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Seizures dynamics in ^a neural field model of cortical-thalamic circuitry

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The focus of this study is to explore the mechanisms during seizure behavior using ^a ^physiologically motivated by corticothalamic circuity. The model is based on the assumption that, the inhibitory projects from thalamus reticular nucleus (TRN) to specific relay nuclei (SRN) are mediated by GABA_A and GABA_B receptors which react different time scales in synaptic transmission. Secondly, we include the effects of slow modulation on the threshold current of TRN population that were found to generate bursting behavior. Our model can reproduce healthy and pathological dynamics including wake, spindle, deep sleep, and also seizure states. In addition, contour maps are used to explore the transition of different activity states. It is worthy to point out seizure duration is significantly affected by ^a time-varying delay as illustrated in our numerical simulation. Finally, ^a reduced model ignoring the cerebral cortex mass can also capture the feature of spike wave discharge as generated in the full network.

seizure, spike wave, neural field, modeling, corticothalamic

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1 Introduction

Epilepsy is one of the most common neurological disorders characterized by recurrent seizures which affects up to 1%–5% of people all over world as reported [\[1\]](#page-9-0). The recurrent seizures are synchronized, paroxysmal, and excessive discharges of neural populations, and they are classified into two main categories: partial and generalized [\[2\]](#page-9-0). In essence, ^a partial seizure can remain confined to its focus, or can migrate to other brain areas, causing ^a secondary generalized seizure. Therefore, seizures are generated by synchronized bursting of ^a group of cortical neurons, leading to increased coherence in the recorded signal. Generalized seizures are pathological brain rhythms across most or all of the cortical regions and are associated with loss of cognitive activity, while absence and tonic-clonic seizures are two common generalized seizures in human. In clinical, these transient seizure events can be observed by the electroencephalogram (EEG) [\[3\]](#page-9-0). Electrode probes measure the mean local field potential generated by synchronous activity of large assemblies of pyramidal cells as well as excitatory and inhibitory interneurons located in the vicinity of the probe contact [\[4\]](#page-9-0).

The cause of epilepsy and its effects on cerebral functions is so complex that it is ^a formidable task to conceive ^a single framework to characterize all the pathophysiological changes in epilepsy at molecular, cellular and neuronal network level [\[5\]](#page-9-0). Considering seizures can be viewed as ^a time involving or dynamical disease [\[6,7\]](#page-9-0), mathematical modeling can be

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conducted to obtain new insights into epileptic seizures [\[8–11\]](#page-9-0). Indeed they have been successfully used to gain in-depth knowledge and generate novel hypotheses related to the cellular and network level brain mechanisms of epileptic seizure, and as ^a tool to guide the prediction of impending seizure and alternative therapeutic treatment [\[12,13\]](#page-9-0).

Neural mass models initiated by Freeman [\[14,15\]](#page-9-0) and then used in mathematical models are so successful in modeling [\[16–18\]](#page-9-0) and produce brain rhythm observed in macroscopic measurements such as EEG [\[19–22\]](#page-10-0). It is known that the thalamus and cortical interactions are found to ^play an essential role in the generation of epilepsy spike-wave discharge [\[23,24\]](#page-10-0) as ^a classical symptom of absence seizure [\[25–27\]](#page-10-0). Therefore ^a macro scale model using neural field theory based on corticothalamic system has been often used and modified and enhanced in seizure [\[28–31\]](#page-10-0). This class of neural field model describes the dynamical behaviors of large interacting groups of neurons in cortex and thalamus, which are referred as two neural layers [\[32–34\]](#page-10-0). Thalamic componen^t is assumed to consist of two neural masses, an excitatory mass of SRN and ^a inhibitory mass of TRN. Similarly, the cortical componen^t incorporates both ^a mass of excitatory pyramidal cell and inhibitory interneurons.

Here we study the seizure behavior through enhancing Freeman's corticothalamic neural field model, which describes the relevant brain activity through ^a second order delay differential equation. Our model has been able to reproduce many properties of EEG observation in ^a ^physiologically ^plausible parameter region.

2 Methods

The cortico-thalamic circuity studied here is ^a mean field neural model since the variables represen^t the local mean value of ^a ^physiological process [\[35,36\]](#page-10-0). It is based on several dynamical variables within three masses: ^a mass of cortical pyramidal neurons, cells in SRN and cells in TRN as illustrated schematically in Figure 1.

Combining some realistic assumptions extracted from theoretical studies and experimental clinical data [\[37–39\]](#page-10-0), especially crucial works in refs. [\[40,41\]](#page-10-0) which enable the partial differential equation (PDE) description of the cortical propagation and ^a reduction to an ordinary differential equation (ODE) model, our model can be described by ^a system of delay differential equations:

$$
\frac{\mathrm{d}\dot{V}_e}{\mathrm{d}t} = \alpha \beta \left[v_{ee} \varphi_e + v_{ei} S_e (V_e) + v_{ei} S_s (V_s) - \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) \dot{V}_e - V_e \right],
$$
\n
$$
\frac{\mathrm{d}\dot{V}_r}{\mathrm{d}t} = \alpha \beta \left[v_{re} \varphi_e + v_{rs} S_s (V_s) - \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) \dot{V}_r - V_r \right],
$$
\n
$$
\frac{\mathrm{d}\dot{V}_s}{\mathrm{d}t} = \alpha \beta \left[v_{se} \varphi_e + v_{sv} S_n (V_r) + v_{sv} S_n (V_r (t - \tau)) \right]
$$

$$
-v_{sn}\varphi_n - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\dot{V}_s - V_s,
$$

$$
\frac{d\varphi}{dt} = \gamma_e^2 \left[S_e(V_e) - \frac{2}{\gamma}\dot{\varphi}_e - \varphi_e\right],
$$

$$
\frac{dX}{dt} = -\frac{1}{\tau_x}(X - X_x),
$$

$$
\frac{dH}{dt} = -\frac{1}{\tau_x}(H - 3X),
$$

where

$$
\frac{dV_e}{dt} = \dot{V}_e, \quad \frac{dV_r}{dt} = \dot{V}_r, \quad \frac{dV_s}{dt} = \dot{V}_s, \quad \frac{d\varphi_e}{dt} = \dot{\varphi}_e,
$$
\n
$$
S_{e,s}(v) = S_{ra}(v) \frac{Q_{\text{max}}}{1 + e^{(-\frac{v - \theta}{\sigma})}}, \quad S_{rb}(v) = \frac{Q_{\text{max}} - r}{1 + e^{(-\frac{v - \theta}{\sigma})}},
$$
\n
$$
\theta_r = \frac{I_c - 3I_bX + (I_b - I_a)H}{\mu},
$$
\n
$$
I_a = I_b - g_H(V_{\text{eff}} - V_K),
$$
\n
$$
I_b = \frac{g_x(V_{\text{eff}} - V_K)}{3}.
$$

Each of the neural mass (cortex, TRN, SRN) is described by its average membrane potential *^Ve,r,s*, while cortical neurons are described by the sequence firing ratio $\phi_{e,r,s}$ to take into account long range connections among cortical cells, and external inputs into SRN are represented by ϕ_n . The parameter ^ν*ab*(*i,e,r,s,n*) denotes the weighting of inputs via synapse from population *^b* onto population *^a*.

In addition, the *^X*-*^H* system is slow currents modulating the bursting behavior in TRN via changes to the firing rate. Sigmoid function *^S*, *^S^r* is ^a good approximation for the firing

Figure ¹ (Color online) Schematics diagram of principle neural fields and loops within the corticothalamic model, representing three neural masses as the main components of the thalamocortical dynamics. The lines represen^t connections between different masses, the mass response are ϕ_e (excitatory and inhibitory), ϕ_r (inhibitory) and ϕ_s (excitatory), while ϕ_n represents subcortical input, including ^a noise component.

rate produced by the voltage at each population *^Ve,r,s*, which generates outgoing activity propagated to other population [\[42\]](#page-10-0). *^Q*max is the maximal firing rate at which the sigmoid firing function saturates, the threshold voltage θ is relative to the threshold current I_θ and the conductance per unite area μ . The *^X*-*^H* system acts as ^a feedback system due to the following principle: when the firing rate of SRN is high, it shifts the sigmoid to the right, and when the firing rate of SRN is low, the sigmoid is shifted to the left. This mechanism is ^a key point to modulate bursting between seizure episodes. More explanation and default values can be found in refs. [\[43,44\]](#page-10-0).

The pos^t synaptic membrane conductance of SRN neurons for both AMPA (excitatory) and GABA (inhibitory) from the TRN cell population is responsible for sustaining and modulating the firing activity. Particularly, inhibition in TRN neurons has been found to be ^a crucial componen^t in the development of spike wave activity, and thus we focus our attention on modeling two important receptors ligand-gated GABA^A and the secondary messenger-gated GABA_B [\[37\]](#page-10-0). Moreover, GABA_B mediated synapses, unlike GABA_A synapses, activate G-proteins which in turn act as the secondary messengers and initiate the opening of ion channels. The inhibitory post-synaptic potential mediated by GABA_B receptors has a much slower time scale than those mediated by GABA_A, then we expect if TRN neurons fires, the GABA_B mediating connection to the SRN will be delayed compared to others or not. Hence, we incorporate ^a time delayed connection from TRN to SRN populations in our model.

³ Simulation results

As we know, the mean field models have been used to describe the brain mass activity recorded by EEG signals and then try to explore the mechanism from normal states to pathology states. Since EEG detection is operated in the cortex layer and is ^a summation of activity from ^a group of neuron scalp within the adjacent region, so we assume ^a functional relationship between cerebral cortex variable ^ϕ*^e* and the scalp EEG voltage. Accordingly, we mainly focus on ϕ_e as an approximation of EEG signals.

3.1 EEG pattern

Essentially, our proposed model can reproduce normal states and also seizure states as other corticothalamic neural field models have already realized. Under different excitatory and inhibitory synapses, activity of each cell population in cortex-thalamus circuit undergoes significant changes. In our model simulation, varying connection strength between each population, such as *^ν*s*e*, *^νrs*, and firing ratio of cortex denoted by ^ϕ*^e* behaves differently, including normal states and also seizure states.

The comparison of different brain states wake, spindle, sleep, seizure, produced by our model can be outlined in [Figure](#page-3-0) 2. We can have ^a simple and brief explanation: in normal state, the firing rate is lower as shown in [Figure](#page-3-0) 2(a), (c) and (e) while in seizure state the firing state is much higher as shown in [Figure](#page-3-0) $2(g)$. More specially, wake state has the lowest firing ratio of cortex denoted by ϕ_e lower than 5Hz, while spindle state and deep sleep state are around 7–9Hz. This implies that magnitude of firing rate is one distinguishing feature among different states.

Another remarkable characteristics is demonstrated by [Figure](#page-3-0) 2(b), (d), (f) and (h), in which the frequency of firing ratio in seizure state alternating with obvious oscillation and other normal states seems to smoothly decrease in the spectrogram. One point should be pointed that the main frequency is around integral multiple of 3Hz which is consistent with EEG sequence, since during active seizures high amplitude spikes in the EEG alternate with slow positive waves around 3Hz.

3.2 Seizure dynamics

Coupling strength between cortex and thalamus cells is found to be crucial for generating different states, so in this section we focus on the seizure pattern. Results demonstrate that periodic patterns that closely resemble the EEG data recorded during human seizure can be carried out through different route. Consistent to experimental finding, seizure patterns can be induced by modulation of varying synaptic transmission as shown in [Figure](#page-4-0) 3.

[Figure](#page-4-0) 3(a) and (b) are presented the seizure initiation through corticothalamic excitation where we linearly increase v_{se} to observe the effects of continuous changes of ϕ_e . There is a gradual increase in the amplitude of ϕ_e presented as abnormal seizure state when *^νse* climbs to reach seizure state, and on ramping down of v_{se} , ϕ_e also declines back to the original normal level.

Similarly, seizure onset by TRN inhibition denoted by *^νsr* can also be found in [Figure](#page-4-0) 3(c) and (d). With the increase of inhibition strength *^νsr* from TRN to SRN, seizure onset activity appears, and then disappears when inhibition strength is set back. Furthermore, it is pointed that the strength of bursting variable *^g^X* and *^g^H* have predominant effect on T-type current which is involved in bursting dynamics of thalamic neurons [\[45\]](#page-10-0). Therefore, we study the activities modulated by *^g^X* with the ratio $g_H = 4gx$ fixed and show the seizure phenomenon due to *^g^H* by upward adjustment of T-current displayed in [Figure](#page-4-0) 3(e) and (f).

The seizure onset by shifting of GABAergic activity towards longer timescale in thalamus appears different that illustrated in [Figure](#page-4-0) 3(g) and (h). We change *^α* with ^a fixed ratio = 4α to observe the variations of firing ratio ϕ_e , where ^a lower value of *^α* means ^a longer timescale.

Besides, it is arguably that there are four classical firing states observed in brain activity as shown in [Figure](#page-5-0) 4. Our modeling can successfully repea^t these four state: low firing

Figure 2 (Color online) Four classical activity states produced by our model. (a) Sequential firing ratio of the normal wake state; (b) frequency spectrum analysis of the normal wake state; (c) sequential firing ratio of the normal spindle state; (d) frequency spectrum analysis of the normal spindle state; (e) sequential firing ratio of the deep sleep; (f) frequency spectrum analysis of the deep sleep state; (g) sequential firing ratio of the seizure state; (h) frequency spectrum analysis of the seizure state. For the convenience of analysis, in the spectrum graphs vertical axis shows the log_{10} scale of the power spectral density.

Figure ³ (Color online) Seizure state produced by varying model parameters. (a) Sequential firing ratio arising from increasing excitatory strength *^νse*; (b) time series of parameter *^νse*; (c) sequential firing ratio arising from increasing the inhibitory strength *^νsr*; (d) time series of parameter *^νse*; (e) sequential firing ratio arising from elevating activation of ^T channels by increasing *^νgX*; (f) time series of parameter *^gX*; (g) sequential firing ratio arising from decreasing dendritic *^α*; (h) time series of parameter *^α*.

Figure 4 (Color online) Four classical rhythm induced under different set of parameters in our model. (a) Low firing state; (b) spike-wave state; (c) simple oscillation; (d) saturation state.

(Figure 4(a)), spike wave (Figure 4(b)), simple oscillation (Figure $4(c)$), and saturation (Figure $4(d)$) under different parameter ranges. The ^phenomenon will be further discussed in the next section.

3.3 Seizure evolution

We first study the effects of *^νse* on the brain activity with the assumption that changes of *^νse* obey linear function. ^A revolution of rising in the firing ratio can be easily observed from [Figure](#page-6-0) 5(a), (b), (c) and (d). As we escalate *^νse* to high levels, the amplitude of firing ratio in cortex mass increases in size from low firing state $(< 5 Hz$) which can be classified as normal state, then evolves into the seizure state where the am^plitude is above 20Hz and finally reaches the maximum firing ratio $Q_{\text{max}} = 250 \text{ Hz}$ as the limit set in our model.

Recently, ^a number of studies on the mean field models have pointed that time delay *^τ* also ^plays ^a significant role in the dynamics of spikes. Here the effect of time delay is also studied in abnormal states, which demonstrates that time delay has grea^t effects on the wave forms of ^ϕ*^e*. [Figure](#page-6-0) ⁶ depicts the wave transformation of seizure states as the time delay altering, including simple oscillation to spike wave [\(Figure](#page-6-0) 6(a) and (b)) and also to saturation state ([Figure](#page-6-0) $6(c)$ and (d)).

3.4 State contour

Based on the analysis presented above, we illustrates that the coupling strength *^νse* is ^a crucial model parameter in the cortex activity. Likewise, data confirms the fact that time delay from RTN to SRN transmission is ^a key factor mediating the behavior of ^ϕ*^e*.

To investigate this further, we employ two parameter continuations to draw ^a contour ^plane of *^νse* and *^τ*. This kind of activity maps is an effective method to study the variation of firing activity resulted from two parameters expressed by coordinate axes. Four typical dynamical states are depicted by four colors as explained in the caption of [Figure](#page-7-0) 7, namely, each region within one color can be classified as one from these four states.

It is noted that, along the horizontal coordinate of [Figure](#page-7-0) 7(a), it is confirmed that the coupling strength is in charge of firing rate: from low firing rate to saturation point. However, from the vertical point of view, time delay also affects the wave form in seizure states. Within ^a certain range of *^νse*, no matter the changes of time delay, it maintains in the same activity status, while what has changed is the wave form of seizure state.

As displayed in [Figure](#page-7-0) 7(b), this kind of ^phenomenon occurs in the contour grap^h of inhibitory connection *^νsr* and time delay *^τ* between two GABAergic channels. Comparing [Figure](#page-7-0) 7(a) and (b) have ^a slight variation on the state sequence, bigger strength of *^νsr* means lower firing state while smaller inhibitory strength leads to saturation.

When considering the effects of both g_X and α with τ , situations become not quite the same as that of synapse strength

Figure ⁵ (Color online) Effect of cortico-thalamus excitation on activity states under different *^νse* values, while other model parameters are same fixed. (a) Normal low firing state; (b) pathological state; (c) seizure state; (d) saturation state.

Figure ⁶ (Color online) Effect of time delay. Different *^τ* under same *^νse* results in different dynamical state. (a) Simple oscillation state; (b) spike-wave state; (c) simple oscillation state; (d) saturation state.

Figure 7 (Color online) Contour graph in two parameter panel. (a) $v_{se} - \tau$; (b) $v_{sr} - \tau$; (c) $gx - \tau$; (d) $\alpha - \tau$. Region in green color for low firing rate state, red for saturation state, and deep blue for simple oscillation state, light blue for spike-wave state. Dotted regions are not classified due to the margin error.

 v_{se} and v_{sr} . The two-dimensional map in (v_{sr} , τ)-space given in Figure 7(c) implies the spike-wave state can hardly exist and will still unlikely arise no matter when either time delay or conductance g_X increases. Situations obtained in the (a, τ) contour ge^t more complex due to the unclear margin caused by simulation error as revealed in Figure 7(d). But it is certain that when time delay is small, firing rate remains low whatever the value of the receptor offset time *^α* and *^β* are.

3.5 Uncertain time-delay

In Figure 7(a), we know that time delay does not affect much on the states if time delay is fixed already, while *^νse* makes ^a grea^t difference on the states. For ^a further study, here we propose ^a random time delay *^τ* instead of fixed time delay which is used in all the above analysis.

Sequence diagraph and spectrogram are ^plotted in [Figure](#page-8-0) ⁸ (a)–(h) which demonstrates random time delay has unique effects on activity states. Unlike fixed time delay, time-varying time delay which may vary randomly within ^a time range can manage seizure state at normal states. Comparing [Figure](#page-8-0) 8(b) and (h), the dominant frequency is gradually down to ^a normal level lower than 5Hz as the maximum of time delay increases. In this situation, increasing time delay can suppress the seizure duration.

3.6 Reduced network

Research both in vivo and in vitro experiments has demonstrated that spike wave activity is first initiated SRN and then propagated to the cortex and finally is induced in the TRN [\[35,36,40,46\]](#page-10-0). As mentioned in ref. [\[18\]](#page-10-0), periodic signal can simulate the excitatory drive to the TRN as an approximation to the full mode when seizure state is generated. In this section, we try ^a reduced model using sinusoidal input to replace the effect of ^ϕ*^e* in the completed network.

From [Figure](#page-9-0) ⁹ we may make ^a conclusion, this reduced network can emulate the full network model, since spike-wave state and simple oscillation state can be found under some appropriate parameter conditions when studying dynamical behavior of SRN denoted by ϕ_s and TRN denoted by ϕ_r .

4 Conclusion

We have developed ^a typical and widely used mean-field model to study the clinical EEG data in seizure patients and extensively simulated the dynamical behavior in the cortex-thalamus circuitry. Besides four dynamical states including normal activity and pathological seizure activity were successfully modeled by changing some parameter value in our presen^t model, we mainly studied the generation

Figure 8 (Color online) Suppression of time-varying delay on dynamical state. (a) Time series of seizure state; (b) spectrogram of seizure state; (c) time series of suppressed seizure duration; (d) spectrogram of suppressed seizure duration; (e) time series of transient attack; (f) spectrogram of transient attack; (g) time series of eliminated seizure; (h) spectrogram of eliminated seizure.

Figure 9 (Color online) Comparison of reduced network and full model. (a) Spike wave form of SRN in simple network; (b) simple oscillation of SRN in simple network; (c) spike wave form of TRN in full network; (d) simple oscillation of TRN in full network.

of seizure state and seizure evolution under different brain connections.

Our results demonstrate that model parameters related to synaptic connection, time delay, conductance, and also slow variable have grea^t effects on the activity state of brain, which implies that multiple pathways can be provided to control and even eliminate seizure spike wave in theoretical model. Based on the simulated results of uncertain time delay illustrated in the Section 3.5, time-varying time delay can also be used to explore for monitoring seizure states. As addition, ^a reduced work is proved to presen^t the main feature of seizure behavior in genera^l full model, which provides an alternative choose for some situation such as theoretical analysis.

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