

Multi-objective robust design optimization of a novel negative Poisson's ratio bumper system

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Negative Poisson's ratio (NPR) structure has outstanding performances in lightweight and energy absorption, and it can be widely applied in automotive industries. By combining the front anti-collision beam, crash box and NPR structure, a novel NPR bumper system for improving the crashworthiness is first proposed in the work. The performances of the NPR bumper system are detailed studied by comparing to traditional bumper system and aluminum foam filled bumper system. To achieve the rapid design while considering perturbation induced by parameter uncertainties, a multi-objective robust design optimization method of the NPR bumper system is also proposed. The parametric model of the bumper system is constructed by combining the full parametric model of the traditional bumper system and the parametric model of the NPR structure. Optimal Latin hypercube sampling technique and dual response surface method are combined to construct the surrogate models. The multi-objective robust optimization results of the NPR bumper system are then obtained by applying the multi-objective particle swarm optimization algorithm and six sigma criteria. The results yielded from the optimizations indicate that the energy absorption capacity is improved significantly by the NPR bumper system and its performances are further optimized efficiently by the multi-objective robust design optimization method.

negative Poisson's ratio structure, bumper system, multi-objective robust design optimization, parameterized model, crashworthiness

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1 Introduction

With the increasing of car ownership and road traffic injuries, crashworthiness has become one of the most important aspects of the vehicle design. Some statistical analysis show that frontal collision, which account for 49%, has the highest proportion of the total accidents [1]. The bumper system is first contacted in the frontal collision, its performance has important influence on the crashworthiness behavior of

the vehicle and it is one focus of the automobile safety design. The bumper system is mainly applied to absorb the collision energy and ease the impact to protect the passengers and car body. To achieve better protection, the bumper system should absorb energy as much as possible when the accident happens. Therefore, the study on the design optimization of bumper system plays an important role in improving the crashworthiness. Tanlak et al. [2] conducted detailed studies on geometrical optimizations of the bumper beam and the crashworthiness was improved significantly. By increasing the plate thickness and adding reinforced parts, the energy absorbing capacity of the bumper system was promoted remark-

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ably by Xu et al. [3]. Hilmann et al. [4] optimized the crashworthiness of the bumper system successfully by applying the genetic algorithm. Sohn et al. [5] studied the energy absorbing performance of a bumper bracket and better crashworthiness behavior can be obtained by the hydraulic molding bumper bracket. The studies above improved the crashworthiness of the bumper system significantly and a better protection of the passenger and car body can be supplied. However, there is a great demand for further improving the crashworthiness of the bumper system with the increasing travel speed and it is challenging to make further improvement based on the existing technology. By comparing to traditional cellular structure, better performances in light weight and high energy absorption capacity can be obtained by NPR structure [6]. To improve the performance of the bumper system effectively and make it adjust to the increasing car speed, a novel NPR bumper system is first proposed in this work by combining the front anti-collision beam, crash box and NPR structure. The detailed studies of the NPR structure will be conducted in Section 2.

The design cycle of the vehicle systems is also one of the key factors of the product competitiveness and cost, thus developing high performance systems in an efficient way is meaningful. Computer aided engineering are applied by a lot of car companies to reduce time and cost, however it needs lots of calculations to achieve a satisfactory result and it is challenging or even impossible to find the optimal design. With the development of optimization theory, the combination of optimization algorithms and finite element methods are applied more and more to improve the performances. Zhang [7] optimized the vehicle crashworthiness by integrating stepwise regression model and multi-objective genetic algorithm. Xu et al. [8] optimized the thin-walled structure to improve the crashworthiness based on adaptive response surface methodology. The studies promoted the structure design greatly. However, there are fluctuations for material characteristics, crash speed, production precision etc., in practical production and working environment. The uncertain factors are normally ignored in the optimization process. Due to the fluctuations of uncertain factors, the quality of the components will decreased significantly or even leads to failure. Increasing safety factor is an effective way to solve this problem, however, it will leads to over-designed structure and the production cost will be increased greatly. To better solve the problem, six sigma robust optimization design method was first proposed and applied by Motorola Inc. to raise product quality and the method are being widely applied in vehicle industry [9]. The robust optimization design was conducted successfully in vehicle side impact crash simulation by Koch et al. [10] in 2004. Sun et al. [11] carried out the six sigma robust optimization in vehicle design by integrating the design of experiment, response surface method and Monte-Carlo simulation technique. Qian et al. [12] per-

formed the multi-objective six sigma robust optimization of energy absorption and peak force based on the approximation models of the responses. The designs of the vehicle structure are promoted remarkably by these studies. However, the optimization methods in the above studies are less effective in optimizing the shape, size and topology of the parts at the same time. The performances of the NPR bumper system are greatly depending on the parameters of the NPR structure, shape and thicknesses of the plate. To make further improvement and achieve the rapid design of NPR bumper system while considering perturbation induced by parameter uncertainties, a multi-objective robust design optimization method is proposed in this work. The full parametric model of the traditional bumper system is first built based on the parametric model knowledgebase. The parametric model of the NPR bumper system is then established by combining the full parametric model of the traditional bumper system and the parametric model of the NPR structure. By applying the MOPSO algorithm, the multi-objective robust optimization design of the NPR bumper system can be obtained based on the approximate models, which are built by integrating the OLHS, six sigma criteria and dual response surface method.

The remaining part of this paper is organized as follows. The performances of the NPR bumper system are detailed studied in Section 2 by comparing to traditional bumper system and aluminum foam filled bumper system. Section 3 describes the process of multi-objective robust design optimization for NPR bumper system. The optimal solutions were obtained and detailed discussed in Section 4. The conclusions of the paper are outlined in Section 5.

2 Performances studies of the NPR bumper system

Due to the limitation of the design space, the energy absorption capacity of the bumper system is hard to make further improvement based on the existing technology. To push the limit of energy absorption capacity, a novel NPR bumper system is first proposed in this work. The traditional bumper system is composed of anti-collision beam, left crash box and right crash box. Based on the traditional bumper system, the NPR bumper system is assembled by combining the traditional bumper system and NPR structure filled core. Since the anti-collision beam, left crash box and right crash box are hollow, NPR structure is applied to fill the energy absorption structure. The NPR structure filled core is assembled by anti-collision beam filled core, left crash box filled core and right crash box filled core. The NPR structure filled core is constructed by NPR cell structure repeating in three directions. Since special concave shape is contained in the NPR cell structure, negative Poisson's ratio property can be obtained by the NPR structure filled core. It should be noted that the length, width and height of the anti-collision beam

are 1160, 38 and 133 mm and those of the left and right crash box are 202, 63 and 117 mm, respectively. To study the performances, the FEM model of the NPR bumper system is first established, shown as Figure 1(a). 324784 elements and 440838 nodes are contained in the FEM model. Figure 1(b) is the enlarged view of the model of the NPR cell structure. The material of the traditional bumper system is SAPH440 and its elastic modulus is 210 GPa, the Poisson's ratio is 0.3, and the density is 7850 kg/m³. The material of the NPR structure is aluminum. The elastic modulus of aluminum is 70 GPa, the Poisson's ratio is 0.3, and the density is 2700 kg/m³.

To test and verify the performances of the NPR bumper system, the comparisons between the performances of the NPR bumper system and traditional bumper system are conducted and detailed discussed in this work. Since the aluminum foam filled material has been widely applied in the vehicle industry to improve the crashworthiness, it is also applied in the bumper system to make the comparisons. It should be noted that the aluminum foam filled bumper system is assembled by combining the traditional bumper system and aluminum foam filled core. The six degrees of freedom (DOF) of the rear-end of the left crash box and right crash box are fixed. The front end of the anti-collision beam is impacted by a rigid wall with a constant speed of 5 m/s, the mass of the rigid wall is 800 kg and the crushing displacement is 160 mm. Nonlinear explicit finite element method in Ls-dyna software package is applied for the calculation. A note about the performances studies is that all the three different bumper systems are analyzed in the same conditions. The energy absorption processes of the three different bumper systems in the collision are shown as Figure 2.

It can be seen from the Figure 2 that the maximum energy can be absorbed by the NPR bumper system during the collision and the energy absorbed by the aluminum foam filled bumper system is larger than that of the traditional bumper system. The differences between the absorbed energy by the

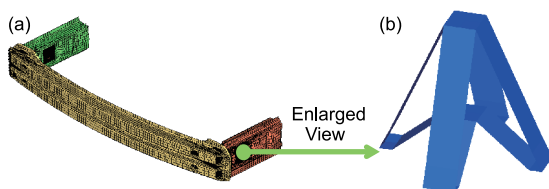


Figure 1 (Color online) FEM model of the NPR structure system. (a) FEM model of NPR bumper system; (b) NPR cell structure.

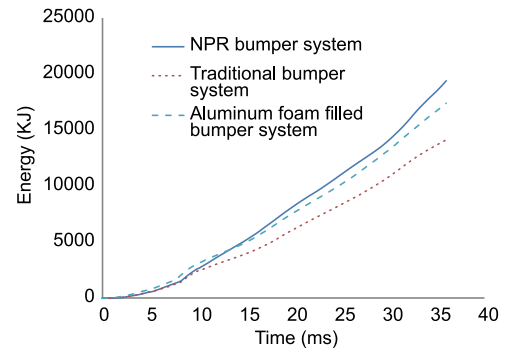


Figure 2 (Color online) Energy absorption processes of the three different bumper systems under constant impact speed.

three different bumper systems are small at the beginning of the collision and they increase with time. The collision energy with 19415, 14143 and 17413 kJ is absorbed by the NPR bumper system, traditional bumper system and aluminum foam filled bumper system at the end of the crash, respectively. Thus it can be concluded that the NPR bumper system has the strongest energy absorption capacity among the three different bumper systems.

Since foam-filled structures are added based on the traditional bumper system, the mass of the NPR bumper system and aluminum foam filled bumper system are larger than that of the traditional bumper system separately. To make the detailed comparisons, the parameters of the three different bumper systems are listed in Table 1.

Specific energy-absorption (SEA), which indicates the absorbed energy per unit mass, is an important indicator of crashworthiness for different structures [13], shown as follows:

$$SEA = \frac{EA}{M}, \quad (1)$$

where EA is the energy absorbed by the structure, M is the total mass of the structure. When higher collision energy is absorbed by the structure and the structure has smaller mass, a larger SEA can be obtained. Based on the definition, the larger is the SEA, the better is the energy absorption capacity of the structure. The SEA of the three different bumper systems in the collision processes are shown in Figure 3. The SEA values of the three different bumper systems are also listed in Table 1.

It can be seen from Figure 3 that the NPR bumper system has the largest SEA at the end of the collision process. Although the mass of NPR bumper system is larger, the SEA of

Table 1 Parameters of the three different bumper systems under constant impact speed

	Mass (kg)	Energy absorption (kJ)	SEA (kJ/kg)
NPR bumper system	5.68	19415	3418
Traditional bumper system	4.39	14143	3222
Aluminum foam filled bumper system	5.92	17413	2941

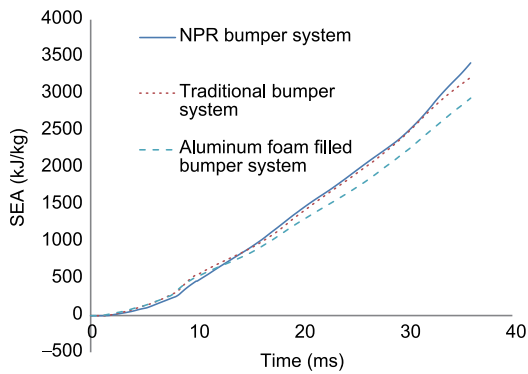


Figure 3 (Color online) SEA of the three different bumper systems under constant impact speed.

the NPR bumper system is still higher than that of the traditional bumper system. The SEA of the aluminum foam filled bumper system is smaller than that of the traditional bumper system although more energy is absorbed and crashworthiness can be enhanced by the aluminum foam filled bumper system. Thus it can be concluded from the above analysis that stronger energy absorption capacity and larger SEA can be obtained by the NPR bumper system and it can be widely applied in the vehicle design to improve the crashworthiness.

3 Process of multi-objective robust design optimization

Although better energy absorption capacity can be achieved by the NPR bumper system, its performances are affected by different design parameters and vary with the fluctuations of uncertain factors. To realize the efficient design of NPR bumper system while considering the influence of uncertain factors, a multi-objective robust design optimization method is proposed.

3.1 Parametric model of NPR bumper system

The geometry of NPR bumper system changes with design parameters. NPR structure filled core and traditional bumper system are the main components of the NPR bumper system, the parametric models of the two components are built separately based on their characteristics and these two parametric models are then integrated to construct the parametric model of NPR bumper system. It is inefficient to build the FEM model by the traditional modeling method when the shape of the parts is changed. In order to achieve the rapid modeling and simulation analysis, a parametric model knowledgebase is first built in the full parametric geometry engine software SFE-CONCEPT. Parametric cross-sections and parametric assemblies are contained in the sub-libraries. The full parametric model of the traditional bumper system is then built based on the parametric model knowledgebase in this work. The design parameters of the cross section are

selected as the design parameters. In the full parametric model, the original assembly relationships and the geometric continuity can be always maintained. When the design parameters change, the size, shape and topology of the bumper system can be modified quickly by changing the cross sections and the model can be established efficiently to be analyzed.

The NPR structure filled core is generated by NPR cell structure repeating in three directions and the NPR cell structure is the basic unit of the NPR structure filled core. The geometry of the NPR structure changes with the design parameters and it is challenging to build the FEM model efficiently when the parameters change. Since the NPR cell structure is symmetric and inerratic, the shape and topology are determined by 6 design parameters, shown as Figure 4. The angle between the long inclined cell wall and the symmetry axis is θ , while the angle between the short inclined cell wall and the symmetry axis is φ . l_1 is the length of the long inclined cell wall. The width of long inclined cell wall and short inclined cell wall is the same, W is the width of the cell wall. The thickness of the long inclined cell wall and short inclined cell wall are T_1 and T_2 respectively. In this work, the parametric model of the NPR structure is established by Matlab code. Based on the relationship among the parameters of the NPR cell structure, the element can be generated adaptively and assembled automatically to construct the model. Thus the FEM model of the NPR structure can be generated in high-efficiency by changing the design parameters in the Matlab code.

3.2 Multi-objective robust design optimization model

OLHS method, dual response surface model, six sigma criteria and MOPSO are integrated in this work to build the multi-objective robust design optimization model and conduct the design optimization. Since the accuracy of the dual response surface model is greatly affected by the choice of sample points, the OLHS method is applied to improve the accuracy [14]. It should be noted that the design variables of the structure will be arranged into two independent arrays to establish the dual response surface models. Noise factors will be put in the outer array to consider the uncertainties and the control factors will be arranged in the inner array. A cross

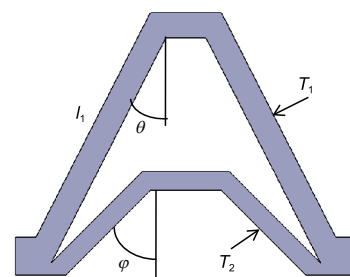


Figure 4 (Color online) Design parameters of the NPR basic structure.

product array will be generated by repeating at the every single sample points. The detailed introduction of the inner array and outer array can be found in the review references [15]. Unlike the traditional response surface model, which is only used for constructing model of the mean response value without considering variance, the dual response surface model can establish models of the mean response value and the standard deviation at the same time [16]. The quadratic polynomial dual response surface models are given as follows:

$$\begin{cases} y_{\mu} = \hat{y}_{\mu} + \varepsilon_{\mu} \\ = b_0 + \sum_{e=1}^n b_e x_e + \sum_{e=1}^n b_{ee} x_{ee}^2 + \sum_{e < f} b_{ef} x_e x_f + \varepsilon_{\mu}, \\ y_{\sigma} = \hat{y}_{\sigma} + \varepsilon_{\sigma} \\ = c_0 + \sum_{e=1}^n c_e x_e + \sum_{e=1}^n c_{ee} x_{ee}^2 + \sum_{e < f} c_{ef} x_e x_f + \varepsilon_{\sigma}, \end{cases} \quad (2)$$

where y_{μ} and y_{σ} are the mean and standard deviation of the true responses, \hat{y}_{μ} and \hat{y}_{σ} are the response models of the mean and standard deviation, ε_{μ} and ε_{σ} are the errors of the response models, x_e and x_f denote the design variables. b and c are the unknown coefficients which can be obtained by the least square method. The squared correlation coefficient (R^2) is applied in this work to verify the accuracy of the response surface models after the surrogate models are established [17], shown as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^t (y_i - \tilde{y}_i)^2}{\sum_{i=1}^t (y_i - \bar{y})^2}, \quad (3)$$

where y_i is the response value obtained from finite element analysis, \tilde{y}_i is the predicted result of mean or standard deviation response from the dual response surface models, while \bar{y} is the mean value of y_i and t is the number of the sampling points.

To reduce product defects and enhance the robustness of the NPR structure, six sigma criteria is applied in this work. Based on the traditional multi-objective design optimization model, the multi-objective robust design optimization model which has considered the perturbation induced by parameter uncertainties are shown as

$$\begin{cases} \min \{f_1(Y_{\mu 1}(x), Y_{\sigma 1}(x)), f_2(Y_{\mu 2}(x), Y_{\sigma 2}(x)), \dots, \\ f_N(Y_{\mu N}(x), Y_{\sigma N}(x))\}, \\ \text{s.t. } g_{\mu j}(x) + 6g_{\sigma j}(x) \leq 0, \\ x_L + 6x_{\sigma} \leq x_{\mu} \leq x_U - 6x_{\sigma}, \end{cases} \quad (4)$$

where $Y_{\mu N}(x)$ and $Y_{\sigma N}(x)$ are the mean values and standard deviation of the N th objective respectively, $g_{\mu j}(x)$ and $g_{\sigma j}(x)$ are the mean and standard deviation of the j th constraint respectively, x_{μ} and x_{σ} are represent the mean and standard deviation of design variable, while x_L and x_U denote the lower and

upper bounds of design variable. After the multi-objective robust optimization models are established, the MOPSO algorithm [18], which exhibits fast convergence and well-distributed Pareto front, is adopted to find the optimal solutions.

3.3 Numerical optimization procedure

Based on the analysis above, the multi-objective robust design optimization process of the NPR bumper system can be summarized as the following steps. The full parametric model of the traditional bumper system is first built based on the parametric model knowledgebase and the parametric model of the NPR structure is established based on the relationship among the parameters of the NPR cell structure. The parametric model of the bumper system is then constructed by combining the full parametric model of the traditional bumper system and the parametric model of the NPR structure. OLHS method, dual response surface model and six sigma criteria are integrated to build the multi-objective robust design optimization model with considering perturbation induced by parameter uncertainties. The MOPSO algorithm is at last applied to find the multi-objective robust optimal design solutions for the NPR bumper system. The detailed multi-objective robust optimization process of the NPR bumper system is given as Figure 5.

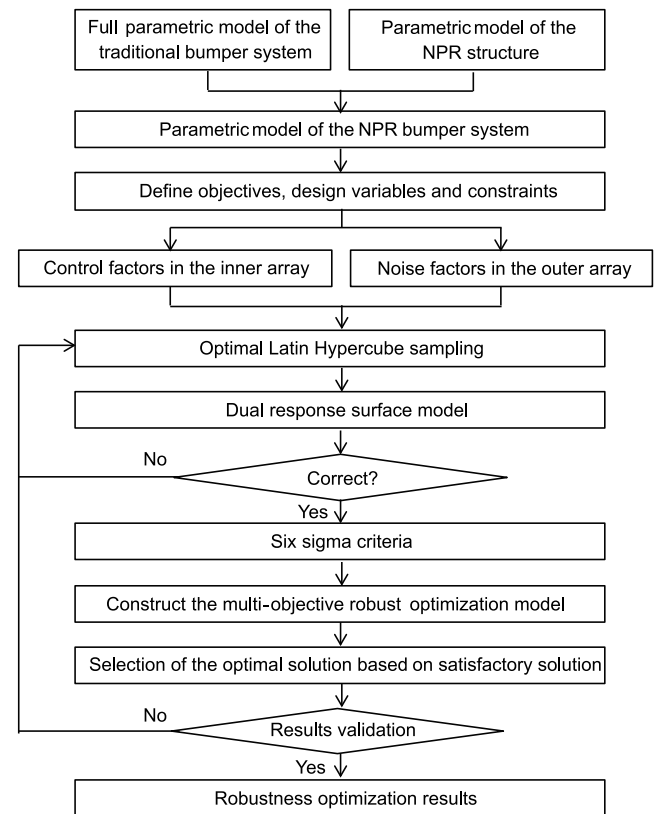


Figure 5 Multi-objective robust optimization process of the NPR bumper system.

4 Optimization result and discussion

The studies in Section 2 show that the energy absorption capacity of the NPR bumper system is improved significantly by comparing to the traditional bumper system and aluminum foam filled bumper system. The performances of the NPR bumper system are determined by the material and design parameters. To further strengthen the performances, it is important to find the optimal structure. During the collision, the bumper system is expected to absorb energy as more as possible to protect the passengers. However, the improvement in energy absorption often leads to increase in structure weight. To reduce emissions and production cost, the weight of the structure should be controlled strictly. The first peak crushing force (PCF) is meaningful for the passenger’s survival rate and it should be restricted in the collision to reduce the injuries. Thus, the energy absorbed by the NPR bumper system (U) and the mass of the structure (M) are selected as the objectives in this work, while the PCF is chosen as the constraint. The size, shape and topology of the structure play key roles in the performances of NPR bumper system and structure weight. The thickness of anti-collision beam (T_3), the width and height of the beam cross section (W_2 and H), the six design parameters of the NPR cell structure are selected as the design variables ($\theta, \varphi, l_1, W_1, T_1$ and T_2). Since the material properties are affected by the rolling process, the density (ρ), Young’s modulus (E) and yielding stress (σ) are chosen as random parameters. The fluctuation are in the ranges of $\rho = (7700, 7900) \text{ kg/m}^3$, $E = (198, 202) \text{ GPa}$ and $\sigma = (213, 217) \text{ MPa}$, respectively.

Based on the number and type of the parameters, the control factors, which are sampled with orthogonal design, are arranged in the inner array with 200 sample points and the noise factors are arranged in the outer array with 4 sample points. To simulate the variability which is caused by the uncertainties of the noise factors, the experiments in the inner array are repeated at the 4 sample points of the outer array, thus the total number of simulations in the work is 800. It should be noted that OLHS is applied to generate the sample points to explore the design space for its proven efficiency. The surrogate models of the energy absorption and PCF are then built by applying the dual response model in the quadratic polynomials. Since the mass of the structure is a linear function of the thickness of the parts, the dual response model of the mass is constructed linearly. After the surrogate models are constructed, their accuracies are assessed by the squared correlation coefficient (R^2). The accuracy assessments for the response surface models are listed in Table 2. It can be concluded that the accuracy requirements are satisfied and the

surrogate models can be applied in the following optimization process.

In order to consider the uncertainties, all the design variables are assumed to be distributed normally and the standard deviations are $[0.01, 0.01, 0.01]$. By combining the six sigma criteria, the multi-objective robust optimization model of the NPR bumper system can be given as follows:

$$\begin{cases} \min : (f_1, f_2) \\ f_1 = -\lambda U_\mu^2 + (1 - \lambda)U_\sigma^2, \\ f_2 = \lambda M_\mu^2 + (1 - \lambda)M_\sigma^2, \\ \text{s.t. } PCF_\mu + 6PCF_\sigma \leq 101, \\ D^L + 6D_\sigma \leq D_\mu \leq D^U - 6D_\sigma, \end{cases} \quad (5)$$

where D are the design variables. After the multi-objective robust optimization model of the NPR bumper system is established, MOPSO algorithm is then applied to find the Pareto optimal fronts. The parameters of the MOPSO algorithm in this work are given as listed in Table 3.

Multi-objective deterministic optimization of the NPR bumper system without taking into account the perturbations of design variables and parametric noise is also conducted in this work to make the comparisons. The MOPSO algorithm with the same parameters is applied to find the optimization results. The Pareto optimal fronts obtained from the deterministic and robust optimization are compared, shown as Figure 6.

Every point in the Figure 6 represents one optimal solution obtained from the deterministic and robust optimization. The optimal solution obtained from the two different optimization processes can obtain the higher performances than the original design, which means that the NPR bumper system is optimized successfully. By comparing the Pareto optimal fronts

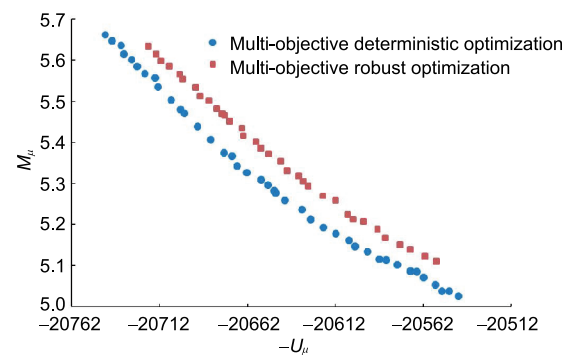


Figure 6 (Color online) Pareto optimal fronts obtained from the deterministic and robust optimization.

Table 2 Accuracy assessment of the response surface models

Coefficient	U_μ (%)	U_σ (%)	M_μ (%)	M_σ (%)	PCF_μ (%)	PCF_σ (%)
R^2	99.08	99.52	99.76	99.58	99.12	99.64

from the deterministic and robust optimization, it can be concluded that the performances of the optimal solution obtained from the multi-objective robust optimization are worse than those of the optimal solution obtained from the multi-objective deterministic optimization. However, the Pareto of the multi-objective robust optimization is more stable and has the lower probability to violate constraint. It can be also seen that the two objectives, energy absorption and mass, conflict with each other and there is a trade-off between them. If the NPR bumper system has the larger mass, a stronger energy absorption capacity can be also obtained. The designer has to make the decision based on the design emphasis.

To select the most satisfactory solution, the satisfaction function S , shown as eq. (6), is applied in this work to find the proper solution for the design [19]

$$S = \frac{(-U) - (-U)_{\min}}{(-U)_{\max} - (-U)_{\min}} + \frac{M - M_{\min}}{M_{\max} - M_{\min}}, \quad (6)$$

where U_{\max} and U_{\min} are the maximum and minimum values of energy absorption among the Pareto optimal fronts, M_{\max} and M_{\min} are the maximum and minimum values of mass among the Pareto optimal fronts. By calculating the satisfactory solution, two optimal solutions are selected respectively from the two different Pareto optimal fronts. The design variables and performances of the selected optimal solutions are listed in Tables 4 and 5 to make the comparisons.

The optimal results based on the deterministic optimization and robust optimizations are also verified by the FEM analysis. It can be seen that there is litter error between the results of surrogate models and traditional FEM models. The optimal results have sufficient accuracies. It can be also concluded that the performances of the NPR bumper system are

improved successfully by the deterministic optimization and robust optimization. Although the performances of the optimal solution obtained by the multi-objective robust optimization is worse than those of the multi-objective deterministic optimization, the perturbations induced by parameter uncertainties have been considered by the multi-objective robust optimization and the standard derivations of the objective functions is lower than those of the multi-objective deterministic optimization. Therefore, the robustness of the multi-objective robust optimization is much better, which will make the optimum design much practical in vehicle design, than that of the multi-objective deterministic optimization.

5 Conclusions

(1) A novel NPR bumper system is proposed in this work by combining NPR structure and the traditional bumper system to promote the energy absorption capacity. The detailed studies conducted between the NPR bumper system, traditional bumper system and aluminum foam filled bumper system shown that the energy absorption capacity are enhanced significantly by the NPR bumper system. The NPR bumper system can be widely used in vehicle design to improve the

Table 3 Parameters of MOPSO algorithm

MOPSO Parameters	Value
Population size	100
External archive	100
Inertial weight	0.70
Global learning coefficient	1.50
Personal learning coefficient	1.50

Table 4 comparison of design variables with the deterministic and robust optimization

Design variable	Original design	Value obtained by deterministic optimization	Value obtained by robust optimization
θ	0.35	0.41	0.46
φ	0.54	0.62	0.69
l_1	12.00	13.43	14.01
W_1	1.50	1.37	1.53
W_2	50.00	53.41	52.64
T_1	0.80	0.86	0.87
T_2	0.80	0.66	0.72
T_3	1.50	1.47	1.41
H	100.00	95.62	96.48

Table 5 Comparison of performances with the deterministic and robust optimization

	U_{μ}	U_{σ}	M_{μ}	M_{σ}	PCF_{μ}	PCF_{σ}
Original design	19415	0.543	5.68	0.176	121.48	0.449
Multi-objective deterministic optimization	20682	0.463	5.36	0.171	113.37	0.237
Verified deterministic optimization by FEM	20689	0.461	5.41	0.172	114.61	0.238
Multi-objective robust optimization	20645	0.401	5.39	0.164	115.29	0.157
Verified robust optimization by FEM	20652	0.406	5.43	0.162	117.34	0.156

vehicle crashworthiness and promote passive security.

(2) The multi-objective robust design optimization method of the NPR bumper system is proposed to achieve the rapid design while considering perturbation induced by parameter uncertainties. The parametric model of the bumper system is first constructed by combining the full parametric model of the traditional bumper system and the parametric model of the NPR structure. OLHS, dual response surface model, six sigma criteria are integrated to build the multi-objective robust optimization model and MOPSO is applied to achieve the Pareto optimal fronts. The optimization results are also verified by comparing to those of the multi-objective deterministic optimization. The detailed studies in this work show that the performances of the NPR bumper system are further improved successfully while the robustness is promoted significantly at the same time. The work also serves a good example for other application and optimization of NPR structure in automotive industries.

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