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Constructal entransy dissipation rate minimization of a rectangular body with nonuniform heat generation

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Based on construtal theory, a nonuniform heat generation problem in a rectangular body is investigated in this paper. Entransy dissipation rate (EDR) is taken as the optimization objective. The optimal body shapes with constant and variable widths of the high conductivity channel (HCC) are derived. For the rectangular first order assembly (RFOA) with constant cross-section HCC, the shape of the RFOA and width ratio of the HCCs are optimized, and the double minimum EDR is obtained. The heat transfer performance of the RFOA becomes worse when the nonuniform coefficient increases. For the RFOA with variable cross-section HCC, the EDR of the RFOA can be minimized for four times. Compared the optimal construct based on minimum EDR of the RFOA with that based on minimum maximum temperature difference, the shape of the former optimal construct is tubbier, and the average temperature difference is lower. In the practical design of electronic devices, when the thermal safety is ensured, the constructal design scheme of the former optimal construct can be adopted to improve the global heat transfer performance of an electronic device.

constructal theory, entransy theory, nonuniform heat generation, rectangular body, generalized thermodynamic optimization

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1 Introduction

Inserting high conductivity channel (HCC) into the heat conduction area is one of the effective ways to dissipate the internal heats of the electronic devices. Bejan [1] first applied constructal theory [2–8] into the HCC design of a rectangular electronic device, and assembled the HCCs into a tree-shaped pathway. The optimal distribution of the HCCs in the rectangular body was obtained, and the peak temperature of the electronic device was reduced. The work mentioned above provided a basic performance optimization method for the heat conduction problem in engineering. Inspired by this work, many scholars implemented constructal optimizations of the heat conduction electronic devices in rectangular [9–18], square [19–23], triangular [24,25] and disc [26–30] areas, respectively. Besides heat conduction electronic devices, constructal theory has been also applied to the optimal designs of heat exchangers [31–33], cavities [34,35], heat sources [36–38], tubes [39,40], micro-channels [41,42], and iron and steel production process [43–52], etc.

To describe heat transfer ability of an object, a new physical quantity, named as "entransy", was proposed by Guo and Li et al. [53,54]

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$$E_{vh} = \frac{1}{2}Q_{vh}U_h = \frac{1}{2}Q_{vh}T, \quad (1)$$

where Q_{vh}, U_h and T are the heat capacity at constant volume, thermal potential and temperature, respectively. From eq. (1), the entransy dissipation rate (EDR) $\dot{E}_{vh\phi}$ of an object can be written by

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi}\mathrm{d}v = \int_v |\dot{q}\cdot\nabla T|\,\mathrm{d}v = \int_v |k(\nabla T)^2|\,\mathrm{d}v, \quad (2)$$

where v, $\dot{q}$ and ∇T are the volume, thermal current density vector and temperature gradient, respectively. The objective of maximum temperature difference (MTD) can lead to the reduction of peak temperature, but it can only reflect the local heat transfer performance when the heat transfer system is multi-dimensional one. The objective of EDR can lead to the reduction of average temperature difference, and it reflects the global heat transfer performance of a multi-dimensional heat transfer system. Therefore, when the researched object is multi-dimensional heat transfer system, the objective of EDR is more suitable to be taken as optimization objective. Henceforth, many scholars introduced entransy theory [55–60] into the optimizations of various heat transfer problems [61–77], which greatly promoted the development of entransy theory.

In the constructal designs of heat conduction bodies, Chen et al. [68] firstly applied entransy theory into the constructal optimization of the heat conduction problem. They found that new optimal construct and lower average temperature difference of the high order assembly could be derived by EDR minimization. Wei et al. [69], Xiao et al. [70,71], Chen et al. [72] and Feng et al. [73] further optimized the heat conduction bodies with triangular [69], tapered [70], sectorial [71,72] and cylindrical [73] elements based on entransy theory, respectively, and provided some new guidelines different from those obtained based on MTD minimization. Wei et al. [74] and Wu et al. [75] optimized the rectangular heat conduction body based on global optimization method, and effectively reduced the average temperature difference compared with the typical local design method. Moreover, Feng et al. [73,76,77] further optimized the heat conduction bodies with rectangular [73,76] and triangular [73,77] elements at micro and nanoscales, respectively, and obtained optimal constructs of the bodies different from those at convectional scale.

The heat conduction problems mentioned above all belong to uniform heat generation ones. Actually, the heat generation in the heat conduction body is always non-uniform one. Ruiz et al. [78] and Cetkin [79] considered different heat generation rates at different areas of a rectangular body. Cetkin and Oliani [80] and Assad [81] further built non-uniform heat generation (NUHG) models in the cube and rectangular bodies, and considered the linear and exponential functions of the NUHGs in the bodies, respectively. Moreover, the NUHG models with heat generations changing in planar two-dimensional directions, cylindrical and sphere three-dimensional directions were built by Vessakosol [82], Gaikwad and Ghadle [83] and Pawar et al. [84], respectively. The investigations of NUHG problems have been implemented by many scholars, but the combination of NUHG problem and entransy theory is rare. In this paper, a heat conduction model with linear function of the NUHG will be considered, and constructal theory and entransy theory will be introduced in the optimization of the model. The optimal constructs of a rectangular body with different cross-section HCCs will be obtained after constructal optimization. Performance comparisons of the bodies obtained based on MTD and EDR minimizations will be implemented.

2 Constructal optimization of rectangular element with nonuniform heat generation

A rectangular element (RE) with nonuniform heat generation is shown in Figure 1. The RE's area is $A_0\ (=H_0\times L_0)$, and the internal heat generation rate $q'''(y)$ varies along the y axis. The thermal conductivity in the heat generation area is k_0. A HCC (width D_0, thermal conductivity k_p) is inserted in the k_0 material, which is used to dissipate the internal heat more effectively. The RE is adiabatic from the surroundings except for the segment M_0, and the temperature of this segment keeps constant at the value of $T_{\min}$. The HCC fraction in the RE can be calculated as: $\phi_0 = D_0/H_0$. For the fixed ϕ_0, the shape of RE will vary when the ratio H_0/L_0 changes.

When the parameters along the third-dimension of the model are assumed to be kept at constants, the two-dimensional heat conduction equations in the k_0 and k_p materials can be, respectively, given as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q'''(y)}{k_0} = 0, \quad (3)$$

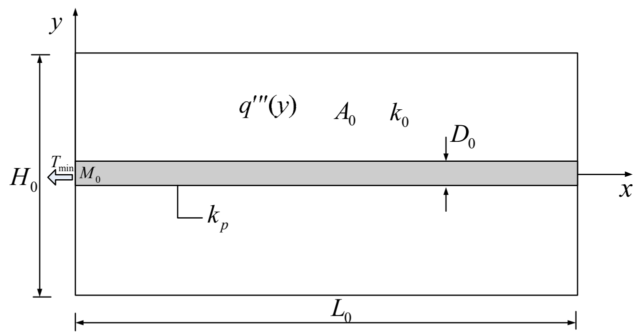

Figure 1 Rectangular element with nonuniform heat generation.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0. \tag{4}$$

The boundary conditions of the symmetrical model are given as follows:

$$T=T_{\min},\quad x=0, 0<y<D_0/2, \tag{5}$$

$$\frac{\partial T}{\partial x}=0\ \begin{cases} x=L_0, 0\le y\le H_0/2, \\ x=0, D_0/2\le y\le H_0/2, \end{cases} \tag{6}$$

$$\frac{\partial T}{\partial y}=0\ \begin{cases} y=0, 0\le x\le L_0, \\ y=H_0/2, 0\le x\le L_0, \end{cases} \tag{7}$$

According to eq. (2), the EDR $\dot{E}_{vh\phi 0}$ of the RE can be given as

$$\begin{aligned}\dot{E}_{vh\phi 0}&=2\int_0^{L_0}\int_{D_0/2}^{H_0/2}k_0\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y\\ &+2\int_0^{L_0}\int_0^{D_0/2}k_p\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\mathrm{d}x\mathrm{d}y.\end{aligned} \tag{8}$$

The corresponding dimensionless entransy dissipation rate (DEDR) is defined as

$$\tilde{\dot{E}}_{vh\phi 0}=\frac{\dot{E}_{vh\phi 0}}{(q'''A_0)^2/k_0}. \tag{9}$$

From eqs. (3)–(9), the DEDR is function of the ratio H_0/L_0. One can implement constructal optimization of the RE by taking the DEDR as optimization objective and H_0/L_0 as optimization variable, respectively. For the fixed internal heat generation rate q''', area A_0 and thermal conductivity k_0, the smaller the DEDR, the lower the average temperature difference and the better the global heat transfer performance of the RE.

To simplify the NUHG problem, the internal heat generation rate $q'''(y)$ in eq. (3) is assumed to be a linear function along the y axis, that is: $q'''(y)=q'''_0\cdot(p+1-2py/H_0)$, , where q'''_0 and p are the constant and nonuniform coefficient of the NUHG model, respectively. Eqs. (3)–(7) will be solved based on COMSOL Multiphysics. The unit size of standard grid is adopted in the calculations, and the structure with $p=2$, $\tilde{k}=200$, $\phi_0=0.1$ and $H_0/L_0=0.5$ is chosen to validate the independence of the grid. The DEDRs of the RE are $\tilde{\dot{E}}_{vh\phi 0}=0.180769$ and $\tilde{\dot{E}}_{vh\phi 0}=0.180771$, respectively, for the standard and refined grids. The percentage error of the DEDRs between the two grid modes is 0.0011%, which illustrates that the unit size of standard grid satisfies the requirement of grid independence.

Figure 2 shows the effect of the nonuniform coefficient p on the characteristic of the DEDR $\tilde{\dot{E}}_{vh\phi 0}$ versus the ratio H_0/L_0 with $\tilde{k}=200$ and $\phi_0=0.1$. From Figure 2, for the fixed p, with the increase in H_0/L_0, $\tilde{\dot{E}}_{vh\phi 0}$ increases first and then decreases; $\tilde{\dot{E}}_{vh\phi 0}$ and H_0/L_0 have their minimum value $(\tilde{\dot{E}}_{vh\phi 0,m})$ and optimal value $((H_0/L_0)_{opt})$, respectively. When $p=0$, the optimal shape of the RE and minimum DEDR are $(H_0/L_0)_{opt}=0.4614$ and $\tilde{\dot{E}}_{vh\phi 0,m}=0.0564$, respectively. According to ref. [51], the optimal results of the RE based on analytical solution are $(H_0/L_0)_{opt}=2/(\tilde{k}\phi_0)^{1/2}=0.4472$ and $\tilde{\dot{E}}_{vh\phi 0,m}=1/[3(\tilde{k}\phi_0)^{1/2}]=0.0745$, respectively. Therefore, the difference of the optimal shapes based on the two methods is small, but that of the minimum DEDR is slightly big. When the shape of the RE is slender enough, the heat transfer along x axis in the k_0 material can be ignored. The two-dimensional heat transfer model is simplified into one-dimensional one in this case, and the differences of the optimal results obtained by the two methods become small.

3 Constructal optimization of first order assembly with nonuniform heat generation

A rectangular first order assembly (RFOA) with nonuniform heat generation is shown in Figure 3. The area of RFOA is $A_1(=H_1\times L_1)$, and it is composed of a number (n) of the REs. The internal heat generation rate $q'''(y)$ varies along the y axis, and the heat is collected by the first or-

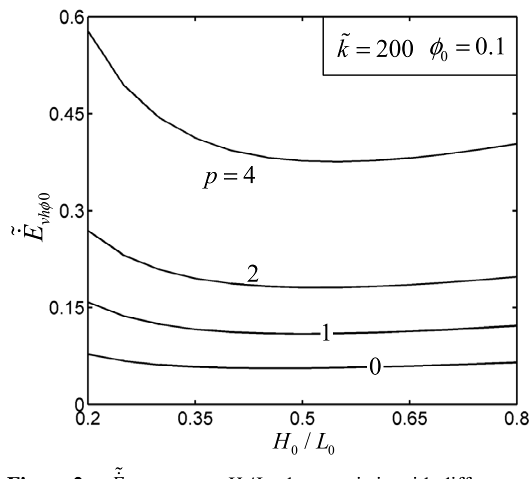

Figure 2 $\tilde{\dot{E}}_{vh\phi 0}$ versus H_0/L_0 characteristic with different p.

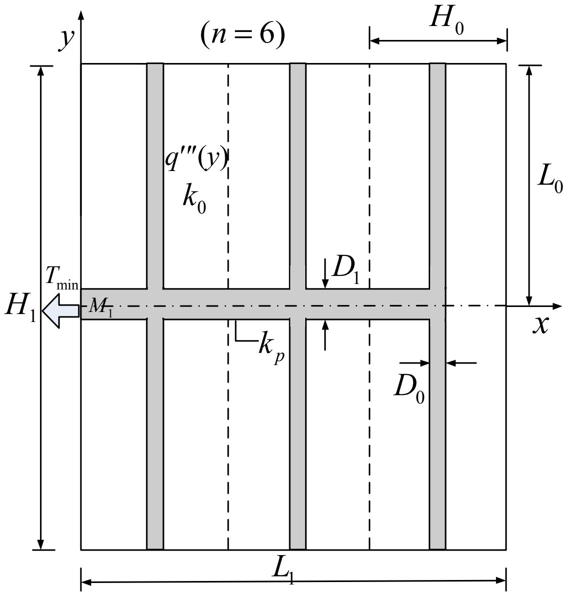

Figure 3 RFOA with nonuniform heat generation.

der HCC with a width of D_1. The RFOA is adiabatic from the surroundings except for the segment M_1, where the temperature ($T_{\min}$) is kept at constant.

The area A_p of the HCC in the RFOA is: $A_p = nD_0(H_1 - D_1)/2 + D_1[(n-1)/nL_1 + D_0/2]$. Therefore, the HCC fraction of the RFOA can be calculated as

$$\phi_1 = \frac{nD_0(H_1 - D_1)/2 + D_1[(n-1)/nL_1 + D_0/2]}{A_1}. \quad (10)$$

For the fixed ϕ_1, the shape of the RFOA will vary when the ratio H_1/L_1 changes.

The EDR of the RFOA can be given as

$$\dot{E}_{vh\phi 1} = \int_{\Omega_1} k_0 \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \mathrm{d}\Omega_1 + \int_{\Omega_2} k_p \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \mathrm{d}\Omega_2, \quad (11)$$

where Ω_1 and Ω_2 are the areas of k_0 and k_p materials, respectively.

The DEDR of the RFOA can be defined as

$$\tilde{\dot{E}}_{vh\phi 1} = \frac{\dot{E}_{vh\phi 1}}{(q''' A_1)^2 / k_0}. \quad (12)$$

For the specified thermal conductivity ratio $\tilde{k}$, HCC fraction ϕ_1 and RE's number n, the DEDR $\tilde{\dot{E}}_{vh\phi 1}$ of the RFOA can be minimized by optimizing H_1/L_1 and D_1/D_0, respectively.

Figure 4 shows the effect of D_1/D_0 on the characteristic of the DEDR $\tilde{\dot{E}}_{vh\phi 1}$ versus the ratio H_1/L_1. From Figure 4, for the fixed D_1/D_0, $\tilde{\dot{E}}_{vh\phi 1}$ decreases first and then increases; $\tilde{\dot{E}}_{vh\phi 1}$ and H_1/L_1 have their minimum value ($\tilde{\dot{E}}_{vh\phi 1,m}$) and optimal value ($(H_1/L_1)_{opt}$), respectively. When the width ratio D_1/D_0 increases, $\tilde{\dot{E}}_{vh\phi 1,m}$ decreases first and then increases; $\tilde{\dot{E}}_{vh\phi 1,m}$ and D_1/D_0 have their minimum value ($\tilde{\dot{E}}_{vh\phi 1,mm}$) and optimal value ($(D_1/D_0)_{opt}$), respectively.

Figure 5 shows the effect of the nonuniform coefficient p on the optimal results ($(H_1/L_1)_{opt}$, $(D_1/D_0)_{opt}$ and $\tilde{\dot{E}}_{vh\phi 1,mm}$) of the RFOA with $n = 6$. From Figure 5, when the nonuniform coefficient p increases, $(H_1/L_1)_{opt}$, $(D_1/D_0)_{opt}$ and $\tilde{\dot{E}}_{vh\phi 1,mm}$ all increase. The shape of the RFOA becomes tubbier in this case. Because the heat generation becomes more nonuniform, the nonuniformity of the RFOA's temperature gradient field also increases. This will lead to the increases in EDR and average temperature difference, and the corresponding global heat transfer performance becomes worse.

As shown in Figure 6, a RFOA with variable cross-section HCC and NUHG is further considered in this paper. In this RFOA, the widths of the first order HCC are different, and are signed D_{11}, D_{12} and D_{13}, respectively. When $\tilde{k}$, ϕ_1 and n are specified, the DEDR of the RFOA can be minimized for four times ($\tilde{\dot{E}}_{vh\phi 1,mmmm}$) by optimizing H_1/L_1, D_{11}/D_{12}, D_{12}/D_{13} and D_{13}/D_0, respectively.

Figure 7 shows the optimal constructs of the RFOA with

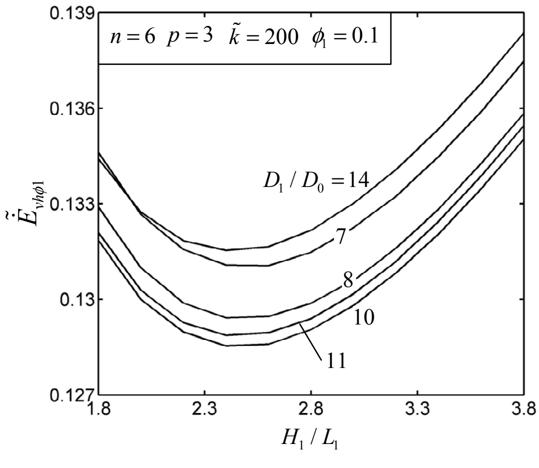

Figure 4 $\tilde{\dot{E}}_{vh\phi 1}$ versus H_1/L_1 characteristic with different D_1/D_0.

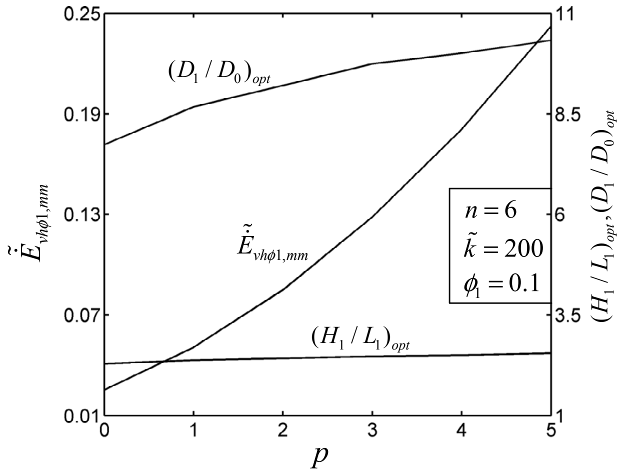

Figure 5 Effect of p on the optimal construct of the RFOA.

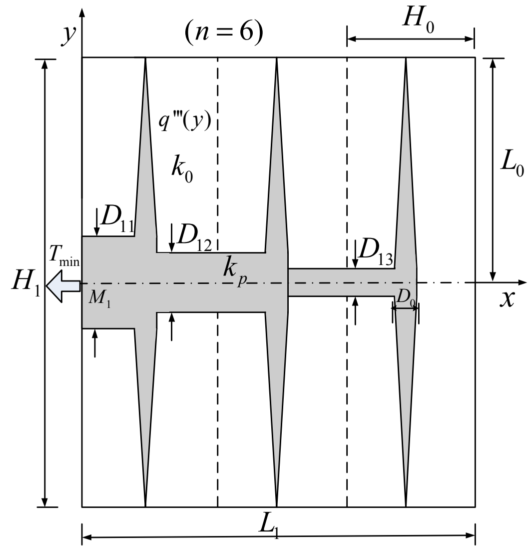

Figure 6 RFOA with variable cross-section HCC and nonuniform heat generation.

variable cross-section HCC based on the minimizations of MTD and EDR, respectively. From Figure 7(a), the optimal construct of the RFOA with variable cross-section HCC based on minimum EDR is $(H_1/L_1)_{opt}=3.0522$, $(D_{11}/D_{12})_{opt}=1.6903$, $(D_{12}/D_{13})_{opt}=1.9582$ and $(D_{13}/D_0)_{opt}=2.7770$,, respectively. The corresponding MTD and EDR are $\Delta\tilde{T}_{1,E}=0.0757$ and $\tilde{\dot{E}}_{vh\phi1,mmmm}=0.0998$, respectively. From Figure 7(b), the optimal construct of the RFOA with variable cross-section HCC based on minimum MTD is $(H_1/L_1)_{opt}=2.7544$, $(D_{11}/D_{12})_{opt}=1.2958$, $(D_{12}/D_{13})_{opt}=1.5329$ and $(D_{13}/D_0)_{opt}=$ 2.9242, respectively. The corresponding dimensionless MTD and EDR are $\Delta\tilde{T}_{1,mmmm}=0.0739$ and $\tilde{\dot{E}}_{vh\phi1,T}=0.1024$, respectively. Compared the optimal construct based on minimum EDR of the RFOA with that based on minimum MTD, the shape of the former optimal construct is tubbier, the EDR is decreased by 2.54%, but the MTD is increased by 2.44%. Therefore, in the practical design of electronic device, when the thermal safety is ensured, the constructal design scheme of the former optimal construct can be adopted to improve the global heat transfer performance of an electronic device.

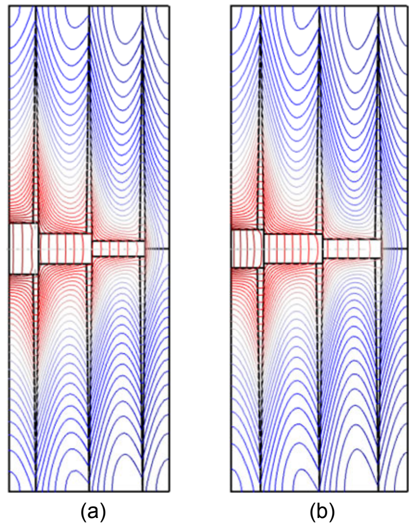

Figure 7 (Color online) Comparisons of the optimal constructs of the rectangular first order assemblies with discrete variable cross-section high conductivity channels: (a) minimum entransy dissipation rate (b) minimum maximum temperature difference

4 Conclusions

A NUHG problem in a rectangular area is investigated in this paper. EDR is taken as the optimization objective, and HCCs with constant and variable widths are considered. The optimal results with minimum EDR are derived. The results show that:

(1) For the RE, the difference of the analytical and numerical solutions depends on the shape of the RE. For the RFOA with constant cross-section HCC, the shape of the RFOA and width ratio of the HCCs are optimized, and the double minimum EDR ($\tilde{\dot{E}}_{vh\phi1,mm}$) is obtained. $\tilde{\dot{E}}_{vh\phi1,mm}$ increases when the nonuniform coefficient p increases. In this case, the global heat transfer performance of the RFOA becomes worse.

(2) For the RFOA with variable cross-section HCC, the DEDR of the RFOA can be minimized for four times ($\tilde{\dot{E}}_{vh\phi1,mmmm}$) by optimizing H_1/L_1, D_{11}/D_{12}, D_{12}/D_{13} and D_{13}/D_0, respectively. Compared the optimal construct based on minimum EDR of the RFOA with that based on minimum MTD, the shape of the former optimal construct

is tubbier, the EDR is decreased by 2.54%, but the MTD is increased by 2.44%. The latter optimal construct improves the local heat transfer performance of the two-dimensional heat conduction body, and the thermal safety of the RFOA is ensured. The former optimal construct makes the temperature gradient field of the two-dimensional heat conduction body more uniform and the average temperature difference lower, and the corresponding global heat transfer performance of the RFOA is improved. Therefore, in the practical design of electronic device, when the thermal safety is ensured, the constructal design scheme of the former optimal construct can be adopted to improve the global heat transfer performance of an electronic device.

The RE and RFOA with NUHG and constant temperature heat sink are considered in this paper. Actually, the internal structure of the rectangular body can be more complex, and the other boundary conditions and thermal stress problem obviously exist in the NUHG body. Moreover, EDR is taken as the optimization objective in this paper, and some other optimization objectives, such as entropy generation rate and multi-objective, can be further considered. Therefore, one can built higher order assembly of the rectangular body with more boundary conditions, and consider thermal stress performance to further carry out multi-objective constructal designs [55–87] of the electronic devices.

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