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# Distributed consensus for multiple Euler-Lagrange systems: An event-triggered approach

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Distributed consensus problems for multiple Euler-Lagrange systems are addressed on the basis of event-triggered information in this study. Distributed consensus protocols are first designed in terms of two event-triggered scenarios: a decentralized strategy and a distributed strategy. Sufficient conditions that guarantee the event-triggered consensus for multiple Euler-Lagrange systems are then presented, with the associated advantages of reducing controller update times. It is shown that the Zeno behavior of triggering time sequences is excluded for both strategies. Finally, multiple Euler-Lagrange systems that consist of six two-link manipulators are considered to illustrate the effectiveness of the proposed theoretical algorithms.

**consensus, event-triggered strategy, Euler-Lagrange system, distributed control**

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## 1 Introduction

Distributed cooperative control in multi-agent systems is an interesting field of research that has attracted considerable attention from a wide range of scientific communities. The motivation of multi-agent cooperative control is to guarantee that a group of autonomous agents will coordinate with each other via local communications to complete some challenging tasks, with associated advantages such as higher robustness, reduced communication costs, and greater efficiency [1–9]. One of the most important and fundamental research issues in the area of cooperative control for multi-agent systems is the consensus problem, which involves guaranteeing that a group of agents to achieve agreement on a common value based only on the information of their interactions.

As an important branch of cooperative control, distributed cooperative attitude control for multiple Euler-Lagrange systems has been studied and has yielded useful results, to name a few. In [10], a behavioral approach was used for attitude synchronization, and a passivity-based approach was then applied to derive a control law without the need for angular velocity measurement. A decentralized scheme for the spacecraft formation problem was then proposed that used a virtual structure approach and behavior-based control in [11,12]. Subsequently, distributed control laws based on graph theory approaches were established for the attitude synchronization problems that were described in terms of Euler parameters fashion in [13]. Distributed consensus tracking problems or attitude coordination problems for multiple Euler-Lagrange systems were also studied in the presence of model uncertainties, external disturbances, time delays, parameter uncertainties, and unknown nonlinear dynamics [14–19]. In addition, nonlinear contraction analysis was used to analyze the global exponential stability of cooperative tracking control laws for both translational and

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rotational dynamics in the Lagrange form in [20]. Recently, distributed leaderless and model-independent consensus algorithms were proposed and analyzed in [21]. In [22], local attitude synchronization problems of multiple Euler-Lagrange systems were addressed without restricting their final motion, and the communication topology was relaxed to enable it to be directed, variable, and uniformly connected. More recently, distributed attitude containment control and finite-time tracking problems for multiple Euler-Lagrange systems were addressed in [19,23–25]. Note that the designed coordinated protocols in the above literature [10–25] were all continuously updated; however, this is unnecessary and wasteful from the perspectives of both the communication load and the service life of the controller. This has inspired us to explore some more practical coordinated protocols that only update at some specific moments while maintaining satisfactory performance for the whole system.

Allowances for continuous measurements and updates are ideal assumptions, and it is more realistic for agents to interact intermittently at specific sampling instants. Therefore, two common sampling methods are often used in practical applications. One is the time-triggered method, which involves the traditional approach of sampling at pre-specified time intervals. While this method has been applied extensively because of its ease and simplicity, it may lead to higher system costs, because sampling occurs at a fixed rate, regardless of whether or not it is really necessary. The other is the event-triggered approach, which updates the control action only when certain specific events occur. These events are triggered at time points when the ratio of the norm of a specific measurement error to the norm of a state-dependent function exceeds an event threshold over time. Event-triggered control offers certain advantages over time-triggered control in terms of reduced communications and sensor energy savings. Distributed control based on the event-triggered approach has thus recently received considerable research attention. In [26,27], based on the deterministic event-triggered strategy that was introduced in [28], distributed consensus algorithms for first-order multi-agent systems were proposed and a lower bound was provided for the minimum inter-event interval to exclude Zeno behavior. Also, event-triggered control was addressed in both networked control systems and wireless sensor/actuator networks [29,30]. In [31], event-triggered communication was studied for the cooperative control problem of heterogeneous multi-agent systems on the basis of passivity analysis. Along with the same design framework given in [27], important works on event-triggered cooperative consensus for multi-agent systems include [32–34], to name a few. Recently, a novel control strategy for multi-agent coordination with event-based broadcasting was presented, networks of single-integrator agents with and without communication delays and networks of double-integrator agents were analyzed in [35]. Also, the distributed rendezvous problem for first-order multi-agent systems with event-triggered controllers was investigated using a combinational approach in [36]. More recently, the average consensus problem was developed based on distributed event-based algorithms with sampled-data event detection; the highlight of this scenario is that the lower bound of the minimum inter-event interval is naturally provided by the synchronous sampling period [37]. In [38], the decentralized event-triggered cooperative control problem with limited communication was discussed for multi-agent systems with first-order integrator dynamics; the ideas in this case can be used to consider practical scenarios where the agents can only exchange quantized measurements.

Motivated by the above observations, this paper investigates distributed consensus problems for multiple Euler-Lagrange systems based on event-triggered information under undirected communication topologies, such that a team of Euler-Lagrange systems is driven into a common constant orientation with zero angular velocity. To the best of the authors' knowledge, the proposed algorithm is the first event-triggered algorithm that guarantees the distributed consensus for networked Euler-Lagrange systems. Using Lyapunov stability theory, sufficient conditions are obtained to enable event-triggered consensus to be achieved. Then, the Zeno behavior is excluded by proving that the triggering time sequences do not converge to a finite time point. When compared with the existing literature on distributed cooperative control for multiple Euler-Lagrange systems with the designed controllers operating in real time [10–25], it is assumed in this paper that the controller only updates at specific sampling instants as time passes, which can effectively reduce the overall energy costs. In contrast to the existing results on the distributed event-triggered consensus problem, where the agent dynamics are represented by first- or second-order integrators [26,27,31–38], the event-triggered consensus problem for multiple Euler-Lagrange systems is investigated, while considering more complex dynamics in real applications.

The rest of this paper is organized as follows. Some preliminary aspects and the model formulation are given in Section 2. The main results are then presented and proved in Section 3. Simulation results are presented for verification of the theoretical results in Section 4. Finally, our conclusions are drawn in Section 5.

The following notations are used throughout this paper. Let $\mathbf{R}$ and $\boldsymbol{N}$ be the sets of real and natural numbers, respectively. Let $\mathbf{R}^n$ and $\mathbf{R}^{n\times n}$ be the $n$-dimensional real vector space and the $n\times n$ real matrix space, respectively. $\boldsymbol{A}^{\mathrm{T}}$ denotes the transpose of a matrix $\boldsymbol{A}$. For a symmetric matrix $\boldsymbol{A}$, $\boldsymbol{A}>0$ ($\geqslant 0$) means that $\boldsymbol{A}$ is positive (semi-)definite. $\otimes$ denotes the Kronecker product. For a vector $\boldsymbol{x}$, $\|\boldsymbol{x}\|$ indicates the Euclidean norm of that vector. For a matrix $\boldsymbol{B}$, $\bar{\sigma}(\boldsymbol{B})$ ($\underline{\sigma}(\boldsymbol{B})$) represents the maximum (minimum) singular value of matrix $\boldsymbol{B}$.

## 2 Preliminaries

### 2.1 Graph theory

Consider a network that consists of $N$ Euler-Lagrange systems. Let a weighted undirected graph $G=(V,E,\boldsymbol{A})$ describe the communication topology among the agents, where $V=\{1,\cdots,N\}$ is the set of nodes, $E\subseteq V\times V$ is the set of edges, and $\boldsymbol{A}=[a_{ij}]\in\mathbf{R}^{N\times N}$ is the adjacency matrix with the nonnegative elements $a_{ij}=a_{ji}$. The set of neighbors of node $i$ is denoted by $N_i=\{j\in V:(j,i)\in E\}$ and $|N_i|$ denotes the cardinality of $N_i$. In addition, the degree of the vertex $i$ is defined as $\deg(i)=\sum_{j=1,j\neq i}^{N}a_{ij}$. Specifically, an edge is denoted by a pair of nodes $(i,j)$ in $G$ that corresponds to an information link between agent $i$ and agent $j$, which means that agent $i$ and agent $j$. can communicate with each other, i.e., $j$ belongs to the communication set $N_i$ of agent $i$, and vice versa. As is customary, self-loops are not allowed, i.e., $a_{ii}=0$ for all $i=1,\cdots,N$, and $a_{ij}>0$ if and only if $(i,j)\in E$. Note that $\boldsymbol{A}$ is symmetric. Let the Laplacian matrix $\boldsymbol{L}=[l_{ij}]\in\mathbf{R}^{N\times N}$ associated with $\boldsymbol{A}$ be defined as $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$ and $l_{ij}=-a_{ij}$, $i\neq j$. Note that $\boldsymbol{L}$ is symmetric positive semi-definite. 0 is a simple eigenvalue of $\boldsymbol{L}$ with the associated eigenvector $1_N$, where $1_N$ is the $N\times1$ vector with each entry being 1, and all other eigenvalues of $\boldsymbol{L}$ are positive if and only if $G$ is connected.

### 2.2 Model formulation

Consider a team of $N$ Euler-Lagrange systems indexed by $V=\{1,\cdots,N\}$. The attitude dynamics of agent $i$ is given by

$$\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\ddot{\boldsymbol{\sigma}}_i+\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i=\boldsymbol{\tau}_i, i\in V, \tag{1}$$

where $\boldsymbol{\sigma}_i\in\mathbf{R}^n$ is the vector of the generalized coordinates, $\boldsymbol{\tau}_i$ is the vector of the torques produced by the actuators, $\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\in\mathbf{R}^{n\times n}$ is the symmetric positive-definite inertia matrix, $\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i\in\mathbf{R}^n$ is the vector of the Coriolis and centrifugal torques, and $\dot{\boldsymbol{M}}_i(\boldsymbol{\sigma}_i)-2\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)$ is a skew-symmetric matrix. In addition, there exist $k_{\underline{m}},k_{\overline{m}},k_c$ such that $0<k_{\underline{m}}\leqslant\|\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\|\leqslant k_{\overline{m}}$ and $\|\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\|\leqslant k_c\|\dot{\boldsymbol{\sigma}}_i\|$ $(i=1,\cdots,N)$, where $k_c>0$.

**Lemma 1** [39]. If a differentiable function $f(t)$ satisfies $f(t),\dot{f}(t)\in L_\infty$ and $f(t)\in L_p$ for some value of $p\in[1,\infty)$, then $f(t)\to0$ as $t\to\infty$.

## 3 Main results

For system (1), two kinds of event-triggered cooperative control strategies are developed in this study.

(1) Decentralized event-triggered cooperative control. The state measurement errors $\boldsymbol{e}_i^{\boldsymbol{\sigma}_i}(t)$ and $\boldsymbol{e}_i^{\dot{\boldsymbol{\sigma}}_i}(t)$ are introduced in the following. Let $\boldsymbol{e}_i(t)=(\boldsymbol{e}_i^{\boldsymbol{\sigma}_i\mathrm{T}}(t),\boldsymbol{e}_i^{\dot{\boldsymbol{\sigma}}_i\mathrm{T}}(t))^{\mathrm{T}}$. In this case, a sequence of event times $t_k^i, k\in\boldsymbol{N}$, exists for each agent $i$ according to a decentralized event-triggered condition $f_i(\boldsymbol{e}_i(t_k^i))=0$. The cooperative control law for agent $i$ is updated at both its own event times $t_k^i, k\in\boldsymbol{N}$, and at the latest event times $t_{k'}^j, k'\in\boldsymbol{N}$, of its neighbor $j\in N_i$, i.e.,

$$\boldsymbol{\tau}_i(t)=\boldsymbol{\tau}_i(t_k^i,\{t_{k'}^j, j\in N_i\}), t\in[t_k^i,t_{k+1}^i), \tag{2}$$

where $k'\triangleq\arg\min_{t\geqslant t_r^j, r\in N}\{t-t_r^j\}$.

(2) Distributed event-triggered cooperative control. In this context, a sequence of event times $t_k^i, k\in\boldsymbol{N}$ exists for each agent $i$ according to a distributed event-triggered condition $f_i(\boldsymbol{e}_i(t_k^i),\{\boldsymbol{e}_{ij}(t_k^i)\mid j\in N_i\})=0$, where $\boldsymbol{e}_i(t)$ is as defined above and $\boldsymbol{e}_{ij}(t)$ will be introduced later. For this case, the cooperative control law for agent $i$ will be updated at its own event times $t_k^i, k\in\boldsymbol{N}$, i.e.,

$$\boldsymbol{\tau}_i(t)=\boldsymbol{\tau}_i(t_k^i), t\in[t_k^i,t_{k+1}^i). \tag{3}$$

### 3.1 Distributed cooperative consensus based on decentralized event-triggered control

In this subsection, a decentralized event-triggered function $f_i(\boldsymbol{e}_i(t))$, which depends only on the information of agent $i$, is designed to achieve consensus with reduced communications. In this case, each agent will update its control input at its own event times and at the latest event times of its neighbors according to (2). In retrospect, the event times for each agent $i\in V$ are denoted by $t_k^i, k\in\boldsymbol{N}$ and are obtained from the triggering rule $f_i(\boldsymbol{e}_i(t))=0, i\in V$. Similarly, the next event time is detected by $t_{k+1}^i=\inf\{t>t_k^i: f_i(\boldsymbol{e}_i(t))=0\}$.

#### 3.1.1 Distributed controller design

Under this condition, the state measurement errors for agent $i$ are defined as

$$\boldsymbol{e}_i^{\boldsymbol{\sigma}_i}(t)=\boldsymbol{\sigma}_i(t_k^i)-\boldsymbol{\sigma}_i(t),$$

$$\boldsymbol{e}_i^{\dot{\boldsymbol{\sigma}}_i}(t)=\dot{\boldsymbol{\sigma}}_i(t_k^i)-\dot{\boldsymbol{\sigma}}_i(t), t\in[t_k^i,t_{k+1}^i), i\in V. \tag{4}$$

For each agent $i$, a distributed consensus protocol is designed based on a planned event-triggered update rule as follows:

$$\boldsymbol{\tau}_i(t) = -\sum_{j=1}^{N} a_{ij}[\boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_j(t_{k'}^j)] - \boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i),$$
$$t \in [t_k^i, t_{k+1}^i),\ i \in V, \tag{5}$$

where $k' \triangleq \arg\min_{t \geqslant t_r^j, r\in N}\{t - t_r^j\}$, which indicates that $t_{k'}^j$ is the latest event time for agent $j$ for $t \in [t_k^i, t_{k+1}^i)$. Therefore, each agent will take the latest state update value of each of its neighboring agents into account in its control law. In other words, the consensus control law for agent $i$ is updated at both its own event times $t_k^i$ and the latest event time instants $t_{k'}^j$ of its neighbor $j \in N_i$. Moreover, $\boldsymbol{K}_i \in \mathbf{R}^{n\times n}$ is symmetric positive definite.

As a result, under the distributed consensus control protocol (5), the closed loop system becomes

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t)) = \dot{\boldsymbol{\sigma}}_i(t) - \dot{\boldsymbol{\sigma}}_j(t),\\
&\frac{\mathrm{d}}{\mathrm{d}t}\dot{\boldsymbol{\sigma}}_i(t) = -\boldsymbol{M}_i^{-1}(\boldsymbol{\sigma}_i)\{\boldsymbol{C}_i(\boldsymbol{\sigma}_i, \dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i(t)\\
&\qquad + \sum_{j=1}^{N} a_{ij}[\boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_j(t_{k'}^j)] + \boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i)\}.
\end{aligned} \tag{6}$$

For notational convenience, the time argument can be dropped if it does not cause confusion.

Let $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_1^{\mathrm{T}}, \cdots, \boldsymbol{\sigma}_N^{\mathrm{T}})^{\mathrm{T}}$, $\dot{\boldsymbol{\sigma}} = (\dot{\boldsymbol{\sigma}}_1^{\mathrm{T}}, \cdots, \dot{\boldsymbol{\sigma}}_N^{\mathrm{T}})^{\mathrm{T}}$, $\boldsymbol{e}^{\boldsymbol{\sigma}} = (\boldsymbol{e}_1^{\boldsymbol{\sigma}_1\mathrm{T}}(t), \cdots, \boldsymbol{e}_N^{\boldsymbol{\sigma}_N\mathrm{T}}(t))^{\mathrm{T}}$, and $\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}} = (\boldsymbol{e}_1^{\dot{\boldsymbol{\sigma}}_1\mathrm{T}}(t), \cdots, \boldsymbol{e}_N^{\dot{\boldsymbol{\sigma}}_N\mathrm{T}}(t))^{\mathrm{T}}$. Based on the definition of $k'$, $\boldsymbol{\sigma}_j(t_{k'}^j) = \boldsymbol{\sigma}_j(t) + \boldsymbol{e}_j^{\boldsymbol{\sigma}_j}(t)$. Then, system (6) can be written in a compact form as

$$\boldsymbol{M}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} = -\boldsymbol{C}(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} - (\boldsymbol{L}\otimes\boldsymbol{I}_n)(\boldsymbol{\sigma} + \boldsymbol{e}^{\boldsymbol{\sigma}}) - \boldsymbol{K}(\dot{\boldsymbol{\sigma}} + \boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}), \tag{7}$$

where $\boldsymbol{M}(\boldsymbol{\sigma}) = \mathrm{diag}[\boldsymbol{M}_1(\boldsymbol{\sigma}_1), \cdots, \boldsymbol{M}_N(\boldsymbol{\sigma}_N)]$, $\boldsymbol{C}(\boldsymbol{\sigma}, \dot{\boldsymbol{\sigma}}) = \mathrm{diag}[\boldsymbol{C}_1(\boldsymbol{\sigma}_1, \dot{\boldsymbol{\sigma}}_1), \cdots, \boldsymbol{C}_N(\boldsymbol{\sigma}_N, \dot{\boldsymbol{\sigma}}_N)]$, and $\boldsymbol{K} = \mathrm{diag}[\boldsymbol{K}_1, \cdots, \boldsymbol{K}_N]$.

In retrospect, the controller (5) updates at the time instants $t_k^i, k \in \boldsymbol{N}$, which are determined by an event-triggered rule proposed in the following subsection.

*3.1.2 Decentralized event-triggered rule*

For each agent $i \in V$, the decentralized event-triggered function is designed as follows:

$$f_i(\boldsymbol{e}_i(t)) = \|\boldsymbol{e}_i(t)\| - \rho_i\|\dot{\boldsymbol{\sigma}}_i(t)\| - \varepsilon_i(t), i \in V,$$

where $\varepsilon_i(t) = \mu_i e^{-\overline{\varepsilon}_i(t-t_0)}$ with $\mu_i > 0$ and $0 < \overline{\varepsilon}_i < 1$, $\rho_i = \sqrt{\frac{a\chi_i\phi_i}{2\varphi_i}}$ with $0 < \chi_i < 1$, $\phi_i = \underline{\sigma}(\boldsymbol{K}_i) - a(|N_i| + \overline{\sigma}(\boldsymbol{K}_i)/2) > 0$, $\varphi_i = \max(|N_i|, \overline{\sigma}(\boldsymbol{K}_i)/2) > 0$, and $0 < a < \underline{\sigma}(\boldsymbol{K}_i)/[2(|N_i| + \overline{\sigma}(\boldsymbol{K}_i))]$.

When the condition $f_i(\boldsymbol{e}_i(t)) < 0$ is violated over time, an event will be triggered based on

$$f_i(\boldsymbol{e}_i(t)) = 0, i \in V. \tag{8}$$

The event times are obtained by examining $f_i(\boldsymbol{e}_i(t)) = 0$, $k \in \mathbf{N}$, for each agent $i \in V$. Without loss of generality, assume here that $t_0^i = t_0$ $(i \in V)$. The cooperative protocol (5) updates at $t_k^i$ and then remains constant until the next event time $t_{k+1}^i$ occurs. When an event is triggered, the measurement errors are reset to zeroes, because at this moment one has $\boldsymbol{e}_i^{\boldsymbol{\sigma}_i}(t_k^i) = \boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_i(t_k^i) = 0$ and $\boldsymbol{e}_i^{\dot{\boldsymbol{\sigma}}_i}(t_k^i) = 0$, which means that $f_i(\boldsymbol{e}_i(t)) < 0$ is effective again. In other words, the role of the designed event-triggered rule (8) is to ensure that $f_i(\boldsymbol{e}_i(t)) \leqslant 0$ always holds.

**Remark 1.** Note that both $\varphi_i$ and $\phi_i$ are related to the number of neighbors of agent $i$ and to the gain matrix $\boldsymbol{K}_i$. The function $\varepsilon_i(t)$ is introduced to fully exclude the Zeno behavior. When the event-triggered rule (8) is detected, each agent $i$ simply needs to collect its own information $\boldsymbol{e}_i(t)$ and $\dot{\boldsymbol{\sigma}}_i(t)$, which means that the agents do not communicate with their neighbors between any two consecutive event instants. In other words, the proposed event-triggered strategy (8) is effective in reducing the communication burden.

*3.1.3 Consensus analysis*

Based on the above developments, sufficient conditions can then be concluded to achieve consensus, subject to decentralized event-triggered communications.

**Theorem 1.** For multiple Euler-Lagrange systems (1), assume that the undirected communication topology $G$ is connected. Then, the consensus problem of the network (1) with the distributed cooperative control law (5) and the decentralized event-triggered rule (8) is solved under any initial conditions $\boldsymbol{\sigma}_i(0)$, $\dot{\boldsymbol{\sigma}}_i(0)$, $i \in V$.

*Proof.* Consider the following Lyapunov function candidate for the system (7):

$$V(t) = \frac{1}{2}\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{\sigma} + \frac{1}{2}\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}}. \tag{9}$$

It follows that $\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{\sigma} = 1/2\sum_{i=1}^{N} a_{ij}\|\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j\|^2$, because $G$ is undirected. Note that $G$ is connected and $\boldsymbol{M}(\boldsymbol{\sigma})$ is symmetric positive-definite, which means that $V(t)$ is symmetric positive-definite with respect to $\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j$ and $\dot{\boldsymbol{\sigma}}_i$. Note that the system (6) with states $\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j$ and $\dot{\boldsymbol{\sigma}}_i$ is nonautonomous because of the dependence of $\boldsymbol{M}_i$ and

$\boldsymbol{C}_i$ on $\boldsymbol{\sigma}_i$. This leads to the nonavailability of LaSalle's invariance principle. However, using Lemma 1, which is a special case of Barbalat's Lemma [40], the problem can be solved.

Taking the time derivative of $V(t)$ along the trajectories of (7) yields

$$\begin{aligned}
\dot{V}(t) &= \boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\dot{\boldsymbol{\sigma}} + \dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}} + 1/2\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\dot{\boldsymbol{M}}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}} \\
&= \boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\dot{\boldsymbol{\sigma}} - \dot{\boldsymbol{\sigma}}^{\mathrm{T}}[\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}} + \boldsymbol{K}(\dot{\boldsymbol{\sigma}} + \boldsymbol{e}^{\dot{\sigma}}) \\
&\quad + (\boldsymbol{L}\otimes\boldsymbol{I}_n)(\boldsymbol{\sigma} + \boldsymbol{e}^{\sigma})] + 1/2\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\dot{\boldsymbol{M}}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}} \\
&= -\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{K}\dot{\boldsymbol{\sigma}} - \dot{\boldsymbol{\sigma}}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{e}^{\sigma} - \dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{K}e^{\dot{\sigma}} \\
&\leqslant -\sum_{i=1}^{N}\underline{\sigma}(\boldsymbol{K}_i)\|\dot{\boldsymbol{\sigma}}_i\|^2 - \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{\boldsymbol{\sigma}}_i^{\mathrm{T}}(\boldsymbol{e}_i^{\sigma_i} \\
&\quad - \boldsymbol{e}_j^{\sigma_j}) - \sum_{i=1}^{N}\dot{\boldsymbol{\sigma}}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{e}_i^{\dot{\sigma}_i} \\
&\leqslant -\sum_{i=1}^{N}\underline{\sigma}(\boldsymbol{K}_i)\|\dot{\boldsymbol{\sigma}}_i\|^2 - \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{\boldsymbol{\sigma}}_i^{\mathrm{T}}\boldsymbol{e}_i^{\sigma_i} \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\dot{\boldsymbol{\sigma}}_i^{\mathrm{T}}\boldsymbol{e}_j^{\sigma_j} - \sum_{i=1}^{N}\dot{\boldsymbol{\sigma}}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{e}_i^{\dot{\sigma}_i} \\
&\leqslant -\sum_{i=1}^{N}\underline{\sigma}(\boldsymbol{K}_i)\|\dot{\boldsymbol{\sigma}}_i\|^2 + \sum_{i=1}^{N}a(|N_i| + \frac{\bar{\sigma}(\boldsymbol{K}_i)}{2})\|\dot{\boldsymbol{\sigma}}_i\|^2 \\
&\quad + \sum_{i=1}^{N}\frac{|N_i|}{a}\|\boldsymbol{e}_i^{\sigma_i}\|^2 + \sum_{i=1}^{N}\frac{\bar{\sigma}(\boldsymbol{K}_i)}{2a}\|\boldsymbol{e}_i^{\dot{\sigma}_i}\|^2 \\
&\leqslant -\sum_{i=1}^{N}\phi_i\|\dot{\boldsymbol{\sigma}}_i\|^2 + \sum_{i=1}^{N}\frac{\varphi_i}{a}\|\boldsymbol{e}_i\|^2,
\end{aligned}$$

where the inequality $|\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}| \leqslant a/2\|\boldsymbol{x}\|^2 + 1/(2a)\|\boldsymbol{y}\|^2$ has been used for any $\boldsymbol{x},\boldsymbol{y}\in\mathbf{R}^n$, where $a>0$. Also, the equation

$$\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}/(2a)\|\boldsymbol{e}_j^{\sigma_j}\|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}/(2a)\|\boldsymbol{e}_i^{\sigma_i}\|^2$$

has been applied because the communication topology is symmetric.

Then, based on the decentralized event-triggered rule (8), the condition $\|\boldsymbol{e}_i(t)\|^2 \leqslant \dfrac{a\chi_i\phi_i}{\varphi_i}\|\dot{\boldsymbol{\sigma}}_i(t)\|^2 + 2\varepsilon_i^2(t)$ is always satisfied. It therefore follows that the time derivative of $V(t)$ satisfies

$$\dot{V}(t) \leqslant -\sum_{i=1}^{N}(1-\chi_i)\phi_i\|\dot{\boldsymbol{\sigma}}_i\|^2 + \sum_{i=1}^{N}\frac{2\varphi_i}{a}\varepsilon_i^2(t). \tag{10}$$

To move on from this, integrating (10) for any $t>0$ yields

$$\begin{aligned}
V(t) &+ \sum_{i=1}^{N}(1-\chi_i)\phi_i\int_0^t\|\dot{\boldsymbol{\sigma}}_i(\tau)\|^2\,\mathrm{d}\tau \\
&\leqslant V(0) + \sum_{i=1}^{N}\frac{2\varphi_i}{a}\int_0^t\varepsilon_i^2(\tau)\mathrm{d}\tau \\
&\leqslant V(0) + \sum_{i=1}^{N}\frac{\varphi_i\mu_i^2}{a\bar{\varepsilon}_i}.
\end{aligned} \tag{11}$$

Therefore, it follows from (11) that $V(t)$ is bounded, which implies that both $\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j$ and $\dot{\boldsymbol{\sigma}}_i$ are also bounded according to (9), i.e., $\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j$, $\dot{\boldsymbol{\sigma}}_i \in L_\infty$. Then, by returning to (11) again, it follows that

$$\int_0^t\|\dot{\boldsymbol{\sigma}}_i(\tau)\|^2\,\mathrm{d}\tau \leqslant \frac{1}{(1-\chi_i)\phi_i}(V(0) + \sum_{i=1}^{N}\frac{\varphi_i\mu_i^2}{a\bar{\varepsilon}_i}).$$

The above inequality means that $\dot{\boldsymbol{\sigma}}_i \in L_2$. Also, remembering that $\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j$, $\dot{\boldsymbol{\sigma}}_i \in L_\infty$, along with the properties of $\boldsymbol{M}_i(\boldsymbol{\sigma}_i)$ and $\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)$ in subsection 2.2, one has that $\ddot{\boldsymbol{\sigma}}_i$ is also bounded ($\ddot{\boldsymbol{\sigma}}_i \in L_\infty$) according to (6). In light of Lemma 1, one has $\lim_{t\to\infty}\dot{\boldsymbol{\sigma}}_i = 0$, which in turn implies that $\lim_{t\to\infty}\ddot{\boldsymbol{\sigma}}_i = 0$. In terms of (6) once more, one has $\lim_{t\to\infty}a_{ij}(\boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_j(t_{k'}^j)) = 0$, or equivalently, $\lim_{t\to\infty}a_{ij}(\boldsymbol{\sigma}_i(t) - \boldsymbol{\sigma}_j(t)) = 0$, which means that $\lim_{t\to\infty}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{\sigma} = 0$ as $t\to\infty$, and as a result, $\boldsymbol{\sigma}_i \to \boldsymbol{\sigma}_j$ as $t\to\infty$, $i,j\in V$. Therefore, the multiple Euler-Lagrange systems described in (1) asymptotically reach consensus under the designed event-triggered cooperative law. This completes the proof.

**Definition 1.** (Zeno Triggering) An event-triggered scheme induces Zeno behavior if the event times $t_k^i(k\in\mathbf{N}, i\in V)$ converge to a finite $t^*$ as $k\to\infty$.

**Remark 2.** When Zeno behavior occurs, the designed control strategy may become very dangerous and may even make the entire system collapse. Hence, to ensure that the designed cooperative policy (5) behaves well, it is essential to exclude Zeno triggering in the following subsection.

### *3.1.4 Exclusion of Zeno behavior*

In the following, an analysis of the minimum inter-event interval $\inf_{k\in\mathbf{N}}\{t_{k+1}^i - t_k^i\}$ for each agent $i$ is carried out to eliminate the Zeno phenomenon.

**Theorem 2.** For the multiple Euler-Lagrange systems described in (1) with the distributed control protocol (5) and the decentralized event-triggered rule (8), assume that the undirected communication topology $G$ is connected. Then, for each agent $i\in V$, if $t_k^i$ exists, then no Zeno behavior occurs for all $t > t_k^i$.

*Proof.* For any $t > t_k^i$, taking the time derivative of $\|\boldsymbol{e}_i(t)\|$ gives

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}(\|\boldsymbol{e}_i(t)\|) &= \frac{\boldsymbol{e}_i^{\mathrm{T}}(t)\dot{\boldsymbol{e}}_i(t)}{\|\boldsymbol{e}_i(t)\|} \leqslant \|\dot{\boldsymbol{e}}_i^{\sigma_i}(t)\| + \|\dot{\boldsymbol{e}}_i^{\dot{\sigma}_i}(t)\| \\
&= \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\| + \|\ddot{\boldsymbol{\sigma}}_i(t)\| \\
&\leqslant \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\| + \|\boldsymbol{e}_i^{\dot{\sigma}_i}(t_k^i)\| \\
&\quad + \|\boldsymbol{M}_i^{-1}(\boldsymbol{\sigma}_i)[\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i(t)
\end{aligned}$$

$$+\sum_{j=1}^{N}a_{ij}[\boldsymbol{\sigma}_i(t_k^i)-\boldsymbol{\sigma}_j(t_{k'}^j)]+\boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i)]\Big\|$$
$$\leqslant\left\|\dot{\boldsymbol{\sigma}}_i(t_k^i)\right\|+\left\|\boldsymbol{e}_i(t)\right\|+k_{\underline{m}}^{-1}[k_c\left\|\dot{\boldsymbol{\sigma}}_i(t)\right\|^2+$$
$$+\left\|\sum_{j=1}^{N}a_{ij}[\boldsymbol{\sigma}_i(t_k^i)-\boldsymbol{\sigma}_j(t_{k'}^j)]+\boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i)\right\|]$$
$$\leqslant\left\|\boldsymbol{e}_i(t)\right\|+\Pi_i,$$

where

$$\Pi_i\triangleq k_{\underline{m}}^{-1}k_ck_{\dot{\sigma}}^2+\left\|\dot{\boldsymbol{\sigma}}_i(t_k^i)\right\|$$
$$+k_{\underline{m}}^{-1}\left\|\sum_{j=1}^{N}a_{ij}[\boldsymbol{\sigma}_i(t_k^i)\boldsymbol{\sigma}_j(t_{k'}^j)]+\boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i)\right\|>0.$$

Note that $\left\|\boldsymbol{M}_i^{-1}(\boldsymbol{\sigma}_i)\right\|\leqslant k_{\underline{m}}^{-1}$ and $\left\|\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i(t)\right\|\leqslant k_c\left\|\dot{\boldsymbol{\sigma}}_i(t)\right\|^2$, and that $k_{\dot{\sigma}}$ is a positive number such that $\left\|\dot{\boldsymbol{\sigma}}(t)\right\|\leqslant k_{\dot{\sigma}}\ (i\in V)$ based on the bounded property of $\dot{\boldsymbol{\sigma}}(t)$ that was obtained in the proof of Theorem 1. Then, one can obtain

$$\left\|\boldsymbol{e}_i(t)\right\|\leqslant\Pi_i(e^{t-t_k^i}-1).$$

To move on, a sufficient condition for $f_i(\boldsymbol{e}_i(t))\leqslant 0$ is given as follows:

$$\left\|\dot{\boldsymbol{e}}_i^{\sigma_i}(t)\right\|+\left\|\dot{\boldsymbol{e}}_i^{\dot{\sigma}_i}(t)\right\|\leqslant\frac{\rho_i}{1+\rho_i}\left\|\dot{\boldsymbol{\sigma}}(t_k^i)\right\|+\frac{1}{1+\rho_i}\varepsilon_i(t).$$

In retrospect, $\varepsilon_i(t)=\mu_ie^{-\bar{\varepsilon}_i(t-t_0)}$, and it thus follows that the next event time $t_{k+1}^i$ is obtained when

$$\left\|\dot{\boldsymbol{e}}_i^{\sigma_i}(t_{k+1}^i)\right\|+\left\|\dot{\boldsymbol{e}}_i^{\dot{\sigma}_i}(t_{k+1}^i)\right\|\leqslant\frac{\rho_i}{1+\rho_i}\left\|\dot{\boldsymbol{\sigma}}(t_k^i)\right\|+\frac{\mu_i}{1+\rho_i}e^{-\bar{\varepsilon}_i(t_{k+1}^i-t_0)}.$$

Consequently, because $\left\|\dot{\boldsymbol{e}}_i^{\sigma_i}(t)\right\|+\left\|\dot{\boldsymbol{e}}_i^{\dot{\sigma}_i}(t)\right\|\leqslant\sqrt{2}\left\|\boldsymbol{e}_i(t)\right\|$, one has

$$\frac{\rho_i}{1+\rho_i}\left\|\dot{\boldsymbol{\sigma}}(t_k^i)\right\|+\frac{\mu_i}{1+\rho_i}e^{-\bar{\varepsilon}_i(t_{k+1}^i-t_0)}\leqslant\sqrt{2}\Pi_i(e^{t_{k+1}^i-t_k^i}-1).\quad(12)$$

If $\lim_{k\to\infty}t_k^i=t^*<\infty$, then according to (12) one has

$$\frac{\mu_i}{1+\rho_i}e^{-\bar{\varepsilon}_i(t_{k+1}^i-t_k^i)}\leqslant\sqrt{2}\Pi_ie^{-\bar{\varepsilon}_i(t^*-t_0)}(e^{t_{k+1}^i-t_k^i}-1),$$

which implies that $0<\frac{\mu_i}{1+\rho_i}\leqslant 0.$ This contradiction shows that $\lim_{k\to\infty}t_k^i=\infty$. On the other hand, since $0<\bar{\varepsilon}_i<1$, then according to (12) one can derive

$$\frac{\mu_i}{1+\rho_i}e^{-\bar{\varepsilon}_i(t_{k+1}^i-t_k^i)}\leqslant\sqrt{2}\Pi_i(e^{t_{k+1}^i-t_0}-e^{t_k^i-t_0}),$$

which implies that $t_{k+1}^i-t_k^i$ has a positive lower bound $\tau_k^i$, i.e., $\inf_{k\in\mathbf{N}}\{t_{k+1}^i-t_k^i\}=\inf_{k\in\mathbf{N}}\{\tau_k^i\}=\tau^i>0$. Otherwise, if $t_{k+1}^i\to t_k^i$, then the above inequality cannot be satisfied. Since $\Pi_i$ is bounded by $k_{\dot{\sigma}}+k_{\underline{m}}^{-1}[k_ck_{\dot{\sigma}}^2+k_{\boldsymbol{\sigma}}\deg(i)+k_{\dot{\sigma}}\bar{\sigma}(\boldsymbol{K}_i)]$ with $\left\|\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j\right\|\leqslant k_{\boldsymbol{\sigma}}\ (\forall i,j\in V,\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j\in L_\infty)$, while $\mu_i>0$ and $0<\bar{\varepsilon}_i<1$ can be chosen such that the left side of the inequality above is sufficiently large. From the above, Zeno behavior is excluded for any agent $i$ according to Definition 1. This completes the proof.

### 3.2 Distributed cooperative consensus based on distributed event-triggered control

In this subsection, a distributed event-triggered scheme is developed based on local information associated with agent $i$ and its neighbors. In this case, the control input for each agent $i\in V$ simply updates at its own event times $t_k^i$, $k\in\mathbf{N}$, which are computed by a triggered law $f_i(\boldsymbol{e}_i(t),\{\boldsymbol{e}_{ij}(t)\mid j\in N_i\})=0$ that is provided in the following.

#### *3.2.1 Distributed controller design*

In this case, the distributed consensus control strategy based on the distributed event-triggered rule for the system (1) is designed as

$$\tau_i(t)=-\sum_{j=1}^{N}a_{ij}[\boldsymbol{\sigma}_i(t_k^i)-\boldsymbol{\sigma}_j(t_k^i)]-\boldsymbol{K}_i\dot{\boldsymbol{\sigma}}_i(t_k^i),$$
$$t\in[t_k^i,t_{k+1}^i),\ i\in V.\qquad(13)$$

Using the distributed consensus protocol (13), the closed loop system becomes

$$\begin{aligned}\boldsymbol{M}_i(\boldsymbol{\sigma}_i)\ddot{\boldsymbol{\sigma}}_i(t)=&-\boldsymbol{C}_i(\boldsymbol{\sigma}_i,\dot{\boldsymbol{\sigma}}_i)\dot{\boldsymbol{\sigma}}_i(t)-\boldsymbol{K}_i[\dot{\boldsymbol{\sigma}}_i(t)+\boldsymbol{e}_i^{\dot{\sigma}_i}(t)]\\&-\sum_{j=1}^{N}a_{ij}[\boldsymbol{\sigma}_i(t)-\boldsymbol{\sigma}_j(t)+\boldsymbol{e}_i^{\sigma_i}(t)-\boldsymbol{e}_{ij}^{\sigma_j}(t)],\end{aligned}\qquad(14)$$

where $\boldsymbol{e}_{ij}^{\sigma_j}(t)=\boldsymbol{\sigma}_j(t_k^i)-\boldsymbol{\sigma}_j(t)$, $i\in V$, $\boldsymbol{e}_{ij}^{\sigma_j}\neq 0$ if $j\in N_i$, and otherwise $\boldsymbol{e}_{ij}^{\sigma_j}=0$.

Then, the system (14) can be written in a compact form as

$$\begin{aligned}\boldsymbol{M}(\boldsymbol{\sigma})\ddot{\boldsymbol{\sigma}}=&-[\boldsymbol{C}(\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}})\dot{\boldsymbol{\sigma}}+(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{\sigma}+\boldsymbol{K}\dot{\boldsymbol{\sigma}}\\&+(\boldsymbol{D}\otimes\boldsymbol{I}_n)e^{\sigma}-(\boldsymbol{E}\otimes\boldsymbol{I}_n)\hat{e}^{\sigma}+\boldsymbol{K}e^{\dot{\sigma}}],\end{aligned}\qquad(15)$$

where $\boldsymbol{D}=\mathrm{diag}[\deg(1),\cdots,\deg(N)]$, $\boldsymbol{E}=\mathrm{diag}[a_1^{\mathrm{T}},\cdots,a_N^{\mathrm{T}}]$ with $\boldsymbol{a}_i=(a_{i1},\cdots,a_{i(i-1)},a_{i(i+1)},\cdots,a_{iN})^{\mathrm{T}}$, $\boldsymbol{e}^{\sigma}$ and $\boldsymbol{e}^{\dot{\sigma}}$ are defined as shown above, and $\hat{\boldsymbol{e}}^{\sigma}=(\tilde{\boldsymbol{e}}_1^{\sigma_1\mathrm{T}},\cdots,\tilde{\boldsymbol{e}}_N^{\sigma_N\mathrm{T}})^{\mathrm{T}}$ with $\tilde{\boldsymbol{e}}_i^{\sigma_i}=(\boldsymbol{e}_{i1}^{\sigma_1\mathrm{T}},\cdots,\boldsymbol{e}_{i(i-1)}^{\sigma_{i-1}\mathrm{T}},\boldsymbol{e}_{i(i+1)}^{\sigma_{i+1}\mathrm{T}},\cdots,\boldsymbol{e}_{iN}^{\sigma_N\mathrm{T}})^{\mathrm{T}}$.

#### *3.2.2 Distributed event-triggered rule*

For each agent $i\in V$, a distributed event-triggered function is proposed as follows:

$$f_i(\boldsymbol{e}_i,\{\boldsymbol{e}_{ij}\mid j\in N_i\})=\deg(i)\left\|\boldsymbol{e}_i^{\sigma_i}\right\|+b_i\left\|\tilde{\boldsymbol{e}}_i^{\sigma_i}\right\|+\bar{\sigma}(\boldsymbol{K}_i)\times\left\|\boldsymbol{e}_i^{\dot{\sigma}_i}\right\|-\alpha_i\left\|\dot{\boldsymbol{\sigma}}_i\right\|-\theta_i(t),$$

where $b_i=\sqrt{\sum_{j\in N_i}a_{ij}^2}$, $\theta_i(t)=\nu_i e^{-\bar{\theta}_i(t-t_0)}$ with $\nu_i>0$ and $0<\bar{\theta}_i<1$, and $\alpha_i$ is a triggering gain that is to be determined later.

Then, a controller update task is forced at $t_k^i$ in (13), when a corresponding event is determined by detecting

$$f_i(\boldsymbol{e}_i(t_k^i),\{\boldsymbol{e}_{ij}(t_k^i)\mid j\in N_i\})=0,\ i\in V. \tag{16}$$

As long as the event is triggered at $t_k^i$, then the measurement errors $\boldsymbol{e}_i(t_k^i)$ and $\boldsymbol{e}_{ij}(t_k^i)$ are automatically reset to zeroes. As a result, $f_i(\boldsymbol{e}_i,\{\boldsymbol{e}_{ij}\mid j\in N_i\})\leqslant 0$ is satisfied again.

**Remark 3.** Note that the above distributed event-triggered function above is entirely in relation to local information. When the event-triggered rule (16) is examined as time passes, each agent requires to gather the information $\boldsymbol{e}_i$ and $\dot{\boldsymbol{\sigma}}_i$ of its own, along with $\boldsymbol{e}_{ij}$ of its neighbor $j\in N_i$. Moreover, the term $\theta_i(t)$ plays an important role in excluding Zeno behavior.

*3.2.3 Consensus analysis*

The following notations will be used in this analysis:

$\alpha^{\max}=\max_{1\leqslant i\leqslant N}\{\alpha_i\}$, $\deg^{\max}=\max_{1\leqslant i\leqslant N}\{\deg(i)\}$,
$\deg^{\min}=\min_{1\leqslant i\leqslant N}\{\deg(i)\}$, $a^{\max}=\max_{1\leqslant i\leqslant N}\{b_i\}$,
$a^{\min}=\min_{1\leqslant i\leqslant N}\{b_i\}$, $\bar{\sigma}^{\max}=\max_{1\leqslant i\leqslant N}\{\bar{\sigma}(\boldsymbol{K}_i)\}$,
$\bar{\sigma}^{\min}=\min_{1\leqslant i\leqslant N}\{\bar{\sigma}(\boldsymbol{K}_i)\}$,

and

$$\beta=\max\left\{\frac{\deg^{\max}}{\deg^{\min}},\frac{a^{\max}}{a^{\min}},\frac{\bar{\sigma}^{\max}}{\bar{\sigma}^{\min}}\right\}.$$

Hereinafter, a sufficient condition will be provided to achieve consensus of the network (1) with the distributed event-triggered information.

**Theorem 3.** For the multiple Euler-Lagrange systems (1), assume that the communication topology $G$ is undirected and connected. Then, under the distributed cooperative control law (13) and the distributed event-triggered rule (16), with the triggering gain $\alpha_i$ satisfying $0<\alpha_i<\sqrt{2}(2\underline{\sigma}(\boldsymbol{K}_i)-1)/(12\beta)$, the consensus problem of the network (1) is solved for any set of initial conditions $\boldsymbol{\sigma}_i(0)$, $\dot{\boldsymbol{\sigma}}_i(0)$, $i\in V$.

*Proof.* Consider the following Lyapunov function candidate again

$$V(t)=\frac{1}{2}\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{L}\otimes\boldsymbol{I}_n)\boldsymbol{\sigma}+\frac{1}{2}\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}}.$$

Taking the time derivative of $V(t)$ along the trajectories of (15) gives

$$\begin{aligned}\dot{V}(t)&=-\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{K}\dot{\boldsymbol{\sigma}}-\dot{\boldsymbol{\sigma}}^{\mathrm{T}}(\boldsymbol{D}\otimes\boldsymbol{I}_n)\boldsymbol{e}^{\boldsymbol{\sigma}}-\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}\\&\quad+\dot{\boldsymbol{\sigma}}^{\mathrm{T}}(\boldsymbol{E}\otimes\boldsymbol{I}_n)\hat{\boldsymbol{e}}^{\boldsymbol{\sigma}}\\&\leqslant-\sum_{i=1}^N\underline{\sigma}(\boldsymbol{K}_i)\left\|\dot{\boldsymbol{\sigma}}_i\right\|^2+\left\|\dot{\boldsymbol{\sigma}}\right\|(\deg^{\max}\left\|\boldsymbol{e}^{\boldsymbol{\sigma}}\right\|\\&\quad+a^{\max}\left\|\hat{\boldsymbol{e}}^{\boldsymbol{\sigma}}\right\|+\bar{\sigma}^{\max}\left\|\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}\right\|),\end{aligned}$$

where the following inequalities have been used: $\|\boldsymbol{D}\|\leqslant\deg^{\max}$, $\|\boldsymbol{E}\|\leqslant a^{\max}$, and $\|\boldsymbol{K}\|=\bar{\sigma}(\boldsymbol{K})=\bar{\sigma}^{\max}$.

In the following, the analysis focuses on treating with the term $\deg^{\max}\left\|\boldsymbol{e}^{\boldsymbol{\sigma}}\right\|+a^{\max}\left\|\hat{\boldsymbol{e}}^{\boldsymbol{\sigma}}\right\|+\bar{\sigma}^{\max}\left\|\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}\right\|$. Because the event-triggered rule (16) is always triggered to ensure that $f_i(\boldsymbol{e}_i,\{\boldsymbol{e}_{ij}\mid j\in N_i\})\leqslant 0$, i.e.,

$$\deg(i)\left\|\boldsymbol{e}_i^{\sigma_i}\right\|+b_i\left\|\tilde{\boldsymbol{e}}_i^{\sigma_i}\right\|+\bar{\sigma}(\boldsymbol{K}_i)\left\|\boldsymbol{e}_i^{\dot{\sigma}_i}\right\|\leqslant\alpha_i\left\|\dot{\boldsymbol{\sigma}}_i\right\|+\theta_i(t).$$

Then,

$$\begin{aligned}\sum_{i=1}^N\deg^2(i)\left\|\boldsymbol{e}_i^{\sigma_i}\right\|^2&\leqslant 2\sum_{i=1}^N\alpha_i^2\left\|\dot{\boldsymbol{\sigma}}_i\right\|^2+2\sum_{i=1}^N\theta_i^2(t)\\&\leqslant 2(\alpha^{\max})^2\sum_{i=1}^N\left\|\dot{\boldsymbol{\sigma}}_i\right\|^2+2\sum_{i=1}^N\theta_i^2(t).\end{aligned}$$

In addition, it yields that

$$\deg^{\max}\left\|\boldsymbol{e}^{\boldsymbol{\sigma}}\right\|\leqslant\frac{\sqrt{2}\alpha^{\max}\deg^{\max}\left\|\dot{\boldsymbol{\sigma}}\right\|}{\deg^{\min}}+\frac{\sqrt{2}\deg^{\max}}{\deg^{\min}}\left(\sum_{i=1}^N\theta_i^2(t)\right)^{1/2}.$$

In a similar trace, one can get

$$a^{\max}\left\|\hat{\boldsymbol{e}}^{\boldsymbol{\sigma}}\right\|\leqslant\frac{\sqrt{2}\alpha^{\max}a^{\max}\left\|\dot{\boldsymbol{\sigma}}\right\|}{a^{\min}}+\frac{\sqrt{2}a^{\max}}{a^{\min}}\left(\sum_{i=1}^N\theta_i^2(t)\right)^{1/2}.$$

And

$$\bar{\sigma}^{\max}\left\|\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}\right\|\leqslant\frac{\sqrt{2}\alpha^{\max}\bar{\sigma}^{\max}\left\|\dot{\boldsymbol{\sigma}}\right\|}{\bar{\sigma}^{\min}}+\frac{\sqrt{2}\bar{\sigma}^{\max}}{\bar{\sigma}^{\min}}\left(\sum_{i=1}^N\theta_i^2(t)\right)^{1/2}.$$

Thus, it follows that

$$\begin{aligned}&\deg^{\max}\left\|\boldsymbol{e}^{\boldsymbol{\sigma}}\right\|+a^{\max}\left\|\hat{\boldsymbol{e}}^{\boldsymbol{\sigma}}\right\|+\bar{\sigma}^{\max}\left\|\boldsymbol{e}^{\dot{\boldsymbol{\sigma}}}\right\|\\&\leqslant 3\sqrt{2}\beta\alpha^{\max}\left\|\dot{\boldsymbol{\sigma}}\right\|+3\sqrt{2}\beta\left(\sum_{i=1}^N\theta_i^2(t)\right)^{1/2}.\end{aligned}$$

Following the above developments above, $\dot{V}(t)$ then satisfies

$$\begin{aligned}\dot{V}(t)&\leqslant-\sum_{i=1}^N(\underline{\sigma}(\boldsymbol{K}_i)-(3\sqrt{2}\beta\alpha^{\max}+1/2))\left\|\dot{\boldsymbol{\sigma}}_i\right\|^2\\&\quad+9\beta^2\sum_{i=1}^N\theta_i^2(t).\end{aligned}$$

The subsequent proof is very similar to that of Theorem 1, and thus is omitted here. This completes the proof.

*3.2.4 Exclusion of Zeno behavior*

Similarly, an analysis of the minimum inter-event interval $\inf_{k\in\mathbf{N}}\{t_{k+1}^i - t_k^i\}$ for each agent $i$ is provided to exclude the Zeno behavior.

**Theorem 4.** For the multiple Euler-Lagrange systems described in (1) with the distributed control protocol (13) and the distributed event-triggered rule (16), assume that the undirected communication topology $G$ is connected. Then, no Zeno behavior occurs for each agent $i \in V$.

*Proof.* For any $t > t_k^i$, one has

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\boldsymbol{e}_i^{\dot{\sigma}_i}\|) \leqslant \|\dot{\boldsymbol{e}}_i^{\dot{\sigma}_i}\| \leqslant \|\ddot{\boldsymbol{\sigma}}_i\| \leqslant k_{\underline{m}}^{-1} k_c k_{\dot{\sigma}}^2 + k_{\underline{m}}^{-1} \left\| \sum\nolimits_{j=1}^N a_{ij}[\boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_j(t_k^i)] + \boldsymbol{K}_i \dot{\boldsymbol{\sigma}}_i(t_k^i) \right\|,$$

where $k_{\underline{m}}^{-1}$, $k_c$, $k_{\dot{\sigma}}$ are the same as those used in subsection $D_1$. Denoting $w_k^i \triangleq k_{\underline{m}}^{-1} k_c k_{\dot{\sigma}}^2 + k_{\underline{m}}^{-1} \left\| \sum\nolimits_{j=1}^N a_{ij}[\boldsymbol{\sigma}_i(t_k^i) - \boldsymbol{\sigma}_j(t_k^i)] + \boldsymbol{K}_i \dot{\boldsymbol{\sigma}}_i(t_k^i) \right\| > 0$, it therefore follows that

$$\|\boldsymbol{e}_i^{\dot{\sigma}_i}\| \leqslant w_k^i (t - t_k^i).$$

Furthermore, one also obtains that

$$\|\boldsymbol{e}_i^{\sigma_i}\| \leqslant \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\|(t - t_k^i) + \tfrac{w_k^i}{2}(t - t_k^i)^2,$$

and

$$\|\boldsymbol{e}_{ij}^{\sigma_j}\| \leqslant \|\dot{\boldsymbol{\sigma}}_j(t_{k'}^j)\|(t - t_k^i) + \tfrac{\vartheta_{kk'}^{ij}}{2}(t - t_k^i)^2,$$

where

$$\vartheta_{kk'}^{ij} \triangleq k_{\underline{m}}^{-1} k_c k_{\dot{\sigma}}^2 + k_{\underline{m}}^{-1} \left\| \sum\nolimits_{l=1}^N a_{jl}[\boldsymbol{\sigma}_l(t_{k'}^j) - \boldsymbol{\sigma}_j(t_{k'}^j)] + \boldsymbol{K}_j \dot{\boldsymbol{\sigma}}_j(t_{k'}^j) \right\| > 0,$$

with $t_{k'}^j$ denoting the latest controller update time of neighbor $j \in N_i$, i.e., $t_{k'}^j \triangleq \inf_{t \geqslant t_l^j, l\in N}\{t - t_l^j\}$, $t \in [t_k^i, t_{k+1}^i)$. Consequently, this yields

$$\|\tilde{\boldsymbol{e}}_i^{\sigma_i}\| = \left( \sum_{i=1}^N \|\boldsymbol{e}_{ij}^{\sigma_i}\|^2 \right)^{1/2} \leqslant \eta_{kk'}^i (t - t_k^i) + \vartheta_{kk'}^i (t - t_k^i)^2,$$

where

$$\eta_{kk'}^i = \sqrt{2 \sum_{j\in N_i} \|\dot{\boldsymbol{\sigma}}_j(t_{k'}^j)\|^2}$$

and

$$\vartheta_{kk'}^i = \sqrt{\frac{1}{2} \sum_{j\in N_i} \left(\vartheta_{kk'}^{ij}\right)^2} > 0.$$

In conclusion, one has

$$\|\boldsymbol{e}_i^{\sigma_i}\| + \|\tilde{\boldsymbol{e}}_i^{\sigma_i}\| + \|\boldsymbol{e}_i^{\dot{\sigma}_i}\| \leqslant (\tfrac{w_k^i}{2} + \vartheta_{kk'}^i)(t - t_k^i)^2 + (w_k^i + \eta_{kk'}^i + \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\|)(t - t_k^i). \quad (17)$$

On the other hand, a sufficient condition is proposed to guarantee that $f_i(\boldsymbol{e}_i(t), \{\boldsymbol{e}_{ij}(t) \mid j \in N_i\}) \leqslant 0$, $t \in [t_k^i, t_{k+1}^i)$:

$$\|\boldsymbol{e}_i^{\sigma_i}\| + \|\tilde{\boldsymbol{e}}_i^{\sigma_i}\| + \|\boldsymbol{e}_i^{\dot{\sigma}_i}\| \leqslant \tfrac{\alpha_i}{\gamma_i}\|\dot{\boldsymbol{\sigma}}_i(t_k^i)\| + \tfrac{1}{\gamma_i}\theta_i(t),$$

where $\gamma_i = \max\{\deg(i), b_i, \bar{\sigma}(\boldsymbol{K}_i) + \alpha_i\} > 0$. In this context, the time of the next event $t_{k+1}^i$ is obtained when

$$\|\boldsymbol{e}_i^{\sigma_i}(t_{k+1}^i)\| + \|\tilde{\boldsymbol{e}}_i^{\sigma_i}(t_{k+1}^i)\| + \|\boldsymbol{e}_i^{\dot{\sigma}_i}(t_{k+1}^i)\| = \tfrac{\alpha_i}{\gamma_i}\|\dot{\boldsymbol{\sigma}}_i(t_k^i)\| + \tfrac{1}{\gamma_i}\theta_i(t_{k+1}^i).$$

Consequently, when combined with (17), one has

$$\begin{aligned}\frac{\alpha_i}{\gamma_i}\|\dot{\boldsymbol{\sigma}}_i(t_k^i)\| + \frac{1}{\gamma_i}\theta_i(t_{k+1}^i) &= \|\boldsymbol{e}_i^{\sigma_i}(t_{k+1}^i)\| + \|\tilde{\boldsymbol{e}}_i^{\sigma_i}(t_{k+1}^i)\| + \|\boldsymbol{e}_i^{\dot{\sigma}_i}(t_{k+1}^i)\| \\ &\leqslant \left(\frac{w_k^i}{2} + \vartheta_{kk'}^i\right)(t_{k+1}^i - t_k^i)^2 + (w_k^i + \eta_{kk'}^i \\ &\quad + \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\|)(t_{k+1}^i - t_k^i).\end{aligned}$$

Let $q_1 \triangleq \frac{w_k^i}{2} + \vartheta_{kk'}^i$, $q_2 \triangleq w_k^i + \eta_{kk'}^i + \|\dot{\boldsymbol{\sigma}}_i(t_k^i)\|$ with $w_k^i > 0$, $\eta_{kk'}^i \geqslant 0$, and $\vartheta_{kk'}^i > 0$. It thus follows that

$$\frac{\nu_i}{\gamma_i} e^{-\bar{\theta}_i(t_{k+1}^i - t_0)} \leqslant q_1(t_{k+1}^i - t_k^i)^2 + q_2(t_{k+1}^i - t_k^i).$$

Note the facts that $x^2 \leqslant e^x - 1$ and $x \leqslant e^x - 1$ for $x \geqslant 0$. One then has

$$\frac{\nu_i}{\gamma_i} e^{-\bar{\theta}_i(t_{k+1}^i - t_0)} \leqslant (q_1 + q_2)(e^{t_{k+1}^i - t_k^i} - 1). \quad (18)$$

If $\lim_{k\to\infty} t_k^i = t^* < \infty$, then according to (18) one has

$$\frac{\nu_i}{\gamma_i} e^{-\bar{\theta}_i(t_{k+1}^i - t_k^i)} \leqslant e^{-\bar{\theta}_i(t^* - t_0)}(q_1 + q_2)(e^{t_{k+1}^i - t_k^i} - 1),$$

which implies that $0 < \frac{\nu_i}{\gamma_i} \leqslant 0$. This contradiction indicates that $\lim_{k\to\infty} t_k^i = \infty$. Also, because $0 < \bar{\theta}_i < 1$, then according to (18), one can derive

$$\frac{\nu_i}{\gamma_i} e^{-\bar{\theta}_i(t_{k+1}^i - t_k^i)} \leqslant (q_1 + q_2)(e^{t_{k+1}^i - t_0} - e^{t_k^i - t_0}),$$

which implies that $t_{k+1}^i - t_k^i$ has a positive lower bound $\tau_k^i$, i.e., $\inf_{k\in\mathbf{N}}\{t_{k+1}^i - t_k^i\} = \inf_{k\in\mathbf{N}}\{\tau_k^i\} = \tau^i > 0$. Otherwise, if $t_{k+1}^i$

$\to t_k^i$, then the inequality above cannot be satisfied. Since $q_1+q_2$ is bounded by

$$k_{\underline{m}}^{-1}\left[\frac{(3+\sqrt{2\deg^{\max}})(k_c k_{\tilde{\sigma}}^2+k_\sigma \deg^{\max}+k_{\tilde{\sigma}}\bar{\sigma}^{\max}}{2}\right.$$
$$\left.+(1+\sqrt{2\deg^{\max}})k_\sigma k_{\underline{m}}\right],$$

while $v_i>0$ and $0<\bar{\theta}_i<1$ can be chosen such that the left side of the inequality above is sufficiently large. Accordingly, no Zeno behavior occurs for any agent $i$, according to Definition 1. This completes the proof.

## 4 Simulations

In this section, some numerical results are given to verify the effectiveness of the above proposed theoretical analysis. A system consists of six two-link manipulators presented in Figure 1 [41] is considered, whose dynamics can be explicitly written as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\ddot{\sigma}_{i1}\\ \ddot{\sigma}_{i2}\end{bmatrix}+\begin{bmatrix}-h\dot{\sigma}_{i2} & -h(\dot{\sigma}_{i1}+\dot{\sigma}_{i2})\\ h\dot{\sigma}_{i1} & 0\end{bmatrix}\begin{bmatrix}\dot{\sigma}_{i1}\\ \dot{\sigma}_{i2}\end{bmatrix}$$
$$=\begin{bmatrix}\tau_{i1}\\ \tau_{i2}\end{bmatrix}, i=1,\cdots,6,$$

where $M_{11}=a_1+2a_3\cos\sigma_{i2}+2a_4\sin\sigma_{i2}$, $M_{12}=M_{21}=a_2+a_3\cos\sigma_{i2}+a_4\sin\sigma_{i2}$, $M_{22}=a_2$, and $h=a_3\sin\sigma_{i2}-a_4\cos\sigma_{i2}$ with $a_1=I_1+m_1l_{c1}^2+I_e+m_el_{ce}^2+m_el_1^2$, $a_2=I_e+m_el_{ce}^2$, $a_3=m_el_1l_{ce}\cos\sigma_e$, and $a_4=m_el_1l_{ce}\sin\sigma_e$.

Furthermore, the undirected communication topology $G$ satisfies $a_{12}=a_{13}=a_{24}=a_{26}=a_{35}=a_{46}=1$. In the simulation, one uses $m_1=1$, $l_1=1$, $m_e=2$, $\sigma_e=30°$, $I_1=0.12$, $l_{c1}=0.5$, $I_e=0.25$, $l_{ce}=0.6$. Also, the gain matrices are chosen as $\boldsymbol{K}_i=\text{diag}(10,10), i=1,\cdots,6$.

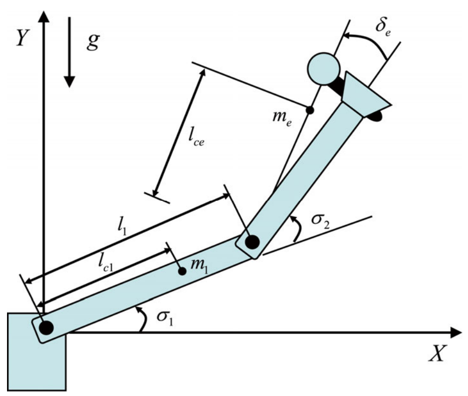


**Figure 1** (Color online) An articulated two-link manipulator.

### 4.1 Decentralized event-triggered cooperative consensus

For this case, one chooses $\chi_1=0.2$, $\chi_2=0.3$, $\chi_3=0.3$, $\chi_4=0.2$, $\chi_5=0.2$, $\chi_6=0.4$, and $a=0.8591$. By computation, one has $\phi_1=1.1341$, $\phi_2=0.2750$, $\phi_3=1.1341$, $\phi_4=1.1341$, $\phi_5=1.9932$, $\phi_6=1.1341$, $\varphi_i=2.5$, $i=1$, $3,\cdots,6$, and $\varphi_2=3$. According to the decentralized event-triggered scheme (8), the error bound of each agent $i\ (i=1,\cdots,6)$ is denoted as $Q_i(t)=\sqrt{\frac{a\chi_i\phi_i}{2\varphi_i}}\|\dot{\boldsymbol{\sigma}}_i(t)\|+\varepsilon_i(t)$ with $\varepsilon_i(t)=5e^{-0.1t}$. Evolutions of the error bound $Q_i(t)$ and the measurement error $\|\boldsymbol{e}_i(t)\|$ of each agent are illustrated in Figure 2, the corresponding event times of each agent are presented in Figure 3. The orientation and angle velocity trajectories of all agents are provided in Figure 4. Moreover, positive lower bounds $\tau^i$ of the interevent intervals of each agent $i$ are 1.9, 1.864, 1.92, 1.9, 2.07, and 1.8 seconds, respectively. Hence, the minimum inter-event interval of the whole system is 1.8 seconds.

### 4.2 Distributed event-triggered cooperative consensus

In this regard, one chooses $\alpha_i=0.6964\ (i=1,\cdots,6)$ by some calculations. In the distributed event-triggered rule (16), the error bound of each agent $i\ (i=1,\cdots,6)$ is denoted as $M_i(t)=\alpha_i\|\dot{\boldsymbol{\sigma}}_i(t)\|+\theta_i(t)$ with $\theta_i(t)=50e^{-0.01}$, while the weighted sum of measurement error is denoted as $E_i(t)=\deg(i)\|\boldsymbol{e}_i^{\sigma_i}\|+b_i\|\tilde{\boldsymbol{e}}_i^{\sigma_i}\|+\bar{\sigma}(\boldsymbol{K}_i)\|\boldsymbol{e}_i^{\dot{\sigma}_i}\|$. Evolutions of the error

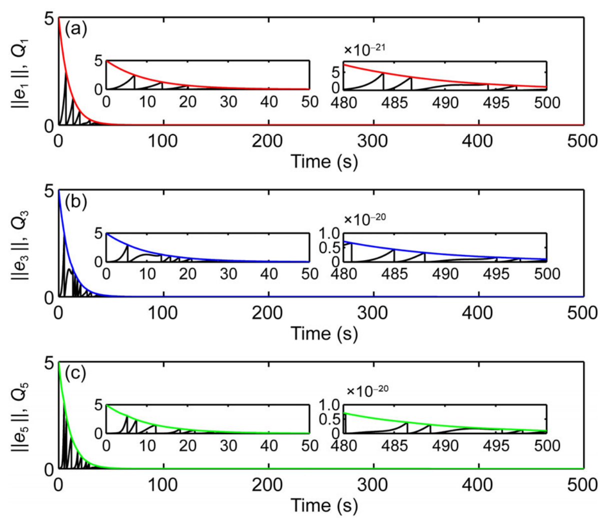


**Figure 2** (Color online) Evolutions of $Q_i(t)$ (colorized solid line) and $\|e_i(t)\|$ (black solid line)) for agent $i$=1 (a), 3 (b), 5(c) in the decentralized case.

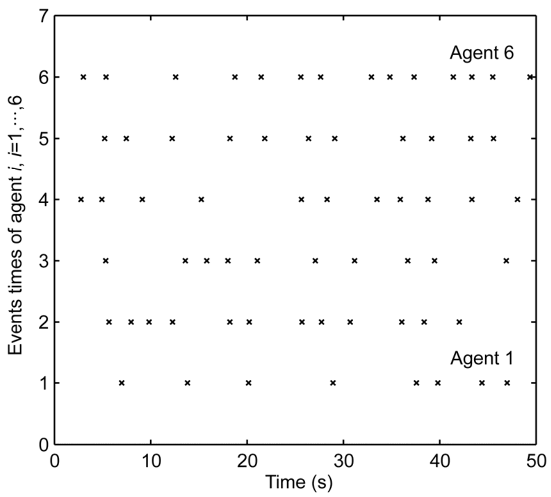


**Figure 3** Events times of all agents in the decentralized case.

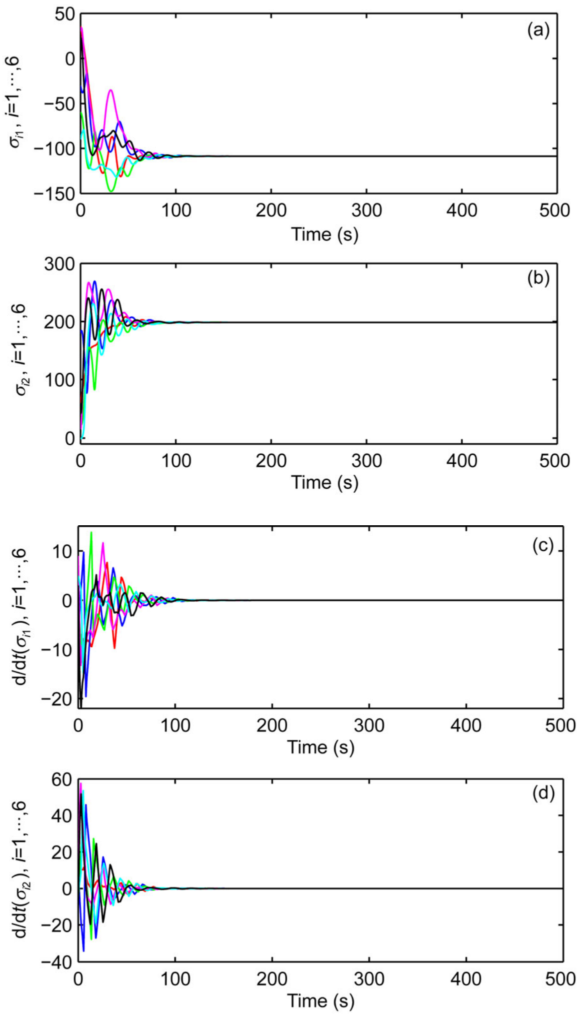


**Figure 4** For decentralized case, (a) consensus of the orientation trajectories $\sigma_{i1}$ of all agents; (b) consensus of the orientation trajectories $\sigma_{i2}$ of all agents; (c) consensus of the velocity trajectories $\dot{\sigma}_{i1}$ of all agents; (d) consensus of the velocity trajectories $\dot{\sigma}_{i2}$ of all agents.

bound $M_i(t)$ and the measurement error $E_i(t)$ of each agent $i$ are illustrated in Figure 5, the corresponding event times of each agent are shown in Figure 6. In addition, the orientation and angle velocity trajectories of all agents are presented in Figure 7. Also, positive lower bounds $\tau^i$ of the inter-event intervals of each agent $i$ are 0.66, 0.66, 0.545, 0.56, 0.87, and 0.56 seconds, respectively. Therefore, the minimum inter-event interval of the whole system is 0.545 seconds.

According to Figures 3 and 6, the numbers of event times in the first 50 seconds of each agent $i$ for decentralized and distributed cases are 8, 12, 10, 11, 11, 14 and 14, 25, 15, 28, 9, 15, respectively. Note that the latter case generates more events, which means more communications are needed according to (16). The numbers of the controller update times of each agent $i$ for decentralized and distributed cases are 30, 45, 29, 37, 21, 37 and 14, 25, 15, 28, 9, 15, respectively. It turns out that the controller designed based on decentralized

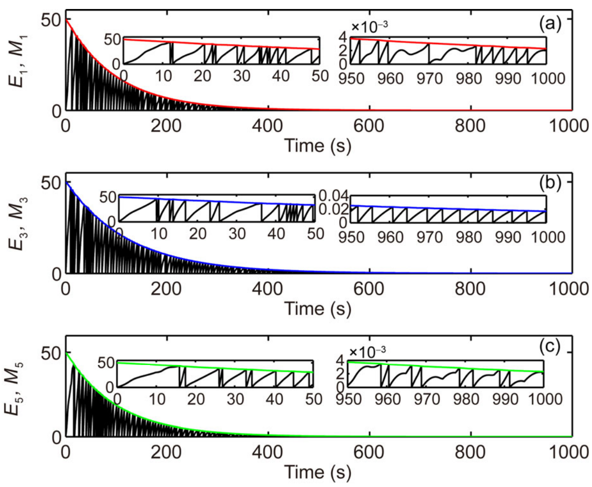


**Figure 5** Evolutions of $M_i(t)$ (colorized solid line) and $E_i(t)$ (black solid line) for agent $i$=1 (a), 3 (b), 5 (c) in the distributed case.

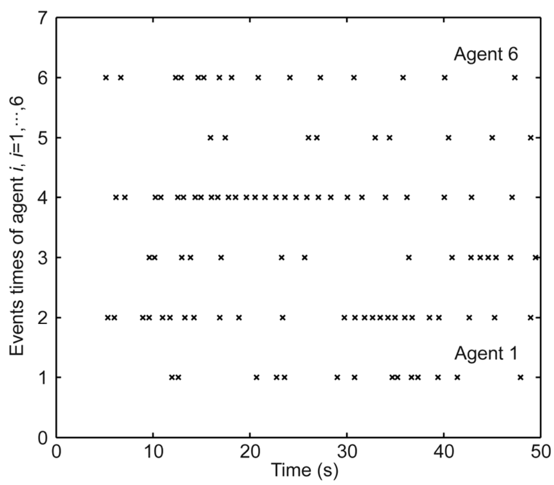


**Figure 6** Events times of all agents in the distributed case.

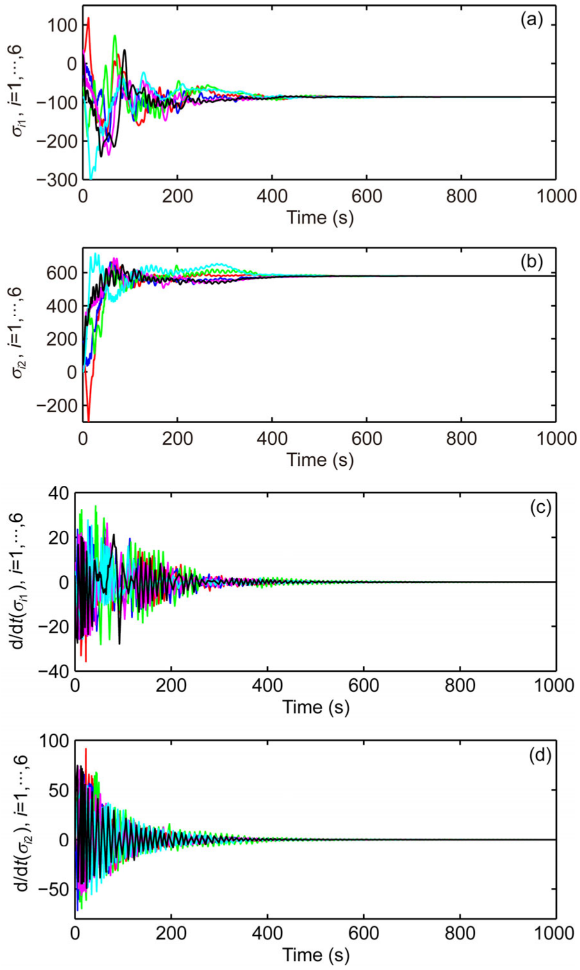


**Figure 7** For distributed case, (a) consensus of the orientation trajectories $\sigma_{i1}$ of all agents; (b) consensus of the orientation trajectories $\sigma_{i2}$ of all agents; (c) consensus of the velocity trajectories $\dot{\sigma}_{i1}$ of all agents; (d) consensus of the velocity trajectories $\dot{\sigma}_{i2}$ of all agents .

event-triggered scheme (8) has more update times. These results are in accordance with intuition: firstly, the distributed event-triggered strategy (16) designed for each agent $i$ obtains more information of the overall system via communication, while the decentralized event-triggered case (8) only uses its own information; in addition, the cooperative control policy (5) for each agent $i$ is updated at its own event times and the latest event-times of its neighbors, while the cooperative protocol (13) for each agent $i$ is just updated at its own event times. It indicates that a trade-off between communication frequencies and controller update times needs to be taken into account. Specifically, if one wants to design an event-triggered strategy and to reduce communication frequencies as far as possible, decentralized approach is a better choice. Whereas, when few controller update times are concerned, maybe distributed scheme is more appropriate.

## 5 Conclusion

In this study, two event-based consensus strategies have been proposed and studied for multiple Euler-Lagrange systems under fixed and undirected communication topologies. Rigorous analyses of the convergence results for the proposed protocols have been addressed by using tools from graph theory and Lyapunov stability theory. Sufficient conditions have been derived such that the multiple Euler-Lagrange systems can reach consensus as long as an appropriated cooperative control law is designed with a reasonably updated rule. Also, Zeno behavior is excluded for the triggering time sequences. The simulation results show that a trade-off between the communication frequencies and the controller update times must be taken into account to enable selection of a suitable strategy. Overall, larger numbers of available communication frequencies, require fewer controller update times, and vice versa. In the future, we will focus on the consensus behaviors of multiple Euler-Lagrange systems without using absolute velocity information.

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