

Structured trajectory planning of collision-free lane change using the vehicle-driver integration data

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In this paper, the structured trajectory planning of lane change in collision-free road environment is studied and validated using the vehicle-driver integration data, and a new trajectory planning model for lane change is proposed based on linear offset and sine function to balance driver comfort and vehicle dynamics. The trajectory curvature of the proposed model is continuous without mutation, and the zero-based curvature at the starting and end points during lane change assures the motion direction of end points in parallel with the lane line. The field experiment are designed to collect the vehicle-driver integration data, such as steering angle, brake pedal angel and accelerator pedal angel. The correlation analysis of lane-changing maneuver and influencing variables is conducted to obtain the significant variables that can be used to calibrate and test the proposed model. The results demonstrate that vehicle velocity and *Y*-axis acceleration have significant effects on the lane-changing maneuver, so that the model recalibrated by the samples of different velocity ranges and *Y*-axis accelerations has better fitted performance compared with the model calibrated by the sample trajectory. In addition, the proposed model presents a decreasing tendency of the lane change trajectory fitted MAE with the increase of time span of calibrating samples at the starting stage.

intelligent vehicle, lane change, trajectory planning, vehicle-driver integration

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1 Introduction

With the development of connected vehicle (CV), the study of intelligent vehicle (IV) has started to focus on automated driving. Using the information from CV environment, the IV can make its own decision to control the travelling trajectory, for example the lane changing. It is well known that lane-changing maneuver of IV is a complex process that involves the interaction between driver behaviors and vehicle dynamics. For IV, the lane changing is one of important driving behaviors, and lane change trajectory planning is an extremely significant issue. Unsafe and uncomfortable

lane-changing maneuver is a key factor for traffic accidents and traffic congestion [1, 2], it is very worthwhile to conduct trajectory planning of IV to perform smooth, safe and convenient lane change.

The past decades have witnessed ambitious research in lane change trajectory planning, and some discrete optimization schemes have been proposed to achieve the optimal lane change trajectory through minimizing a given cost function. However, the trajectory should be planned dynamically, so that the optimization is a time-consuming process. Furthermore, it is difficult to apply the optimized trajectory for lane change control due to the trajectory complexity [3]. In order to obtain simple lane change trajectory, some parametric models, such as the curvature polynomials of arbitrary order, are used to generate the trajectory. Nota-

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bly, the trajectory should satisfy the smooth requirement of lane-changing maneuver. Meanwhile, it should be reasonable and possible for practical execution [4]. Unfortunately, the curvature jerks of most trajectories generated by curvature polynomials are non-continuous, and the smooth performance of motion path faces with the same challenge [5].

To seek a feasible trajectory that maintains smooth travelling of IVs in lane change process, some basic principles for lane change trajectory planning are introduced [6]: a) the trajectory is continuous; b) the first and second derivatives of the trajectory are continuous and bounded; c) the trajectory can be generated easily and quickly; d) the trajectory should be reasonable and possible for practical execution; e) the trajectory should avoid rough output from the lane change controller. To satisfy the requirements, the comprehensive parametric model is used to generate the lane change trajectory by referring to the advantages of previous trajectory planning. For example, Yang et al. [7] proposed a lane change trajectory planning model by integrating linear offset and sine function, and overcame the defects of non-continuous curvature jerks. However, the proposed model did not consider the feeling of drivers for lane changing trajectory.

To solve the issue above-mentioned, a new structured trajectory planning model is introduced using the field data of vehicle-driver integration in this paper. The main contributions of this paper are as follows: (1) a trajectory planning model suitable for collision-free road environment is proposed, which satisfies the requirements of continuous curvature, the zero-based curvature at the starting and end points during lane change and continuous jerk curve. In addition, the proposed model based on linear offset and sine function over other methods has the advantage of simplicity, which provides an alternative method to control the lane change trajectory of IV. (2) driver's decision to change lane is associated with driver characteristics, driver attitudes (such as aggressive behavior) and depends on many factors. In order to reflect the impact of driver characteristics on the lane change trajectory, the trajectory planning model is validated using the vehicle-driver integration data collected from the field experiment, which benefit the balance driver comfort and vehicle dynamics; (3) only a few samples are used to calibrated the proposed model to obtain the lane change trajectory curve, therefore the lane change trajectory planning model does not consume much computation time, and yield much smooth trajectories with a lower computational cost.

2 Literature review

Over the past year, a large number of research achievements in lane change trajectory planning have been competed to assist driver achieve the smooth and comfortable lane-changing maneuver. At present, these studies mainly

focus on advanced driver assistance system (ADAS), IV trajectory tracking control [8], vehicle collision avoidance system (CAS) [9], and so on[10].

Considering the complexity of lateral and longitudinal lane change maneuver, lane change safety is a persistent concerned topic for ADAS and vehicle CAS. Butakov et al. [11] developed an ADAS that took both the dynamics and characteristics of individual vehicle/driver system into account during lane change. These characteristics were captured by a hidden Markov model, and the parameters of the model could be adjusted online to fit the individual vehicle/driver response during lane change. Using a full vehicle dynamics model to collect the information of steering angles etc., a lane change trajectory with a lower lateral acceleration profile and a smaller maximum lateral acceleration was proposed to reduce the time of performing an obstacle avoidance maneuver [12]. The simulation case study of a limited lateral acceleration with constrained direct trajectory optimization showed that the proposed trajectory optimization technique required less time than that of trapezoidal acceleration profile for lane-changing maneuver. According to analysis and evaluation on the lane change trajectory of various types including the circular and cosine type, the constant-speed offset type and the ladder type, a lane change trajectory model under multiple barriers was proposed by designing an optimal controller for lateral stability of vehicles [13]. In order to account for the practical requirements of IVs in safe handling of dynamic highway and inner city scenarios, semi-reactive trajectory generation method was proposed, and the method realized multiple objectives such as velocity keeping, merging, following and stopping, as well as reactive collision avoidance by means of optimal control strategies [3].

To achieve smooth trajectory control of lane change process, a long-time optimal and simple tracking trajectory is extremely vital. In view of this, various curves have been proposed to fit the lane change trajectory, such as circular, harmonic, polynomial and line segments. Rossi et al. [14] introduced a geometrical tangent curve to control the lane change trajectory by assigning some waypoints on the path. Vale et al. [15] developed a line segment trajectory that took the flexibility of vehicles into account, and provided smooth paths that could maximize the clearance to obstacles. This line guidance approach involved necessary maneuvers and was capable of maximizing the common parts of different paths. On the other hand, trajectory planning method based B-spline curves were designed with consideration of the non-holonomic constrained conditions including obstacle avoidance, curvature continuity, rotational speed, and so on [16]. Hidalgo et al. [17] introduced a lane change trajectory planner based Bezier curve, and endowed the vehicles with anti-collision property. Further, to enhance the active safety and realize the automation of IVs on the highway, a lane change trajectory was planned and tracked for lane-changing maneuvers on curved road. The longitudinal

and lateral coupling and the difference of curvature radius between the outside and inside lane were considered, and the Lyapunov theory was applied to evaluate the stability of the proposed tracking controller [18]. In general, since the curves have the advantages of continuous curvature and simplicity, many scholars are devoting much attention to describing the lane changing trajectory planning based on various curves. However, the acquisition of trajectory usually consumes much computation time.

Besides the above-mentioned achievements, driver feelings during lane-changing maneuver are attracting more and more attention in cooperative path planning. A cooperative driving architecture was presented to enable direct vehicles to adapt their motion to surrounding traffic situation by utilizing information obtained from other vehicles and infrastructure in the vicinity [19]. Considering driver experience, a cooperative controller for lane-changing maneuver was proposed to perform cooperative adaptive cruise control by communicating with several involved vehicles [20]. In addition, Feng et al. [5] also designed a lane change trajectory planning and tracking control model based on the cooperative vehicle infrastructure system. In this model, polynomial curve was adopted to describe the trajectory planning issues, and collision detection was mapped into a parameter space by infinite dynamic circles.

3 Lane change trajectory planning model

According to the basic principles and related studies based on curves for the above-mentioned lane change trajectory planning, four requirements should be satisfied to balance lane-changing maneuver safety and driver comfort during lane change [7]:

- 1) The trajectory curvature during vehicle lane change is continuous, and no mutation occurs;
- 2) The trajectory curvature at the starting and end points is equal to zero, which assures the motion direction of two end points in parallel with the lane line;
- 3) The trajectory of lane change is smooth and the maximum of trajectory is less than a threshold, namely the lateral acceleration during vehicle lane change should be less than the acceleration of driving comfort;
- 4) The jerk curve is continuous during lane-changing maneuver.

In this study, an integrated lane change trajectory planning model is proposed considering the zero-based lateral acceleration of linear offset function and the smooth of sine function. For the collision-free lane change scenario, the host vehicle performs a lane change from the starting point (x_s, y_s) to the end point (x_e, y_e) with a certain longitudinal velocity, as shown in Figure 1. As shown in Figure 1, three parameters, D , L and A , are introduced to reflect the impact of driver characteristics and vehicle dynamics on driver's decision to change lane. D , L and A is related to driver types

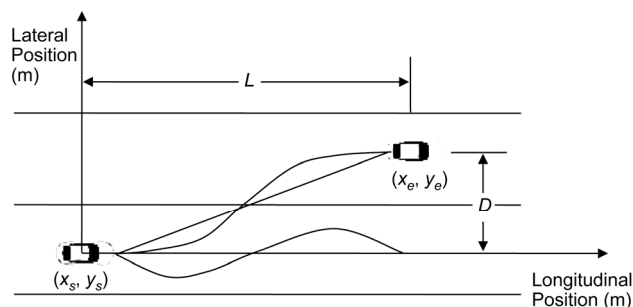


Figure 1 Collision-free lane change scenario.

and emergence level of driver's lane change.

The initial trajectory equation is:

$$y(x) = x + \sin x, \quad (1)$$

where x is the longitudinal displacement, m; $y(x)$ is the lateral displacement, m.

Considering actual road conditions, the integrated lane change trajectory planning model can be expressed as:

$$y(x) = \frac{D}{L}x + A \sin\left(\frac{2\pi}{L}x - \pi\right),$$

$$x \in (0, L), t \in (0, L/\dot{x}), \quad (2)$$

where D and L respectively denote the lateral and longitudinal distances between the starting and end points during lane change, m; A is the amplitude of sine function, which indicates the emergence level of driver's lane change, m.

It is noticed that the values of D , L and A is different for different driver types and vehicle dynamics. Aggressive drivers may be more likely to adopt the emergent maneuver relative to conservative drivers during lane change process. Figure 2 illustrates the lane change process of different drivers. As can be seen in Figure 2, aggressive drivers are apt to completing the lane change in a shorter longitudinal distance L and a bigger amplitude A through a bigger steering angle relative to conservative drivers. Obviously, this lane-changing maneuver is unsafe and uncomfortable for drivers. In order to reflect the impact of driver characteristics and vehicle dynamics on the process of changing lane comprehensively, the vehicle-driver integration data collected from the field experiment is used to calibrate three parameters, D , L and A , of lane change trajectory planning

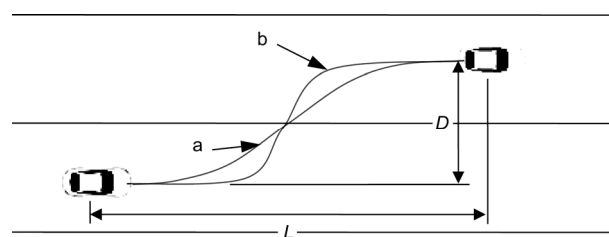


Figure 2 Lane change process of (a) conservative drivers and (b) aggressive drivers.

model.

The first, second and third derivative functions of Eq.(2) can be figured out as:

$$\begin{cases} \dot{y}(x) = \frac{D}{L} + \frac{2\pi A}{L} \cos\left(\frac{2\pi}{L}x - \pi\right), \\ \ddot{y}(x) = -\frac{(2\pi)^2 A}{L^2} \sin\left(\frac{2\pi}{L}x - \pi\right), \\ \dddot{y}(x) = -\frac{(2\pi)^3 A}{L^3} \cos\left(\frac{2\pi}{L}x - \pi\right). \end{cases} \quad (3)$$

Then, the trajectory curvature K during vehicle lane change can be expressed as:

$$K = \frac{|\ddot{y}(x)|}{[1 + \dot{y}(x)^2]^{3/2}}. \quad (4)$$

When $x = 0$ or $x = L$, i.e., the vehicle is located at the starting point or end point, the curvature K is equal to 0. At this moment, the curvature radius becomes infinitely large, which means the vehicle travels along a straight line.

The proposed model not only satisfies the requirement of continuous curvature in lane change trajectory, but also acquires zero-based curvature at the starting and end points of the lane change process. Moreover, the jerk curve is also continuous.

4 Datasets and experimental results

4.1 Data preparation

In order to validate the effectiveness of the proposed model, field experiment was carried out on a two-lane two-way road at the campus in Beijing Jiaotong University. The length of the experiment road is 1.4 km, and the width of each lane is 3.5 m, as shown in Figure 3.

The experiment vehicle is an automatic transmission of QIRUI MPV V5, which is 1.8 m in width and 4.7 m in length. It weighs approximately 1543 kg and can accommodate seven persons. The experiment vehicle provides two



Figure 3 (Color online) Field experiment road.

information acquisition methods: CAN Bus and various sensors. CAN Bus can acquire the velocity of the experiment vehicle, and the sensors mounted on the experiment vehicle can acquire the GPS data and the information of steering angle, accelerator pedal angle, brake pedal angle and three-axis acceleration. Figure 4 shows the experiment vehicle configuration.

Lane-changing maneuver is a comprehensive interaction of driving behavior and vehicle dynamics, so that the trajectory planning should be calibrated using the vehicle-driver integration data. In order to collect actual sample data using the sensors, field experiment was conducted on the experiment road. The experiment vehicle travelled 20 circles within a velocity range of 10–40 km/h during the field experiment, and the driver freely performed lane-changing maneuver. The samples of lane change were extracted from the field experiment. Since some disturbances, such as pedestrian and other vehicles, occurred in the field experiment, the samples of incomplete lane change were abandoned. Finally, 15 samples were used for further study.

During the experiment, the real time vehicle information was collected by a computer, and the data of the collected experiment vehicle were recorded in the following format:

\$15-07-30 16:04:07.400, 010.1, 6.5, 005. 0, 080. 5, 116333828, 39948048, 347, 00704, -1090, 04066.

The meaning of each section in the recorded data is described in Table 1.

In order to obtain actual lane changing trajectory, a funnel-type water bottle was placed in the front of the vehicle, and a camera was used to record the trajectory during lane change according to the watermark. The recorded trajectory curve is shown in Figure 5.

4.2 Model validation

All 15 samples were used to calibrate the lane change trajectory planning model. In order to obtain the significant factors for lane-changing maneuver, correlation analysis of L, D, A and the variables collected during lane change was carried out, and the results are shown in Table 2. It can be seen that the vehicle velocity, the variation ratio of steering angle and the Y-axis acceleration have significant correla-



Figure 4 (Color online) Experiment vehicle configuration.

Table 1 Description of each section in the collected experiment sample data

No.	Content	Description	Source
1	15-07-30 16:04:07.400	Data and time, the accuracy is 0.1 s	GPS
2	010.1	Velocity, km/h	GPS and CAN Bus
3	6.5	Displacement of brake pedal, degree	Brake pedal angle sensor
4	005.0	Displacement of accelerator pedal, degree	Accelerator pedal angle sensor
5	080.5	Steering angle, degree	Steering angle sensor, the measurement range is 1080°
6	116333828	Longitude, 10 ⁻⁶ degree	
7	39948048	Latitude, 10 ⁻⁶ degree	GPS
8	347	Traveling direction, degree	
9	00704*9.8/4096	X-axis acceleration, m/s ²	Three axle acceleration, the measurement range is 2G
10	-1090*9.8/4096	Y-axis acceleration, m/s ²	
11	04066*9.8/4096	Z-axis acceleration, m/s ²	

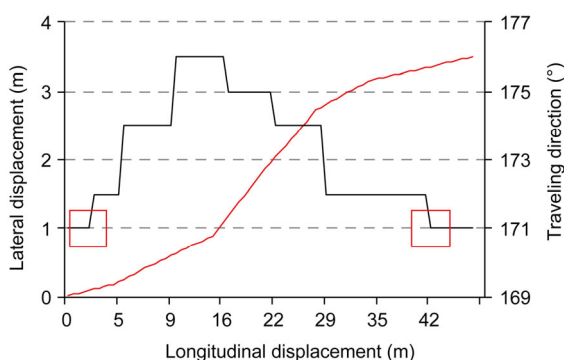


Figure 5 (Color online) The recorded lane change trajectory (red line) and vehicle traveling direction (black line) (the red box denotes the starting and end points of lane change).

Table 2 Correlation analysis of L , D , A and influencing variables

	Vehicle velocity	Variation ratio of Displacement of brake pedal	Variation ratio of Displacement of acceleration pedal	Variation ratio of steering angle	X-axis acceleration	Y-axis acceleration
L	0.871**	0.009	0.317	0.042	0.372	-0.629*
D	-0.02	0.168	0.007	0.038	-0.118	0.401
A	0.29	-0.029	-0.01	0.609*	-0.343	-0.129

**, Correlation is significant at the 0.01 level (2-tailed).

*, Correlation is significant at the 0.05 level (2-tailed).

tion with lane-changing maneuver. Hence, these three factors can comprehensively characterize the driving behaviors and vehicle dynamics during lane change. Consequently, they are selected as the candidate variables to calibrate the model.

Table 3 lists the parameter estimation of linear regression of lateral displacement L . The fitted equation of lateral displacement L is

$$L = 33.549 + 1.591 \times x_1 - 6.971 \times x_2, \quad (5)$$

Table 3 Parameter estimation of linear regression of lateral displacement L

Model	Coefficient	S.D.	t value	Sig.
Constant	33.549	18.646	1.788	0.101
velocity	1.591	0.314	5.062	0.000
Y-axis acceleration	-6.971	3.648	-1.897	0.084

where x_1 and x_2 denote the vehicle velocity and Y-axis acceleration respectively.

Similarly, the parameter estimation of linear regression of the amplitude A can be obtained as shown in Table 4. The fitted equation of amplitude A is

$$A = 0.162 + 0.035 \times x_3, \quad (6)$$

where x_3 denotes the variation ratio of steering angle.

For the lane of 3.5 m in width, the lane change trajectory planning model (Initial Model) is

$$y(x) = \frac{3.5}{33.549 + 1.591x_1 - 6.971x_2} \cdot x + (0.162 + 0.035x_3) \cdot \sin\left(\frac{2\pi}{33.549 + 1.591x_1 - 6.971x_2} \cdot x - \pi\right). \quad (7)$$

In order to evaluate the effectiveness of Initial Model, the fitting results were tested using three performance indexes of MAE, MAPE and RMSE. The 15 samples are divided into three groups according to their velocity, and the comparison results of the fitted values and actual values in different velocity ranges are shown in Table 5. It can be seen that there is certain difference of the fitted results for different velocity ranges, of which the trajectory planning for the velocity range [10, 20] has the worst fitted performance.

In order to further analyze the effect of velocity, the lane change trajectory model is recalibrated using the samples in the velocity range of 10~20 km/h. The revised model based on velocity is used to fit the samples in the same velocity range, and the results are shown in Table 6.

As can be seen in Table 6, the predicted results of the re-

Table 4 Parameter estimation of linear regression of the amplitude A

Model	Coefficient	S.D.	t value	Sig.
Constant	0.162	0.046	3.526	0.004
Variation ratio of steering angle	0.035	0.013	2.663	0.021

Table 5 Comparison results of the fitted values and actual values

Velocity range	Mean	S.D.	MAE (m)	MAPE (%)	RMSE
10-20	17.39	1.25	0.22	0.34	0.26
20-30	24.40	2.12	0.23	0.28	0.26
30-40	34.85	1.21	0.09	0.27	0.10

Table 6 Fitted results of the revised model based on velocity

Model	MAE(m)	MAPE(%)	RMSE
Initial model	0.22	0.34	0.26
Revised model based on velocity	0.10	0.21	0.11
Savings (%)	54.55	38.24	57.70

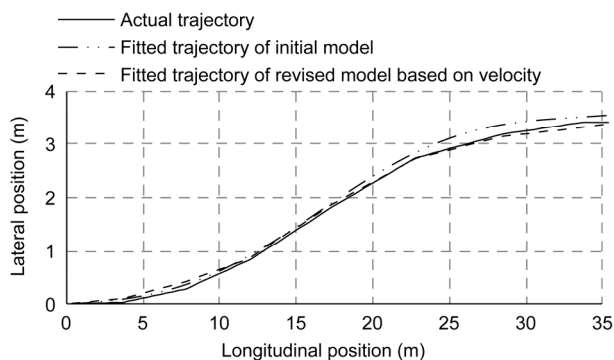
vised model show significant improvement compared with those of the initial model, and the savings of MAE is up to 54.55%. Figure 6 illustrates the fitted results of the initial model and the revised model based on velocity.

Therefore, velocity range is a vital influencing factor for lane change trajectory planning, and the trajectory calibrated according to different velocity ranges can be used to plan the IV path.

Similarly, the fitting performance of the Initial Model can be analyzed using Eq. (7) and the experiment sample data of Y-axis acceleration, and the results are shown in Table 7. It can be seen that there is also certain difference of the fitted results for different Y-axis accelerations, of which the trajectory in Y-axis acceleration range of [3.5–4.5] shows the worst prediction result. Hence, the lane changing trajectory model is recalibrated using the samples with Y-axis acceleration of 3.5~4.5 m/s².

The revised trajectory model based on Y-axis acceleration is used to fit the samples in the same Y-axis acceleration range, and the predicted results are shown in Table 8. As can be seen, the fitted results of the revised model based on Y-axis acceleration show significant improvement compared with those of the initial model, and the savings has a lower value than that of velocity classification.

In order to compare the effects of velocity range and Y-axis acceleration on the fitted results, all samples are selected as the fitted trajectory, and three revised models that were calibrated based on three velocity ranges are used to fit the lane change trajectory of corresponding samples respectively. Then, the mean value of MAE, MAPE and RMSE of the three models is used as the MAE of the revised model based on velocity. Similarly, the mean value of MAE, MAPE and RMSE of the revised models based on acceleration classification can also be obtained. The results are

**Figure 6** Fitted results of the initial model and the revised model based on velocity.**Table 7** Fitted results of the Initial Model based on Y-axis acceleration

Y-axis acceleration	Mean	S.D.	MAE(m)	MAPE(%)	RMSE
2.5-3.5	3.37	0.27	0.21	0.23	0.24
3.5-4.5	4.15	0.18	0.22	0.34	0.26

Table 8 Fitted results of the revised model based on Y-axis acceleration

Y-axis acceleration	MAE(m)	MAPE(%)	RMSE
3.5-4.5 initial model	0.22	0.34	0.26
3.5-4.5 revised model	0.21	0.28	0.25
Savings (%)	4.55	17.65	3.85

shown in Table 9. It can be seen that the fitted performance of the revised model based on velocity show significant improvement compared with that of the revised model based on Y-axis acceleration. Hence, when lane change trajectory planning is studied, the effect of velocity should be considered carefully.

For a structured lane change trajectory planning model, time span of calibrating the lane change model is a vital factor for the generation of trajectory. The calibrated trajectory using the data within two minutes at the initial stage of lane change contains more information than that using the data within one minute. Accordingly, the predicted results of the former show bigger errors compared with those of the latter. Table 10 shows the fitted results in different time spans.

Table 9 Comparison of the fitted performance of the revised models based on velocity and Y-axis acceleration

Model	MAE(m)	MAPE(%)	RMSE
Initial model	0.22	0.30	0.25
Revised model based on velocity	0.19	0.25	0.22
Savings (%)	13.64	16.67	12.00
Revised model based on Y-axis acceleration	0.20	0.27	0.24
Savings (%)	9.09	10.00	4.00

Table 10 Fitted results in different time spans

Time Span (s)	L prediction value (m)	A prediction value(m)	MAE (m)	MAPE (%)	RMSE
1.0	56.18	0.47	0.20	0.13	0.27
1.5	56.76	0.46	0.18	0.12	0.25
2.0	61.35	0.43	0.06	0.04	0.09
2.5	61.56	0.42	0.05	0.04	0.08
3.0	62.43	0.41	0.04	0.05	0.05
3.5	63.31	0.39	0.03	0.06	0.04
4.0	63.72	0.37	0.04	0.07	0.05
4.5	63.78	0.37	0.04	0.07	0.05
5.0	63.60	0.38	0.04	0.06	0.04
5.5	63.50	0.39	0.04	0.06	0.04
6.0	63.41	0.40	0.04	0.05	0.04
6.5	63.36	0.40	0.04	0.05	0.04

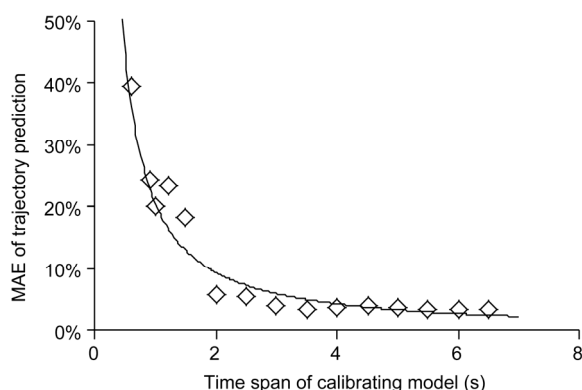


Figure 7 Fitted MAE of trajectory prediction by calibrating models with different time spans.

Figure 7 shows the fitted MAE of trajectory prediction by calibrating models with different time spans. As can be seen, the fitted MAE shows a decreasing tendency with the increase of time span.

5 Conclusions

In summary, a lane change trajectory planning model is proposed based on linear offset and sine function to balance driver comfort and vehicle dynamics, and then it is validated using the vehicle-driver integration data from the field experiment. Some conclusions can be drawn as follows:

(1) The integrated trajectory model based on linear offset function and smooth sine function satisfies the requirements of the continuous curvature of lane change trajectory, the zero-based curvature at the starting and end points and the continuous jerk curve. The proposed model can be used to plan the lane change trajectory, and assure the safety of lane-changing maneuver and driver comfort during lane change.

(2) Vehicle velocity is a vital influencing factor for planning the lane change trajectory, and the proposed model should be calibrated according to different velocity ranges in order to obtain better trajectory planning.

In our study, it is found that the determination of starting point during lane change has some effects on the prediction accuracy. Therefore, more work should be conducted in future to identify the lane change intention and determinate the starting point of lane change more accurately.

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