

Damage identification method of girder bridges based on finite element model updating and modal strain energy

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A timely and accurate damage identification for bridge structures is essential to prevent sudden failures/collapses and other catastrophic accidents. Based on response surface model (RSM) updating and element modal strain energy (EMSE) damage index, this paper proposes a novel damage identification method for girder bridge structures. The effectiveness of the proposed damage identification method is investigated using experiments on four simply supported steel beams. With Xiabaishi Bridge, a prestressed continuous rigid frame bridge with large span, as the engineering background, the proposed damage identification method is validated by using numerical simulation to generate different bearing damage scenarios. Finally, the efficiency of the method is justified by considering its application to identifying cracking damage for a real continuous beam bridge called Xinyihe Bridge. It is concluded that the EMSE damage index is sensitive to the cracking damage and the bearing damage. The locations and levels of multiple cracking damages and bearing damages can be also identified. The results illuminate a great potential of the proposed method in identifying damages of real bridge structures.

damage identification, response surface model, finite element model updating, element modal strain energy damage index, girder bridge

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1 Introduction

The technique of damage identification based on model updating has developed fast in recent years [1–5]. For example, Kaneva et al. [6] proposed a damage detective and location method based on finite element (FE) model updating technique, and the results indicated that this method could identify structural small changes, as well as the changes' locations. Weng et al. [2] combined the structural system identification method and the FE model updating technique to study the earthquake simulation shaking table

experiments for a 1:4 scale six-storey steel frame structure and a two-storey concrete frame structural model; the results showed that this method could identify not only the damage locations, but also the damage levels. Weber and Paultre [7] conducted ambient vibration testing and FE model updating for a three-dimensional truss tower model in the laboratory; by combining model linearization and regularization technique, as well as considering numerous simulative and model updating details, they further indicated that the tower damage could be identified after the model updating without ignoring the details. Farrar et al. [8,9] summarized several issues about structural health monitoring (SHM) and damage diagnose/prognostic, and stated that

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the damage identification based on model updating was one of the most effective methods to meet the challenge of SHM.

When structural damage occurs, its mechanical properties will change, usually expressed as local structural stiffness degradation [10]. For example, the modal strain energy (MSE) may change in the damaged area and thus MSE can be applied to early damage identification. Doebbling et al. [11] and Stubbs and Kim [12] applied MSE damage index to numerical simulation of the continuous girder damage identification. Shi et al. [13] proposed the MSE change rate in the damaged area as a damage index. Heam and Testa [14] indicated that the ratio of the structural element modal strain energy (EMSE) to the total kinetic energy was an inherent property of structural eigenvalues. Previous studies indicated that MSE is very sensitive to structural damage location, and therefore suitable for structural damage identification [13,15]. However, these existing studies focused almost on either numerical modeling or simulation and there is relatively scarce discussion on the application of the EMSE index to real-world structural damage identification. To apply the EMSE damage index, we need to establish the FE models both before and after damage, which is relatively easily implementable for numerical simulation. Unfortunately, it is more difficult to tackle the actual bridge structures since the corresponding EMSE index-based damage identification requires a relevantly accurate FE model as a prerequisite.

The basic idea of response surface model (RSM) method is to build an input-output model by utilizing a small number of data sets even for large-scale structures. This method was firstly proposed by Box and Wilson [16] and has found diverse applications in model updating and damage identification of structures [17]. For example, Ren et al. [18,19] investigated FE model updating by both dynamic and static RSM methods; Fang and Perera [20] employed D-optimum design and first-order RSM to predict dynamic response and damage identification for both intact and damaged systems. Furthermore, the proposed method was confirmed by numerical simulation example calculation, model experiment of a reinforced concrete frame, and I-40 Bridge testing results. With Xiabaishi Bridge, a large span continuous rigid frame bridge, as an engineering background and with the data its SHM system as a basis, Zong et al. [21] combined the Center Composite Design and third-order RSM to complete the FE model updating. Empirical studies indicated that the RSM-based FE model updating can achieve high accuracy.

In this paper, a structural damage identification method is proposed by combining the third-order RSM-based FE model updating and the EMSE damage index. The feasibility and effectiveness of the proposed method are verified using physical model testing, numerical simulation and actual bridge analysis.

2 Damage identification method based on RSM updating and EMSE index

2.1 EMSE damage index

The structural local damage can be expressed by stiffness decreasing in principle. It will inevitably cause changes in dynamic characteristic, e.g., the structural natural frequency and vibration mode. For beam structures, the MSE of the j th element at the i th order mode can be expressed by eqs. (1) and (2) [14].

$$MSE_{ij} = \frac{1}{2} \int_j (EI)_j (\phi_i''(x))^2 dx, \quad (1)$$

$$MSE_{ij}' = \frac{1}{2} \int_j (EI)_j' [(\phi_i''(x))']^2 dx, \quad (2)$$

where i is the order, MSE_{ij} and MSE_{ij}' are the MSE of the j th element at the i th order mode before and after damage, respectively. We also use the notation $(EI)_j$ and $(EI)_j'$ to denote the j th element's stiffness before and after damage, respectively. Moreover, by $[(\phi_i''(x))]$ and $[(\phi_i''(x))']$ we mean the mode of vibration before and after damage, respectively. Thus, the damage index β_{ij} which is of the j th element at the i th order mode can be expressed by

$$\beta_{ij} = \frac{|MSE_{ij} - MSE_{ij}'|}{MSE_{ij}}. \quad (3)$$

It had been proved that the EMSE of the first four order modal shapes is most sensitive to damage [22]. In order to adequately enhance the sensitivity of the structural damage index, a total damage index is defined as the algebraic sum of the first four EMSE damage indices. The equation is shown as follows:

$$\beta_j = \beta_{1j} + \beta_{2j} + \beta_{3j} + \beta_{4j}, \quad (4)$$

where β_j , β_{1j} , β_{2j} , β_{3j} and β_{4j} are the total damage index and the first four mode order damage indices (index 1 to index 4 for short) of the j th element.

2.2 Third-order RSM and model updating

The third-order polynomial function shown as eq. (5) is adopted to establish RSM in this paper.

$$\begin{aligned} \hat{y} = & \varphi_0 + \sum_{i=1}^m \varphi_i x_i + \sum_{i=1}^m \varphi_{ii} x_i^2 + \sum_{i=1}^m \varphi_{iii} x_i^3 + \sum_{i=1}^m \sum_{\substack{j \\ (i < j)}}^m \varphi_{ij} x_i x_j \\ & + \sum_{i=1}^m \sum_{\substack{j \\ (i \neq j)}}^m \varphi_{ij} (x_i)^2 x_j + \sum_{i=1}^m \sum_{\substack{j \\ (i < j < k)}}^m \sum_k \varphi_{ijk} x_i x_j x_k + \varepsilon, \end{aligned} \quad (5)$$

where \hat{y} is the response characteristic, the x_i ($i=1,2,\dots,k$)

is the selected design parameters for variance analysis, and $x_i \in [x_i^l, x_i^u]$, ($i \in (1, k)$) with x_i^l and x_i^u the upper limit and lower limit of x_i respectively; $x_i x_j$ and $x_i x_j x_k$ denote the interaction effects among the i, j and k ; ε is the error item and $\varphi_0, \varphi_i, \varphi_{ii}, \varphi_{iii}, \varphi_{ij}, \varphi_{ijj}, \varphi_{ijk}$ are regressive coefficients of corresponding parameters respectively, determined by the least-squares estimation.

The procedure to build polynomial RSM method can be summarized as Figure 1. In the process of the FE model updating based on high-order (third-order) RSM method, the D-optimum design is employed to design experiments and to obtain sample points [23]. The detailed RSM-based updating process can be seen in refs. [24,15].

2.3 Damage identification method based on RSM updating and EMSE damage index

Using the RSM-based updating method, an accurate and effective FE model of both the intact and damaged actual structures can be constructed. The EMSE of the undamaged and damaged structures can also be calculated by the FE model. Furthermore, the element damage index can be obtained. Consequently, the structural damage locations and levels can be confirmed. Structural damage identification process based on the RSM updating technique and the EMSE damage index is shown in Figure 2.

3 Damage identification of physical models

3.1 Experimental models and FE model updating

Four simply supported steel *H*-section beams with different damage locations were made in laboratory. The beam numbers, sizes and damage locations are illustrated in Figure 3.

Each damage gap is 0.1 m in width and 0.05 m in height.

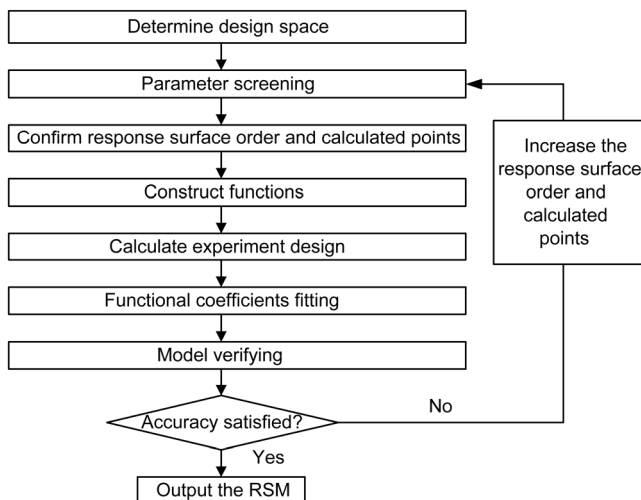


Figure 1 Flowchart of model updating based on polynomial RSM.

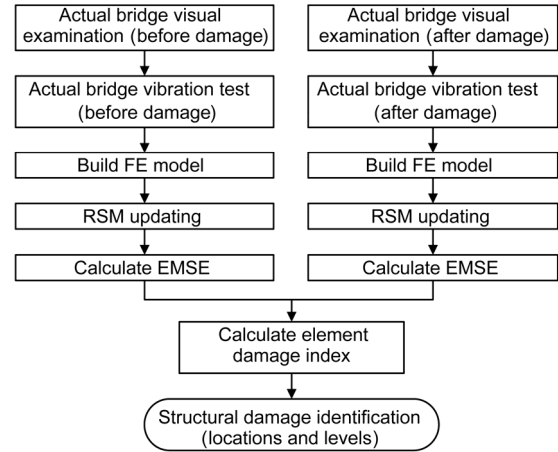


Figure 2 Flowchart of damage identification based on RSM model updating and EMSE index.

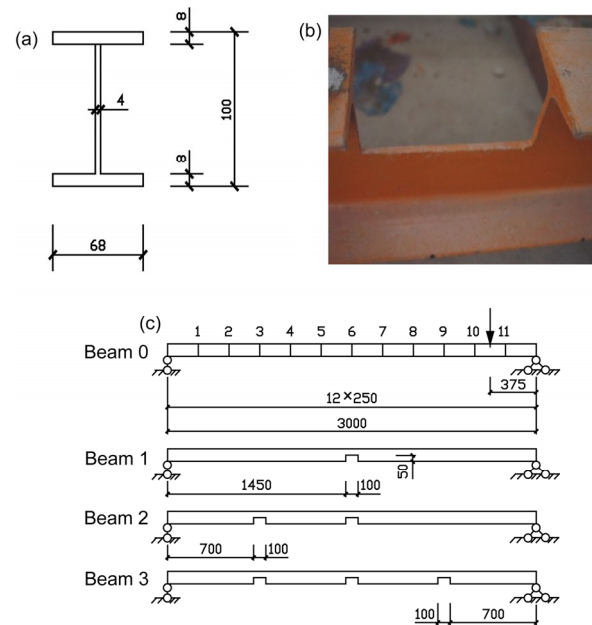


Figure 3 (Color online) Measurement and damage details of the steel beam models. (a) Transverse cross section (unit: mm); (b) damage gap; (c) damage location of the steel beam (unit: mm).

The ambient vibration test is shown in Figure 4, with the sampling frequency 600 Hz and sampling time 20 min in each mode. Initial FE models were developed for the physical models as a set of beam elements. Each FE model has



Figure 4 (Color online) Ambient vibration test of the steel beam model.

60 elements and 61 nodes and Figure 5 exhibits an example for an FE model of Beam 3. The initial material properties are modulus of elasticity $E=2.1 \times 10^5$ MPa, density $\rho=7800$ kg/m³.

The first three order natural frequencies are selected as response characteristics. Damage parameters are chosen as I_1, I_2, I_3 , which are cross sectional moments of inertia of the damaged locations at 1/4, 1/2, and 3/4 of the beam span, respectively. The third-order polynomial RSM in eq. (5) and D-optimal method are employed for experimental design. The parameter values after model updating are listed in Table 1. Afterwards, comparison between the measured and updated frequencies, as well as the MAC values (see in Table 2), is obtained by FE model calculation with the updated parameters replacing the nominal ones. It can be seen from Table 2 that the updated parameters retain their physical meaning, and the updated frequencies are relatively consistent with the measured ones with a maximum of 6.3% error and a minimum of 90% MAC values. Results also indicate that the RSM based FE model updating can establish accurate and effective FE model for both the undamaged and damaged structures with different damage locations.

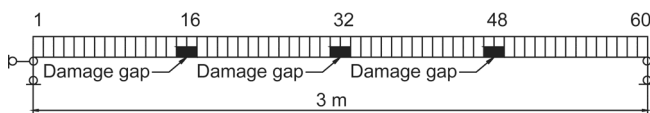


Figure 5 Finite element model of Beam 3.

3.2 Crack damage identification results

Figures 6–8 illustrate the behavior of the first four EMSE damage indices and their summation called total index, which are calculated according to eqs. (1)–(4), respectively. Figure 6 indicates that the EMSE damage index 4 has good identification effect for single crack damage in Beam 1, for which the damaged EMSE changes with rate larger than 900%, and the undamaged EMSE changes with rate approximately to 50%. The damage total index of EMSE attains the maximum change in the damage location 1/2 of the beam, where the rate of change reaches at 2700%. Meanwhile, the rates of change at the locations 1/4 and 3/4 of the beam get large interferences and the values also reach more than 1500%. Other elements' reactions are relevantly stable. It can be concluded that the damage indices of different orders of ESME may have different sensitivities.

Similar phenomena also occur in Figures 7 and 8. The damage index 4 and the damage total index can determine the multiple crack damage locations well, and the damage total index is more sensitive to the changes of EMSE. Combined with the FE model updating, the proposed damage indices can not only detect the damaged location, but also identify the degradation of element stiffness in the damaged location d location. Specifically, from Table 1 we can see that the maximum error of the degradation of element stiffness is less than 12%. From the aforementioned illustration, it is apparent that the proposed damage identification method based on the RSM updating and EMSE is effective in localizing single and multiple damages.

Table 1 Comparison between nominal and updated parameters of three damaged beams (unit: $\times 10^{-6} \text{ m}^4$)

Parameter	Beam 1			Beam 2			Beam 3		
	I_1	I_2	I_3	I_1	I_2	I_3	I_1	I_2	I_3
Nominal value	2.23	0.105	2.23	0.105	0.105	2.23	0.105	0.105	0.105
Updated value	2.092	0.117	2.130	0.121	0.114	2.041	0.093	0.117	0.113
Updating rate (%)	-6.1	11.4	-4.5	15.2	8.6	-8.5	-11.4	11.4	7.6

Table 2 Comparison between the measured and updated frequencies

Beam number	Modal order	Measured frequency (Hz)	Updated frequency (Hz)	Relevant error (%)	MAC (%)
Beam 0	1st	33.617	32.779	-2.5	93.1
	2nd	135.031	130.78	-3.1	91.4
	3rd	293.029	293.02	0	95.5
Beam 1	1st	20.713	22.018	6.3	95.1
	2nd	131.019	130.04	-0.7	93.4
	3rd	231.180	232.69	-0.7	90.9
Beam 2	1st	18.903	19.455	2.9	96.7
	2nd	85.941	90.666	5.5	94.3
	3rd	202.541	203.07	0.3	90.8
Beam 3	1st	17.017	17.288	1.6	94.7
	2nd	66.693	67.135	0.7	94.9
	3rd	148.331	149.68	0.9	90.2

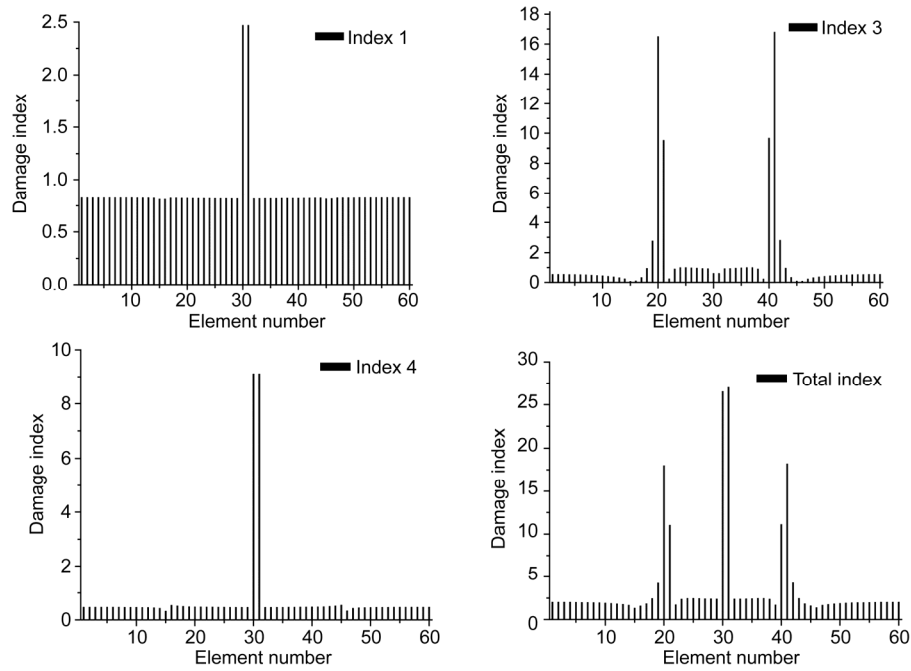


Figure 6 Damage indices of Beam 1.

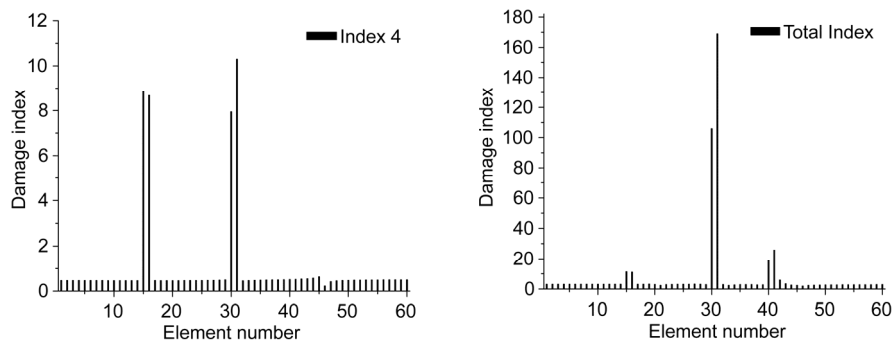


Figure 7 Damage indices of Beam 2.

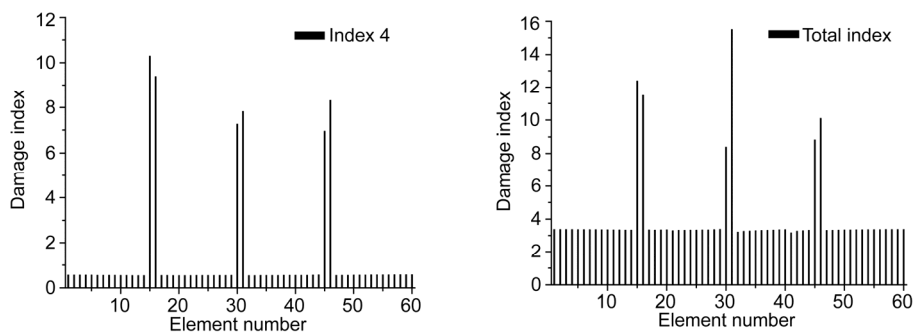


Figure 8 Damage indices of Beam 3.

4 Bearing damage identification of Xiabaishi Bridge

4.1 FE model updating of Xiabaishi Bridge

Xiabaishi Bridge (Figure 9), located in Fujian Province of

China, is a prestressed continuous rigid frame bridge with spans 145+2×260+145 m. The SHM system of the bridge was installed in 2007 and has worked very well so far [25]. The initial three-dimensional FE model of Xiabaishi Bridge was established based on ANSYS platform. The concrete box girders (including flange, web, and diaphragms) were



Figure 9 (Color online) Xiabaishi Bridge.

modeled using Solid45 elements, the reinforcements were modeled by Link8 elements and the pavement concrete was modeled by the concentrated mass element (Mass 21). Bridge bearings were modeled by a set of Combin14 spring elements to connect the superstructure, the piers and the platform. The prestress effects of internal prestressing tendons on vibration frequencies were neglected [17]. The initial material characteristics are listed as follows: the main box girder is of C60 concrete strength grade with the initial elastic modulus 3.65×10^4 MPa; the main pier is of C50 concrete strength grade with the initial elastic modulus 3.50×10^4 MPa; the concrete density of the main bridge is 2450 kg/m^3 , and the Poisson ratio is 0.167. The FE model has 29250 elements and 45975 nodes in total.

The first six vertical (V_1 to V_6), first two transverse (T_1 and T_2) and first longitudinal (L_1) modal frequencies were employed as response values. The updated parameters are listed in Table 3, where R_{v1} , R_{t1} and R_{l1} are vertical, transverse and longitudinal spring stiffness of the support bearings for both ends of the main bridge, respectively, and R_{v2} , R_{t2} and R_{l2} are vertical, transverse and longitudinal spring stiffness of the bridge, respectively. The multiple of the initial concrete elastic module (3.65×10^4 MPa) is symbolically represented by N . The particular updating process of Xiabaishi Bridge using high-order RSM can be found in ref. [26]. Table 4 lists the comparison of the measured frequencies with those from the updated FE model of Xiabaishi Bridge. The relative errors after updating are all less than 3.7%, which illustrate that the updated FE model can relatively reflect the actual condition of Xiabaishi Bridge in the

Table 3 Parameter ranges and updated values of Xiabaishi Bridge FE model

Parameter	Maximum value	Minimum value	Updated value
R_{v1}	24	4	15.455
R_{v2}	32	4	25.19
R_{t1}	15	1	4.449
R_{t2}	15	0.05	1.202
R_{l1}	80	8	31.997
R_{l2}	20	1	3.582
N	1.25	0.85	1.244

Note: The unit of R_{v1} , R_{v2} , R_{t1} is $\times 10^7$ N/m, the unit of R_{t2} , R_{l1} , R_{l2} is $\times 10^6$ N/m.

design space of the parameters.

4.2 Bearing damage identification of Xiabaishi Bridge

The bearing damages of Xiabaishi Bridge were simulated according to the updated FE models. For the complicated structural components and a large number of model elements, the bearing damage conditions were subdivided as follows. Condition 1: R_{t1} and R_{t2} are both reduced by 1%. Condition 2: R_{t1} and R_{t2} are both reduced by 3%. Condition 3: R_{t1} and R_{t2} are both reduced by 5%. Condition 4: R_{t1} and R_{t2} are both reduced by 10%. Condition 5: R_{t1} and R_{t2} are both reduced by 20%. Condition 6: R_{t1} and R_{t2} are both reduced by 20%, and R_{v1} and R_{v2} are both reduced by 10%.

After simulating the damage, each order frequency of the bridge model was substituted into RSM to calculate the updated parameters based on Goal Attainment Method in MATLAB 10.0. The results are listed in Table 5. Thereby, the FE models under different damage conditions of Xiabaishi Bridge are established. Each order EMSE damage index of each damage condition is calculated using eqs. (1)–(4) through the updated FE models of Xiabaishi Bridge under different bearing damages. For example, results for condition 1 and condition 6 are respectively illustrated in Figures 10 and 11.

It can be seen from Figure 10 that the change rate of each order EMSE damage index at the damaged elements is much larger than that of other elements for tiny and single

Table 4 Comparison between the measured and updated model frequencies of Xiabaishi Bridge

Modal order	Measured frequency (Hz)	Initial frequency (Hz)	Updated frequency (Hz)	Error before updating (%)	Updated relative error (%)
T_1	0.418	0.363	0.409	-13.158	-2.153
T_2	0.527	0.483	0.526	-8.349	-0.188
V_1	0.659	0.573	0.658	-13.050	-0.146
V_2	0.813	0.743	0.838	-8.610	3.114
V_3	1.261	1.1	1.249	-12.768	-0.967
V_4	1.404	1.22	1.366	-13.105	-2.728
V_5	1.675	1.456	1.615	-13.075	-3.612
V_6	1.892	1.74	1.946	-8.034	2.854
L_1	1.684	1.813	1.714	7.660	1.793

Table 5 Comparison between the actual and updated parameter values

Damage condition		R_{v1}	R_{v2}	R_{h1}	R_{h2}	R_{h1}	R_{h2}	N
Intact	Actual value	15.455	25.19	4.449	1.202	31.997	3.582	1.244
	Updated value	15.455	25.19	4.449	1.202	31.678	3.582	1.244
1	Updated value	15.347	25.14	4.448	1.2	31.361	3.579	1.243
	Error (%)	0.72	0.21	0.02	0.17	1.01	0.08	0.08
2	Actual value	15.455	25.19	4.449	1.202	31.398	3.582	1.244
	Updated value	15.447	25.21	4.449	1.204	31.397	3.581	1.246
	Error (%)	0.05	0.08	0	0.17	0	0.08	0.16
3	Actual value	15.455	25.19	4.449	1.202	30.397	3.582	1.244
	Updated value	15.456	25.208	4.437	1.202	30.367	3.581	1.244
	Error (%)	0	0.07	0.03	0	0.1	0.02	0
4	Actual value	15.455	25.19	4.449	1.202	28.798	3.582	1.244
	Updated value	15.444	25.172	4.45	1.202	28.789	3.582	1.244
	Error (%)	0.07	0.07	0.03	0	0.03	0	0
5	Actual value	15.455	25.19	4.449	1.202	25.597	3.582	1.244
	Updated value	15.347	25.16	4.45	1.202	25.607	3.583	1.243
	Error (%)	0	0.12	0.02	0	0.04	0.02	0.08
6	Actual value	13.941	25.19	4.449	1.202	25.597	3.582	1.244
	Updated value	13.942	25.191	4.449	1.204	25.579	3.579	1.244
	Error (%)	0.01	0	0	0.13	0.07	0.06	0

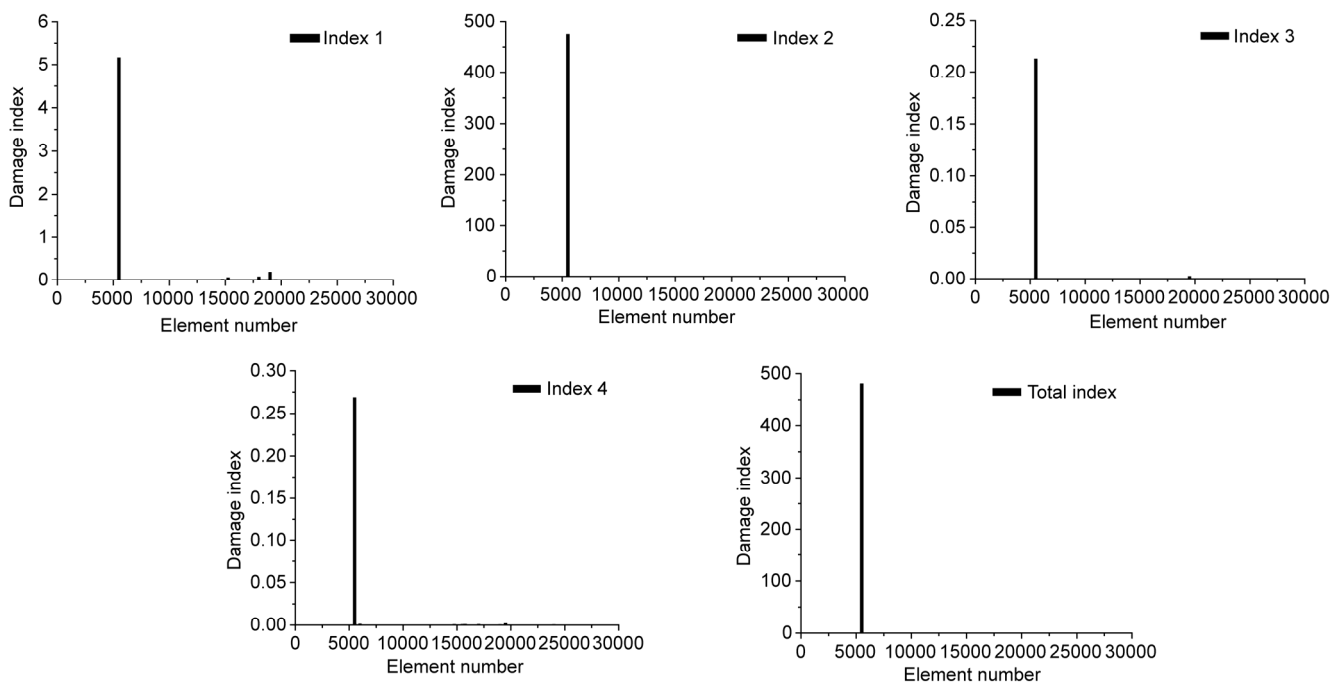


Figure 10 Damage indices of condition 1 under bearing damage.

bearing damage condition. However, the changes of EMSE at undamaged locations are very close to 0. Although the EMSE damage indices in different orders change with different rates, the damage location can be effectively identified. Both EMSE damage index 2 and the total index are rather sensitive, changing with rates surpassing 47000%.

For larger damage and multiple bearing damage conditions (see Figure 11), the change rates of damage index 1, index 2, index 3, index 4, and damage total index in the

damage locations are 82%, 100%, 17%, 20% and 220%, respectively.

It can be demonstrated that the damage index 2 and the damage total index are also much more prominent. In the case of multiply damage locations, the EMSE of some undamaged elements has also changed. The biggest change rate of the undamaged element among the total indices reaches 50%. However, the change rates of the EMSE with respect to most undamaged elements are still small and can

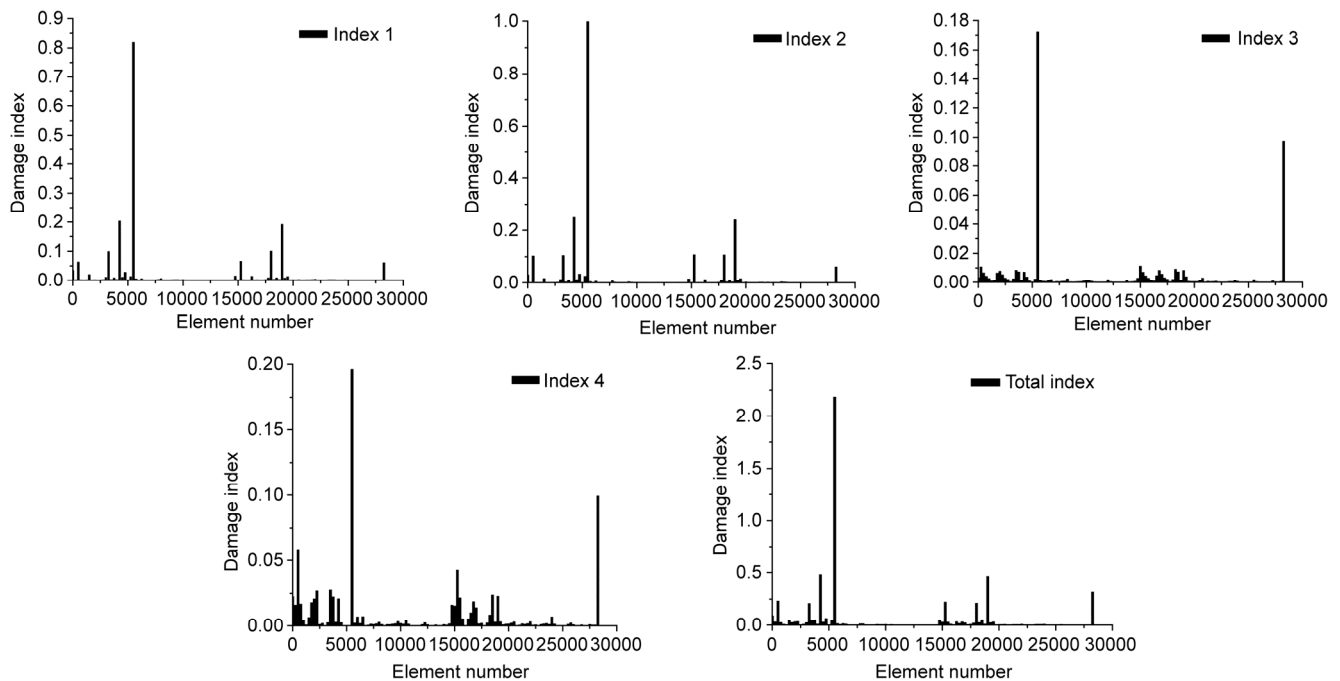


Figure 11 Damage indices of condition 6 under bearing damage.

be very close to 0%.

By comparing condition 1 with condition 6, the change rates of each order EMSE damage index are different. It reveals that each order EMSE damage index has diverse sensitivities to bearing damage, in which the damage index 2 and damage total index are more sensitive to bearing damage.

5 Actual bridge crack damage identification

5.1 Engineering background

Xinyihe Bridge, shown in Figure 12, is located at Jiangsu Province of China as a part of Beijing-Shanghai National Expressway. The overall length of this bridge is 2168.20 m with 12 bays (here a bay represents 6 spans between two expansion joints), namely 72 spans totally. Each span has four partially prestressed concrete box girders, and simply-supported continuous system was employed in the bridge superstructure. The bridge was put into use in 2001. According to the inspection report in 2013, the bridge has different types of cracks. Figure 13 shows several typical cracks in different locations of the box girders.

5.2 FE model updating of Xinyihe Bridge

In July 2013, ambient vibration testing was conducted on the right part of the ninth bay of Xiyihe Bridge. Figure 14 shows the FE model. Firstly, based on the results of appearance inspection and ambient vibration testing, the third-

order RSM model was utilized to update the initial FE model. In order to identify the current working status and the cracks both on the web and bottom, the stiffness was modified to obtain a true FE model which can reflect the real cracking situation. As shown in Table 6, seven parameters were chosen to be updated. Among them, E_0 represents good girder parts, E_1 represents the girder web with more cracks, E_2 represents the girder web with fewer cracks, E_3 represents girder bottom with well-distributed cracks, K_1 is the transversal spring stiffness of joints, K_2 is the transversal spring stiffness of middle supports and K_3 is longitudinal spring stiffness of the joints and supports. In the process of the FE model updating, the first four vertical, first two transversal, and the first longitudinal modal frequencies of the bridge structure were employed as response values. The initial and updated parameters are indicated in Table 7. The comparisons between the measured frequencies from the ambient vibration testing and the calculated frequencies from the updated FE model, as well as their MAC values are listed in Table 8. It can be seen from Table 8 that the relative errors are all less than 5%, and the MAC values are all above 89%. Results justify the use of the updated FE model in damage identification.

5.3 Crack damage identification of Xinyihe Bridge

Based on the aforementioned updated Xinyihe Bridge model, the EMSE of the first four vertical order modes can be obtained by ANSYS software. Therefore, the damage indices $-\beta_{1j}$, β_{2j} , β_{3j} , β_{4j} , β_j of Xinyihe Bridge can also be calculated by comparing the initial model. Since there



Figure 12 (Color online) Overall view of Xinyihe Bridge.

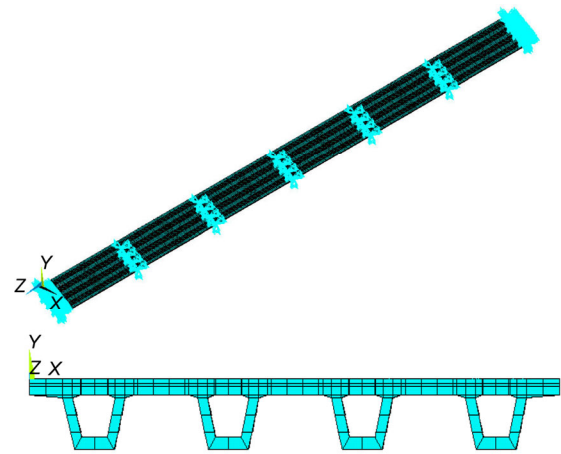


Figure 14 (Color online) Finite element model of Xinyihe Bridge.



Figure 13 (Color online) Typical types of cracks of Xinyihe Bridge. (a) Vertical web crack; (b) diagonal web crack; (c) transverse crack at girder bottom; (d) longitudinal crack at girder bottom; (e) vertical crack at the end of crossgirder; (f) transverse wet joint crack.

Table 6 The updated parameters of Xinyihe Bridge

Parameter	Location and/or damage status
E_0	Good status girders
E_1	Web of 2nd, 3rd, 5th spans (with more cracks)
E_2	Web of 4th span (with fewer cracks)
E_3	Bottom board of 2-3, 2-4, 5-1, 5-2, 5-3 and 5-4 (with well-distributed cracks)
K_1	Joints at both ends
K_2	Middle supports
K_3	Joints and supports

Note: $i-j$ means the j th box girder of the i th span.

are many elements in the two bridge models, here we chose a row of 10 elements in each span of the first box web to calculate the EMSE damage indices. According to the crack distribution sketch for the Xinyihe Bridge's ninth bay (Figure 15), the first and sixth spans both have good web. As a

Table 7 Comparison of parameter values before and after updating

Parameter	$E_0I (\times 10^{11})$ (N m ²)	$E_1I (\times 10^{11})$ (N m ²)	$E_2I (\times 10^{11})$ (N m ²)	$E_3I (\times 10^{11})$ (N m ²)	$K_1 (\times 10^6)$ (N/m ²)	$K_2 (\times 10^6)$ (N/m ²)	$K_3 (\times 10^6)$ (N/m ²)
Initial value	13.62	13.62	13.62	13.62	0.60	0.60	4.0
Updated value	15.71	12.39	13.38	9.51	0.27	0.48	1.74
Change (%)	15.3	-9.0	-1.7	-30.2	-55.0	-20.0	-56.5

Table 8 Comparison between the measured and updated frequencies

Vibration direction	Modal order	Measured frequency (Hz)	Updated frequency (Hz)	Relevant error (%)	MAC (%)
Vertical	1st order	2.870	2.882	0.42	90.55
	2nd order	3.164	3.091	-2.31	89.46
	3rd order	3.670	3.638	-0.87	90.32
	4th order	4.248	4.270	0.52	89.38
Transversal	1st order	0.919	0.874	-4.90	92.48
	2nd order	1.450	1.471	1.45	91.03
Longitudinal	1st order	1.302	1.281	-1.61	91.23

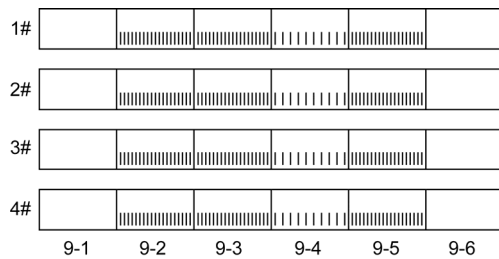


Figure 15 Crack distribution sketch of the ninth bay of Xinyihe Bridge.

comparison, the web of fourth span has slightly more cracks, whereas the second, third and fifth spans have the most cracks.

We now use the actual damage status and field testing data to investigate the sensitivity of EMSE to crack damages. The results are shown in Figure 16. As can be seen from Figure 16, although each order damage index cannot distinguish damage units from the undamaged units well, the damage total index can still reflect the general characteristic of the web cracking damage for the ninth bay in Xinyihe Bridge. Consequently, the proposed damage identification method based on the FE model updating and EMSE can identify damages cases, for which there are large damage areas and small damage levels in a real bridge.

6 Conclusions

In this paper, a damage detection method based on RSM and EMSE is proposed, and indoor beam model experiments, real bridge damage simulation and testing are em-

ployed to verify the proposed method. The following conclusions may be extracted from the analysis.

(1) FE model can be updated by third-order RSM method, with each parameter possessing its own physical significance. Maximum error of the updated frequencies for the simply supported steel beam model is less than 6.3%, while those for Xiabaishi Bridge and Xinyihe Bridge are less than 3.7% and 5%, respectively. The results indicate that the RSM-based updating can be employed to establish relatively accurate FE models for bridge structures.

(2) From the model experiments on four steel beams, it can be concluded that the damage index 4 and the total index are more sensitive to the crack damage, and therefore effective in locating both single and multiple damages. The element stiffness degradation in the damage locations can also be identified by incorporating RSM model updating technique.

(3) Bearing damage simulation analysis of Xiabaishi Bridge reveals that the damage index 2 and the damage total index have better sensitivities to the bearing damage. Without noise interference, the bearing damage locations as well as the levels can be clearly identified based on model updating method and EMSE damage index.

(4) Crack damage identification results on the Xinyihe Bridge suggest that the proposed damage total index can reflect the general characteristics of the cracking damage, and thus identify the damage in large area and small level of a real bridge.

(5) The proposed damage method can efficiently identify damage locations and levels of both cracking and bearing damage. It has promising potential for damage detection application to real engineering structures.

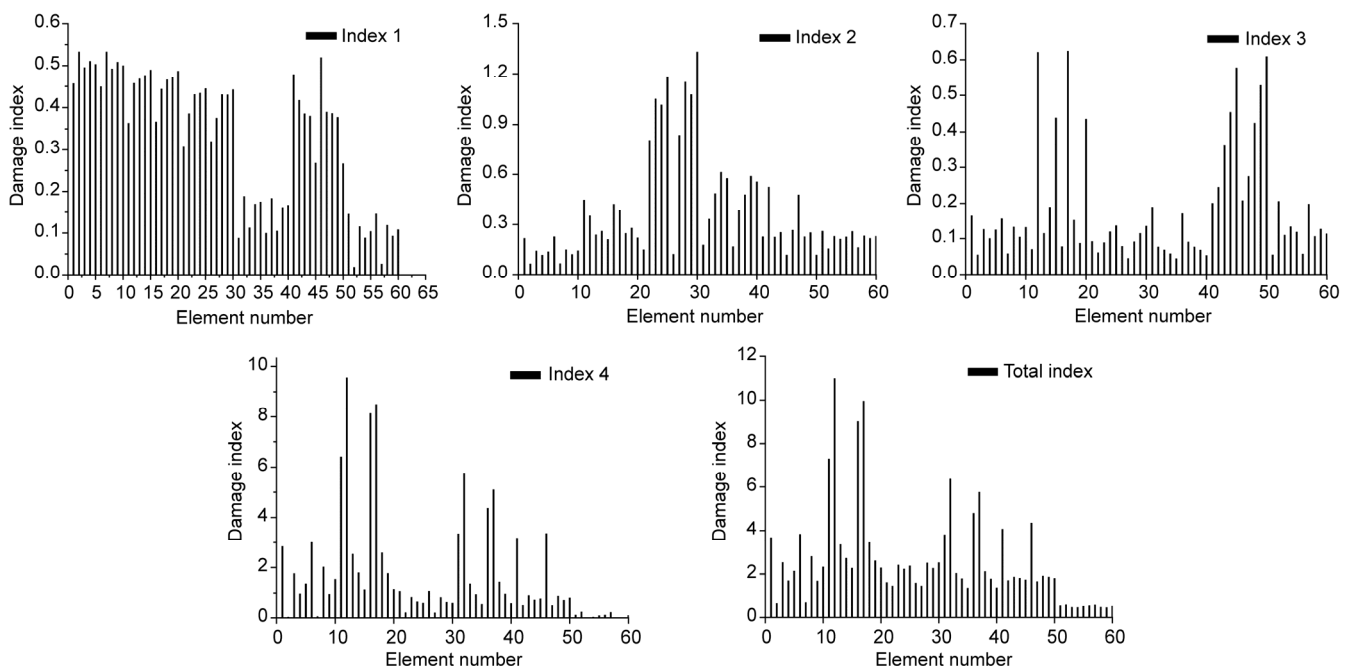


Figure 16 Damage indices of Xinyihe Bridge.

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