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Parameters identification of chaotic systems based on artificial bee colony algorithm combined with cuckoo search strategy

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Artificial bee colony (ABC) algorithm is motivated by the intelligent behavior of honey bees when seeking a high quality food source. It has a relatively simple structure but good global optimization ability. In order to balance its global search and local search abilities further, some improvements for the standard ABC algorithm are made in this study. Firstly, the local search mechanism of cuckoo search optimization (CS) is introduced into the onlooker bee phase to enhance its dedicated search; secondly, the scout bee phase is also modified by the chaotic search mechanism. The improved ABC algorithm is used to identify the parameters of chaotic systems, the identified results from the present algorithm are compared with those from other algorithms. Numerical simulations, including Lorenz system and a hyper chaotic system, illustrate the present algorithm is a powerful tool for parameter estimation with high accuracy and low deviations. It is not sensitive to artificial measurement noise even using limited input data.

chaotic systems, parameter estimation, swarm intelligence, ABC, CS, local search

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1 Introduction

Nonlinear phenomenon is one of the most popular topics in the past decades. Hou and Chen [1] applied super-harmonic responses analysis method to identify the crack faults in rotor system. Zheng et al. [2] also used the nonlinear theory to research molecular dynamics, which fully illustrates the necessity of studying nonlinear. Besides, Chaos theory as a fundamental branch in this subject gains much attention. Inside, since not all dynamical parameters of these systems are usually known, and therefore it is significant to estimate these unknown variables [3].

Until now, variety of optimal techniques are applied to solve parameters identification for the chaotic systems [4–7]. Besides, meta-heuristic-based algorithms like genetic algorithm (GA), differential evolutionary (DE) algorithm, particle swarm optimization (PSO) [8–10] gain their popularity in the application to tackle with this problem. Zheng et al. [2] applied GA to identify Lorenz system with one-dimensional parameter considered. He et al. [11] seemed to be the first to introduce PSO to estimate the parameter of chaotic system. Gao et al. [12] also adopted a novel quantum-behaved PSO to identify parameters of Lorenz system. Later, Sun et al. [13] inserted a variant mechanism into PSO and then applied the improved version to identify the Lorenz and Chen system. Modares et al. [14] estimated the parameters through PSO modified with a nonlinear factor. Alfi and Modares [9] used a novel adaptive PSO combining with an adaptive mutation mechanism and a dynamic inertia weight to solve the problem. Li et al. [15] and Peng et al. [16] applied chaotic ant swarm (CAS) algorithm to deal with chaotic systems as well. Ref. [17] introduced a high-efficiency hybrid quantum-inspired evolutionary algorithm with

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DE to identify the parameters of the Lorenz system. All these methods mentioned have generally achieved a satisfactory result.

Apart from the heuristic algorithms presented above, artificial bee colony (ABC) [18] algorithm aroused much attention. This meta-heuristic algorithm is motivated by the intelligent behavior of honey bees seeking a high quality food source. It has the advantages of simple structure (as simple as PSO and DE), ease of use, and high stability. It has been applied solved many engineering problems, such as structural optimization design [19,20] and structural damage identification [21,22]. In addition, Kang and Li [23,24] used ABC and PSO to optimize support vector regression and then made system reliability analysis for slopes. However, only a few works have been reported to the field of parameters estimation for chaotic systems. Li and Yin [25] introduced a kind of ABC combined with DE operator to identify the parameters of the Chen system. Hu et al. [26] put forward a hybrid ABC algorithm to identify the uncertain fractional-order chaotic systems, in which a new modified search mechanism was used to describe local search.

Despite good identified results in the mentioned references, the models used usually are normal chaotic systems such as Lorenz system [12,27] and Chen system [25], little research can be found about the estimation of hyper chaotic systems. Meanwhile, studies on impacts of artificial noise and number of sample data are rare as well, so this paper is aiming at improving the standard ABC and these two problems mentioned above. As with the method, it is widely acknowledged that these heuristic algorithms are easily to trap the local minimum or slow convergence [9,12,13,28]. In order to balance local search and global search ability of original ABC further, some improvements are also made. Firstly, the local search mechanism of cuckoo search (CS) optimization is introduced into the onlooker bee phase to enhance its exploitation ability. Since something behinds the CS is Lévy flights and this mechanism has been successfully proved to be effective in CS [29,30]. In fact, Lévy flights is a kind of stochastic process subject to power-law distribution and this mechanism can be utilized to describe many behaviors like collecting honey and hunter-gatherer. Introducing this mode into the onlooker bee phase, it means bees can realize detailed research in promising food source. Secondly, the scout bee phase is also modified by the chaotic search mechanism. This improvement is beneficial for algorithm getting rid of local minimum due to the feature (ergodicity) of chaos [31]. Via such change, the ceased solution (trapped in local minimum) can continue to search, which can increase algorithm's exploration ability.

2 Problem formulation

In general, if one does not have a prior knowledge about the chaotic system, then the system identification becomes a difficult problem and we have to choose the system parameters by trial and error. Consequently, the system identification problem is usually reduced to a parameters estimation approach [27].

Considering a n-dimensional chaotic system, given as follows:

$$\dot{\boldsymbol{x}} = F(\boldsymbol{x}, \boldsymbol{x}_0, \boldsymbol{\theta}_0), \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, ..., x_N)^{\mathrm{T}} \in \mathbf{R}^n$ denotes the state vector, $\dot{\boldsymbol{x}}$ is the derivative of $\boldsymbol{x}$ and $\boldsymbol{x}_0$ is the initial value. It should be noted $\boldsymbol{\theta}_0 = (\theta_{10}, \theta_{20}, ..., \theta_{d0})$ is a set of original parameters, which must be estimated.

Supposing we have known the basic information of system (1) in advance, then the calculated system can be presented as

$$\dot{\tilde{\boldsymbol{x}}} = F(\tilde{\boldsymbol{x}}, \boldsymbol{x}_0, \tilde{\boldsymbol{\theta}}), \tag{2}$$

where $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_N)^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of calculated model, and $\tilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_d)^{\mathrm{T}}$ is a serious of identified parameters.

Obviously, there appear disparity between the response acquired from the measurement and those obtained from calculations, therefore we can define the mean squared error (MSE) as the objective function, given as follows:

$$\mathrm{MSE} = \frac{1}{W}\sum_{t=1}^{W}\left\|\boldsymbol{x}_t - \tilde{\boldsymbol{x}}_t\right\|^2, \tag{3}$$

where W represents the length of data adopted for parameter identification, $\boldsymbol{x}_t$ and $\tilde{\boldsymbol{x}}_t$ $(t = 1, 2, ..., W)$ denote state vectors of the original and the estimated systems at time t, respectively. Because of irregular dynamic behavior nature of chaotic systems, the parameter estimation for chaotic systems is always a multidimensional continuous optimization problem, where the decision vector is $\boldsymbol{\theta}_0$ and the optimization goal is to minimize MSE [32]. In addition, when utilizing some traditional techniques to solve this problem, it easily catches in local optimal and difficult to obtain the global optimal parameters due to the complexity of the objective function, so more and more heuristic algorithms are applied to solve parameter identification for chaotic systems. Figure 1 presents the process of parameters estimation as an optimization problem.

3 Algorithm for parameters estimation

3.1 Description of ABC algorithm

The ABC algorithm is motivated by the real behavior of honey bees seeking high quality food sources. The related details can be seen in ref. [19]. In the algorithm, a food source position is defined as a possible solution and the nectar quality of the food source matches the fitness of the relevant solution in optimization process.

The general structure of algorithm is introduced as follows:

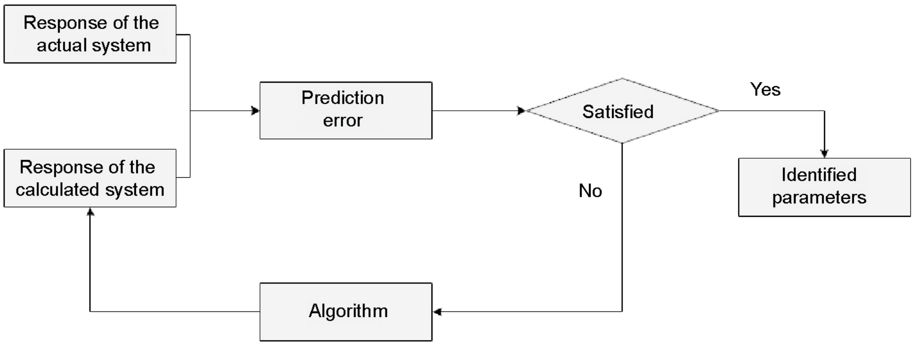

Figure 1 The process of parameters estimation as an optimization problem.

(1) Initialization phase. Food source is expressed as eq. (4) in a random way

$$x_{m,i} = l_i + \text{rand}(0,1) \cdot (u_i - l_i), \tag{4}$$

where m, is the dimension, u_i and l_i represent the upper bound and lower bound of the parameter $x_{m,i}$.

(2) Employed bees phase. Supposing the $\boldsymbol{x}_m$ is a food source that to be exploited and behavior of the employed bee can be simulated in the following equation:

$$\begin{aligned} &v_{m,i} = x_{m,i} + \phi_{m,i} \cdot (x_{m,i} - x_{k,i}),\\ &m = 1,2,...,SN,\ \ m \neq k,\ \ i \in \{1,2,...,D\}, \end{aligned} \tag{5}$$

where $\boldsymbol{x}_k$ is a food source, $\phi_{m,i}$ is a random number from [−1, 1], i is a randomly chosen dimension, D is the dimension number and SN denotes the number of employed bees (it is known that the quantity of the employed bees is the same as the onlooker bees', which is SN). After producing a new candidate source, its profitability is calculated accordingly, and 'greedy selection' is utilized between $\boldsymbol{x}_m$ and $\boldsymbol{v}_m$.

The fitness of each solution in this problem is calculated according to

$$\text{fit}(\boldsymbol{x}_m) = 1/(1+\text{fit}(\boldsymbol{x}_m)). \tag{6}$$

(3) Onlooker bees phase. The employed bees return home and share their food source information with the onlooker bees. They select the food source to exploit relying on the probability value p_m:

$$p_m = \text{fit}(\boldsymbol{x}_m) / \sum\nolimits_{m=1}^{SN} \text{fit}(\boldsymbol{x}_m), \tag{7}$$

after selecting food source, onlooker bees will fly there to exploit better food source. In the original ABC algorithm, the behavior is simulated by eq. (5), then fitness value is calculated applying the 'greedy selection' to produce better food source.

(4) Scout bees phase. limit parameter is used to judge whether the solution is abandoned or not. If the solution couldn't improve after the limit times, as mentioned above, the solution will be given up and eq. (4) will be used to produce a new food source to replace the abandoned one. The limit parameter mentioned in scout phase can be calculated with eq. (8) [18]:

$$l = SN \cdot D. \tag{8}$$

3.2 Improvements to ABC algorithm

3.2.1 The search behavior of onlookers

In real honey bee colonies, an employed bee exploits the food source and then conveys the information to the onlookers by dancing. And an onlooker will observe variety of dances and know more information and finally make a decision, therefore, the behavior of onlookers should be different with that of employers [28]. However, in the original ABC, the same formula (eq. (5)) is adopted to simulate this two honey-collected activities. So in the improved algorithm, the CS local search mechanism is inverted into the onlooker bee phase.

CS is a bio-inspired optimization method that mimics the brood parasitism behavior of many species of cuckoos. The biggest feature of the CS is that it realizes its intensive search mode through Lévy flights [33]. In past, the flight behavior of many animals and insects have been analyzed in various studies which exhibit the important properties of Lévy flights [29]. Furthermore, the moving direction of this flight mode is random, but its step-size is subjected to power-law distribution regular. And thus it frequently moves in a small step-size and may have a large step once in a while, which can guarantee the detailed search further.

In this research, a Mantegna algorithm [30] for a symmetric Lévy stable distribution is applied for producing random step sizes. Here, 'symmetric' means that the step-size might be positive or negative. In this method, the step-size s can be calculated by the following equation:

$$s = \frac{u}{|v|^{1/\beta}}, \tag{9}$$

where β $(0 < \beta \le 2)$ denotes an index, $\boldsymbol{u}$ and $\boldsymbol{v}$ are submitted to normal distributions, that is

$$u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2), \tag{10}$$

where

$$\sigma_u = \left\{ \frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\beta\Gamma[(1+\beta)/2]2^{(\beta-1)/2}} \right\}^{1/\beta}, \quad \sigma_v = 1, \tag{11}$$

where $\Gamma(\cdot)$ denotes the Gamma function and can be acquired based on the following equation:

$$\Gamma(1+\beta) = \int_0^{\infty} t^{\beta} e^{-t} dt. \tag{12}$$

Then the step sizes can be generated based on Lévy distribution to exploit the search area and given as follows:

$$\text{step\_size}(t) = 0.001 \times s(t) \times (\boldsymbol{x}_{\text{best}}(t) - \boldsymbol{x}_k(t)), \tag{13}$$

where t is the iteration counter for Lévy search strategy, $s(t)$ is obtained from eq. (9). Similar with PSO, $(\boldsymbol{x}_{\text{best}} - \boldsymbol{x}_k)$ is social learning component in which $\boldsymbol{x}_{\text{best}}$ is the best solution in the current iteration cycle and $\boldsymbol{x}_k$ is the randomly selected solution within colony and $\boldsymbol{x}_k \neq \boldsymbol{x}_{\text{best}}$. The final solution update equation to simulate the onlookers can be given as follows:

$$x_{ij}^{\text{T}}(t+1) = x_{ij}(t) + \text{step\_size}(t) \times U(0,1), \tag{14}$$

where x_{ij} is an onlooker bee that is going to exploit the food source, $U(0,\ 1)$ is a uniformly distributed random number between 0 and 1 and $\text{step\_size}(t) \times U(0,1)$ is the actual random flights calculated from Lévy distribution. As is mentioned above, parameter t is the times of exploiting food sources, and if a more satisfied place is found, the 'greedy selection' is also applied. With this expression, the learning component and CS search mode are introduced into the second bee phase, which can not only enhance the dedicated search for food sources but also fit real honey-collection behavior for bees.

3.2.2 The search behavior of scouts

In the standard ABC, if a food sources is exploited up to the limit times, the food source will be abandoned and they will restart finding another one randomly. In the improved algorithm, the deserted solution, also the solution trapped in the local minima is utilized to produce chaotic sequence and the best solution will be replaced the original solution (will be abandoned). Compared with random motion, chaos has its own characteristics including randomness, ergodicity and regularity. Among these features, ergodicity can be viewed as an effective way to help the algorithm escape from trapping local minima, so this mechanism is introduced into scout bee phase. Through such improvement, the solution ceased exploitation can continue to local search, which can increase the exploration ability of the algorithm.

The chaotic sequence can usually be produced by the following well-known one-dimensional sine map [34] defined as follows:

$$Z_{k+1} = \sin(\pi \cdot Z_k), \quad Z_k \in (0,1), \quad 0 < a \leq 4, \tag{15}$$

where Z_k is the value of the variable Z at the kth iteration. Usually after several steps iteration, it will lead to chaos phenomena. Supposing the abandoned solution is $x_{\text{abandoned}}$, the maximum iteration number is 300 and then the new solution can be calculated as follows:

$$x_{\text{abandoned},j} = x_{\min,j} + Z_{\text{maxi mum}.k,j} \cdot (u_{\max,j} - l_{\min,j}). \tag{16}$$

4 Numerical simulations

4.1 The Lorenz system

As the most typical chaotic system, Lorenz system is employed as the first numerical example. The phase diagram from x_1x_2-plane, x_1x_3-plane and x_2x_3-plane of the Lorenz are showcased in the Figure 2(a)–(c). Anyone can find the singular attract-or in the central place and observe the untidy feature. The general expression of the chaotic system can be described as follows:

$$\begin{cases} \dot{x}_1 = \theta_1(x_2 - x_1), \\ \dot{x}_2 = (\theta_2 - x_3)x_1 - x_2, \\ \dot{x}_3 = x_1x_2 - \theta_3x_3, \end{cases} \tag{17}$$

where x_1, x_2 and x_3 are the state variables, θ_1, θ_2 and θ_3 are unknown positive constant parameters. To produce the chaos phenomena, the vector parameter $\theta = [\theta_1, \theta_2, \theta_3]^{\text{T}} = [10, 28, 8/3]^{\text{T}}$ and it must be estimated. Furthermore, in order to make comparison with ref. [27], the same searching ranges for this set were: $0 \leq \theta_1 \leq 20$, $0 \leq \theta_2 \leq 50$ and $0 \leq \theta_3 \leq 5$. For the objective function, first, a fourth-order Runge-Kutta method was used to solve the system (eq. (17)) with step length h=0.01 to get a discrete time series of this system at time $0h, 1h, ..., 300h$. And then serious of evolutionary algorithms are applied to identify these parameters.

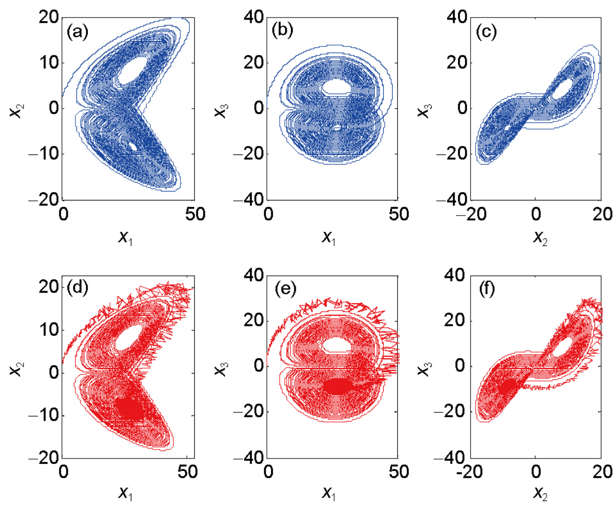

Figure 2 (Color online) The phase diagram of Lorenz system. (a)–(c) Without noise; (d)–(f) with noise.

4.1.1 Parameter settings on algorithm

In this case, for the ABC, colony size is $N_{pop} = 50$; for the CS, according to the ref. [35], colony size is $N_{pop} = 50$, discovery rate is $P_a = 0.25$; for the imperial competitive algorithm (ICA), the related settings is the same as ref. [36]. $N_{pop} = 100$, number of imperial countries is $N_{imp} = 8$, weight coefficient $\xi = 0.1$, possession probability $P_r = 0.2$, angle coefficient $\theta = 0.5$; for the present method, colony size is $N_{pop} = 50$; parameter $t = 15$; $C_{max} = 50$. For the ant colony-particle swarm optimization (ACO-PSO), according to the ref. [27], the colony size is 50, weight factors $\omega_{max} = 0.9$, $\omega_{min} = 0.4$ acceleration constants $c_1 = c_2 = 1.49$ passive congregation coefficient $c_3 = 0.7$ range parameter$d = 0.25$. Since swarm intelligence is a kind of stochastic algorithm, to ensure fairness in comparison of the robustness of the examined algorithms, for each problem the analysis is repeated 30 times independently. The final estimation results of all cases including their averages, standard deviation and the worst result are listed in the Tables 1–4.

4.1.2 Parameter estimation on Lorenz system

After completing calculation, several typical evolving processes for the objective function MSE were carried out. Figure 3 presents the evolution of the objective function of the best solution based on mentioned four techniques. it can be observed that the objective function value from the proposed algorithm is closer to zero than that from the ABC, CS and ICA algorithms, implying that the present algorithm can converge to the global optimum very quickly and also indicating that the identified results from are closer to presetting values. Figure 4 also shows that all estimated parameters obtained by the present algorithm are very close to the true values in all experiments. It also shows that trajectories of the identified parameters asymptotically converge to their actual values. Again, it can be easily observed that it only takes 20 iterations for estimated parameters converging to actual values. Furthermore, Table 1 summarizes the statistical results acquired in the estimation of the parameters θ_1, θ_2 and θ_3 using the mentioned algorithms, for a complete

Table 1 Statistical results obtained for the generalized three-dimensional Lorenz system with the four algorithm used

Statistical result	Algorithm	Parameters θ_1	θ_2	θ_3
Average	ABC	11.1252	26.8145	2.6595
	CS	10.0150	27.9814	2.7071
	ICA	10.0023	27.9980	2.6703
	Present algorithm	**10.0000** a)	**28.0000**	**2.6667**
Best	ABC	9.4839	28.8635	2.6667
	CS	9.9419	28.0717	2.6731
	ICA	9.9979	28.0049	2.6664
	Present algorithm	**10.0000**	**28.0000**	**2.6667**
Worst	ABC	12.4197	26.1719	2.6463
	CS	10.2999	27.5283	2.7832
	ICA	10.0672	27.8991	2.6854
	Present algorithm	**10.0000**	**28.0000**	**2.6667**

a) Bold values indicate that our algorithm is better than other algorithms.

Table 2 Results obtained by several parameters estimation algorithm available in the literature (Lorenz system)

Parameter	PSO [32]		EP [37]	DE [10]		GA [27]		PSO-ACO [27]		Present algorithm	
	Average	Best	Best	Average	Best	Average	Best	Average	Best	Average	Best
θ_1	10.0184	9.9953	10.0162	10.0101	10.0001	10.0033	9.9013	10.0005	10.0000	10.0000	10.0000
θ_2	27.9934	28.0071	27.9961	27.9939	28.0000	28.0011	28.05187	28.0011	28.0000	28.0000	28.0000
θ_3	2.6663	2.6670	2.6659	2.6666	2.6667	2.6673	2.5703	2.6673	2.6666	2.6667	2.6667
MSE	4.18	0.0468	0.0172	3.60×10^{-4}	2.00×10^{-7}	6.34×10^{-3}	Not given	1.20×10^{-5}	1.03×10^{-6}	$\mathbf{1.59\times10^{-9}}$ a)	$\mathbf{1.13\times10^{-9}}$

a) Bold values indicate that our algorithm is better than other algorithms.

Table 3 Statistical results obtained for the generalized hyper chaotic system with four algorithm used

Statistical result	Algorithm	Parameters				MSE
		θ_1	θ_2	θ_3	θ_4	
Average	ABC	4.5842	16.5182	1.1819	0.2270	3.2543
	CS	5.9629	15.6173	1.3282	0.2416	3.7987
	ICA	4.4487	15.5847	0.6330	0.4003	2.0517
	Present algorithm	**4.9999** a)	**15.9999**	**1.0000**	**0.5000**	**2.57×10^{-8}**
Best	ABC	5.0014	13.7785	1.2937	0.5399	0.3119
	CS	4.9042	16.7636	0.9569	0.5126	0.2864
	ICA	5.0781	16.4324	1.0697	0.5201	0.0192
	Present algorithm	**5.0000**	**16.0000**	**1.0000**	**0.5000**	**1.98×10^{-9}**
Worst	ABC	3.8326	24.7651	2.4097	0	7.7600
	CS	9.2616	20.1714	1.8928	0	7.4882
	ICA	4.2433	11.4808	0.4571	0	2.6434
	Present algorithm	**4.9995**	**16.0012**	**0.9996**	**0.5001**	**3.87×10^{-7}**

a) Bold values indicate that our algorithm is better than other algorithms.

Table 4 Statistical results obtained for the generalized hyper chaotic system by the proposed algorithm with different input data

Statistical result	Number of sample data	Parameters				MSE
		θ_1	θ_2	θ_3	θ_4	
Average	300	4.9999	15.9999	1.0000	0.5000	**2.57×10^{-8}** a)
	200	4.9999	15.9999	1.0000	0.5000	3.63×10^{-7}
	100	4.9743	16.0031	1.0006	0.5026	9.53×10^{-6}
	50	5.0009	15.9996	0.9998	0.5004	8.60×10^{-7}
Best	300	5.0000	16.0000	1.0000	0.5000	**1.98×10^{-9}**
	200	5.0000	16.0000	1.0000	0.5000	4.46×10^{-9}
	100	5.0000	16.0000	1.0000	0.4999	1.86×10^{-8}
	50	5.0000	16.0000	1.0000	0.5000	5.52×10^{-9}
Worst	300	4.9995	16.0012	0.9996	0.5001	**3.87×10^{-9}**
	200	4.9992	15.9956	0.9994	0.4998	8.66×10^{-7}
	100	4.9378	15.9986	0.9992	0.5065	4.93×10^{-5}
	50	5.0208	15.9960	0.9983	0.5050	2.74×10^{-5}

a) Bold values indicate that our algorithm is better than other algorithms.

overview of our estimation.

Observing from Table 1, one can find that either the best identified value, the average one or the worst one acquired by proposed algorithm are the most accuracy. In addition, Table 2 also records the results acquired by other meta-heuristic methods in literature, such as: PSO [33], evolutionary algorithm (EA) [38], DE [10] and PSO-ACO [27]. Among these algorithms, in terms of the average value, the outcome obtained by PSO-ACO is $\hat{\boldsymbol{\theta}}$ = [10.0005, 28.0011, 2.6673], which the most satisfactory identified result in reference. While the outcome from the present algorithm is $\hat{\boldsymbol{\theta}}$ = [10.0000, 28.0000, 2.6667]. Furthermore, the MSE got by the suggested method is 1.59×10^{-9}, which is far less than that acquired by other algorithms. In general, the present algorithm shows more competitive optimization ability in dealing with this problem.

4.2 A new 4-D hyper-chaotic system

A more complex hyper-chaotic system is adopted as the second numerical example to test the proposed algorithm further. At least, such mentioned autonomous system should have the following distinguishing features. It is usually dissipative, which has more dimension number and one or more nonlinear terms. The most important thing is that the system has two positive Lyapunov exponents with some given parameters and initial conditions [37]. In brief, compared with the traditional Lorenz system, it becomes more difficult to estimate these original parameters. In this case, according to ref. [37], the governing equation of a new four-dimensional

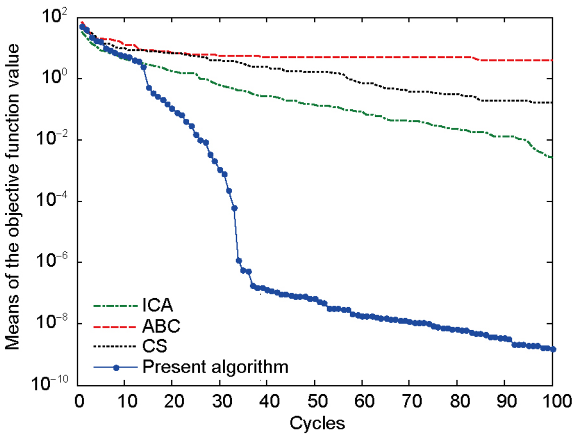

Figure 3 (Color online) Convergence process of means of the objective values by four algorithms used (Lorenz system).

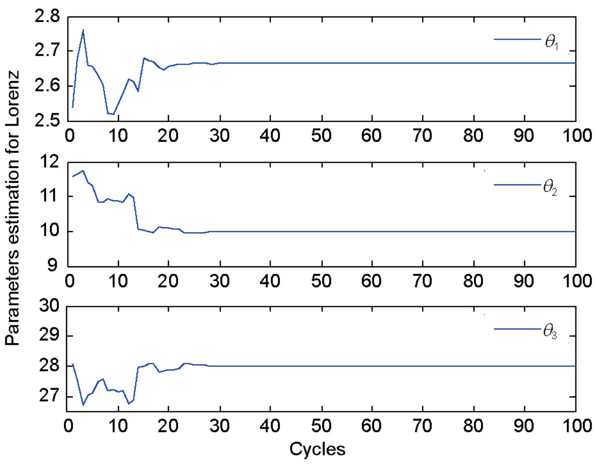

Figure 4 (Color online) Searching process for the Lorenz system with the present algorithm.

continuous autonomous hyper-chaotic system is given as follows:

$$\begin{cases} \dot{x}_1 = \theta_1(x_2 - x_1) + x_4, \\ \dot{x}_2 = x_1x_3 - x_2, \\ \dot{x}_3 = \theta_2 - x_1x_2 - \theta_3x_3, \\ \dot{x}_4 = -\theta_4x_4 - x_2x_3. \end{cases} \tag{18}$$

The control parameters can be set as: $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4]^{\mathrm{T}} = [5, 16, 1, 0.5]^{\mathrm{T}}$. Resulting after 5×10^4 iterations: four Lyapunov exponents are 1.400240, 0.313208, −0.968585 and −3.327758. It implies that there exists a hyper-chaotic attractor and reflects very rich chaotic and hyper-chaotic behaviors. The three-dimensional phase portrait of the hyper-chaotic attractor is illustrated in Figure 5(a). The searching ranges for this set were: $0 \le \boldsymbol{\theta} \le 50$, which is larger than that of Case 1. For the objective function, the same sampled data as the Lorenz system is used for parameter estimation in this example.

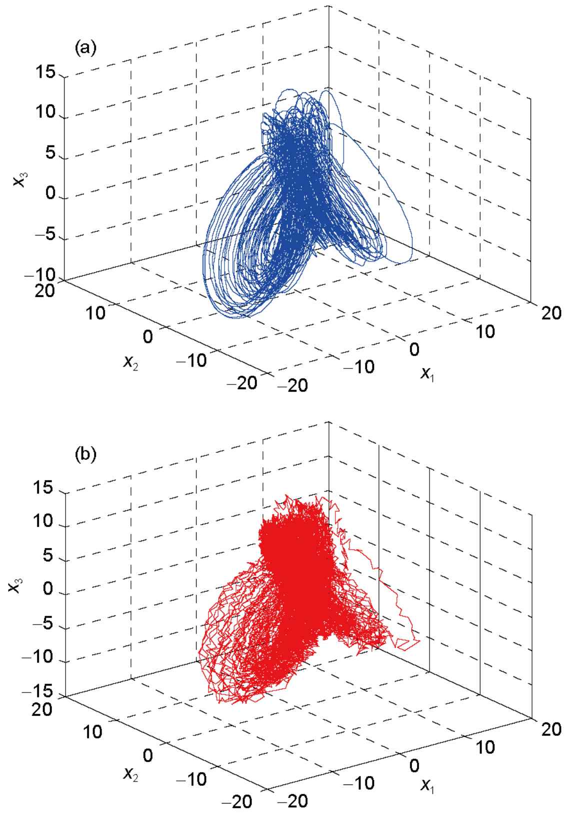

Figure 5 (Color online) Three-dimensional plot of the trajectory in the x_1-x_2-x_3 space. (a) Without noise; (b) with noise.

Parameter estimation on hyper-chaotic system: Similar with Case 1, typical evolutionary processes for the objective function MSE were carried out. Figure 6 records the local evolution (from 50th cycle to 100th cycle) of the objective function based on mentioned four techniques. This picture shows that the value of the proposed algorithm decreases to zero very fast while the other three methods trap in local minima, which means the present algorithm can still acquire a satisfied estimated results in this case and the other techniques may obtain the results with some big errors. Figure 7 exhibits the evolutionary process of all estimated parameters. It only needs 50 cycles' for the hybrid algorithm to converge, adequately implying its high efficiency. The statistical outcomes of the best objective function value, the average , the best and the worst of identified parameters are listed in Table 3. From observing this table, it can be found the values acquired by the proposed algorithm is much better than ABC, CS and ICA, which fully demonstrate the high accuracy of the present method further.

4.3 The influence of the sample data

In this part, the influence of different input data is investigated. Because for the 4-D hyper chaotic system, the other three mentioned algorithms cannot obtain a rational estimated result, only the present method is adopted to calculate. Besides the previous 300 data $(0h, 1h, \ldots, 300h)$, we also

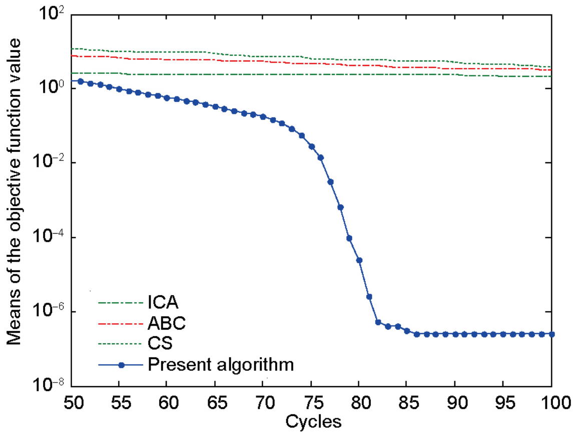

Figure 6 (Color online) Convergence process of means of the objective values by four algorithms used (Hyper chaotic system).

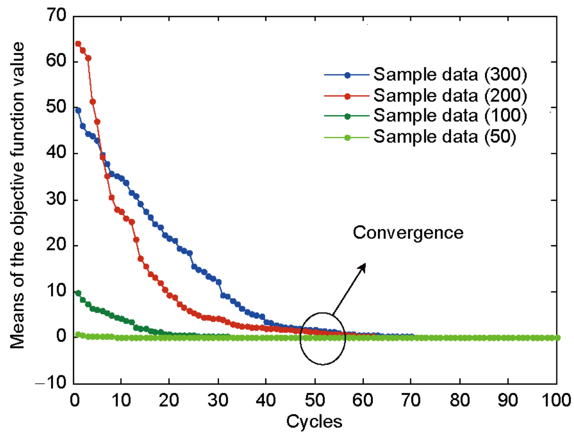

Figure 8 (Color online) Convergence process of means of the objective values by the proposed algorithm with different input data (Hyper chaotic system).

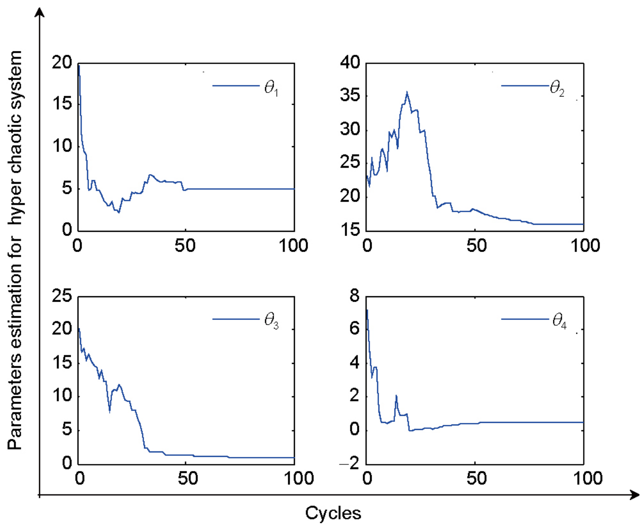

Figure 7 (Color online) Searching process for the Lorenz system with the present algorithm.

introduced 200 data $(0h, 1h, ..., 200h)$, 100 data $(0h, 1h, ..., 100h)$, and 50 data $(0h, 1h, ..., 50h)$, to calculate. Figure 8 shows the evolutionary process based on the four group data. One can find different input has a distinct influence on the initial iteration, but for all situations, it cost nearly 50 iterations for the algorithm to converge to zero, which indicates these four situations can all obtain a relative accuracy estimated outcomes. Table 4 records the final identified results of different input data. As with average results, the best is coming from the 300 data used while the worst is the 100 data. In brief, this four group input data can obtain a satisfied estimated outcome, but the deviation of the 300 data is the smallest.

4.4 The influence of the noise

In practical measurements of responses in systems, there can be a possibility of errors due to measurement noise or/and modeling error. In order to account for these errors in the measurements; uniformly distributed random noise [35,39,40] can be added to the simulated responses data with zero mean and a variance of 1. The noise for responses can be incorporated by using the following equation:

$$\overline{f_i} = f_i(1 + n_l(2 \cdot \text{rand} - 1)), \quad (19)$$

where $\overline{f_i}$ is the ith measured response with noise, f_i is the ith measured response without noise, n_l is the noise level for responses (e.g., 0.01 refers to a 1 percentage noise level). In this study, in order to simulate the experimental responses in a realistic way, 10% random noise is added to the analytic responses. Furthermore, from observing Figures 2(d)−(f) and 5(b), one can point that singular attract-tors of two phase diagram become more obscure and the untidy feature is more prominent, which greatly increase the difficulty for identification.

Parameter estimation with noise: Because the present algorithm can acquire a more competitive estimated result, so in this case, only the proposed algorithm is adopted to calculate. Figure 9 exhibits the evolutionary processes of parameters of the Lorenz system, respectively. It can be observed that the algorithm could still obtain results without much deviation; even the data used is polluted by artificial noise. Furthermore, Figure 10 shows the relative estimation errors from 30 runs of the proposed algorithm with each single run executing 100 iterations. The estimation error can be calculated as follows:

$$\text{error} = |\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}| / \boldsymbol{\theta}, \quad (20)$$

where $\hat{\boldsymbol{\theta}}$ denotes the estimated vector, while $\boldsymbol{\theta}$ represents the pre-assumed vector. The mean estimated errors for these three parameters maintain at ten of the negative power. This fully illustrates the robustness of the proposed algorithm.

For the 4-D hyper chaotic system, the same noise level (10%) is considered. Similar with the Lorenz, Figure 11 also

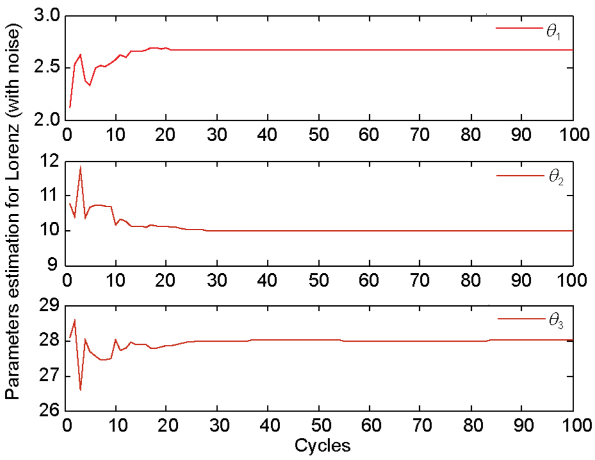

Figure 9 (Color online) Searching process for the Lorenz system with the present algorithm (with noise).

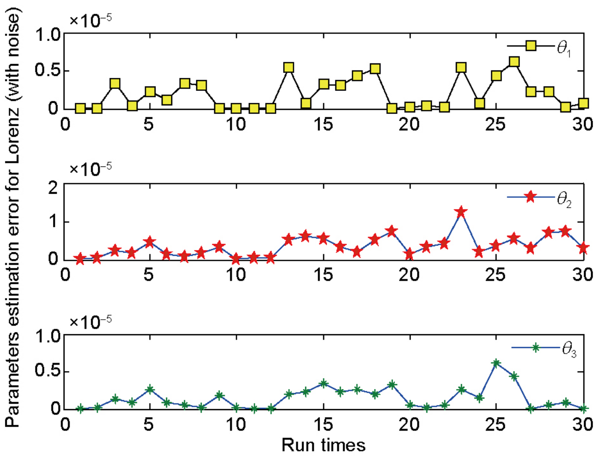

Figure 10 (Color online) The relative estimation error of Lorenz system with each run (with noise).

shows the relative error of each estimated parameter of each run. The final results are $\hat{\theta}$ = [5.0079, 16.0468, 1.0064, 0.5032] (300 input data), $\hat{\theta}$ = [5.0169, 16.5089, 1.0488, 0.5116] (200 input data), while the results from 100 and 50 input data are $\hat{\theta}$ = [5.0244, 16.1729, 1.0942, 0.5242] and $\hat{\theta}$ = [5.1012, 16.3949, 1.1341, 0.5087] respectively. It can be observed that the outcome of 300 data is the most accuracy without much deviation, which presents that the more input data is beneficial to enhance the robust of the algorithm. On the other hand, although the same noise level is introduced, the identified result of the Lorenz is closer to the assumed value. This may be attributed to the strong nonlinear feature of the 4-D hyper chaotic system.

5 Conclusion

In this work, parameter estimation for chaotic systems (Lorenz and a 4-D hyper chaotic system) is formulated as

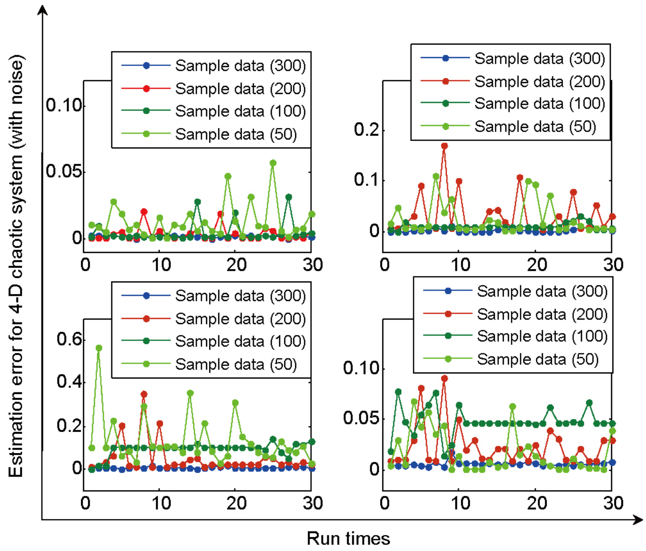

Figure 11 (Color online) The relative estimation error of 4-D hyper chaotic system with each run (with noise).

multidimensional optimization problem, and the proposed hybrid artificial bee colony algorithm is implemented to solve these problems on three-dimensional chaotic systems.

Based on the results and discussion presented in this study, the following main conclusions are given:

(1) Numerical simulations show that the present algorithm can estimate the parameters for chaotic systems with low deviations. Compared with other evolutionary algorithms, the proposed method can acquire a more accuracy result with less iteration. This result can be attributed to the Lévy walk and chaotic search mechanism.

(2) For the different input data, one can find this difference sample data mainly affect the initial iteration and it seems has no significant influence on global convergence. As with final estimated results, the result coming from 300 data used is the most accuracy and less deviation, which fully presents that the number of input data has a notable effect on the stability of the algorithm.

(3) Even the input data is contaminated with artificial noise, the present algorithm could still acquire a good identified result, especially for the Lorenz system, the error is particularly small. Moreover, for the 4-D hyper chaotic system, the identified result from 300 input data is also the best. This illustrates the more sample data can enhance the robust of the algorithm further. The future work is to apply the proposed hybrid algorithm to other chaotic systems and make this method as a powerful tool for various numerical optimization problems in physics.

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