A survey on fabrication, control, and hydrodynamic function of biomimetic robotic fish

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Understanding and replicating the locomotion principles of fish are fundamental in the development of artificial fishlike robotic systems, termed robotic fish. This paper has two objectives: (1) to review biological clues on biomechanics and hydrodynamic flow control of fish swimming and (2) to summarize design and control methods for efficient and stable swimming in robotic fishes. Our review of state-of-the-art research and future-oriented new directions indicates that fish-inspired biology and engineering interact in mutually beneficial ways. This strong interaction offers an important insight into the design and control of novel fish-inspired robots that addresses the challenge of environmental uncertainty and competing objectives; in addition, it also facilitates refinement of biological knowledge and robotic strategies for effective and efficient swimming.

fish swimming, robots, robotic fish, hydrodynamics, motion control

1   Introduction

Over millions of years of evolution, fishes have developed a variety of morphological and structural features for moving through water with high efficiency, speed, and maneuverability [1–3]. In particular, fishes can survive in a range of extreme areas including deep-sea water areas, high mountain springs, and other harsh environments [4]. The biological principles underlying fish swimming have inspired man-made underwater systems for decades. To date, much effort has been devoted to the design and development of artificial fishlike robotic systems (i.e., robotic fish), mainly involving hydrodynamic analysis, mechanical design, control methods, and physical tests. The main impetus behind this endeavor is achievement of enhanced swimming performance in propulsive efficiency, speed, acceleration, maneuverability, and stealth over the conventional rotary propeller used in ship or underwater vehicles. Robotic fish has been applied in areas such as pollution detection, water quality monitoring, underwater exploration, oceanic supervision, and fishery conservation [5–9].

As indicated by ichthyologists, there exist many propulsion modes in swimming fish, most of which fall into two categories according to the body part utilized for propulsion: body and caudal fin (BCF) propulsion, and median and paired fin (MPF) propulsion [1]. The latter can further be subdivided into pectoral fin (PF) propulsion and undulation fin (UF) propulsion. A mainstream viewpoint holds that none of these modes of fish swimming is superior to the others because each species of fish has well evolved for its habitat [10]. More recent evidence suggests that fish rely on multiple control surfaces including caudal fin, pectoral fin, pelvic

fin, dorsal fin, anal fin, and body to achieve fast and maneuverable propulsion [11]. Undoubtedly, the use of multiple fins and flexible bodies significantly contributes to efficient propulsion, especially low-speed maneuverability and high-speed stability. The dynamically configurable multiple control surfaces offer an integrative paradigm to create next-generation bio-inspired underwater vehicles. On the one hand, robotics theories and experiments are applied to investigate the locomotion principles of biological fish, serving as a tool to decode the secrets of highly maneuverable, stable, and energy-efficient movements [12]. Discoveries and updated knowledge, on the other hand, provide better guidance for the development of high-performance autonomous underwater vehicles (AUVs) with stronger adaptability and robustness compared to conventional robots in complex, confined, and unstructured environments. Although only a few robotic fish-based real-world applications have been presented, owing to low overall efficiency and deficient inner space, cross-discipline efforts from biomechanics, fish physiology, material, fabrication techniques, and neural control are bridging the gap between fish-inspired biology and engineering.

The objective of this paper is to provide a comprehensive introduction to the state-of-the-art fabrication, control, and hydrodynamic function of robotic fish, which will shed light on locomotion generation, control, and deployment of biomimetic AUVs. In particular, compared with previous review papers [13–17], more emphasis is placed on the interaction of fish-inspired biology and engineering within the framework of fish swimming propelled by multiple control surfaces. It should be noted that, as biomimetic robotic fish is a fast-growing interdisciplinary area primarily interwoven with biology and robotics, it is not possible to discuss every published article. Thus, only typical examples are presented here, unless otherwise stated.

2　Biological studies on fish fins

Ray-finned fish possess fin rays that support multiple fins (Figure 1). The muscle at the fin ray base can be used to actively control the curvature of the fin ray and lead to complex 3D deformation of the fin surface [18]. These soft and controllable surfaces are frequently reported to function during various swimming modes and are widely believed to play a vital role in enhancing swimming performance. Biological investigation of fish fins can significantly expand our understanding of the high-efficiency, high-maneuverability performance of fishes and enlighten the design of future robotic fish. Thus, the hydrodynamic functions of some types of fins are summarized in Table 1 as a general overview.

2.1　Biomechanics of pectoral fins

Although the morphology and locomotion patterns of pectoral fins vary between different types of fishes, in general,
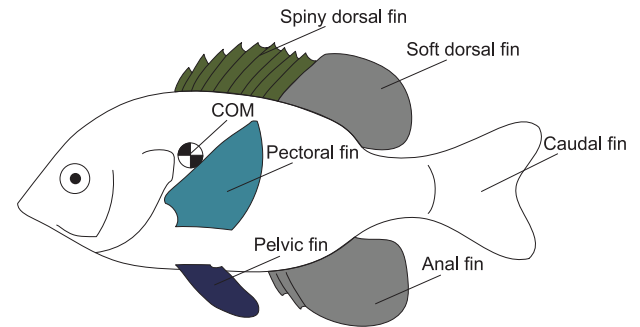
Figure 1　(Color online) Morphology and position of the fins of a bluegill sunfish. This type of ray-finned fish has complete fins and is often used in biology studies. The fin rays of the spiny dorsal fin are stiff and sharp and can bend only slightly. Therefore, the spiny dorsal fin can only perform erect or fold motion. The rays of the other fins, such as pectoral fin and caudal fin, are soft, flexible, segmented, and even branched, which enables complex 3D deformation. Note that the morphology and position of the fins vary with the species of fish. Some fish species may have even lost some types of fin during evolution.

pectoral fins play a critical role in body posture adjustment for all types of fishes that have a high aspect body ratio shape. For some fishes, pectoral fins are also the primary source of thrust in cases such as steady swimming at low speed and backward swimming [19,20]. As illustrated in Figure 2, the pectoral fins of ray-finned fish always possess complex three-dimensional (3D) motion. The formation of such a complex structure may be due to both passive transformations by the hydrodynamic load and active fin shape control by intrinsic muscles [18]. On the one hand, the highly flexible tissues, such as the fin membrane and the segmented rays, are liable to deform and bend under hydrodynamic force. On the other hand, the tendons at the ray base can contract to actively actuate the bilaminar structure of the rays, resulting in a bending motion or a change in fin ray stiffness. Apart from the regulation of a single fin ray, most fishes can control the fin base angle about the body, thus enabling control of the attack angle [21,22]. The significance of actively modulating fin shape, orientation, and stiffness is to change the direction of thrust and to switch propulsion mode between drag-base and lift-base mechanisms [23].

Two factors need to be considered when investigating the kinematics and hydromechanical repertoire of pectoral fins: the species of fish and their water environment. First, under different circumstances, pectoral fins are used for various purposes. For example, during braking, the ventrally positioned paired fins of trout will perform a spanwise rotation in the opposite direction and exhibit mediolateral and dorsoventral excursions that generate force posteroventrally, rotating the body about the center of mass (COM) to the weak side. However, during hovering and steady swimming, no such phenomenon can be observed [22]. When experiments shifted from laboratory to natural water, things were a little different. Most of the fishes in a natural water environment were found to beat paired fins during steady swimming [24]. Drucker and

Table 1 Summary of a fish fin's hydrodynamic function

Behaviors	Effect of movement patterns		Effect of morphology
	Movement patterns	Locomotor effects	
Pectoral fin	Reorienting the whole fin surface in the angle of the base	Altering the direction of the force	The position of the pectoral fins has a vital impact on the direction of the force. For example, the pectoral fins of the bluegill sunfish are located in a high position and can easily direct the force through the COM, which is favorable for body stability
	Curling up the distal margin	Generating force directed forward in braking behavior	
	Elevation and depression of the posterior fin margin	Initiating the pitching motion	
	Alternately beat with complex area control during one complete cycle	Generating thrust or reverse momentum jet in forward or backward swimming. Reducing the COM oscillation during steady swimming	
	Enlarging the fin area	Enhancing the drag to brake or thrust in low-speed forward or backward swimming	
	Actively touching the obstacles	Sensing and moving the obstacles	
Dorsal and anal fins	Moving the trailing edge alternately	Generating lateral wake jets that have important consequences for stability and maneuvering. Generating thrust or reverse momentum jets for forward or backward swimming	No intensive study has focused on the relative position of the dorsal and anal fins
	Performing out-of-phase motion relative to the caudal fin	Altering the fluid environment around the tail	
	Depression	Reduce drag	
	Erection	Enlarging the fish's lateral area and increasing the added momentum	
	Contracting the bilateral muscle at the fin ray base	Stiffening the fin to boost the ability of resisting water load	
Caudal fin	Flat shape	Steady swimming	The shape of the tail has a significant impact on the direction of the reaction force. Heterocercal tails generate thrust with a tilted angle corresponding to the horizontal plane, whereas homocercal tails do not

Where the Behaviors column lists:

Pectoral fin:
(1) Braking
(2) Vertical movement
(3) Backward swimming
(4) Low-speed steady swimming
(5) Moving through obstacles

Dorsal and anal fins:
(1) Steady swimming
(2) Backward swimming
(3) Moving through obstacles
(4) Various maneuvers such as C-start

Caudal fin:
(1) Steady swimming
(2) Braking
(3) Acceleration
(4) Glide

Lauder argued that the current is always accompanied with large-scale turbulence in natural streams, which requires that a fish recruit its paired fins to correct heading and body posture [22]. This contradicts the observation in a circulating water channel. Second, pectoral fins with diverse morphologies show different hydrodynamic functions. Take sturgeon and bluegill sunfish for example. The pectoral fins of the former are ventrally positioned and have little effect on the surrounding flow during steady swimming under low speed [25]. However, for the later, whose pectoral fins are vertically positioned, the pectoral fins provide most of the thrust force under steady swimming at low speed. In fact, no body undulation is formed and only pectoral fins move when swimming speed is less than one body length/s (i.e., BL/s) [19–21].

Except for the vital function in propulsion, maneuver, and correcting attitude, it has also been discovered that pectoral fins contribute to environmental sensing. Flammang and Lauder [26] showed that when bluegill sunfish approach obstacles, their pectoral fins did not steer clear of them; instead, they actively touched these barriers. Further, following the
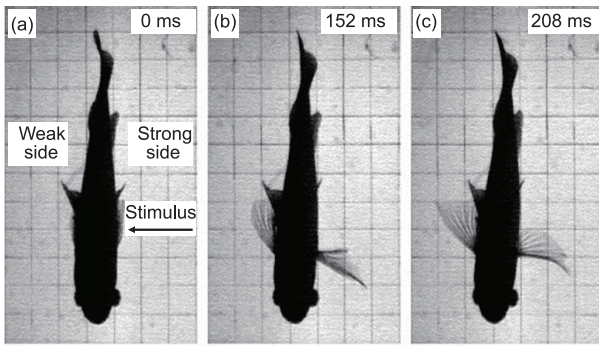
Figure 2 Ventral view of a bluegill sunfish performing turning maneuver with only pectoral fins. The fish was stimulated on one side while performing steady swimming. The pectoral fins that were originally adducted against the body flared immediately and inclined the head to the other side. The movements of the pectoral fins were not mirrored. In the figure above, the fin on the same side of the body as the turning stimulus is termed the "strong-side" fin, while the contralateral fin is termed the "weak-side" fin. Adapted with permission from ref. [21].

removal of a previously present light source, an increase in touch was observed. Their study demonstrates that fish can use their pectoral fins to help perceive their environment in complex underwater conditions.

2.2 Biomechanics of dorsal and anal fins

As opposed to pectoral fins, dorsal and anal fins are less agile and can scarcely act alone. In most cases, these fins coordinate together to achieve a certain function, as indicated in Figure 3. With the help of 2D DPIV (digital particle image velocimetry) experiments on bluegill sunfish and trout, researchers have demonstrated that the upstream fins (i.e., dorsal fin and anal fin) can affect not only the flow structures around the fish itself but also the flow downstream [27–29]. It has been demonstrated that when a fish is performing steady swimming, the trailing edge of the soft dorsal fin and the lead-

ing edge of the caudal fin cooperate and exhibit out-of-phase locomotion pattern. Studies conducted on trout have shown that its soft dorsal fin can direct the flow and generate a strong lateral jet in a direction that is not parallel but perpendicular to the fish's propulsion direction [29]. This flow structure with strong lateral velocity component can enhance the vortex shed by the caudal fin and hence increase thrust. In many fish species, the anal and dorsal fins can mirror the position of each other and have similar locomotion patterns; therefore, predictions and conclusions on dorsal fins can also be applied to anal fins [30,31].

In addition to the dorsal-caudal fin co-work system, a cooperative relationship also exists between the dorsal and anal fins. As previously mentioned, the locomotion of the dorsal and anal fins generates a lateral flow jet. The direct consequence of the lateral jet is generation of lateral force and torque relative to the body axis. Studies have shown that during steady swimming, a bluegill sunfish moves its dorsal and anal fins simultaneously, which counterbalances the torque generated by each fin. The locomotion of these two fins may not be in phase in some fish species, which may be due to the asymmetry of the fish body [27]. While the body of a needlefish is symmetric, its dorsal and anal fin motion shows no phase lag locomotion [32].

The dorsal fins of most teleost fishes have separated into two parts. In addition to the soft dorsal fin mentioned above, there also exists a spiny dorsal fin positioned ahead of the soft one that is supported by stiff fin rays. A popular theory is that the emergence of this type of thorny structure is closely related to the fishes' self-defense mechanism [33]. The fin rays of the spiny dorsal fin cannot curve laterally as can be accomplished by that of the soft dorsal fin. The spiny fin can spread and fold the fin membrane to change the fin area exposed to the flow field. It is surmised that the function of
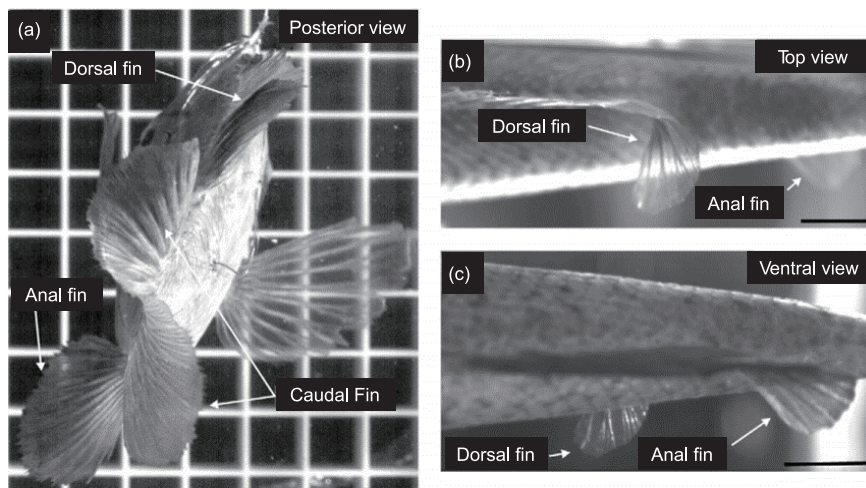

Figure 3 Median fin movement of a bluegill sunfish during maneuvering. (a) The pectoral fins, anal fin, dorsal fin, and caudal fin all work together to enlarge the area towards the oncoming flow, thereby boosting the drag for braking. (b) and (c) The fish curves its dorsal and anal fins to produce lateral force momentum during turning maneuver. Adapted with permission from ref. [27].

the erected fin is to cancel the lateral force and torque that may induce COM oscillation in the horizontal plane and rolling about the body axis, hence enhancing the stability of the fish. The locomotion of the spiny dorsal fin is often observed to accompany the motion of other fins. Studies conducted on yellow perch show that the spiny/soft dorsal and anal fins have different cooperative relationships under different swimming states. During steady swimming, these three fins are erect at low speed (0.5 BL/s), whereas they are folded against the body surface at relatively high speed. In the beginning phase of C-start, all the fins are immediately erect [27,30,31]. One potential explanation is that, when swimming at a low velocity, keeping the body balanced has top priority, whereas at high swimming speed, erected fins may do more harm than good as drag induced by an expanded surface requires more energy to compensate. For C-start locomotion, most of the fish body would involve thrust generation in an instant, which requires a large wetted area to produce high force therefore to escape.

2.3 Biomechanics of fish caudal fin

Some researchers have estimated that the thrust generated by the caudal fin accounts for approximately 90% of the overall thrust produced by a fish [1]. Although this assessed value does not reflect many factors such as the locomotion pattern, the swimming speed, and the effect of multi-fin, and may thus not reflect the fact precisely, it is undeniable that the caudal fin of a fish is one of the largest sources of its propulsion. To date, numerous studies related to fish caudal fins such as caudal fin musculoskeletal system [20,34], kinematics [35,36], and hydrodynamics [37,38] have been conducted. Nowadays, more studies are increasingly being conducted by applying physical models. Some key factors, such as the caudal fin shape, fin membrane thickness, fin ray stiffness, and movement patterns, are being systematically investigated using these biomimetic robots [39–43]. For experiments that are difficult to perform, such as investigation of propulsion performance under different fluid mediums and the pressure distribution on the fin surface, simulation models are widely utilized [44–47].

For the convenience of simplification, most of the studies conducted to date modeled the caudal fin as a flapping plate or foil, in which the caudal fin performing heave and pitch motions was usually rigid or possessed a certain flexibility. However, such a simplified model is not a replica of a real fish, which can change its morphology under different conditions. As shown in Figure 4, ray-finned fishes have an intrinsic muscle system, separate from the posterior axial body musculature, which can control the caudal fin rays and achieve complex 3D surface deformations [26,34,36]. It has also been observed that a bluegill sunfish tends to "cup" its caudal fin during steady swimming while forming an "S" shape in the trailing edge during braking. In addition, during
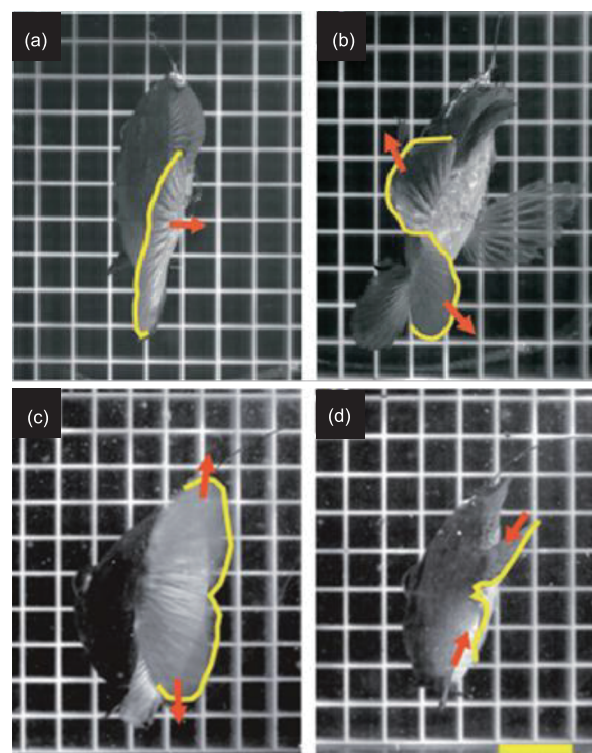
Figure 4 (Color online) Caudal fin 3D modulation of a bluegill sunfish during maneuvering. (a) The fish flattens the caudal fin during steady swimming. (b) The fish bends the caudal fin into an "S" shape to enhance drag during braking. (c) The fish enlarges the caudal fin surface when performing acceleration. (d) The caudal fin is depressed during gliding. Adapted with permission from ref. [36].

the burst and the coast state, the fish folds its caudal fin in the second phase [26,36].

The 3D movements of the caudal fin have also fascinated some engineers and roboticists. Consequently, a series of robotic models have been employed to conduct bio-fluid experiments aimed at revealing the mechanism behind 3D movements. For example, Esposito et al. [43] developed a physical model proportionally enlarged from an adult bluegill sunfish. The developed model can imitate five specific motions that are frequently observed: flapping, cupping, W shape, undulation, and rolling. Their results show that 3D motions, especially for undulation motion, can produce a significant lift force of the same magnitude as the thrust. Further, the force magnitude increases with the raised frequency. Inspired by this study, Ren et al. [48,49] considered the influence of the heave and pitch motions of the peduncle and the oncoming flow speed. They demonstrated that the phase between peduncle and fin ray motions could significantly affect the propulsion force and wake structure, with the underlying assumption that the fish could achieve vectoring thrust by coordinating peduncle and fin ray movements. In addition, they discovered that the lift force decreases distinctly at high flow speeds. This discovery helps to explain why the tail of high-speed fish is

relatively rigid and lacks 3D locomotion, whereas the tail of fish requiring high maneuverability is soft and flexible.

The above discussion focused on the homocercal caudal fin in bony fishes, in which the upper tail lobe mirrors the lower portion. 3D motions break the symmetry in morphology and achieve asymmetry in hydrodynamic functions [34]. For some fishes, such as sharks, the morphology of the caudal fin is no longer symmetric (heterocercal caudal fin). This characteristic causes the generated thrust to deviate from the body axis and is of great significance for body attitude correction [50]. Owing to lack of fin ray structures, a shark's caudal fin cannot achieve the complex 3D deformation achievable by ray-finned fish. However, its intrinsic muscle system can actively tune the caudal fin stiffness, which allows sharks to produce thrust virtually continually [51].

3 Robotic fish model with fins

Similar to the classification of propulsion modes of real fishes, robotic fishes can be generally divided into two categories.

The first type is BCF fish, which undulates its body and caudal fin to generate thrust and can achieve high swimming speeds. This type of robotic fish is the most common [17]. The second category is MPF fish, which depends on its paired or median fins for propulsion and possesses extraordinary maneuverability. The most common design for the BCF fish is to use multiple joints to fit the fish body curve, in which the rotating joint is always actuated by a small servo motor [52–57]. Although this type of design has been widely applied, its disadvantage is apparent: the body curve is discrete and cannot fully replicate the continuous undulation of a real fish. Marchese et al. [58] adopted another approach to achieve continuous body bending by constructing a body with soft materials. They constructed the fish trunk from silicone rubber molded with multiple inner chambers. Then, when the chambers are inflated with high-pressure compressed air, the body bends laterally on one side, which mimics the continuous bending shape of a real fish to a significant extent. However, because the undulation pattern is "pre-programmed" during the fabrication process, multiple swimming modes (such as anguilliform, carangiform, and thunniform) cannot be achieved on one platform.

All the robotic fishes discussed above try to mimic the body undulation of the real fish in a 2D plane, irrespective of the locomotory mechanism employed. However, the body of a fish possesses 3D morphology with variable body cross-section and multiple fins that move in complex structures [59]. Some robotic fishes are equipped with simple pectoral fins to help the fishes rise, sink, roll, turn, or swim backward [60,61]. However, the type of rigid plates employed, which can only adjust the attack angle, are not as pliant as those of a real fish. While the morphology of the robotic fish in ref. [54] is similar to a real fish with multiple fins, the function of its fins is mainly for decoration and cannot be regarded as functional fins. To date, no BCF robotic fish has successfully combined flexible control surface with undulatory body. However, the significance of multiple fins in improving the BCF swimming performance of robotic fishes is attracting an increasing amount of attention.

A series of bioinspired experimental robotic models have been developed to facilitate investigation of the influence of movable fins on fish swimming. These robotic prototypes investigate the hydrodynamic influence of the frequency and amplitude [62], multi-fin locomotion [63], fin surface 3D motion [49], fin surface area variation [64], fin shape [65], and stiffness variation [66], respectively. For convenience of engineering implementation and conducting experiments, the actuators of these devices are often arranged outside the body and cables are widely used to transmit power. Most of these models cannot swim freely; in general, they are fixed in a circulating flow channel or towed in a water tank during experiments.

MPF robotic fishes can be grouped into two types. The first type depends on an undulatory ribbon-like median fin to produce propulsion, mimicking bony fishes such as *Apteronotus Albifrons* and *Gymnarchus Niloticus* [67]. The second type flaps its paired pectoral fins to mimic a sort of cartilaginous fish called manta ray. Many robotic fishes have been developed using fin-based propulsion, including freeswimming robots and physical experimental models. Studies on median fin propulsion of robotic fishes can be found in ref. [68–71]. Studies on paired fin propulsion of robotic fishes can be found in ref. [72–74]. Inspired by undulatory fins in marine animals, a group from ETH has developed an underwater vehicle with four fin-like propellants that can realize multiple locomotion modes agilely [75]. The fins of the MPF robotic fish above are supported by flexible or multi-joint fin rays. The fin membrane is made of elastic fabric and can be stretched to some extent. By coordinating the motion of each fin ray, the In addition to the traditional robotic fin design, a structure termed "The Fin Ray Effect" was developed by engineers from FESTO (Festo A G, Co. K G. Available: https://www.festo.com/rep/en_corp/assets/pdf/Aqua_ray_en.pdf, December 12, 2016) based on the functional anatomy of the vivo fish and has been used to build the pectoral fins of the Aqua Ray. Driven by a water-hydraulic actuator, the robot can achieve smooth and high-efficiency movement.

In traditional underwater vehicle designs, motor and air cylinder are often adopted as the power source; gear, link, and joint are always used for power transmission. However, these components are limited when been applied in a compact space. Furthermore, they are too "rigid" to achieve flexible motions. Smart materials, with their ability to perform flexible and complex movements, bestow advantages in un-

derwater robot applications [76–78]. At least three types of smart materials have been successfully applied in underwater robot fabrication: SMA, IPMC, and lead zirconate titanate (PZT). The characteristics and application of these materials in underwater robots are reviewed in ref. [79]. These studies demonstrate that smart materials can competently serve as actuators for underwater robots. They can make mechanisms that are much more compact and significantly reduce the size of robots, while enabling their driving ability to be comparable to traditional actuators.

The fish fin possesses a high degree of freedom and can deform its surface into a complex structure. It is difficult to achieve such a complex locomotion by traditional mechanisms in a small space. Some previous researchers turned to smart materials to fabricate artificial biomimetic fins. For instance, Zhang et al. [78] developed a flexible and lightweight robotic pectoral fin inspired by a Koi Carp, which is driven with SMA wires. The SMA wires are embedded on the two opposite sides of a plastic plate. When a pulse width modulation current is delivered through SMA wires, the plate can bend under the contraction of the wire. Two plastic plates were arranged in series to construct a two-DOFs fin ray. This artificial pectoral fin is compact in mechanism and can achieve four typical movement patterns observed in vivo fish. Similar techniques have also been applied by Wang et al. [77], who embedded the SMA wires in flexible bending segments to actuate the motion of a micro-robot fish. The flexible materials used to build the bending segments can store the elastic energy at the bending phase and release the power at the return movement, which contributes to energy saving. Hubbard et al. [80] and Palmre et al. [81] applied IPMC to construct biomimetic pectoral fins. The IPMC is patterned in a monolithic actuator, which can achieve multiple bioinspired locomotion behaviors. Chen et al. [76] used a synthesis technique to fabricate a hybrid IPMC membrane actuator. The actuator consists of separated IPMC beams, which can be actuated individually to achieve complex 3D motion of the robotic manta ray. Compared to the fins driven by SMA, the output force, torque and motion range of the IPMC-driven fin are much more limited. A more detailed summary is given in Table 2.

4 Kinematic and hydrodynamic models of fish fins

Much effort has been invested toward building kinematic or dynamic models of movable fins. In this section, we review some of the biological and robotic studies conducted. The models not only provide us a better understanding of the extraordinary swimming ability of fishes, but also help us realize effective control of robotic fins.

4.1 Kinematic models of fish fin

Rigid rods or strips with some flexibility are widely used as fin rays in robotic fins, and the kinematic description of this type of robotic fin surface is straightforward [43,67,68,83,84]. Consider the modeling method used in robotic undulatory fin study, for instance; the surface structure of the fin can be modeled as a ruled surface in 3D space [68,69], as shown in Figure 5(a). In this model, the fin is regarded as a ruled surface with zero thickness. The fin baseline and the fin ray are considered to be directrix and generatrix, respectively. The position of an arbitrary point P on the fin ray can be expressed as follows:

$$p(r,s,t) = b(s,t) + rd(s)c(s,t)\,(0 \le s \le L), \tag{1}$$

where $b(s,t)$ is the directrix function, $c(s,t)$ is a time-varying unit vector of the fin ray at $b(s,t)$, $d(s)$ is the length of the fin ray, r is a value between zero and one, and $rd(s)$ can be used to indicate the position of a point on an individual fin ray. Each fin ray is frequently actuated individually with a motor to form the fin trailing edge into a certain wave. When the wave transmits from anterior to posterior rays, the rotated angle of each fin ray can be expressed as follows:

$$\theta(s,t) = \theta_0(s)\sin\left(\frac{2\pi}{\lambda}s + \frac{2\pi}{T}t + \phi\right), \tag{2}$$

where ϕ is the phase lag between the first fin ray and the fin ray at position $b(s,t)$, and λ and T are the wavelength and the period, respectively.

4.2 Dynamic models of fish fin

Dynamic models can be formulated by reasonable simplification and force analysis based on kinematic models. A good example of a dynamic model of a robotic fin can be found in ref. [84,86]. As illustrated in Figure 5(b), a dynamic model is based on a two-ray unit and can be easily extended to consider the condition where multiple two-ray units are arranged serially. In particular, the following requirements are assumed to be satisfied:

(1) The two-ray unit is restricted by a slide rail and can only move along it;

(2) The fin ray is rigid and cannot be bent;

(3) The fin membrane is tautly attached to the fin rays and can withstand hydrodynamic loads without buckling.

When the fin unit is flapping in the water, its dynamic equation can be expressed as follows:

$$L(q,\dot{q},q) + \Gamma(q_1 - q_2) + \phi(q,\dot{q}) = \tau. \tag{3}$$

The dynamic equation is established under a generalized coordinate system $q = [q_1 \quad q_2 \quad q_3]$. The three coordinate components represent the displacement of the unit and the rotated angle of each fin ray, respectively. $\tau = [\tau_0 \quad \tau_1 \quad \tau_2]$ is a generalized force: τ_0 is the force applied to the slide pair on the rail by the moving fin unit; τ_1 and τ_2 indicate the input torque of

Table 2   Typical robotic fishes with controllable fins[a]

Prototype	Biological counterpart	Mode	Actuation	DOFs	Characteristics	Organization
PDAC device [62]	Bluegill sunfish	BCF	Cable driven	4–5	Integrating multiple fins and the caudal peduncle motion	Drexel University
Robotic caudal fin [82]	Bluegill sunfish	*	Cable driven	6	Experimenter can change the fin rays with various flexibilities	Drexel University
Robotic pectoral fin [82]	Bluegill sunfish	*	Cable driven	15	Each fin ray can be driven up to three actuated DOFs: sweep, in-plane rotation, and curvature control	Drexel University
Robotic caudal fin [49]	Bluegill sunfish	*	Cable driven	7	The heave and pitch motion of the caudal peduncle is incorporated	Beihang University
Bioinspired pectoral fin [76]	Koi carp	*	Shape memory alloy (SMA)	10	Each fin ray was assembled using two SMA-driven plates placed in series	Nanyang Technological University
Robotic knifefish [67]	Ghost knifefish	MPF	Servo motor	32	Can generate various types of waves: inward counter-propagating waves, unidirectional waves, and outward counter-propagating waves	Northwestern University
Robotic batoid ray [76]	Manta ray	MPF	Ionic polymer metal composite (IPMC)	4	Each fin was fabricated by bonding the IPMC beams with PDMS	University of Virginia
Sepios [74]	Ghost knifefish	MPF	Servo motor	9	Realizes high maneuverability with its four undulate fins	Swiss Federal Institute of Technology
Aqua Ray [75]	Manta ray	MPF	Water-hydraulic driven	2	Combines the fin ray effect with fluidic muscle	Festo AG & Co. KG

a) * means that the robotic model mimics only the fin instead of the complete fish.

two rays. $\phi(q,\dot{q})$ represents the generalized force applied to the fin ray joint by the fluid. $\Gamma(q_1 - q_2)$ is the elastic force generated by stretching the fin membrane. $L(q,\dot{q},q)$ incorporates the influence of friction, gravity, and unit inertial force. Before determining the hydrodynamic force acting on the fin ray joints, the force f_n acting on an area element ds is first considered. The direction of force is deemed to be perpendicular to the finite area:

$$f_n = -\frac{1}{2}\rho C_n e_n v_n^2 \mathrm{sgn}(v_n), \tag{4}$$

where ρ, C_n, e_n, and v_n indicate fluid density, friction coefficient, the unit vector perpendicular to the finite area, and the speed of the finite area relative to the fluid, respectively. Integrating the hydrodynamic force acting on the finite areas and using a transformation matrix J to switch the force in generalized coordinate system gives the following:

$$\phi(q,\dot{q}) = \int J f_n \mathrm{d}s. \tag{5}$$

Experimental results agree well with the theoretical prediction. However, both theoretical models and the experiments only incorporate the condition in still water. Following research questions still exist: how well does the model work in more sophisticated underwater environments? What happens when the robotic fin can move freely in more than one direction? Are there any correction terms that should be imported into the equation to satisfy the requirements of more complex underwater environments?

Some robotic fins possess a mechanism that is close to that of real fish fins [87]. In these robots, the fin ray is no longer a rod that can rotate about its joint. Instead, it can bend under the stress at the fin ray base. Alben et al. [18] established a model that shows the relationship between the external force and the bending angle well (shown in Figure 5(c)). The model is based on the following prerequisites:
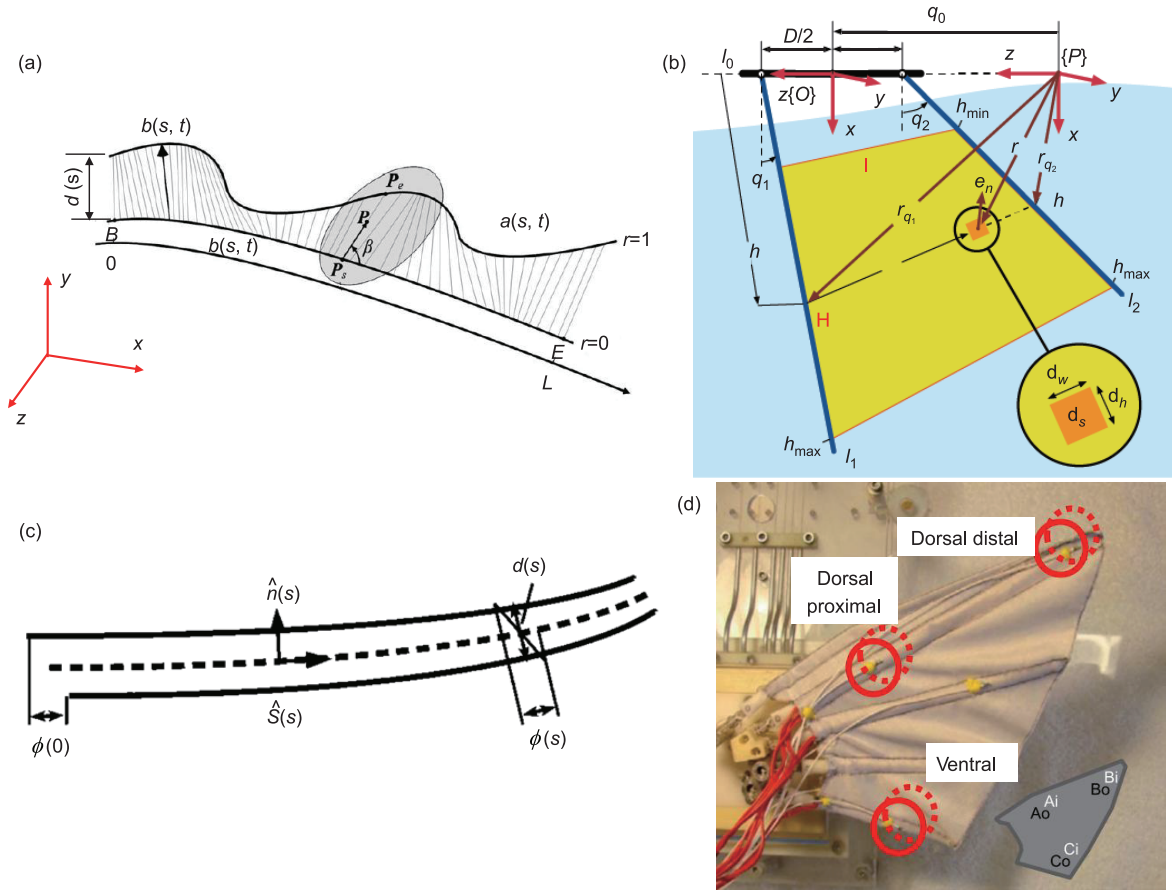
Figure 5 (Color online) Schematic for fin modeling. (a) Undulatory ribbon fin is simplified as a ruled surface with zero thickness. (b) Two-ray unit and the establishment of the generalized coordinates. (c) A simplified model of a single fin ray. (d) Sensors spread on the surface of a robotic pectoral fin. Adapted with permission from ref. [85].

(1) Constitutive equation of elastomer can be applied to the fin ray.

(2) The materials in the bilaminar structure cannot be compressed.

(3) The hydrostatic equilibrium equation of a single fin ray can be expressed as follows:

$$-2\partial_s\left(\frac{B\partial_s(\phi)}{d^2}\right) + \frac{G\phi}{d^2} + \partial_s\left(\frac{M_{ext}}{d}\right) = 0. \tag{6}$$

Here, apart from the bending angle ϕ and the external torque, all of the remaining terms are structural parameters of the fin ray. B, G, and d indicate the bending rigidity, the shear modulus of the inside material, and the diameter of the fin ray, respectively. Experimental results indicate good consistency with the theoretical prediction when constraints are added at the base and distal point of a real fish fin ray.

It should be pointed out that dynamic model is infrequently applied to determine the input of a robotic system directly in most cases. Some terms in dynamic equations, such as the hydrodynamic force acting on the fin surface, are difficult to obtain in real time. One possible alternative is to establish a control model based on values that are easy to measure. For example, inspired by the perception ability of pectoral fins, Jr.Kahn et al. [85] spread multiple pressure/bending sensors on a robotic pectoral fin (Figure 5(d)) and found that the sensor modalities and the placement locations all have a significant impact on force prediction.

5   Motion control of fishlike swimming

Swimming locomotion can generally be categorized according to the temporal features of movements as periodic (or steady) swimming and transient (or unsteady) swimming. The former is characterized by a cyclic repetition of the propulsive movements, whereas the latter often lasts milliseconds and is utilized to catch prey or escape predator [1]. To date, the control mechanisms of fish bodies and fins are not entirely understood. As stated previously, fish swim using multiple body segments and organizing left-right alternations in each segment so as to produce the body wave that propels them through the water. From the perspective of neural control, these rhythmic motor patterns are internally

generated by central pattern generators (CPGs), i.e., central neuronal circuits whose activation can produce rhythmic patterns in the absence of sensory or descend inputs that carry specific timing information [88,89]. Thus, the neural system can generate and control a large variety of motor behaviors via coordination among segmental CPGs. Typically, CPGs have advantages such as rhythmicity, stability, adaptability, and variety [90]. These fascinating properties make CPGs well suited for locomotion control of robots with multiple joints or DOFs and even of hyper-redundant robots. By virtue of the inherent nonlinear dynamic property of the CPG models, jerky changes in oscillating joint angles can be avoided and smooth transition between gaits can be ensured. Notably, CPGs remove the need for trajectory planning and precise knowledge of mechanical system characteristics.

When BCF-type swimming is being imitated, the link-based body wave fitting is intuitively exploited to generate fishlike motions. Biologically, the body wave of a traveling fish appears as a curvature of the spine and muscle with increasing amplitude. A widely used body wave is mathematically described as follows:

$$y_{\text{body}}(x,t) = (c_1 x + c_2 x^2)\sin(kx + \omega t), \qquad (7)$$

where y_{body} represents the transverse displacement of the moving tail, x denotes the displacement along the main axis, k indicates the body wave number ($k = 2\pi/\lambda$), λ is the body wavelength, c_1 is the linear wave amplitude envelope, c_2 is the quadratic wave amplitude envelope, and ω is the body wave frequency ($\omega = 2\pi f = 2\pi/T$). Yu et al. [53] proposed a discretized body wave design method, in which the oscillatory body is discretely designed as a multi-linkage (or N-link) mechanism composed of several oscillating hinge joints actuated by motors. The discrete form of eq. (7) is given by:

$$y_{\text{body}}(x,\kappa) = (c_1 x + c_2 x^2)\sin\left(kx \pm \frac{2\pi}{M}\kappa\right), \qquad (8)$$

where κ denotes the κ-th variable of the sequences $y_{\text{body}}(x,\kappa)$ ($\kappa = 0, 1, \ldots, M-1$) in one oscillation period and M indicates the discrete degree of the overall traveling wave.

In the interest of simplicity, the movements of joints functionally corresponding to control surfaces are considered as oscillatory in a harmonic (sinusoidal) fashion, which can be described as follows:

$$\theta_i(t) = \bar{\theta}_i + A_i\sin(2\pi f_i t + \phi_i)(i = 1, 2, \cdots, N), \qquad (9)$$

where $\theta_i(t)$ indicates the angular position of the i-th control surface at time t, $\bar{\theta}_i$ signifies the angular bias for asymmetric oscillations deviating from the median position, A_i represents the oscillatory amplitude, f_i is the oscillatory frequency, and ϕ_i is the phase difference. As a response to the input data, the swimming speed can be mediated by the magnitude of amplitude (A_i) and frequency (f_i), while the direction is influenced

by the bias (ϕ_i).

However, this body wave-based swimming control should be properly discretized and parameterized for a specific swimming gait, which is often time-consuming and intractable [53,54,91]. Furthermore, the swimming stability of a desired gait and the smooth transition between two different gaits are scarcely guaranteed. Inspired by the neurobiological principles of rhythmogenesis, CPG-based locomotor controllers have been designed and implemented in various swimming robots, such as anguilliform [92,93], carangiform [94,95], ostraciiform/crawling [96], thunniform [97], undulatory fin [98], dolphin-like [99], flapping [100,101], and amphibious [102] swimmers. Among existing CPG models, coupled nonlinear dynamic systems are commonly employed. As the intrinsic stability properties allow for feedback integration, sensory feedback that modulates CPG activity tends to result in an environment-adaptable locomotion. To close the loop around the neural systems, sensory feedback is integrated into the CPG models to eventually affect the output of the model under the hierarchical control architecture to some extent. In particular, feedback loops coupled with learning algorithms and optimization techniques allow the CPGs to find the most effective or efficient output through continuous interaction with the environment [103,104]. Some good examples in which sensory feedback is employed in the CPG control to improve adaptability and autonomy of robotic fish can be found in [91,96,104].

During the implementation of CPG-based swimming control, the core issue in building a CPG model is to find the coupling ways of nonlinear oscillators. Usually, one fin corresponds to one oscillator, and multiple oscillatory fins constitute a network of oscillators. Different types of couplings such as unidirectional, bidirectional, mutual, and the nearest neighbor coupled can be utilized to achieve diversified motions. Determination of the topological structure largely depends on the actual situation of a robot application, because couplings are tightly linked to gait modulation and gait transition in terms of speediness, stability, and smoothness. Offline generation of swimming gaits in conjunction with online optimization or learning offers an effective means to produce multimodal swimming [105–107]. Furthermore, unbalanced thrust forces generated by fins can be used for steering, typically through modulating frequency, amplitude, and phase relation. Thus, powerful and versatile propulsion solutions can be obtained in the context of CPG-based control, involving diverse swimming by different fins, agile turns by altering phase relations, and online gait transitions.

6 Hydrodynamic experimental technologies

With the development of new technologies, novel experimental methods have emerged to help scientists conduct their experiments and understand the underlying mechanism of

fishes. In this section, we review several novel measures developed to investigate the swimming mechanism of fishes that are highly suited for application in robotic experiments. We trust that these methods will enlighten readers who will perform hydrodynamic fish swimming experiments in the future.

6.1 3D wake flow acquisition

DPIV measurement has widely been used in bio-fluid experimental studies to obtain wake flow structures on a certain plane. Before conducting such experiments, trace particles are seeded evenly into the water and a laser sheet is adjusted to a predetermined position to illuminate the particles on this plane. The movements of the particles on the plane are then recorded using a high-speed camera. Using cross-correlation, the velocity field and the wake flow structures are eventually obtained from the image sequence. The results obtained in DPIV experiments are important to estimate the propulsion performance of fishes or a physical model [55,108–112]. However, the structure of the wake flow is 3D; the results from only a 2D plane cannot reflect the overall view and may lead a researcher to make wrong judgments. Undoubtedly, a 3D view of the flow structure can significantly enhance our understanding of the mechanism underlying fish swimming [38,113,114].

A series of experimental technologies, including tomographic PIV, stereo PIV, and defocusing digital PIV, have been developed over the past ten years to reconstruct 3D flow structures. Detailed descriptions of these methods are reviewed in ref. [115,116]. The practice of obtaining the three-dimensional flow structure surrounding a real fish is discussed in ref. [38,114,117]. In contrast to 2D PIV measurements, the light source used in 3D PIV is volume light and the trace particles in the observation area are recorded using multiple high-speed cameras or a camera ar-

ray (Figure 6(a)). The 3D flow structure is then reconstructed from multiple image sequences (Figure 6(b)). However, because of its high cost, this type of measurement is not widely applied in fish biomechanics studies. A more economical choice for obtaining fluid overview is to integrate 2D PIV results on multiple planes. In this method, the light source is a thick light sheet (2–8 mm) rather than a volume light source; the number of cameras can be reduced to two. To track the particle velocities in three directions, the charge-coupled device (CCD) of the high-speed camera is set at a particular angle to the illuminated plane, which meets the requirements of the Scheimpflüg conditions. This guarantees that clear image sequences are obtained. However, this method can only reconstruct the 3D velocity field only on one plane at a time rather than the entire volume space simultaneously [49,113,118,119].

6.2 Fluid experiments using robotic models

Experiments on real fish are often difficult to control. First, animals are intractable, which makes test repeatability difficult to attain. Second, animals interact with their environment in a sophisticated manner, which is difficult to investigate via quantification. Third, an animal is an intricate organic system that involves overly many factors in which we are not interested but which may have a significant impact on observation results. To avoid these disadvantages in biological studies, researchers have turned to bioinspired robots to examine the mechanism underlying fish swimming in depth [12,59,62].

Some bioinspired robots employed in experiments are complex and replicate the morphology and locomotion pattern of its biological counterpart to a high degree, such as the multi-fin fish robot performing fishlike fin motions [62], the glass knife fish robot possessing undulatory median fin [71], and the salamander robot that can transfer gait between gro-
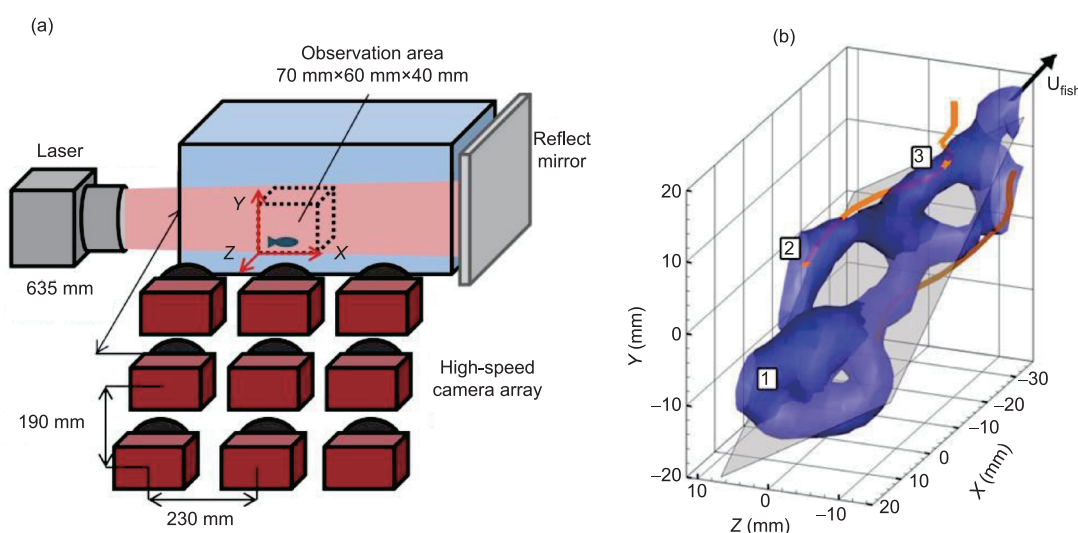
Figure 6 (Color online) Application of 3D PIV in a bio-fluid experiment. (a) Schematic diagram of synthetic aperture PIV experimental setup. (b) 3D reconstruction of the wake flow structure behind a fish's caudal fin. Adapted with permission from ref. [113].

und and water [120]. However, it is not the complexity of a robotic system that determines whether the experimental setup is efficient. Instead, it is whether the robotic model can mimic key elements in animal locomotion and morphology characteristics that matters. Some simple robotic systems can also play a critical role in experimental investigations. For example, the flapping flexible foil, which mimics the 2D body wave of a swimming fish, has been demonstrated to be effective in studies conducted on thrust efficiency and COM oscillation [42,107,121,122], even though this type of model is far from a real fish in appearance.

6.3 Experimental self-propelled swimming methods

Although a host of robotic fishes that can swim freely have been developed successfully [6,56,60,123], their application in investigating the swimming mechanism is limited. The biological relevance of a robotic fish is often regarded as "trivial": it is the swimming performance, such as the cruising speed, maximum navigation range, and the turning radius that garner the most interest. However, such "trivial" characteristics as actively movable fins, flexibility of the biological tissues, and intricate musculoskeletal system, are crucial to a fish and will require deep study in the future. From this viewpoint, robotic systems that cannot propel themselves freely but precisely replicate biological characteristics are more suitable for bio-fluid experimental studies [43,52,55].

Three experimental methods have been applied to help these robots to generate a "forward speed". The first solution is to fix the robotic model in a circulating water channel that can provide stable flow towards the model [43]. However, the problem with this scenario is that because the propulsion of fish is an unsteady process [108,124], the forward speed is not constant with the oscillation of the COM in the vertical and horizontal directions. Furthermore, the stable flow does impede maneuverability studies, in which the flow speed needs to be varied in real time. The second method is to use a towing system to tow the model forward in a water tank [48]. However, because the robot is moved passively, such scenarios still cannot reflect the real freely swimming situation. The third method is to mount the robot on an air track and let the robot propel itself along the rail. This method is not as perfect as it seems: its greatest challenge is the question of how to reduce the influence of the additional mass from the experimental setup. For example, in ref. [55], many apparatus including laser, lens group, and high-speed cameras have to move with the robotic fish to record the wake flow structures in real time. All these pieces of equipment have an additional mass that can significantly influence the swimming performance of the fish.

To overcome the disadvantages besetting traditional experiments, Wen et al. [125] proposed a self-propelled method based on force feedback control. A schematic diagram of the proposed method is shown in Figure 7: F_x is the net thrust
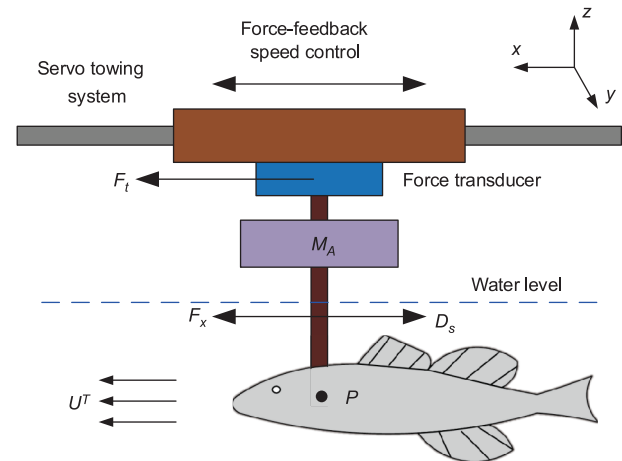
Figure 7 (Color online) Schematic of a self-propelled system. Adapted with permission from ref. [125]. The robotic model is fixed to a towing system that can provide a towing speed along the x-axis. The force transducer, mounted between the towing system and the robotic model, can measure the force generated by the model in real time. The towing system can change the towing speed instantaneously according to the measured force, which guarantees that the model moves at a self-propelled speed.

generated by the robotic fish, i.e., the resultant force of drag coming from the fish body and the thrust produced by the fish locomotion; D_s is the drag of the strut, which can be ignored; U^T is the towing speed at time T; M_A is the additional mass; and F_t is the force acting on the robotic fish from the towing system. When the robotic fish is swimming freely, F_t should be zero. Therefore, the concept underlying this method is to control towing speed U^T actively to ensure that the average F_t in one cycle is close to zero. In this manner, the power needed to accelerate the additional mass is compensated by the servo towing system.

7 Future studies

The evolution morphology of living organisms enables them to adapt to their environments. Many of these amazing creatures are usually delicately designed with composite materials, complex musculoskeletal systems, and nested neural systems, which poses challenges when attempting to translate these biological design into engineering implementations. The fin is considered to be critical in most of the swimming behaviors of fishes, and fin-like robotic mechanisms have already been applied in some underwater vehicles. However, compared to real fish fins, these devices seem clumsy with far fewer DOFs, and controllable fin surface and actuators with lightweight, waterproof properties. There are multiple challenges that need to be overcome before the entire functions of a real fish fin can be realized on an engineering platform. Perhaps one of the major differences between a real fin and a robotic fin is whether it can be used for environmental perception. For example, the pectoral fins of a real fish play a significant role in navigation [26], which is

very difficult to incorporate into engineering prototypes. The question of how to add sensory functions to a robotic fin and use feedback signals to aid navigation is worth exploring. Some researchers have used distributed pressure sensors for thrust prediction on a robotic pectoral fin [80]; however, such a sensor network is still difficult to apply to a small fin owing to limited space.

Second, the question of how to simplify the actuation system is another engineering challenge. To precisely replicate the 3D movement of a ray-finned fish fin, the robotic fin is usually designed with multiple, individual actuators. For example, the Sepios robot has 36 servo motors to control four undulatory fins [75]. These rigid actuators significantly increase the robot's size and weight and raise difficulties in control. Most of the body of a fish is flexible and some parts of the bilateral body muscles are relaxed during swimming to enable undulatory motion and efficient swimming [126]. Researchers have imposed heave and pitch motions on one end of a flexible foil and achieved fishlike undulation [108]. The same principle could also be incorporated into future artificial fin design, utilizing only a few actuators to drive an under-actuated robotic fin system. Taking the actuation frequency, force output, size, and robustness into account, we also consider the fluidic actuator as a favorable choice for robotic fin actuation. Successful application [127–129] of soft actuators demonstrates that soft actuators are sufficiently powerful and reliable, and could be used to provide multi-DOFs and muscle-like movements in the future.

The third challenging issue is fin surface control. More research needs to be conducted to understand the hydrodynamic functions of fish fin under different circumstances to facilitate the construction of better control models. With the rapid development of robotic technology, future work may turn to robotic fish models with both undulatory body and active moving fins. In a self-propelled swimming state, a robotic fish could mimic the locomotion of a real fish, enabling quantitative analysis with the application of multi-axis force transducers. DPIV experiments would also be extensively conducted in the future to measure the complex vortex structures generated by both the soft fin surface and the undulatory body, thereby providing answers to fundamental biological questions. The fin surface and the body need to be controlled simultaneously to achieve harmonic motion and boost overall hydrodynamic swimming performance. A live fish possesses a hierarchical neural system with a complex set of feedback loops as well as a sophisticated sensory system. Mirroring a fish's neural system in a robot via engineering technologies may be a good choice to realize effective fin control.

8 Conclusions

In this paper, we reviewed biological clues for biomechan-

ics and hydrodynamic flow control of fish swimming and summarized design and control methods for efficient and stable swimming in robotic fishes. Specifically, we reviewed state-of-the-art biomechanics studies, integrative theoretical models, and bio-robotic models of robotic fishes with fins. Then, we introduced primary motion control methods to generate fishlike swimming, explored hydrodynamic experimental technologies, and discussed some critical issues and promising research directions.

Fishes possess many interesting structural features of potential significance for locomotion that have yet to be studied. The use of mechanical devices to explore the functioning of these features promises to help unlock some of the secrets of fish swimming performance. Implementing a fishlike flexible underwater bio-robotic system incorporating both undulatory body and multiple soft fins remains a challenging task for future research. This bio-robotic device would strike a balance between advanced multidisciplinary science and cutting-edge engineering, yielding an emerging technology for future underwater vehicles with high thrust efficiency and maneuverability.

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