

A necessary and sufficient stability criterion for networked predictive control systems

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Stability of a networked predictive control system subject to network-induced delay and data dropout is investigated in this study. By modeling the closed-loop system as a switched system with an upper-triangular structure, a necessary and sufficient stability criterion is developed. From the criterion, it also can be seen that separation principle holds for networked predictive control systems. A numerical example is provided to confirm the validity and effectiveness of the obtained results.

networked control system, networked predictive control, stability, network-induced delay, data dropout

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1 Introduction

In recent years, networked control systems (NCSs) whose control loops are closed by real-time networks have received much attention. NCSs have many advantages compared with traditional point-to-point control systems such as efficient resource sharing and energy saving [1,2]. However, NCSs have several disadvantages stemming from an introduction of a real-time network such as network-induced delay, data dropout, data disorder and quantized error, which present challenges to conventional control theories built on ideal assumptions. To solve these problems, a number of control methodologies have been proposed. To mention a few, time-delay system method was used to model and stabilize NCSs in [3–6]. A switched system approach including the switched Lyapunov function approach as well as the average dwell-time approach was adopted to provide an output feedback stabilization, exponential stabilization or

disturbance attenuation of NCSs [7–9]. The jump system approach was also applied to NCSs by modeling the network-induced delay as a Markov chain [10–12]. Some advanced control methods such as intelligent control methods and adaptive control methods were utilized to realize the fault detection, the scheduling and the distributed coordination of NCSs [13–15]. Other methodologies can be found in some good survey papers [16–19] and references therein.

It should be noted that many existing works attempted to design a controller that is sufficiently robust to handle network constraints such as network-induced delay and data dropout rather than actively compensate for them. Recently, a new model-based method called networked predictive control has been proposed to actively compensate for network-induced delay and data dropout [20–28]. This method has been demonstrated to be very effective by substantial simulations and experiments. The compelling fact is that a control performance similar to local control (i.e., there is no network in the system) can be obtained using this method. However, how to analyze stability of a networked predictive control system is a challenging topic which has not been

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completely solved. In existing results, the common Lyapunov function approach [20–27], or the switched Lyapunov function approach [28] are usually applied to obtain a stability condition for the closed-loop system and these methods can only yield some sufficient stability conditions. To the best of authors' knowledge, there is no necessary and sufficient stability condition for a networked predictive control system when taking both the network-induced delay and data dropout into consideration, which motivates this study.

In this paper, a new formulation of the networked predictive control method is presented. The closed-loop system is modeled as a switched system with an upper-triangular structure. A necessary and sufficient stability condition is obtained. It shows that the stability of a networked predictive control system is related to the network-induced delay and the maximum number of successive data dropouts. It also indicates that separation principle holds for networked predictive control systems which means that the state feedback controller and the state observer can be designed independently. Finally, a numerical example is provided to confirm the validity and effectiveness of main results.

2 Preliminaries

Consider the following discrete-time plant:

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

$$y(k) = Cx(k), \quad (2)$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and $y(k) \in \mathbb{R}^p$ are the state vector, the control input vector and the output vector, respectively. A , B and C are constant matrices with appropriate dimensions.

For the sake of simplicity, but without loss of generality, NCSs with random delay and data dropout in the feedback channel are considered in this study as shown in Figure 1. The following assumptions are made [22,25].

Assumption 1. Network-induced delay $\tau(k)$ is bounded by $\tau_1 h \leq \tau(k) \leq \tau_2 h$ with h being the sampling interval. The number of successive data packet dropouts is not larger than N . Where $\tau_2 \geq \tau_1 \geq 0$ and $N \geq 0$ are integers.

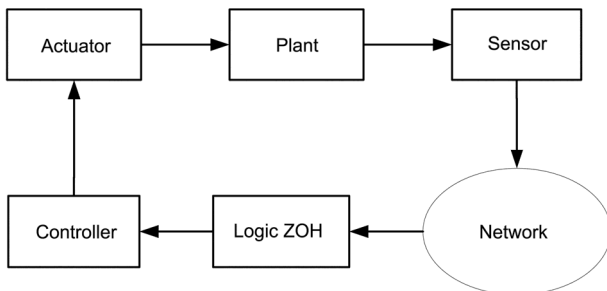


Figure 1 A framework of networked control systems.

Assumption 2. All signals in the system are transmitted with time-stamps and all components in the system are synchronized.

Let $\{t_k = kh \mid k=1,2,\dots\}$ be the sampling instants. The sensor samples the output of the plant at t_k and sends the output of the plant $y(t_k)$ together with its time-stamp to the controller through a network. Due to the random network-induced delay and data dropout, 'packet disorder' will exist, which means that data packets sent earlier (later) arrive later (earlier). Another situation will also exist where either more than one data packet or no data packet arrives at the controller node in one sampling interval. To deal with these situations, a logic zero-order-hold (ZOH) [29] is introduced at the controller node to select and store the latest data packets. The mechanism of the logic ZOH can be described as follows.

Logic ZOH. Step 1: Set $k=0$, $\ell_0 = 0$, and $t_0=0$. Step 2: At sampling instant t_k , ZOH updates its output to $y(t) = y(\ell_k h)$ for $t_k \leq t \leq t_{k+1}$. Step 3: If there are some data packets reaching the ZOH during $t_k \leq t \leq t_{k+1}$, compare their time-stamps and denote the largest one by ϑ . If $\vartheta > \ell_k$, then the ZOH stores $y(\vartheta h)$ and lets $\ell_{k+1} = \vartheta$. Step 4: Let $k=k+1$ and go to step 2.

A data packet that is successfully transmitted from the sensor to the ZOH and subsequently stored by the ZOH is called an effective data packet. The time-stamp sequence of effective data packets is denoted by $\{T_i \mid i=1,2,\dots\}$. The time of an effective data packet is used to update the ZOH is called an updating instant. The updating instant sequence is denoted by $\{S_i \mid i=1,2,\dots\}$. It is clear that the controller uses $y(T_i)$ to develop the control law at the updating instant S_i . To make the above definitions more clear, an illustrative example is shown in Figure 2.

For S_i and T_i , we have the following lemma.

Lemma 2.1. For S_i and T_i , denote $\theta_i = (S_{i+1} - S_i) / h$, $\eta_i = (T_{i+1} - T_i) / h$, the following inequalities hold

$$1 \leq \theta_i \leq \tau_2 - \tau_1 + N + 1, \quad (3)$$

$$1 \leq \eta_i \leq \tau_2 - \tau_1 + N + 1, \quad (4)$$

where τ_2 , τ_1 and N are as defined in Assumption 1.

Proof. Clearly, the minimum of θ_i and η_i is 1. Considering that the maximum of successive data packet dropouts is N , $T_{i+1} - T_i$ reaches its maximum only if $T_i + 1h, T_i + 2h, \dots, T_i + Nh$ are all dropped out, $T_i + Nh + 1h$ suffers the maximum delay, $T_i + Nh + 1h, T_i + Nh + 2h, \dots, T_i + Nh + \tau_2 h - \tau_1 h$ are all not effective data packets and $T_i + Nh + \tau_2 h - \tau_1 h + 1h$ is stored by the ZOH as an effective data packet. In such a case, $T_{i+1} - T_i = \tau_2 h - \tau_1 h + Nh + 1h$. Similarly, we can prove the maximum of $S_{i+1} - S_i$ is $\tau_2 h - \tau_1 h + Nh + 1h$.

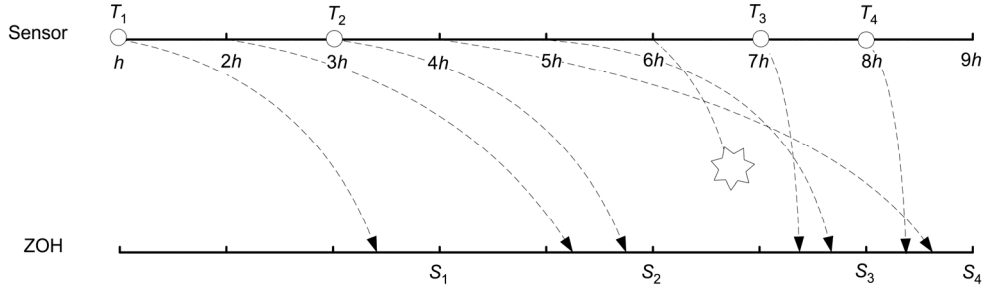


Figure 2 An example to show $\{T_i\}$ and $\{S_i\}$ of the logic ZOH, where \circ denotes an effective data packet.

3 Main results

In this section, we will focus on stability analysis of networked predictive control systems. First, the networked predictive control method is briefly formulated. And then, a necessary and sufficient stability condition for networked predictive control systems is developed.

3.1 Networked predictive control method

From the above discussion, we can see that $y(T_i)$ is the most recent measurement signal available at the sampling time S_i . The following networked predictive control method is proposed to compensate for the network-induced delay and data dropout.

Step 1. Based on the received measurement signal $y(T_i)$, the controller predicts the system's current state by the following iteration

$$\begin{aligned} \hat{x}(T_i + 1h | T_i) &= A\hat{x}(T_i | T_{i-1}) + Bu(T_i) \\ &\quad + L[y(T_i) - C\hat{x}(T_i | T_{i-1})], \\ \hat{x}(T_i + 2h | T_i) &= A\hat{x}(T_i + 1h | T_i) + Bu(T_i + 1h), \\ \hat{x}(T_i + 3h | T_i) &= A\hat{x}(T_i + 2h | T_i) + Bu(T_i + 2h), \\ &\dots \\ &\dots \\ &\dots \end{aligned} \quad (5)$$

$$\hat{x}(S_i | T_i) = A\hat{x}(S_i - 1h | T_i) + Bu(S_i - 1h).$$

Step 2. Based on $\hat{x}(S_i | T_i)$, a feedback control law is

$$u(S_i) = K\hat{x}(S_i | T_i), \quad (6)$$

where L is the observer gain matrix, and K is the controller gain matrix which can be determined by some standard methods such as pole assignment.

Using the above networked predictive control method, the network-induced delay and data dropout in the feedback channel can be well compensated as reported in [22,25].

Remark 3.1. Based on the time-stamp sequence of effective data packet T_i and updating instant sequence S_i , a new formulation of networked predictive control method is proposed. This formulation facilitates the stability analysis of the closed-loop system.

3.2 Stability analysis

From the iteration (5), we can obtain

$$\begin{aligned} \hat{x}(T_i + \beta h | T_i) &= A^\beta \hat{x}(T_i | T_{i-1}) \\ &\quad + \sum_{j=1}^{\beta} A^{j-1} u(T_i + \beta h - jh) \\ &\quad + A^{\beta-1} L(y(T_i) - C\hat{x}(T_i | T_{i-1})), \end{aligned} \quad (7)$$

where $1 \leq \beta \leq S_{i+1} - T_i$.

It is clear that

$$\begin{aligned} x(T_i + \beta h) &= Ax(T_i + \beta h - 1h) + Bu(T_i + \beta h - 1h) \\ &= A^2 x(T_i + \beta h - 2h) + ABu(T_i + \beta h - 2h) \\ &\quad + Bu(T_i + \beta h - 1h) \\ &\dots \\ &\dots \\ &\dots \\ &= A^\beta x(T_i) + \sum_{j=1}^{\beta} A^{j-1} Bu(T_i + \beta h - jh). \end{aligned} \quad (8)$$

Define $e(T_i + \beta h | T_i) = x(T_i + \beta h) - \hat{x}(T_i + \beta h | T_i)$ and subtraction of (7) from (8) yields

$$\begin{aligned} e(T_i + \beta h | T_i) &= A^\beta e(T_i | T_{i-1}) - A^{\beta-1} LCe(T_i | T_{i-1}) \\ &= A^{\beta-1} (A - LC)e(T_i | T_{i-1}). \end{aligned} \quad (9)$$

Therefore,

$$e(T_{i+1} | T_i) = A^{n-1} (A - LC)e(T_i | T_{i-1}). \quad (10)$$

Denote $d_i = S_i - T_i$, from (6) and (9), the closed-loop system can be obtained as follows

$$\begin{aligned} x(S_i + 1h) &= Ax(S_i) + Bu(S_i) \\ &= Ax(S_i) + BK\hat{x}(S_i | T_i) \\ &= (A + BK)x(S_i) - BKe(S_i | T_i) \\ &= (A + BK)x(S_i) \\ &\quad - BKA^{d_i-1} (A - LC)e(T_i | T_{i-1}). \end{aligned} \quad (11)$$

Similarly,

$$\begin{aligned}
x(S_i + 2h) &= Ax(S_i + 1h) + Bu(S_i + 1h) \\
&= (A + BK) x(S_i + 1h) - BKe(S_i + 1h | T_i) \\
&= (A + BK)^2 x(S_i) \\
&\quad - (A + BK)BKA^{d_i-1}(A - LC)e(T_i | T_{i-1}) \\
&\quad - BKA^{d_i}(A - LC)e(T_i | T_{i-1}).
\end{aligned} \tag{12}$$

For an integer $\gamma > 0$, it is clear that

$$\begin{aligned}
x(S_i + \gamma h) &= (A + BK)^\gamma x(S_i) \\
&\quad - (A + BK)^{j-1} BKA^{d_i+\gamma-j-1} \\
&\quad \times (A - LC)e(T_i | T_{i-1}).
\end{aligned} \tag{13}$$

Therefore, we can obtain

$$\begin{aligned}
x(S_{i+1}) &= (A + BK)^{\theta_i} x(S_i) - \sum_{j=1}^{\theta_i} (A + BK)^{j-1} \\
&\quad \times BKA^{d_i+\theta_i-j-1}(A - LC)e(T_i | T_{i-1}).
\end{aligned} \tag{14}$$

Define a new vector $\xi_i = \begin{bmatrix} x(S_i) \\ e(T_i | T_{i-1}) \end{bmatrix}$, the closed-loop system can be described as

$$\xi_{i+1} = \Xi_i \xi_i, \tag{15}$$

where

$$\Xi_i = \begin{bmatrix} (A + BK)^{\theta_i} & \Xi_i^{12} \\ 0 & A^{\eta_i-1}(A - LC) \end{bmatrix}$$

with

$$\Xi_i^{12} = -\sum_{j=1}^{\theta_i} (A + BK)^{j-1} BKA^{d_i+\theta_i-j-1}(A - LC).$$

It is clear that Ξ_i is switched according to θ_i , η_i and d_i . Therefore, the closed-loop system (15) is a switched system. According to the switched system theory [30], a switched linear system with a block upper-triangular structure is asymptotically stable if and only if each of its block diagonal subsystems is asymptotically stable. Therefore, the closed-loop system (15) is asymptotically stable if and only if eigenvalues of $(A+BK)\theta_i$ and $A^{\eta_i-1}(A-LC)$ are within the unit circle. Clearly, eigenvalues of $(A+BK)\theta_i$ are within the unit circle is equivalent to eigenvalues of $A+BK$ are within the unit circle. Therefore, we have the following stability theorem.

Theorem 3.2. The closed-loop networked predictive control system is asymptotically stable for any delay and data dropout satisfying Assumption 1 if and only if eigenvalues of $A+BK$ and $A^{\kappa-1}(A-LC)$, for all $1 \leq \kappa \leq \tau_2 - \tau_1 + N + 1$ with κ being integers, are within the unit circle.

Remark 3.3. From Theorem 3.2, it can be seen that the state feedback controller and the state observer can be designed independently by guaranteeing eigenvalues of $A+BK$

and $A^{\kappa-1}(A-LC)$, for all $1 \leq \kappa \leq \tau_2 - \tau_1 + N + 1$, are within the unit circle. This property is in accordance with the separation principle.

If $T_{i+1} - T_i = h$, which indicates that there are no packet disorder, no packet dropout and not more than one sensor signal reaching the controller node in a sampling interval, the following corollary can be obtained.

Corollary 3.4. For $T_{i+1} - T_i = h$, the closed-loop networked predictive control system is asymptotically stable if and only if eigenvalues of $A+BK$ and $A-LC$, are within the unit circle.

4 A numerical example

In this section, a numerical example is given to confirm the efficiency and correctness of the proposed main results.

Consider the cart-pendulum system [31] whose simplified and discretized model is as follows.

$$A = \begin{bmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix}, B = \begin{bmatrix} -0.0001 \\ -0.0053 \end{bmatrix}, C = [1 \quad 0].$$

The eigenvalues of matrix A are 1.1329 and 0.8827, which means that the open-loop system is unstable. The feedback control gain matrix K and the observer gain matrix L are designed by the pole assignment method to ensure the closed-loop system without network delay and data dropout is stable. The desired poles of the feedback controller and the observer are [0.6, 0.7] and [0.5, 0.2], respectively. K and L are as follows.

$$K = [846.1460 \quad 119.0538], L = \begin{bmatrix} 1.3156 \\ 14.1481 \end{bmatrix}.$$

The network-induced delay is bounded by $h \leq \tau(k) \leq 3h$ as shown in Figure 3. The maximum number of successive data dropouts is 2 as shown in Figure 4 where '1' indicates that data are transferred to the controller successfully and '0' indicates that data are dropped.

According to Theorem 3.2, eigenvalues of $A+BK$ and $A^{\kappa-1}(A-LC)$, for $1 \leq \kappa \leq 5$ are listed in Table 1. It can be seen that all eigenvalues are within the unit circle. It means that the closed-loop networked predictive control system is asymptotically stable for any network-induced delay and data dropouts satisfying the above assumptions.

Let $x(0) = [0 \quad 0]^T$ and $\hat{x}(0) = [-1 \quad 0]^T$, then the simulation results are given in Figure 5. It can be seen that the closed-loop networked predictive control system is asymptotically stable and the performance of the closed-loop networked predictive control system is quite similar to that of the local closed-loop system (i.e., there is no network in the system).

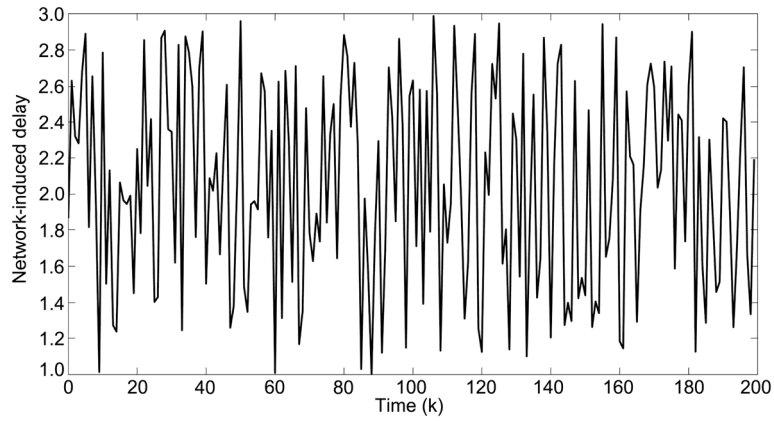


Figure 3 Network-induced delay.

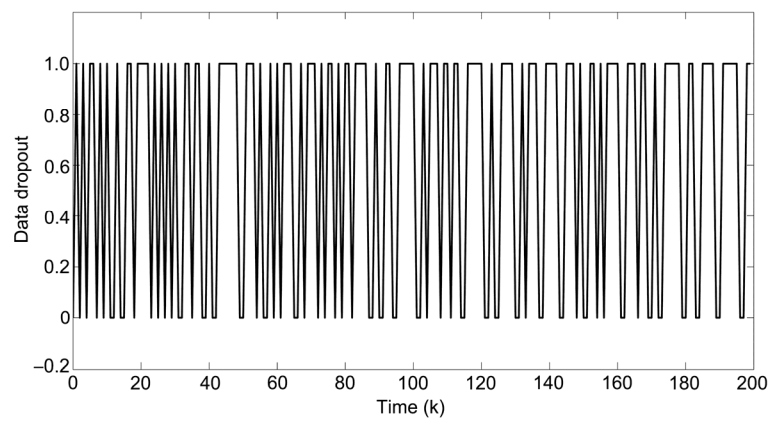


Figure 4 Data dropout.

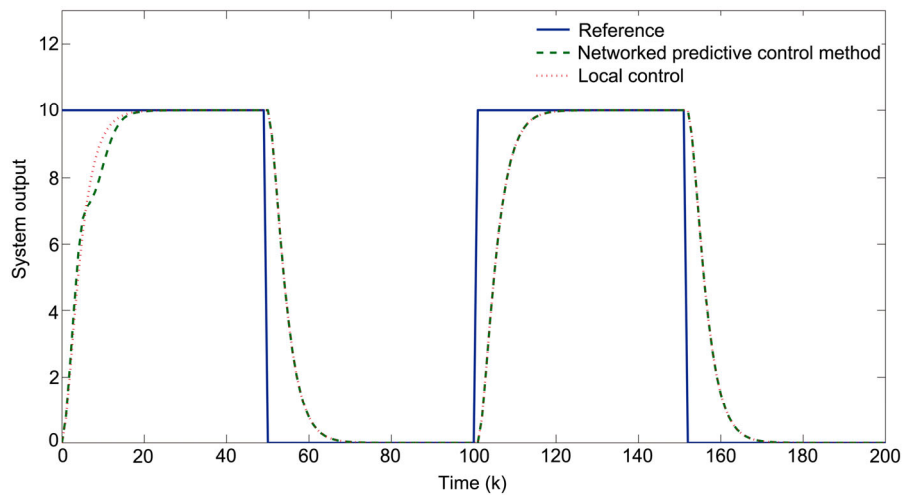


Figure 5 Response of the closed-loop system.

Next, we will consider another case. If the poles of the observer are chosen as $[-0.5, 0.2]$, L can be obtained as

$$L = \begin{bmatrix} 2.3156 \\ 40.9853 \end{bmatrix}.$$

The controller gain matrix remains

$$K = [846.1460 \quad 119.0538].$$

The network-induced delay and data dropout are as shown in Figures 3 and 4, respectively. For this case, the

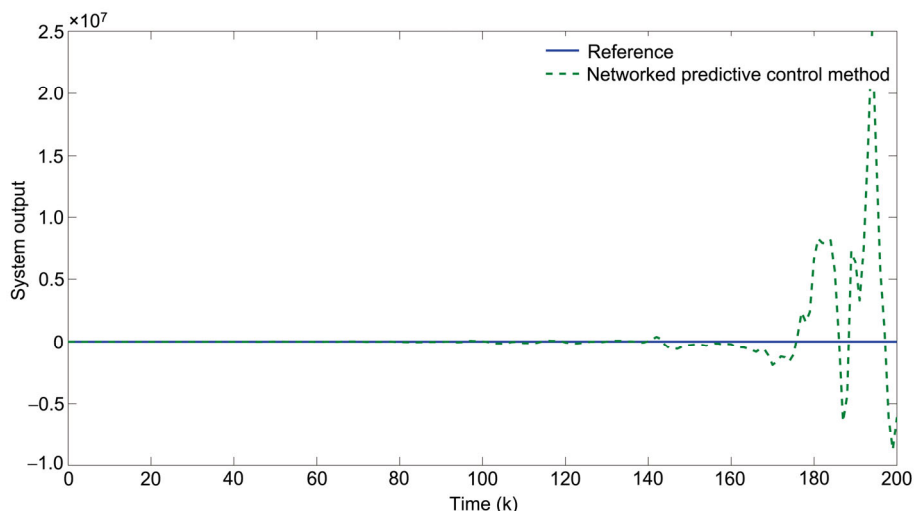


Figure 6 Response of the closed-loop system.

Table 1 Eigenvalues of $A+BK$ and $A^{\kappa-1}(A-LC)$

Matrices	Eigenvalues
$A+BK$	0.6, 0.7
$A-LC$	0.5, 0.2
$A(A-LC)$	$0.1555+0.2754i, 0.1555-0.2754i$
$A^2(A-LC)$	$-0.0367+0.3141i, -0.0367-0.3141i$
$A^3(A-LC)$	$-0.2293+0.2177i, -0.2293-0.2177i$
$A^4(A-LC)$	$-0.7105, -0.1408$

Table 2 Eigenvalues of $A+BK$ and $A^{\kappa-1}(A-LC)$

Matrices	Eigenvalues
$A+BK$	0.6, 0.7
$A-LC$	-0.5, 0.2
$A(A-LC)$	-1.5684, 0.0638
$A^2(A-LC)$	-2.7690, 0.0638
$A^3(A-LC)$	-4.0284, 0.0248
$A^4(A-LC)$	-5.3555, 0.0187

eigenvalues of $A+BK$ and $A^{\kappa-1}(A-LC)$, for $1 \leq \kappa \leq 5$ are listed in the Table 2. It is easy to see that eigenvalues of $A^{\kappa-1}(A-LC)$, $2 \leq \kappa \leq 5$ are all outside unit circle. Let $x(0) = [0 \ 0]^T$ and $\hat{x}(0) = [-1 \ 0]^T$, simulation results are given in Figure 6. It can be seen that the closed-loop networked predictive control system is unstable.

5 Conclusion

In this study, the problem of stability of networked predictive control systems has been investigated. A necessary and sufficient stability criterion has been derived using the switched system method. In this criterion, the relationship between stability of the closed-loop system and some parameters (feedback controller gain matrix K , observer gain matrix L , network-induced delay and data dropout) has been

established. From the stability criterion, it also can be seen that the state feedback controller and the state observer can be designed independently. Finally, a numerical example has confirmed the efficiency and correctness of the obtained results.

In this study, only plants with perfect linear models have been considered. There still exist several challenging issues to be further investigated. For example, when the model of the plant contains some uncertainties or the model of the plant is nonlinear, how to find necessary and sufficient stability conditions for the closed-loop networked predictive control systems is under study.

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