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Basin flood control system risk evaluation based on variable sets

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Flood control system risk evaluation is an effective measure for flood risk management and decisions. In order to make better flood risk decisions and thereby improve social and economic benefits, the flood control risk evaluation index system should be built to quantify and normalize flood risk effectively and efficiently. Because the current evaluation index has the binary miscibility characteristic of fuzziness and clarity, this paper establishes a new flood control system risk evaluation method based on the theory of variable sets (VS). Through a comparison of flood control risk evaluation with variable fuzzy sets (VFS) in the same basin flood control system risk evaluation, it is revealed that the new method, i.e., flood control risk evaluation with variable fuzzy/clear mixture sets (variable sets), will be reasonable in all cases. Finally, in one case study, i.e., the flood control system risk evaluation of Fengman Reservoir Basin, which is located in the southeast central of Jilin Province in China, the risk evaluation levels for each county in the basin as well as the whole flood risk distribution map of the basin could be provided with the new method. This provides useful information for basin flood control planning and design.

variable sets (VS), flood control system, risk evaluation, evaluation indices, flood control risk distribution map

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1 Introduction

Flood disasters are some of the most frequent and worst natural disasters in the world [1,2]. In recent years, hydraulic engineering has mitigated the impacts of floods on human beings and achieved remarkable effects on disaster prevention and reduction [3–5]. However, with the acceleration of climate change, human activities and global integration process, the inducing factors and bearing bodies of hazards are changing dynamically, and the risks are becoming more complicated [6–9]. For this reason, the loss and uncertainty created by future floods will increase rapidly [10–12]. Therefore, we should study flood risks with flexible approaches and methods.

Flood control system risk evaluation is an effective method for flood risk management and decisions [13,14]. However, due to the complexity of flood and engineering system structures, we are still at the very beginning of flood control risk evaluation study. The important components of flood control risk evaluation are evaluation indices and standards, in which the greatest difficulties lie in the fuzzy uncertainty of indices and their weights [15–17]. There are various evaluation methods for flood risk, such as the hydraulic model [18], gray theory [19] and the geographic based method [20,21]. But fuzzy theory has a special advantage for solving fuzzy complicated system issues; therefore, it has become one of the most popular evaluation methods and has been widely used in various research areas [22,23]. Furthermore, many improved models or approaches based on the fuzzy theory have been developed in recent years [15,16,24–26].

Chen et al. [27–29] established engineering fuzzy set theory over ten years ago and since created the variable

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fuzzy sets (VFS) theory. VFS theory has been widely applied and extended in hydrology, water resources and other fields [30,31]. Later, Chen et al. [32,33] extended VFS further to include variable fuzzy/clear sets, which can be abbreviated as variable sets (VS). We have applied this theory in hydrology and water resources engineering with favorable results. Other scholars have done research based on VS theory [34,35].

According to the mixing characteristics of fuzziness and clearness in evaluation indices, a new method of flood control system risk evaluation, which is based on VS, is proposed in this paper. Firstly, the reasonability of the new method is verified through a comparison of flood control risk evaluation with VFS in the same basin flood control system risk evaluation. Then, the new method is applied into the flood control system risk evaluation of the Second Songhua River Basin in the Northeast of China.

2 Background and fundamentals of variable sets

VS theory inherited the philosophy, mathematics and engineering background of VFS. VS's rationales and fundamental theorems were derived from VFS. VS broadens and advances VFS by combining its scope to both fuzzy systems and clear systems. Additionally, VS improves the whole series of theories and operation symbols to be in agreement with actual engineering conditions. Therefore, VS's theories and methods could be developed and applied more widely in different forms of engineering research.

2.1 Variable set theory background

2.1.1 Philosophical background

Everything in nature continuously and eternally evolves until its end. Evolution is a common phenomenon in natural systems. It is inevitable that new things will be generated as old things are eliminated, and the formed transitive or intermediary pattern reflects the real process of evolution.

In evolution, qualitative change takes place in two forms, i.e., sudden qualitative change and gradual qualitative change. Because both sudden qualitative change and gradual qualitative change are outcomes of the opposite unity theorem, the quantity-quality exchange theorem and the negation of the negation theorem, it is meaningful to describe qualitative change quantitatively.

Human society, as an important element in nature, follows its evolution laws. Following dialectics, three basis laws and the definition of difference, dimension, intermediary status, as well as polarization in dialectical materialism and the conception and definition of relative membership function (RMF) have been given. VFS has been established on the basis of RMF. Furthermore, relative difference degree (RDD) has been defined, and VF theory, which is based on RDD and relative difference function (RDF), has been built. In VS, under the conditions of space and time variation, the fuzzy matters, phenomena and concepts are mixed with clear concepts, and any components in the mixture may vary dynamically and interdependently. Therefore, the dialectics basis laws of opposite unity and its exchange during the evolutionary process of matter's movement are the philosophical grounds for VS theory.

2.1.2 Mathematical background

In 1965, Zadeh [36] proposed the concept of fuzzy sets, which broke through the characteristic function of Cantor sets into the interval's membership function [0, 1], and fuzzy mathematics was born. Although fuzzy mathematics has far-reaching significance in mathematics, it focuses on static things, phenomena and concepts, and could not describe their dynamic variability objectively; the same was true with Cantor sets. In fact, things, phenomena and concepts will alter continuously with as time, space and other conditions change in the complicated objective world, and the opposite unity of integration and dialectics between the clarity opposite of Either/Or, as well as the fuzziness opposite of Both-And, could be presented in the changing process. Therefore, research on dynamic fuzzy concepts within static fuzzy sets theory would result in a contradiction between theory and objective, which, to date, has been the main deficiency of fuzzy set theory.

2.1.3 Engineering background

All water bodies in the world, such as moisture in the atmosphere, rivers and lakes on the surface, and underground water in the ground, are changing dynamically all the time. Hydrology focuses on the laws of water body movement and changes, in which many fuzzy things, phenomena and concepts exist, including flood seasons and non-flood seasons, floods and droughts, wet and dry, clean and polluted. Moreover, these fuzzy phenomena and concepts are variable under definite temporal and special conditions. Classic hydrology does not consider the fuzziness and variability of hydrological phenomenon, so this research gap has been addressed by fuzzy hydrology.

With the shift from Cantor sets to Zadeh's fuzzy sets, the basic concept [0, 1] defined in mathematics was broken through. Then, Chen [37] further expanded the properties of research objects. He had expanded the clarity system opposite of Either/Or, i.e., static fuzzy sets proposed by Zadeh, to the fuzziness system opposite of Both-And, i.e., dynamic VFS, and made groundbreaking changes in the mathematical definition. Based on scientific research and experience over many years, Chen [37] found that things, phenomena and concepts will alter continuously with the changes of time, space and other conditions in the complicated objective world, and the opposite unity of integration and dialectics between the clarity opposite of Either/Or and the fuzziness opposite of Both-And could be presented through this changing process. For example, the concept of reservoir design flood level is clear; however, whether or to what extent the reservoir water level is above the design flood level or not in the reservoir flood control operation process is a fuzzy concept. Essentially, reservoir flood control optimal operation is the opposite unity of optimal implementation between clear concepts and fuzzy concepts in various changing conditions. Another example is that a man inside or outside a door are two clarity things opposite of Either-Or, but there must be a transition process from the clarity opposite of Either/Or to the fuzziness opposite of Both-And when someone is transiting from outside to inside the door. Assume that someone has a weight of W kilogram (characteristic value). He has been converted from clarity to fuzziness when he crosses the threshold and the transition from outside to inside the door is being implemented. At the moment that one foot is inside the door and another foot is outside the door, which could be approximated with the denotation that weight $W/2$ is inside the door and $W/2$ is outside the door, his relative membership degrees (RMD) to the collection of inside the door or outside the door are 0.5, respectively. When he has crossed the threshold, he becomes the man outside the door, i.e., he has been converted from fuzziness to clarity. The two cases above illustrate the opposite unity of integration and dialectics between the clarity opposite of Either/Or and the fuzziness opposite of Both-And.

2.2 The theoretical foundations of variable sets

2.2.1 *Opposite unity theorem*

Let U be a universe of discourse (UD), $u \in U$. The contrasting properties of u are represented by $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The two endpoints P_l and P_r of a continuum are defined as 1, 0 or 0, 1. For u in U, a pair of measures are defined as $\mu_{\underset{\wedge}{A}}(u)$ and $\mu_{\underset{\wedge}{A}^c}(u)$ at any point of the continuum, which can be called the opposite relative membership degree (RMD) of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The mapping is defined below:

$$\mu_{\underset{\wedge}{A}}, \mu_{\underset{\wedge}{A}^c} : U \to [0,\ 1], u \mapsto \mu_{\underset{\wedge}{A}}(u), \mu_{\underset{\wedge}{A}^c}(u) \in [0,\ 1]. \quad (1)$$

Eq. (1) is the opposite RMF of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. No matter what kind of change u makes, whether after change or before change, the measured antagonist value summation of u is always equal to 1 and does not change. That is

$$\mu_{\underset{\wedge}{A}}(u) + \mu_{\underset{\wedge}{A}^c}(u) = 1. \quad (2)$$

The dynamic change of the mapping (1) can be expressed by a continuum in Figure 1.

Eq. (2) is called the opposite unity theorem of VS.

Let

$$D_{\underset{\wedge}{A}}(u) = \mu_{\underset{\wedge}{A}}(u) - \mu_{\underset{\wedge}{A}^c}(u). \quad (3)$$

Then $D_{\underset{\wedge}{A}}(u)$ is the opposite RDD of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The

Figure 1 Chart of opposite relative membership function change.

following mapping is defined as the opposite RDF of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$:

$$D_{\underset{\wedge}{A}} : U \to [1, -1],\ u \mapsto D_{\underset{\wedge}{A}}(u) \in [1, -1]. \quad (4)$$

Eq. (4) is expressed on the number-axis as Figure 2.

Adding up eqs. (2) and (3) together, the relationship of RMF and RDF is

$$\mu_{\underset{\wedge}{A}}(u) = \left[1 + D_{\underset{\wedge}{A}}(u)\right] / 2. \quad (5)$$

Let

$$D_{\underset{\wedge}{A}^c}(u) = \mu_{\underset{\wedge}{A}^c}(u) - \mu_{\underset{\wedge}{A}}(u). \quad (6)$$

Then the mapping below is the opposite RDF of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$:

$$D_{\underset{\wedge}{A}^c} : U \to [-1, 1],\ u \mapsto D_{\underset{\wedge}{A}^c}(u) \in [-1, 1]. \quad (7)$$

According to eqs. (3) and (6), an equation is obtained as follows:

$$D_{\underset{\wedge}{A}}(u) = -D_{\underset{\wedge}{A}^c}(u). \quad (8)$$

2.2.2 *Quantity-quality exchange theorem*

If $C(u)$ represents the change of u in UD, the three symbols $C_1(u)$, $C_2(u)$ and $C_3(u)$ express the changes of u when time, space and other conditions change respectively:

$$C(u) = \{C_1(u), C_2(u), C_3(u)\}. \quad (9)$$

Suppose $D_{\underset{\wedge}{A}}(u) \neq 0$, and let $D_{\underset{\wedge}{A}}(C(u))$ express the change of $D_{\underset{\wedge}{A}}(u)$.

If $D_{\underset{\wedge}{A}}(u)$ and $D_{\underset{\wedge}{A}}(C(u))$ satisfy the following inequation:

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) < 0,\ \ D_{\underset{\wedge}{A}}(C(u)) \neq 1, 0, -1, \quad (10)$$

then this change is named as a gradually qualitative change.

If $D_{\underset{\wedge}{A}}(u)$ and $D_{\underset{\wedge}{A}}(C(u))$ satisfy the following inequation:

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) > 0,\ \ D_{\underset{\wedge}{A}}(C(u)) \neq 1, 0, -1, \quad (11)$$

then this change would be the quantity change.

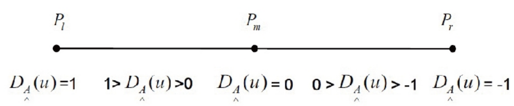

Figure 2 Chart of opposite relative difference function change.

Inequalities (10) and (11) are called the qualitative change and quantity change theorem of VS, otherwise uniformly known as then quantity-quality exchange theorem.

2.2.3 *The negation of the negation theorem*

The change of the value of u from 0 to 1 can be called a period (Figure 3).

Suppose there are N periods (where N is a positive integer and $N \in [1, +\infty]$). Before changing, the original state of $D_{\underset{\wedge}{A}}(u)$ is at the left endpoint P_l. After a whole periodic change ($N = 1$), $D_{\underset{\wedge}{A}}(u)$ is at the right endpoint P_r finally. This says that value of $D_{\underset{\wedge}{A}}(u)$ has changed from 1 to -1 and $D_{\underset{\wedge}{A}}(C(u)) = -1$. The negation of the negation theorem can be expressed as $D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) = 1 \cdot (-1) = (-1)^1$.

If there are several periodic changes ($N > 1$) of $D_{\underset{\wedge}{A}}(u)$ and the final state is at the right endpoint P_r (where N is an odd number) or the left endpoint P_r (where N is an even number), then the process of change of $D_{\underset{\wedge}{A}}(u)$ is called N times negation ($\underset{\wedge}{A}^c$) and there is

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) = (-1)^N. \tag{12}$$

$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) = -1$ when N is an odd number, and $D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) = 1$ when N is an even number. When N=2, there is

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) = 1. \tag{13}$$

Eq. (13) is the mathematic express of the negation of the negation theorem of VS.

2.2.4 *The opposite unity, quantity-quality exchange and the negation of the negation comprehensive theorem*

According to the opposite unity, quantity-quality exchange and the negation of the negation theorem, $D_{\underset{\wedge}{A}}(u)$, the N times evolution process of RDD, can be expressed by a vector as follows:

$$D_{\underset{\wedge}{A}-\underset{\wedge}{A}^c}(u) = (1, 0, -1, 0, 1, 0, -1, \cdots, 1, 0, -1). \tag{14}$$

According the Figure 3, eq. (14) shows the continuum's left endpoint $P_{l1}(\underset{\wedge}{A})$ (the original state of $D_{\underset{\wedge}{A}}(u)$ before the change).

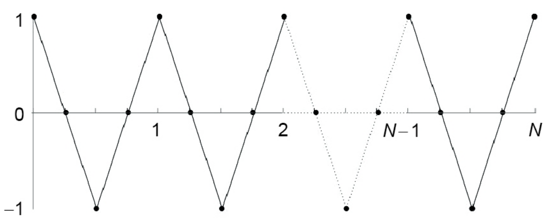

Figure 3 Chart of opposite relative difference function change in negation of negation theorem.

In eq. (14), the first "−1" corresponds to $P_{r1}(\underset{\wedge}{A}^c)$, the right endpoint of $D_{\underset{\wedge}{A}}(C(u))$ after change (the final state of the change when N=1, and the original state P_{l2} of the change when N=2, and so on). Considering the final state of every changing period as the original state of the next period, we can change every "−1" to "1". Then in N period, the changing process, eq. (14), can be

$$D_{\left(\underset{\wedge}{A}-\underset{\wedge}{A}^c\right)^N}(u) = (1, 0, 1, 0, 1, \cdots, 1, 0, 1). \tag{15}$$

Eq. (15) clearly expresses the dynamic process of things, phenomenon and its' opposite property RDD from quantitative change to qualitative change as N period changes, which is called the opposite unity, quantity-quality exchange and the negation of the negation comprehensive theorem.

3 Variable sets method for flood control system risk evaluation

Most evaluation indices for flood control system risk have the characteristics of both the clarity opposite of Either/Or and the fuzziness opposite of Both-And simultaneously. Therefore, this paper proposes the VS method for flood control system risk evaluation on the basis of Chen's VS theory [32]. The method is presented in detail.

Let U be the object sets of flood control risk to be evaluated:

$$U = \{u_1, u_2, u_3, \cdots, u_n\} = \{u_j\},\ j = 1, 2, \cdots, n. \tag{16}$$

The characteristic value matrix of multi-index i ($i = 1, 2, \cdots, m$, m is the sum of indices) could be identified as

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} = (x_{ij}),\ i = 1, 2, \cdots, m. \tag{17}$$

The index standard value interval matrix to be evaluated could be recognized according to the multi-level h ($h = 1, 2, \cdots, c$, c is the sum of levels) and the multi-index m:

$$\boldsymbol{I} = \begin{bmatrix} [a_{11}, b_{11}] & [a_{12}, b_{12}] & \cdots & [a_{1c}, b_{1c}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \cdots & [a_{2c}, b_{2c}] \\ \vdots & \vdots & \ddots & \vdots \\ [a_{m1}, b_{m1}] & [a_{m2}, b_{m2}] & \cdots & [a_{mc}, b_{mc}] \end{bmatrix} = ([a_{ih}, b_{ih}]),\ h = 1, 2, \cdots, c. \tag{18}$$

(1) Suppose level 1 (h=1) represents tiny risk. Regarding index i, the relative membership degree (RMD) of a_{i1}, i.e., the upper bound of the level 1 standard value interval $[a_{i1}, b_{i1}]$, to level 1 is 1 on the basis of physical conception. According to opposite unity theorem, the RMD of a_{i1} to the opposite level 2 is 0, and the RMD of the lower bound c to either level 1 or the opposite level 2 is 0.5. On the basis of the quantity-quality

exchange theorem, b_{i1} is the gradual qualitative change point from level 1 to level 2. If the characteristic value M_{i1} of index i is in the level 1 standard value interval $[a_{i1}, b_{i1}]$ and its RMD to level 1 is 1, M_{i1} is equal to a_{i1}, i.e., $M_{i1} = a_{i1}$.

(2) Suppose level c ($h = c$) represents extreme risk. Regarding index i, the RMD of b_{ic}, i.e., the lower bound of the level c standard value interval $[a_{ic}, b_{ic}]$ to level c is 1, the RMD of b_{ic} to the opposite level $(c-1)$ is 0, and the RMD of upper bound a_{ic} to either level c or the opposite level $(c-1)$ is 0.5, which means a_{ic} is the gradual qualitative change point from level $(c-1)$ to level c, according to physical conception. If the characteristic value M_{ic} of index i is in the level c standard value interval $[a_{ic}, b_{ic}]$ and its RMD to level c is 1, M_{ic} is equal to b_{ic}, i.e., i.

(3) Suppose level l ($h = l$) represents intermediate risk. Regarding index i, if c is an odd number, the intermediate level could be identified as $l = (c+1)/2$, the RMD of upper bound a_{il} or the RMD of lower bound b_{il} to the level l standard value interval $[a_{il}, b_{il}]$ is 0.5. Meanwhile, the RMD of a_{il} to level $(l-1)$ is 0.5 due to the coincidence between a_{il} and $b_{i(l-1)}$, and the RMD of b_{il} to level $(l+1)$ is 0.5 due to the coincidence between b_{il} and $v_h(u) = \left[1 + \sum_{i=1}^{m}[w_i D_{ih}(u)]\right]/2$. If the characteristic value M_{il} of index i is in the intermediate level l standard value interval $[a_{il}, b_{il}]$ and its RMD to level l is 1, M_{il} is equal to the midpoint of the level l value interval, i.e., $M_{il} = (a_{il} + b_{il})/2$. However, if c is an even number, the intermediate level would not exist.

For the multiple indices and multiple levels of the risk evaluation problem, the genetic model of the point value M_{ih}, which meets the three requirements above, can be expressed by

$$M_{ih} = \frac{c-h}{c-1}a_{ih} + \frac{h-1}{c-1}b_{ih},\ h = 1,2,\cdots,l,\cdots,c. \tag{19}$$

Eq. (19) could be expressed as $M_{i1} = a_{i1}$ when $h = 1$, $M_{ic} = b_{ic}$ when $h = c$, and $M_{il} = \frac{a_{il} + b_{il}}{2}$ when $h = \frac{c+1}{2}$. Matrix $\boldsymbol{M}$ could be described according to eq. (19) and the index standard value interval matrix $\boldsymbol{I}$.

$$\boldsymbol{M} = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1c} \\ M_{21} & M_{22} & \cdots & M_{2c} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mc} \end{bmatrix} = (M_{ih}). \tag{20}$$

According to matrix $\boldsymbol{X}$, the index characteristic value vector of the evaluation object u_j could be identified as

$$\vec{x}_j = (x_{1j}, x_{2j}, \cdots, x_{mj}),\ i = 1,2,\cdots,m. \tag{21}$$

Regarding the flood control evaluation object u_j, suppose that the characteristic value x_{ij} of the index i drops in the interval $[M_{ih}, M_{i(h+1)}]$ between level h and $(h+1)$ in matrix $\boldsymbol{M}$, where $h = 1,2,\cdots,(c-1)$. Then the RMD of index i to level h can be calculated by the equations as follows:

$$\begin{aligned} &\mu_{ih}(x_{ij}) = 0.5\left(1 + \frac{b_{ih} - x_{ij}}{b_{ih} - M_{ih}}\right), \\ &x_{ij} \in [M_{ih}, b_{ih}], \\ &h = 1,2,\cdots,(c-1), \end{aligned} \tag{22}$$

$$\begin{aligned} &\mu_{ih}(x_{ij}) = 0.5\left(1 - \frac{b_{ih} - x_{ij}}{b_{ih} - M_{i(h+1)}}\right), \\ &x_{ij} \in [b_{ih}, M_{i(h+1)}], \\ &h = 1,2,\cdots,(c-1). \end{aligned} \tag{23}$$

It is indicated from eqs. (22) and (23) that $\mu_{ih}(x_{ij}) = 1$ and $\mu_{i(h+1)}(x_{ij}) = 0$ if $x_{ij} = M_{ih}$, $\mu_{ih}(x_{ij}) = 0.5$ and $\mu_{i(h+1)}(x_{ij}) = 0.5$ if $x_{ij} = b_{ih}$, $\mu_{ih}(x_{ij}) = 0$ and $\mu_{i(h+1)}(x_{ij}) = 1$ if $x_{ij} = M_{i(h+1)}$ on the basis of the opposite unity theory.

Based on physical conception, the RMD of index i should be 0 to levels smaller than level h or larger than level $(h+1)$, which means:

$$\mu_{i(<h)}(x_{ij}) = 0, \tag{24}$$

$$\mu_{i(>(h+1))}(x_{ij}) = 0, \tag{25}$$

Using eqs. (19)–(25), we can get the RMD matrix of index i to evaluate object u_j to level h:

$$\begin{aligned} &\boldsymbol{\mu}(u_j) = \begin{bmatrix} \mu_{11}(u_j) & \mu_{12}(u_j) & \cdots & \mu_{1c}(u_j) \\ \mu_{21}(u_j) & \mu_{22}(u_j) & \cdots & \mu_{2c}(u_j) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1}(u_j) & \mu_{m2}(u_j) & \cdots & \mu_{mc}(u_j) \end{bmatrix} = \left(\mu(u_j)\right)_{ih}, \\ &i = 1,2,\cdots,m, \\ &h = 1,2,\cdots c. \end{aligned} \tag{26}$$

Using eqs. (5) and (26), we can get the RDD matrix of index i to evaluate object u_j to level h:

$$\begin{aligned} &\boldsymbol{D}_{ih}(u_j) = \begin{bmatrix} D_{11}(u_j) & D_{12}(u_j) & \cdots & D_{1c}(u_j) \\ D_{21}(u_j) & D_{22}(u_j) & \cdots & D_{2c}(u_j) \\ \vdots & \vdots & \ddots & \vdots \\ D_{m1}(u_j) & D_{m2}(u_j) & \cdots & D_{mc}(u_j) \end{bmatrix}, \\ &i = 1,2,\cdots,m, \\ &h = 1,2,\cdots c. \end{aligned} \tag{27}$$

Supposing the weight vector of index i is

$$\begin{aligned} &\vec{w} = (w_1, w_2, \cdots, w_m) = (w_i), \\ &\sum_{i=1}^{m} w_i = 1. \end{aligned} \tag{28}$$

Based on the comprehensive RMD model developed in ref. [30], the multi-indices of the comprehensive RMD nonlinear model of level h could be obtained with the use of the index RDD as follows:

$$v_h(u) = \frac{1}{1+\left[\frac{d_{ih}(u)}{d_{i(h+1)}(u)}\right]^{\alpha}} = \frac{1}{1+\left[\frac{\sum_{i=1}^{m}[w_i(1-D_{ih}(u))]^p}{\sum_{i=1}^{m}[w_i(1+D_{ih}(u))]^p}\right]^{\frac{\alpha}{p}}}. \quad (29)$$

Eq. (29) contains parameters α and p. α is an optimization criterion parameter, e.g., a least-absolute criterion would be used when $\alpha = 1$ and the least square optimization criterion would be used when $\alpha = 2$. p is a distance parameter, e.g., Hamming distance would be used when $p = 1$ and Euclidean distance would be used when $p = 2$. The derivation and detail description of eq. (29) could be seen in ref. [30]. In general use, $\alpha = 1$ and $p = 1$, and eq. (29) could be changed into a simple linear model:

$$v_h(u) = \left[1+\sum_{i=1}^{m}[w_i D_{ih}(u)]\right]/2. \quad (30)$$

We could obtain the basin flood control system risk evaluation result with the rank characteristic value function, which is

$$H(u) = \sum_{h=1}^{c} v_h(u)h, \quad h = 1, 2, \ldots, c. \quad (31)$$

4 Model validation

To validate the VS method for basin flood control system risk evaluation, this paper compares the method proposed in ref. [10] to the VS method for basin flood control system risk evaluation. Thus, the basin flood control system, its evaluation index system, criterion and levels are cited in ref. [10]. As for the evaluation index system establishment, the five indices cited in ref. [10] are: 1) reservoir risk (x_1); 2) dike risk (x_2); 3) flood diversion and storage area risk (x_3); 4) lake risk (x_4); and 5) river control engineering risk (x_5).

The index characteristic value vector of this flood control engineering system is:

$$\boldsymbol{X}' = (x_1, x_2, x_3, x_4, x_5) = (0.189, 0.268, 0.313, 6.2, 0.019). \quad (32)$$

According to the level index standard interval values in ref. [10], the five levels of the index standard value interval matrix would be:

$$\boldsymbol{I}' = \begin{bmatrix} [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.9, 1] \\ [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.9, 1] \\ [0.00, 0.25] & [0.25, 0.50] & [0.50, 0.75] & [0.75, 0.90] & [0.9, 1] \\ [1, 5] & [5, 7] & [7, 9] & [9, 10] & [10, 12] \\ [-0.07, -0.045] & [-0.045, -0.025] & [-0.025, 0.01] & [0.01, 0.045] & [0.045, 0.07] \end{bmatrix}. \quad (33)$$

Using eq. (19) and matrix I', the point value mapping matrix M' could be obtained:

$$\boldsymbol{M}' = \begin{bmatrix} 0 & 0.3125 & 0.625 & 0.863 & 1 \\ 0 & 0.3125 & 0.625 & 0.863 & 1 \\ 0 & 0.3125 & 0.625 & 0.863 & 1 \\ 1 & 5.5 & 8 & 9.75 & 12 \\ -0.07 & -0.04 & -0.0075 & 0.0363 & 0.07 \end{bmatrix}. \quad (34)$$

Using eqs. (22)–(27), the RMD and RDD matrixes of multiple indices to multiple levels could be obtained as follows:

$$\boldsymbol{\mu}_{ih}(u) = \begin{bmatrix} 0.622 & 0.378 & 0.000 & 0.000 & 0.000 \\ 0.356 & 0.644 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.999 & 0.001 & 0.000 & 0.000 \\ 0.000 & 0.767 & 0.233 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.757 & 0.243 & 0.000 \end{bmatrix}, \quad (35)$$

$$\boldsymbol{D}(u) = \begin{bmatrix} 0.244 & -0.244 & -1 & -1 & -1 \\ -0.288 & 0.288 & -1 & -1 & -1 \\ -1 & 0.998 & -0.998 & -1 & -1 \\ -1 & 0.534 & -0.534 & -1 & -1 \\ -1 & -1 & 0.514 & -0.514 & -1 \end{bmatrix}. \quad (36)$$

In order to compare the method proposed in ref. [10] with the VS method, the five-index weighting vector is cited from ref. [10]:

$$\vec{w} = (0.141, 0.276, 0.181, 0.373, 0.03). \quad (37)$$

Using eq. (30), the comprehensive RMD vector of multiple indices to multiple levels could be calculated:

$$\vec{v} = (0.186, 0.698, 0.110, 0.007, 0). \quad (38)$$

The level characteristic value of the basin flood control system risk evaluation could be obtained using eq. (31) as follows:

$$H' = 1.933. \quad (39)$$

According to evaluation result eq. (39), the basin flood control system risk belongs to level two, meaning that the basin flood control system is at a slight risk.

Almost identical to the revaluation result in this paper, the level characteristic value is 1.9 and the basin flood control system risk is at level 2 in ref. [10]. According to the comprehensive analysis of the basin index characteristic value, the revaluation result in this paper is consistent with a basin's actual flood control capacity.

5 Case study

5.1 Study area

This study is applied to the basin within the Fengman Reservoir, which has a storage volume of more than 112×10^8 m^3. The basin is located in the Second Songhua River, situated in the southeast of the Jilin Province in China, which stretches from longitude 125°18′E to 125°15′E and from latitude 41°40′N to 44°05′N, and shown in Figure 4.

The basin drains an area of 42500 km^2, where 13 counties in the Jilin Province are totally or partly located. The population is more than 4 million and the annual GDP exceeds 100 billion yuan. The Second Songhua River has two sources, i.e., Tou-Dao River and Er-Dao River, in which the Er-Dao River is the main source and originates from the Tianchi Volcano in the Changbai Mountains; it has an elevation of 2691 m. The Second Songhua River forms after the Tou-Dao River joins the Er-Dao River. The major tributaries of the Second Songhua River include the Tou-Dao River, the Er-Dao River, the Hui-Fa River, and the Yin-Ma River. The whole study basin distributes to the northwestern slope of Changbai mountains, where its elevation decreases gradually from southeast to northwest, dropping from 2000 m in the source region to 200–500 m in the Fengman Reservoir.

5.2 Evaluation index system

Flood control risk evaluation indices could be identified in terms of disaster-affected bodies, disaster-inducing factors, and disaster-inducing environments generally. Furthermore, evaluation indices can be broken into two categories: self-identified and quantifiable indices with clarity, and fuzzy concepts or uncertain indices with fuzziness. In this paper, a flood control risk evaluation index system that considers both fuzziness and clarity is established, in which the two kinds of indices are evaluated first through two subsystems, and then the basin flood control system risk evaluation result is obtained with the weighing approach. The two subsystems include the certain index subsystem I and the fuzzy uncertainty index subsystem II. The detailed processes are presented as follows.

5.2.1 Subsystem I

(1) Evaluation indices

As mentioned above, flood risk includes three main certain factors: disaster-affected bodies, disaster-inducing factors, and disaster-inducing environments.

Disaster-affected bodies means the characteristics of disaster-affected people. For a disaster-affected body, we use four indices: 1) population density (x_1); 2) GDP per unit area (x_2); 3) rate of old and young people per unit area (x_3); 4) rate of personal auto ownership (x_4). Population density (x_1) and GDP per unit area (x_2) are two important indices to reflect population and economy, while rate of old and young people per unit area (x_3) and the rate of personal auto ownership (x_4) reflect the abilities of escaping and post-disaster reconstruction.

As for disaster-inducing factors, we use four indices: 1) the precipitation amount per year (x_5); 2) R95P (x_6); 3) the maximum precipitation amount in 24 h (x_7); 4) the maximum precipitation amount in three days (x_8). The precipitation amount per year (x_5) could reflect the basin precipitation characteristics of wet, moderate, and dry years; R95P (amount of

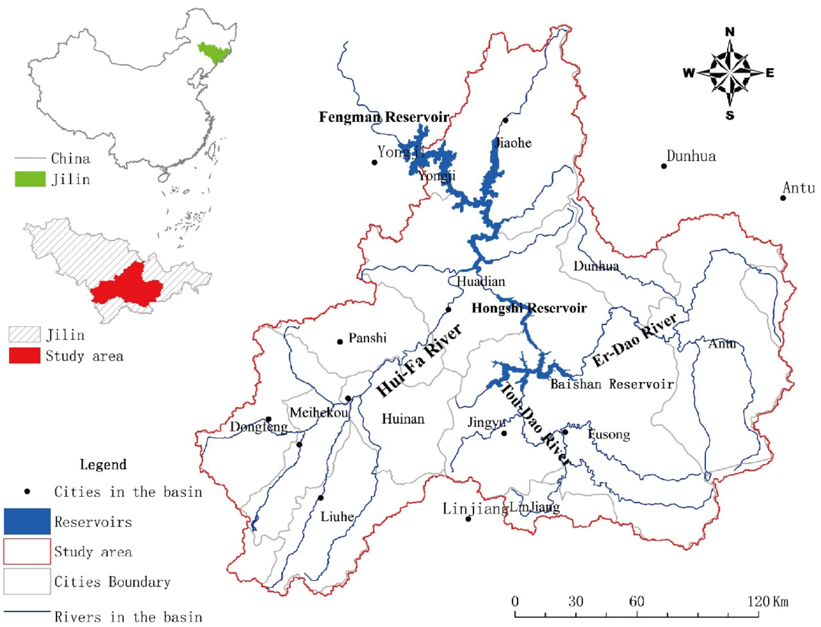

Figure 4 (Color online) Fengman Reservoir Basin.

annual precipitation with daily precipitation exceeding 95th percentile precipitation), which is recommended by the World Meteorological Organization (WMO) to be used as the extreme precipitation monitoring index, is an important factor for reflecting the occurrence of floods; the maximum precipitation amount in 24 h (x_7) could reflect the precipitation intensity of the basin has the largest influence on flood formulation.

A disaster-inducing environment is the external conditions for flood formation and development, including elevation, average slope, river channel, and land use. Plenty of large agricultural counties distribute over the basin above the Fengman Reservoir, and the percentage of cultivated land area is used to reflect land use. Therefore, for disaster-inducing environments, we use four indices: 1) the percentage of farmland area (x_9); 2) drainage density (x_{10}); 3) elevation (x_{11}); 4) slope (x_{12}).

The certain indices and their characteristic values in 2010 for basin flood control system risk evaluation are shown in Table 1.

(2) Evaluation criterion

Regarding each index, based on a statistical analysis of the mean value and standard deviation of its characteristic values, general industry regulations and actual basin conditions, as well as related criteria in different research, the classification standard value point for all twelve indices to five levels (from tiny to extreme, including tiny risk, slight risk, intermediate risk, heavy risk, and extreme risk), are identified and shown in Table 2.

(3) Index weight

Some approaches could be used to determine index weight, including subjective weight, objective weight, and combined subjective and objective weight. An analytical hierarchy process and binary fuzzy comparison are the most widely used methods in subjective weight determination, and the entropy method has been widely used in objective weight determination. Both subjective weight determination and objective weight determination have their own advantages as well as drawbacks. As far as the attribute of weight to reflect the importance of the index concerned, index weight represents the various preferences of evaluators, so there are always reasonable explanations to their different values. Therefore, in this paper, first, we assume that there are no preferences and differences among different indices, and equal weight w_0 is used. Second, based on the three factors of flood risk, the risk preference strategies of different factors are set, and then the expert grading method is used to identify index weights. Therefore, there are three further risk preference strategies: 1) weight w_1 , disaster-inducing factors with the biggest causes to flood risk; 2) weight w_2, disaster-inducing environments with the greatest importance; and 3) weight w_3, disaster-affected bodies with the key to flood risk. The basin flood control system risk evaluation index weights are shown in Table 3.

5.2.2 Subsystem II

On the basis of objective analysis for flood control, anthropic factors are considered in the evaluation index system, including the government capacities of controlling and fighting floods and the psychological pressure of people within the basin when facing flood disaster. These two factors could reflect the comprehensive influences of many non-engineering

Table 1 Basin flood control subsystem I risk evaluation indices and their characteristic values in 2010[a)]

County	x_1 (Person km^{-2})	x_2 (10^4 ¥ km^{-2})	x_3 (Person km^{-2})	x_4 (0.01 car Person^{-1})	x_5 (mm)	x_6 (mm)	x_7 (mm)	x_8 (mm)	x_9 (%)	x_{10} (km km^{-2})	x_{11} (m)	x_{12} (°)
Jiaohe	72.69	215.74	21.83	2.25	902.00	269.20	61.35	72.34	18.00	0.85	454.49	9.31
Yongji	150.02	316.67	42.63	1.69	1008.90	385.90	76.72	93.51	32.00	0.78	379.72	10.20
Huadian	68.73	283.62	19.93	7.08	767.94	241.70	80.82	153.20	17.00	0.81	475.87	10.55
Panshi	136.31	613.47	39.67	5.17	908.85	393.50	65.64	113.23	26.00	1.07	361.41	6.53
Huinan	155.31	293.64	50.47	4.56	1063.29	460.90	54.00	100.59	35.00	0.87	430.78	7.31
Dongf-eng	160.38	368.10	45.89	2.62	1133.56	424.30	66.55	113.40	41.00	0.85	392.07	4.54
Mei-hekou	284.50	829.82	95.31	9.11	1119.58	438.40	67.03	95.51	34.00	0.84	358.88	4.46
Liuhe	111.46	207.11	32.52	6.62	1214.04	461.80	65.83	115.51	27.00	0.86	502.61	9.89
Jingyu	46.89	118.35	15.90	3.20	1068.02	412.80	73.42	110.90	4.00	0.82	655.47	9.30
Dunhua	40.46	89.48	11.89	5.95	773.06	248.90	122.76	209.79	14.00	0.75	738.36	10.90
Antu	29.24	54.27	8.88	2.09	157.86	215.60	48.75	116.88	4.00	0.85	838.96	7.65
Fusong	46.80	166.16	16.04	7.52	909.92	275.80	66.73	109.91	3.00	0.88	865.55	8.13
LinJiang	63.63	208.60	21.29	2.97	1172.63	365.30	54.19	99.85	3.00	0.75	841.49	11.26

a) Data sources: precipitation amount per year, R95P, maximum precipitation amount in 24 h, and maximum precipitation amount in three days are obtained from Hydrological Administration of Jilin Province in China; drainage density, elevation and slope are extracted from the basin DEM (digital evaluation model); percentage of farmland area, rate of old and young per unit area, ratio of personal auto ownership, population density, and GDP per unit area are obtained from the statistical yearbook.

Table 2 Flood control risk evaluation index standard value interval in subsystem I

Evaluation indices	△	(Tiny) Level 1	(Slight) Level 2	(Intermediate) Level 3	(Strong) Level 4	(Special) Level 5
x_1 (Person km^{-2})	60	[0, 50]	[50, 110]	[110, 170]	[170, 230]	[230, 290]
x_2 (10^4 ¥ km^{-2})	200	[0, 100]	[100, 300]	[300, 500]	[500, 700]	[700, 900]
x_3 (Person km^{-2})	20	[0, 10]	[10, 30]	[30, 50]	[50, 70]	[70, 90]
x_4 (0.01 car Person^{-1})	2	[10, 8]	[8, 6]	[6, 4]	[4, 2]	[2, 0]
x_5 (mm)	250	[0, 200]	[200, 450]	[450, 700]	[700, 950]	[950, 1200]
x_6 (mm)	80	[0, 200]	[200, 280]	[280, 360]	[360, 440]	[440, 520]
x_7 (mm)	25	[0, 25]	[25, 50]	[50, 75]	[75, 100]	[100, 120]
x_8 (mm)	30	[0, 75]	[75, 105]	[105, 135]	[135, 165]	[165, 200]
x_9 (%)	10	[0, 5]	[5, 15]	[15, 25]	[25, 35]	[35, 45]
x_{10} (km km^{-2})	0.1	[0, 0.6]	[0.6, 0.7]	[0.7, 0.8]	[0.8, 0.9]	[0.9, 1]
x_{11} (m)	150	[950, 800]	[800, 650]	[650, 500]	[500, 350]	[350, 0]
x_{12} (°)	2	[15, 11]	[11, 9]	[9, 7]	[7, 5]	[5, 0]

Table 3 The basin flood control subsystem I risk evaluation index weights in different preference strategies

Evaluation indices	W_0	W_1	W_2	W_3
x_1	0.0833	0.105	0.0625	0.1575
x_2	0.0833	0.075	0.0625	0.1125
x_3	0.0833	0.075	0.0625	0.1125
x_4	0.0833	0.045	0.0625	0.0675
x_6	0.0833	0.06	0.075	0.045
x_5	0.0833	0.08	0.075	0.06
x_7	0.0833	0.12	0.075	0.09
x_8	0.0833	0.14	0.075	0.105
x_9	0.0833	0.075	0.1125	0.0625
x_{10}	0.0833	0.045	0.1125	0.05
x_{11}	0.0833	0.105	0.135	0.075
x_{12}	0.0833	0.075	0.09	0.0625

measures, such as disaster-affected bodies and disaster-inducing environments, and it is difficult to consider them in the current evaluation index system. For this reason, there has been a close concentration between these two factors. Therefore, two fuzzy indices, which could reflect the basin flood control risk caused by the above two factors, are proposed in subsystem II. The two fuzzy indices include the risk of the government fighting floods in emergencies (y_1) and the risk of people's disaster-affected psychology (y_2).

It is known that the risk of the government fighting floods in emergencies is lowered as the government gains capacity in fighting floods in emergencies, and the risk of people's disaster-affected psychology is lowered as people gain capacity in disaster-affected psychology. In accordance with the five-level evaluation criterion in subsystem I, the two fuzzy indices could be classified into five levels, from tiny to extreme. With the fuzzy recognition method, the classification standard value points of the two fuzzy indices' five levels are ascertained to be [0, 0.2], [0.2, 0.4], [0.4, 0.6], [0.6, 0.8], [0.8, 1]. Based on an integrated consideration of the actual situation in different regions, such as gender and age proportions of population, historical flood disaster, financial revenue and expenditure, emergent mechanisms for flood control, deployable goods and materials, and social mobilization, the fuzzy indices y_1 and y_2 are assigned to each region through expert scoring and comparative analysis, and shown in Table 4. With the binary comparison method, the weight vector of the fuzzy indices y_1 and y_2 are assigned to be (0.7, 0.3). According to the certain index subsystem I-VS method, the rank characteristic value of the fuzzy uncertainty index subsystem II would be obtained.

5.2.3 *Entire system evaluation*

Subsystem I contains the certain indices that are the main factors for basin flood control risk. Limited human activity impacts are considered in subsystem II to some extent, so subsystem I is more accurate than subsystem II. According to the binary comparison method, the weights of subsystem I

Table 4 Flood control risk evaluation indices and values in subsystem II

Fuzzy indice	Jiaohe	Yongji	Huadian	Panshi	Huinan	Dongfeng	Meihekou
y_1	0.51	0.25	0.62	0.51	0.36	0.36	0.56
y_2	0.35	0.35	0.6	0.4	0.25	0.55	0.35
Fuzzy indice	Liuhe	Jingyu	Dunhua	Antu	Fusong	LinJiang	
y_1	0.45	0.31	0.51	0.22	0.29	0.27	
y_2	0.7	0.35	0.3	0.2	0.5	0.3	

and subsystem II are assigned to be 0.6 and 0.4, respectively, and the basin flood control system risk evaluation result could be obtained with entire system evaluation equation as follows:

$$H_{\text{system}} = w_{\text{subsystem I}} \cdot H_{\text{subsystem I}} + w_{\text{subsystem II}} \cdot H_{\text{subsystem II}}. \tag{40}$$

5.3 Variable sets evaluation

According to the VS method for flood control system risk evaluation proposed in this paper, the evaluation indices, evaluation criteria, and evaluation index weights offered above, we take the Jiaohe County as an example to illustrate the evaluation process as follows.

As shown in Table 1, the index characteristic value vectors come from the statistics of measured data. The index characteristic value vector for the flood control risk evaluation of Jiaohe County in 2010 is

$$X = (x_1, x_2, \cdots, x_{12}) \\ = (72.69, 215.74, 21.83, 2.25, 902.00, 269.20, \\ 61.35, 72.34, 18.00, 0.85, 454.49, 9.31).$$

According to the level index standard interval values in Table 2, the five levels index standard value interval matrix would be:

$$\boldsymbol{I} = \begin{bmatrix} [0, 50] & [50, 110] & [110, 170] & [170, 230] & [230, 290] \\ [0, 100] & [100, 300] & [300, 500] & [500, 700] & [700, 900] \\ [0, 10] & [10, 30] & [30, 50] & [50, 70] & [70, 90] \\ [10, 8] & [8, 6] & [6, 4] & [4, 2] & [2, 0] \\ [0, 200] & [200, 280] & [280, 360] & [360, 440] & [440, 520] \\ [0, 200] & [200, 450] & [450, 700] & [700, 950] & [950, 1200] \\ [0, 25] & [25, 50] & [50, 75] & [75, 100] & [100, 120] \\ [0, 75] & [75, 105] & [105, 135] & [135, 165] & [165, 200] \\ [950, 800] & [800, 650] & [650, 500] & [500, 350] & [350, 0] \\ [0, 5] & [5, 15] & [15, 25] & [25, 35] & [35, 45] \\ [0, 0.6] & [0.6, 0.7] & [0.7, 0.8] & [0.8, 0.9] & [0.9, 1] \\ [15, 11] & [11, 9] & [9, 7] & [7, 5] & [5, 0] \end{bmatrix}.$$

Similarly, using eq. (19) and matrix $\boldsymbol{I}$, the point value mapping matrix $\boldsymbol{M}$ could be obtained:

$$\boldsymbol{M} = \begin{array}{c} \begin{matrix} m_1 & b_1 & m_2 & b_2 & m_3 & b_3 & m_4 & b_4 & m_5 & b_5 \end{matrix} \\ \begin{bmatrix} 0 & 50 & 65 & 110 & 140 & 170 & 215 & 230 & 290 & 290 \\ 0 & 100 & 150 & 300 & 400 & 500 & 650 & 700 & 900 & 900 \\ 0 & 10 & 15 & 30 & 40 & 50 & 65 & 70 & 90 & 90 \\ 10 & 8 & 7.5 & 6 & 5 & 4 & 2.5 & 2 & 0 & 0 \\ 0 & 200 & 220 & 280 & 320 & 360 & 420 & 440 & 520 & 520 \\ 0 & 200 & 262.5 & 450 & 575 & 700 & 887.5 & 950 & 1200 & 1200 \\ 0 & 25 & 31.25 & 50 & 62.5 & 75 & 93.75 & 100 & 120 & 120 \\ 0 & 75 & 82.5 & 105 & 120 & 135 & 157.5 & 165 & 200 & 200 \\ 950 & 800 & 762.5 & 650 & 575 & 500 & 387.5 & 350 & 0 & 0 \\ 0 & 5 & 7.5 & 15 & 20 & 25 & 32.5 & 35 & 45 & 45 \\ 0 & 0.6 & 0.625 & 0.7 & 0.75 & 0.8 & 0.875 & 0.9 & 1 & 1 \\ 15 & 11 & 10.5 & 9 & 8 & 7 & 5.5 & 5 & 0 & 0 \end{bmatrix} \end{array}.$$

According to formulation, the RMD and RDD matrix of the twelve indices to five levels would be:

$$\boldsymbol{\mu}(u)=\begin{bmatrix} 0 & 0.915 & 0.085 & 0 & 0\\ 0 & 0.78 & 0.22 & 0 & 0\\ 0 & 0.77 & 0.23 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0.59 & 0.41 & 0 & 0\\ 0 & 0 & 0 & 0.89 & 0.11\\ 0 & 0.046 & 0.954 & 0 & 0\\ 0.52 & 0.48 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0.2 & 0.8 & 0 & 0\\ 0 & 0 & 0.17 & 0.83 & 0\\ 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{D}(u)=\begin{bmatrix} -1 & 0.83 & -0.83 & -1 & -1\\ -1 & 0.56 & -0.56 & -1 & -1\\ -1 & 0.54 & -0.54 & -1 & -1\\ 1 & -1 & -1 & -1 & -1\\ -1 & 0.18 & -0.18 & -1 & -1\\ -1 & -1 & -1 & 0.78 & -0.78\\ -1 & -0.908 & 0.908 & -1 & -1\\ 0.04 & -0.04 & -1 & -1 & -1\\ 1 & -1 & -1 & -1 & -1\\ -1 & -0.6 & 0.6 & -1 & -1\\ -1 & -1 & -0.66 & 0.66 & -1\\ 1 & -1 & -1 & -1 & -1 \end{bmatrix}.$$

Using eqs. (26)–(30), the comprehensive RMD and RDD vector of multiple indices to multiple levels could be calculated, in which the equal weight w_0 is used for each index as an example:

$$v_{\text{Jiaohe}}=(0.29,0.32,0.24,0.14,0.01)\,.$$

Finally, using eq. (31), the level of characteristic value for the flood control risk evaluation of Jiaohe County could be obtained as follows:

$$H_{\text{Jiaohe}}=2.26.$$

The evaluation result shows that the flood control risk of Jiaohe County in 2010 belongs to level 2, which lies between a slight risk level and an intermediate risk level; it is perhaps more closer to a slight risk level. With the same calculation process and different objective risk weight strategies (Table 3), flood control risk evaluations for thirteen counties within Fengman Reservoir Basin in 2010 are completed and shown in Figure 5. For example, the level characteristic value for the flood control risk evaluation of Jiaohe County with the use of different objective risk weight strategies (w_0, w_1, w_2, w_3) are 2.26, 2.2, 2.26 and 2.18, respectively.

5.4 Results and discussion

Figure 5 indicates that there are few differences among various level characteristic values for the basin flood control system risk evaluation with different objective risk weight strategies. Although human social and productive activities influence flood risk, i.e., they are one reason for generating flood risk, the fundamental reasons for basin flood control risk are the objective floods that cause the risks and a series of external circumstances for flood generation. Therefore, it can be seen that basin flood control risk is determined mainly by the objective conditions of the basin, such as regional geographic location, climatic conditions, and underlying surface conditions, which have fewer relationships with objective index weights.

With different risk weight strategies, the flood control risk of Meihekou County in 2010 is between level 3 and level 4 invariably; it inclines to an intermediate risk with three strategies as w_0, w_1, and w_2, and inclines to a heavy risk with strategy as w_3. The evaluation results are related to its population density, GDP per unit area, and characteristic of more people with less land; therefore, there are greater losses to population and economy when floods happen in Meihekou County. Its geographic location and socioeconomic development conditions determine its highest risk levels among the thirteen counties within the Fengman Reservoir Basin, and, especially, risk is more obvious as evaluation strategy inclines to population security. However, the characteristics in Antu County are fewer people and vast lands, general economic development, forests as the majority of land use, and good conditions of the underlying surface. The location of Antu County in the upstream of the basin, and its capacity of natural flood control, determines it to be of least risk and highest security.

In each county where towns are the location in which populations and economies are concentrated generally, the flood control requirement and risk is higher. Therefore, using the center around the town, the risk evaluation result of every point in the basin could be obtained with the IDW (inverse distance weighted) spatial interpolation method, as shown in Figure 5.

Figure 5 illustrates that the flood control risks for the different weight strategies of five counties located in the upstream of the Wudaogou, i.e., Dongfeng, Liuhe, Meihekou, Huinan, and Panshi, are mostly between slight risk and intermediate risk; they are at intermediate risk or even heavy risk in a few regions, which is higher than other regions invariably. Compared to the flood control risks of the above five counties, the flood control risk of the region covering the middle and northwest of Huadian to the east of YongJi are lower, and the flood control risks of the regions located in the upstream of the basin, including Antu County and Fusong County, and middle and upper reaches of Jiaohe are the lowest.

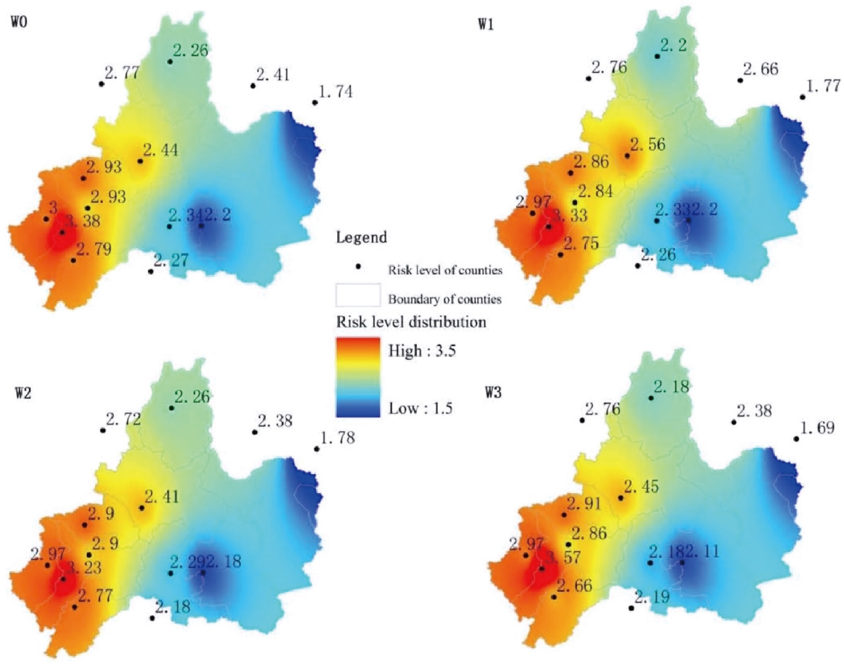

Figure 5 (Color online) Level characteristic values for the basin flood control system risk evaluation with different objective risk weight strategies.

Considering the actual situation of the Fengman Reservoir Basin, flood control risk is between intermediate risk and heavy risk in the regions where the underlying surface and vegetation have been destroyed severely. Farmland area accounts for a larger proportion of the land, drainage density is high, water conservancy projects have been constructed to a large extent, population density is high, socioeconomic development is fast, and the treasure assembly level is higher than in other regions; additionally, the intense social activities of people increase the flood risk to some extent. The flood control risk is between slight risk and intermediate risk in the regions where the general underlying surface conditions are better, the socioeconomic development is slower, the degree of natural environment damage is lower, and the capacity of the underlying surface in water storage and flood control is stronger than in the regions where the flood control risks are between intermediate risk and heavy risk.

6 Conclusion

Because the evaluation indices themselves have the binary miscibility characteristics of fuzziness and clarity, this paper introduces VS theory, including its background and foundations, and establishes a new flood control system risk evaluation method based on VS theory. Through a comparison of flood control risk evaluation with VFS in the same basin flood control system risk evaluation, this paper indicates that the new method will be at the same reasonable level. In the case of the Fengman Reservoir Basin, which is located in the southeast central of Jilin province in China, a evaluation index system that considers objective and anthropic factors is established, the new method is used to evaluate the basin's flood control system risk and obtain the risk evaluation levels for each county in the basin. The whole flood risk distribution map of the basin is obtained with the spatial interpolation method.

The evaluation results are validated in two ways in this paper. First, the results are compared with other similar research results [32,33], and proved to be reasonable. Second, based on the practical experience and basin characteristics, the results are judged in a macroscopic view. Finally, combined with the study basin catastrophic flood in 2010, which caused huge losses to many counties within the basin, e.g., Meihekou and Huadian Counties, the evaluation results of risk levels in different regions are proved to be truthful according to the research used to study the basin for several years.

In general, the evaluation results of the flood control system risk of the Fengman Reservoir Basin coincide with the basin's actual characteristics and better reflect the flood control risk distribution in the basin. Additionally, the revaluation results could provide scientific grounds for the planning and design of the basin flood control and disaster reduction. Furthermore, the whole flood risk distribution map of the basin could be used to make propaganda and educate for the people in different regions, and the basin flood control capacity could be improved in both engineering and non-engineering respects to realize the actual significance of the basin flood control system risk evaluation. Meanwhile, not discounting the fact that lives and properties need to be guaranteed, people's disaster-affected psychology is a new subject that must to be considered in disaster prevention and reduction at present or in the future. Therefore, the evaluation with the new method is conductive for reflecting the ways human get along in nature, which realizes the balance between human and nature in future planning and development and makes flood control risk evaluation more meaningful. In the future, we plan to develop more practical applications for the new flood control system risk evaluation method, e.g., development of basin flood control operations and risk assessment platforms in China supported by National Science and Technology Pillar Program during the Twelfth Five-year Plan Period and expect to obtain more achievements.

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