsymbol{u},\end{cases} \tag{24}$$

The equation can also be expressed as

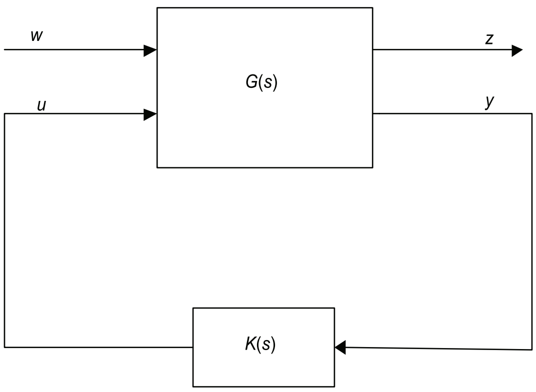

Figure 3 H∞ standard design problem.

$$\boldsymbol{G}(s)=\begin{bmatrix}G_{12}(s) & G_{22}(s)\\ G_{21}(s) & G_{22}(s)\end{bmatrix}=\begin{bmatrix}A & B_1 & B_2\\ C_1 & D_{11} & D_{12}\\ C_2 & D_{21} & D_{22}\end{bmatrix}. \tag{25}$$

The transfer function from w to z is

$$\begin{aligned}T_{zw}&=\mathrm{LTF}(G(s),K(s))\\&=G_{11}+G_{12}K(I-G_{22}K)^{-1}G_{21}.\end{aligned} \tag{26}$$

For a given generalized controlled object $P(s)$, feedback controller $K(s)$ is solved to make the closed-loop transfer function internal stable and meet the condition $\|T_{zw}\|_\infty=\gamma_0$ or $\|T_{zw}\|_\infty\le\gamma\ (\gamma\ge\gamma_0)$.

H∞ mixed sensitivity robust control problem is one of the most typical problems in H∞ control. When applying H∞ control method to design system, the designed problem is usually transformed into a mixed sensitivity problem to solve. It can ensure the robustness of the system and improve the performance of the system.

The augmented system after weighting is shown in Figure 4, where W_1 and W_2 are the weight function; G is the actual object; K is the controller. The sensitivity function can be written as

$$S=(I-KG)^{-1}. \tag{27}$$

Complementary sensitivity function can be written as

$$T=KG(I-KG)^{-1}. \tag{28}$$

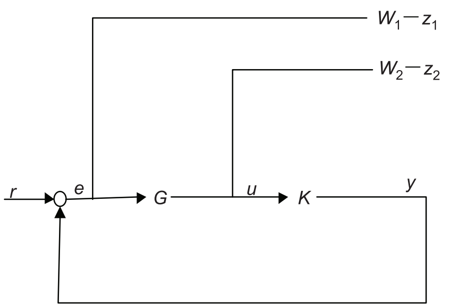

Figure 4 H∞ mixed sensitivity problem.

The sensitivity function S is the transfer function of the system from disturbance to the output, which can reflect the ability of the system to restrain the disturbance. The complementary sensitivity function T is related to the robust stability of the system. Obviously small S produces small e that corresponds good tracking performance and small T produces a small control output. As S+T=I, so we need to make a trade-off in the actual situation by constantly adjusting the weight function to make the system have a better performance.

Mixed sensitivity problem is to find the internal stable control law K, which makes the closed-loop system from disturbance w to the controlled output z to meet the transfer matrix:

$$\min_{K\,\mathrm{stab}}\left\|\begin{matrix}W_1S\\ W_2T\end{matrix}\right\|_\infty\le 1. \tag{29}$$

H2 standard design problem is similar to H∞ standard design problem. The H2 norm reflects the asymptotic variation of the system when the system is disturbed by white noise. H2 performance index is very effective for studying the performance of the system under random factors such as measurement noise and random interference.

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{Ax}+\boldsymbol{B}_1\boldsymbol{w}+\boldsymbol{B}_2\boldsymbol{u},\\ \boldsymbol{z}_\infty=\boldsymbol{C}_\infty\boldsymbol{x}+\boldsymbol{D}_{\infty1}\boldsymbol{w}+\boldsymbol{D}_\infty\boldsymbol{u},\\ \boldsymbol{z}_2=\boldsymbol{C}_2\boldsymbol{x}+\boldsymbol{D}_{21}\boldsymbol{w}+\boldsymbol{D}_{22}\boldsymbol{u},\\ \boldsymbol{y}=\boldsymbol{C}_y\boldsymbol{x}+\boldsymbol{D}_{y1}\boldsymbol{w}+\boldsymbol{D}_{y2}\boldsymbol{u}.\end{cases} \tag{30}$$

As a generalization of H∞ control problem, H2/H∞ mixed problem is one of the important problems in robust control. H2/H∞ mixed control problem is to design a controller to make the closed-loop system stable for the 30 generalized controlled object. The H2/H∞ mixed control problems are divided into three cases to discuss in this paper:

1) $\min\|T_{z\infty w}(s)\|_\infty^2$, $\|T_{z2w}(s)\|_2\le\gamma_1$;
2) $\min\|T_{z2w}(s)\|_2^2$, $\|T_{z\infty w}(s)\|_\infty\le\gamma_2$;
3) $\min\left(a\|T_{z\infty w}(s)\|_\infty^2+b\|T_{z2w}(s)\|_2^2\right)$.

$\|T_{z\infty w}(s)\|_\infty\le\gamma_3$, $a\|T_{z\infty w}(s)\|_\infty^2 : b\|T_{z2w}(s)\|_2^2\approx 1:1$.

The corresponding controllers are called H2/H∞ mixed controllers.

3.4 Design of mixed controller for 4WS vehicle

As is shown in Figure 5, it is 4WS vehicle stability control structure, and the disturbance input of the system is the sensor noise n, the ideal yaw rate r*, the steering angle of front wheel δ_f, the road surface random disturbance d_r and the lateral wind disturbance f_y. $\boldsymbol{G}_0(s)$ is the transfer function from the output torque of the steering motor (T_a) to yaw rate. $\boldsymbol{G}_d(s)$=[G_1,G_2,G_3]. $\boldsymbol{G}_d(s)$ is the transfer matrix from δ_f, d_r and f_y to yaw rate r. z_1, z_2 and z_3 are the three controlled outputs of system, where z_1 is on behalf of the performance of tracking the target and restraining interference, z_2 is on behalf of the system's robust stability and the performance of restraining noise and z_3 is on behalf of the size of the controller output. $\boldsymbol{W}_1$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_3$ are the three weight functions. The selection principles of weight function by mixed sensitivity theory are as follows:

1) Because the order of controller designed by mixed sensitivity control method is the same as the generalized controlled object with weight matrix, it should be reduced as far as possible in order to make the resulting controller possible and easy to implement in engineering. Therefore, W_1 and W_2 are selected as first order.

2) The weight function W_1 reflects the sensitivity function requirements of system and should have low pass characteristics, so it is selected as first order low-pass transfer function. The weight function W_2 reflects the complementary sensitivity function requirements of the system and should have high pass characteristics, so it is selected as first order high-pass transfer function.

3) W_3 is the weight function for control output and can limit the size of the output signal. W_3 is selected as a constant.

According to the stability diagram of the 4WS vehicle shown in Figure 5, we can get the generalized control object of the system.

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{bmatrix} = \begin{bmatrix} W_1 & -W_1G_1 & -W_1G_2 & -W_1G_3 & 0 & -W_1G_0 \\ 0 & W_2G_1 & W_2G_2 & W_2G_3 & 0 & W_2G_0 \\ 0 & 0 & 0 & 0 & 0 & W_3 \\ W_1 & -W_1G_1 & -W_1G_2 & -W_1G_3 & -1 & -W_1G_0 \end{bmatrix} \begin{bmatrix} r^* \\ \delta_{\mathrm{f}} \\ d_{\mathrm{r}} \\ f_{\mathrm{y}} \\ n \\ T_{\mathrm{a}} \end{bmatrix}. \tag{31}$$

Generalized control object is

$$\boldsymbol{G}(s) = \begin{bmatrix} W_1 & -W_1G_1 & -W_1G_2 & -W_1G_3 & 0 & -W_1G_0 \\ 0 & W_2G_1 & W_2G_2 & W_2G_3 & 0 & W_2G_0 \\ 0 & 0 & 0 & 0 & 0 & W_3 \\ W_1 & -W_1G_1 & -W_1G_2 & -W_1G_3 & -1 & -W_1G_0 \end{bmatrix}. \tag{32}$$

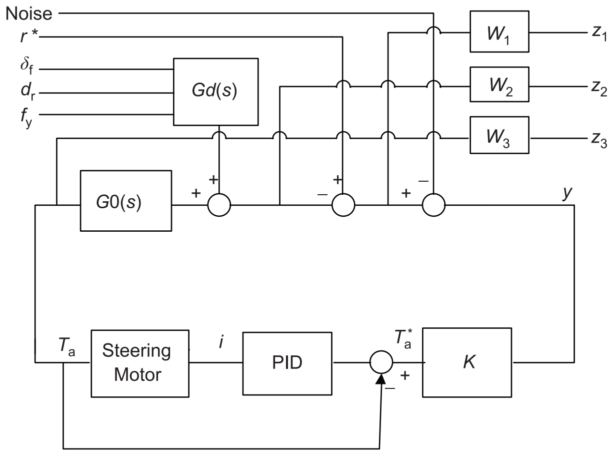

Figure 5 Stability control block diagram of the 4WS vehicle.

In general, good control effect often requires large energies, and the H_2 control system can minimize output energy under the pulse and white noise interference. So H_2 norm of transfer function from disturbance w to the control output u is selected to optimize as another performance index.

Figure 6 is the bode diagram of W_1 and W_2. W_1 is a low pass transfer function, and W_2 is a high pass transfer function.

In the stability control, the ideal output torque of the steering motor is obtained by the robust controller. In order to increase the reliability of the steering motor, a closed loop control is added to the steering motor. The deviation between ideal output torque and actual output torque is the input of the PID controller and the steering motor current i is the output of the PID controller. The current signal of the steering motor is given by the ECU, and the actual output torque is obtained by the steering motor.

3.5 Analysis of simulation results

Based on the matlab/simulink platform, the 4WS vehicle is

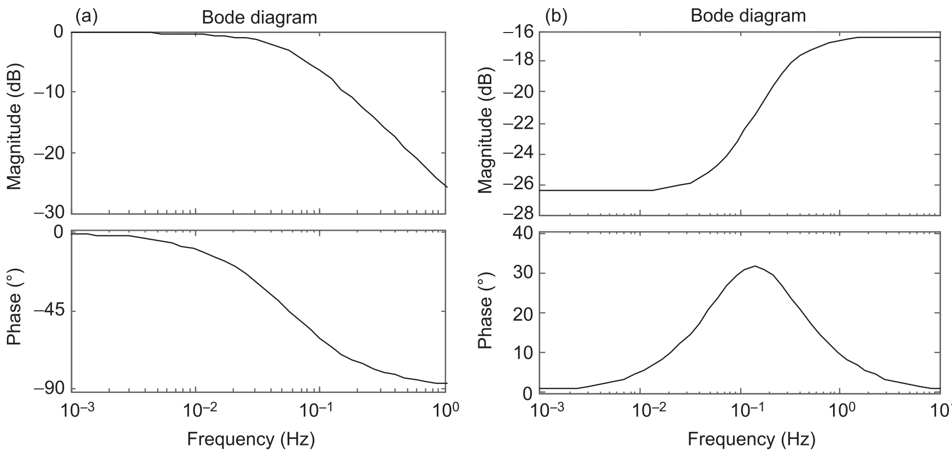

Figure 6 Bode diagram. (a) The bode diagram of W_1; (b) the bode diagram of W_2.

analysed in this paper. H_2/H_∞ mixed controllers are designed in stablity control, respectively analysising three kinds of controllers. The paper selects two typical vehicle working conditions, steering wheel angle step input and double lane change simulation, to verify the 4WS vehicle's yaw rate tracking capability and path tracking performance.

3.5.1 Steering wheel angle step input

The steering wheel angle step input (front wheel angle step input) is a typical condition to study the steady state response of vehicle. In this paper, the steering wheel angle step input (100°) is selected to verify the tracking capability of the designed controller for the yaw rate. The car is running on a good road at a speed of 90 km/h. Figures 7 and 8 depict yaw rate response and sideslip angle response of the 4WS vehicle. Figure 9 shows the output of the designed controller.

As can be seen from Figure 7, the 4WS vehicle can better track ideal yaw rate compared with front wheel steering vehicle. Corresponding to this, the 4WS vehicle has better steering characteristics by rear wheel steering system and can improve the handling stability and flexibility in the process of vehicle running. Three kinds of controls can well track the desired yaw rate and the steady-state error could be neglected. When the system is limited by minimizing H_∞ norm, it presents a very good H_∞ performance. At this time, the peak of the yaw rate response is 0.6 rad/s. When the system is limited by minimizing H_2 norm, it presents a very good H_2 performance. At this time, The peak of the yaw rate response is 0.55 rad/s and the overshoot decreases 8%. H_2/H_∞ control compromises the merits of H_∞ control and H_2 control. The overshoot decreases 4%. As shown in the Figure 8, three cases have little effect on the sideslip angle and they are less than 0.1 rad. Figure 9 shows the output of the controller. H_2 control makes the output decrease 50% compared with H_∞ control while H_2/H_∞ control makes the output decrease 33% in order to balance the H_∞ performance and H_2 performance.

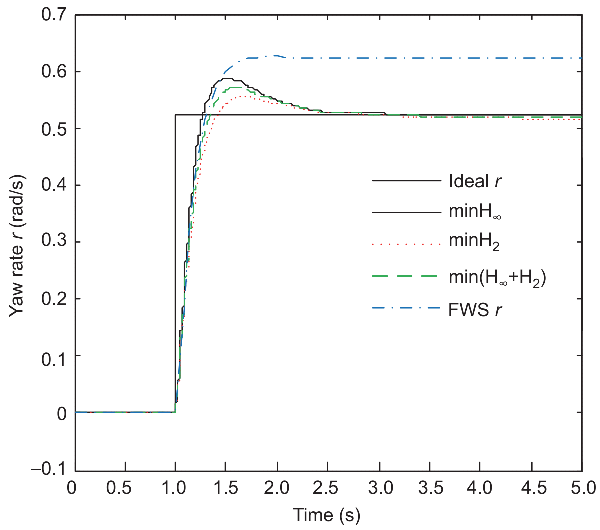

Figure 7 (Color online) Yaw rate response.

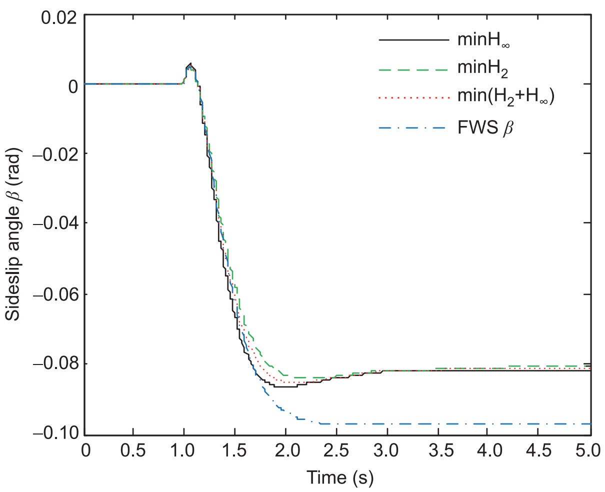

Figure 8 (Color online) Sideslip angle response.

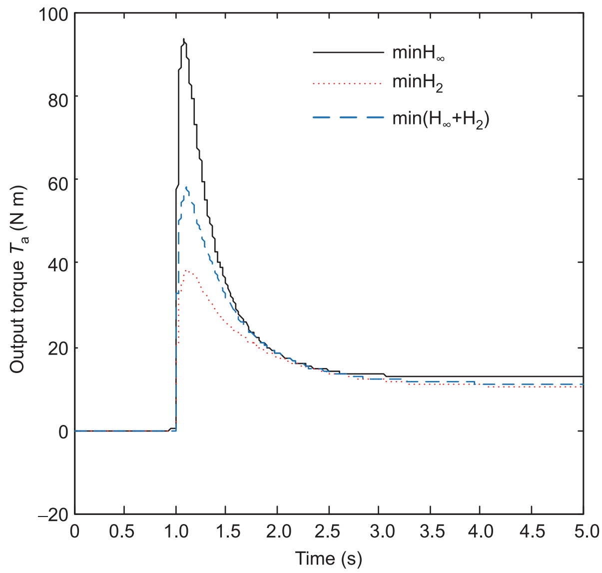

Figure 9 (Color online) Actual output torque of steering motor.

In order to verify the restraining interference effect of the designed controllers, the side wind and road random disturbances are taken in this paper [16]. The lateral wind interference is taken as constant, and the road surface random disturbance is taken as white noise. The simulation results are shown in Figure 10.

In Figure 10, there is only the side wind interference when t is 0 to 1 s. When t is 1 s to 2 s, there is only the road random disturbance. Thus the interference has greater influence on the 4WS vehicle without control. As can be seen from Figure 10, the effect of restraining interference with H_2 control is the worst of all. It is accompanied by a steady state error. H_∞ control makes the system possess a good H_∞ performance with good effect of restraining interference. The

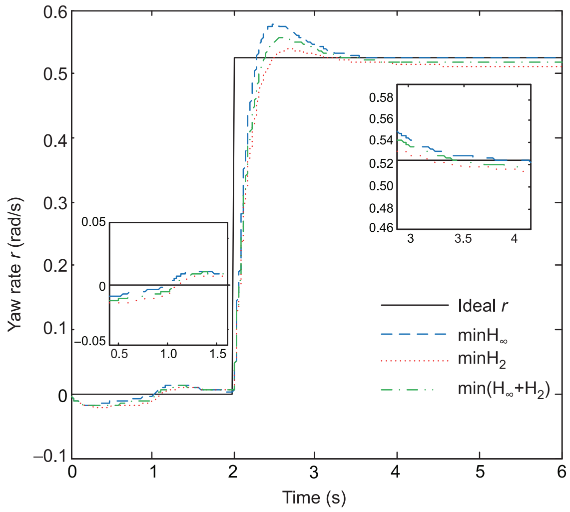

Figure 10 (Color online) Yaw rate under interference.

effect of restraining interference with H_2/H_∞ control is similar to the H_∞ control. In summary, the controllers with the H_∞ control and the H_2/H_∞ control have a good effect of restraining interference.

Figure 11 shows the bode diagram of closed-loop system when the parameters are perturbed. As can be seen from the figure, when the parameters are perturbed, the closed-loop system changes a lot. Figure 12 shows the yaw rate response when parameters are perturbed. When parameters of the system are perturbed, the change of yaw rate response is small, and H_∞ control and H_2/H_∞ control still can well track the ideal yaw rate while H_2 control appears steady state error.

3.5.2 Double lane change simulation

The handing stability of the vehicle is closely related to the operation of the driver. In the study of the stability of the 4WS vehicle, the driver cannot be ignored. Therefore, driver vehicle closed-loop simulation is carried in this paper. The vehicle enters a double lane change at a speed of 15 m/s. The double lane change simulation results are shown in Figure 13.

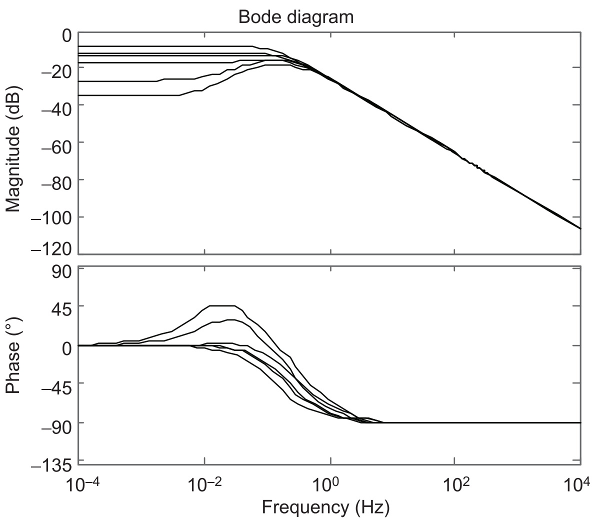

Figure 11 Closed loop system bode diagram under parametric perturbation.

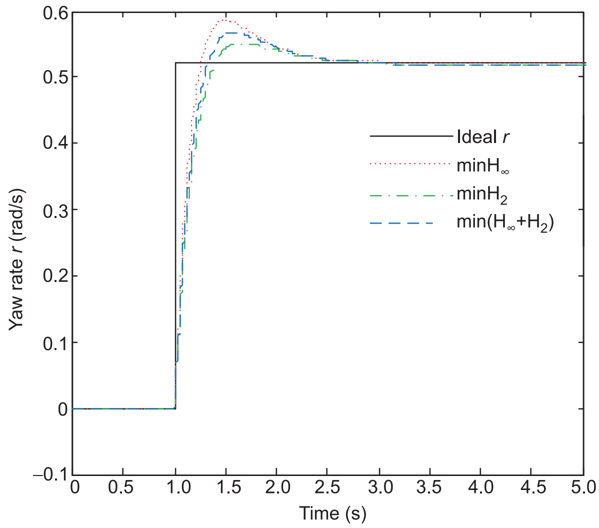

Figure 12 (Color online) Yaw rate response under parametric perturbation.

As can be seen from Figure 13, three kinds of controls can well complete the double lane change simulation. The oscillation of 4WS vehicle with H_∞ control is the smallest because the response time is the shortest. The oscillation of the vehicle with H_2 control is the biggest due to the longest response time. And the oscillation exists only in the back straight path. The vehicle with H_2/H_∞ control is between the two. In summary, the vehicle with three kinds of controls have good path tracking ability.

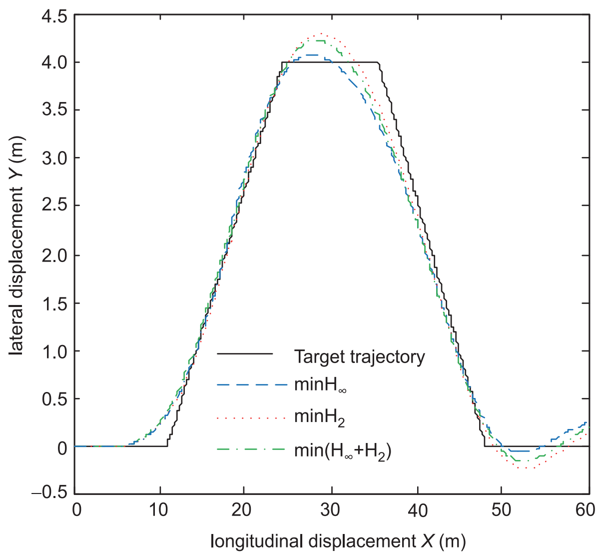

Figure 13 (Color online) Double lane road simulation.

4 Experiment

In order to verify the credibility of simulation results, real vehicle experiment is made in this paper. The test car is a 4WS vehicle and the steering angle of rear wheel is controllable. The steer-by-wire system employs a DC motor acting as steering actuator. In the experiment, the car running on a good road is given a steering wheel step input 90° at a speed of 90 and 30 km/h. H_2/H_∞ control is applied to the experiment. Yaw rate are obtained by sensor. The sample time of yaw rate sensor is 0.01 s. The experiment results are shown in Figure 14.

As can be seen from the Figure 14, 4WS vehicle can well track the ideal yaw rate by the designed controller regardless of low speed or high speed. The experiment result of yaw rate response shows that the control system almost has no steady state error. Compared with front wheel steering vehicle, 4WS vehicle can improve the flexibility at low speed and stability at high speed of vehicle. The experiment can further prove that the proposed control strategy is effective for 4WS vehicle and the designed controller has effect on the stability control of 4WS vehicle. Therefore, 4WS vehicle has better steering characteristics than front wheel steering.

5 Conclusions

1) 4WS vehicle dynamics model is established, including the rear wheel steering by wire system model, tire model, vehicle model and driver model;

2) According to the variable transmission ratio strategy, desired yaw rate is obtained. The yaw rate tracking strategy is applied to 4WS vehicle and rear wheel steering resistance moment is taken into account. Based on the robust control theory, H_2/H_∞ mixed robust controller design is carried to research the stability control of four wheel steering system.

3) The simulation results show that the mixed H_2/H_∞ robust controller design can achieve four wheel steering stability control and the experiment further verify the credibility of simulation results. The designed controller can well track the ideal yaw rate and restrain interference.

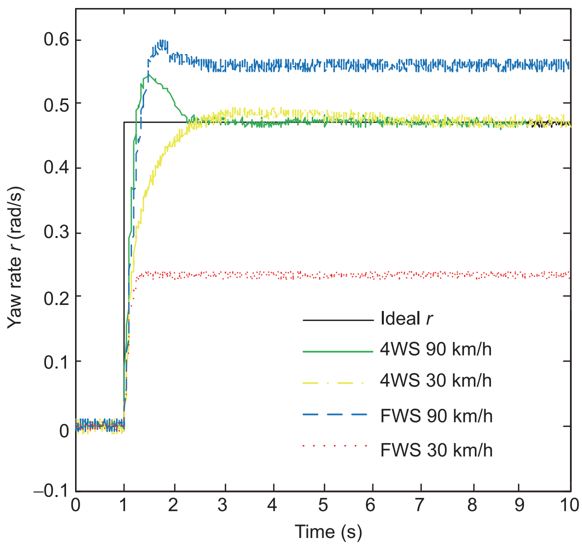

Figure 14 (Color online) Yaw rate of 4WS vehicle.

This work was supported by the National Natural Science Foundation of China (Grant No. 51375007), the Fundamental Research Funds for the Central Universities (Grant No. NE2016002), and Open Foundation of the State Key Laboratory of Fluid Power and Mechatronic Systems (Grant No. GZKF-201605).

1 Chen C, Shu M, Liu R. Virtual-point-based asymptotic tracking control of 4WS vehicles. Int J Control Autom Syst, 2015, 13: 371–378
2 Nonaka K, Oda T. Model predictive sliding mode control for four wheel steering and driving vehicles. IFAC Proc Volume, 2013, 46: 794–799
3 von Vietinghoff A, Kiencke U. Gain scheduling control for combined four wheel steering and individual wheel braking. IFAC Proc Volume, 2007, 40: 327–333
4 Li M, Jia Y, Du J. LPV control with decoupling performance of 4WS vehicles under velocity-varying motion. IEEE Trans Contr Syst Technol, 2014, 22: 1708–1724
5 Li M, Jia Y. Decoupling control in velocity-varying four-wheel steering vehicles with H_∞ performance by longitudinal velocity and yaw rate feedback. Vehicle Syst Dyn, 2014, 52: 1563–1583
6 Yin G D, Chen N, Wang J X, et al. Robust control for 4WS vehicles considering a varying tire-road friction coefficient. Int J Automot Technol, 2010, 11: 33–40
7 Song Z J. The analysis of transient stability for four wheel steering automobile with proportional control. Appl Mech Mater, 2011, 141: 534–538
8 Russell H E B, Gerdes J C. Design of variable vehicle handling characteristics using four-wheel steer-by-wire. IEEE Trans Contr Syst Technol, 2016, 24: 1529–1540
9 Mashadi B, Ahmadizadeh P, Majidi M, et al. Integrated robust controller for vehicle path following. Multibody Syst Dyn, 2015, 33: 207–228
10 Marzbani H, Salahuddin M H A, Simic M, et al. Four wheel steering advantageous for the autodriver algorithm. In: Neves-Silva R, Tshirintzis G A, Uskov V, et al, Eds. Proceedings of Smart Digital Futures 2014. Amsterdam: IOS Press, 2014. 505–512
11 Huang D W, Wang H L, Zhu Z W, et al. Stochastic stability of four-wheel-steering system. Chaos Soliton Fract, 2007, 33: 823–828
12 Zhao W Z, Li Y J, Wang C Y. Robust control of hand wheel torque for active front steering system. Sci China Tech Sci, 2015, 58: 107–116
13 Schramm D, Hiller M, Bardini R. Vehicle Dynamics: Modeling and Simulation. Heidelberg: Springer Publishing Company, 2014. 143–183
14 Zhao W Z, Wang C Y. Mixed H_2/H_∞ road feel control of EPS based on genetic algorithm. Sci China Tech Sci, 2012, 55: 72–80
15 Zhang H, Zhao W Z. Stability control strategy of steer-by-wire system based on LQG/LTR. Sci China Tech Sci, 2017, 60: 844–853
16 Huang D W, Wang H L, Guo Y F. Dynamical behaviors of a four-wheel-steering system under the sense of mean value. Appl Mech Mater, 2013, 419: 244–249