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An experimental study of the incipient bed shear stress partition in mobile bed channels filled with emergent rigid vegetation

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This paper investigates the bed shear stress based on the condition of the incipient motion of sediment in a uniform-flow flume covered with emergent rigid vegetation, which is represented by arrays of circular cylinders arranged in a regular pattern. A total of 148 tests are performed to observe the influence of the vegetation density, bed slope, flow depth and sediment size on the bed shear stress. The tests reveal that when the sediment is in incipient motion, the resistances acting on the flow passing the rigid vegetation contain the vegetation resistance and the bed shear stress. This shear stress could be divided into two parts: the grain shear stress and the shear stress caused by sand dunes, which are the deformed bedform with the sediment incipient motion. An empirical relationship between the shear stress of the sand dune and vegetation density, the Froude number, the apparent vegetation layer velocity is developed.

open channel flows, hydraulic radius, incipient motion, bed shear stress, emergent rigid vegetation

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1 Introduction

A variety of aquatic plants grow in natural rivers. Nevertheless, people artificially grow a variety of plants on the beach and shallow flow areas. Riverbed erosion could be effectively resisted by the protection of aquatic plants. The flow velocity in plant areas is reduced due to the increase of bed roughness, which is beneficial to the sediment resuspension, and thus the flow transparency. Many aquatic plants that improve water quality can absorb and even degrade certain poisonous and harmful pollutants. Aquatic plants also provide rich nutrients and habitats for animal communities in river ecological systems. Consequently, plants play an important role in the health of a river ecological system [1,2].

The flow resistance and characteristics of sediment scouring and deposition are changed by the presence of aquatic plants. Many studies [3–7] have quantitatively studied the influence of vegetation on flow resistance. They suggested that aquatic plants have a decisive effect on the flow resistance and proposed the drag coefficient of plant.

Li and Shen [7] theoretically analyzed the relationship between the vegetation drag coefficient and vegetation density, arrangement, average velocity. They found that the plant arrangement affected the plant drag coefficient. For non-submerged plants, less sediment is transported when plants grow in a staggered arrangement. Fathi-Moghadam and Kouwen [4] investigated the resistance of flexible plants using single isolated pine and cedar tree models. They found that the drag coefficient of plant, which is influenced by plant stiffness, exponentially decreased with the square of velocity. Thompson et al. [8,9] showed that rigid vegetation with different shapes affected the drag coefficient. They also studied the relationship between sand resistance and vegetation resistance using isolated vegetation. Ishika-

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wa et al. [5] experimentally indicated that changes in the vegetation density clearly affected the plant drag coefficient. By summarizing the experimental data, Cheng et al. [3] inferred a theoretical formula for the vegetation drag coefficient and postulated that the arrangement of the vegetation did not significantly affect the drag coefficient.

By examining the sediment transport with vegetated flow, many studies [10–13] have found that the bed resistance cannot be ignored, even when the vegetation is dense. Specht [13] showed that the presence of plants facilitated the formation of a sand dune on the bed surface, and this sand dune was steep along the flow direction for certain experiments. Jordanova and James [11] found that the sediment transport had a compact relationship with the sand resistance, which was computed by subtracting the vegetation resistance from the total resistance. Baspist [10] experimentally determined that the grain resistance was clearly reduced with the presence of the submerged flexible vegetation, and the rate of sediment transport increased. Kothyari et al. [12] computed the ratio of the grain resistance to the total resistance over a large range, and they derived a sediment bed load formula using the sand resistance. They postulated that the sand shear stress, rather than the apparent shear stress, better described the rate of sediment transport. With respect to the incipient motion of the sediment, Tang et al. [14] found that the bedfom had deformed before the sediment reached incipient motion. They proposed that sediments moved on the bed surface before being suspended and transported by the overlying flow, which was adopted to describe the incipient motion of sediment in flows with vegetation.

An increasing amount of information reveals that the presence of vegetation distinctly affects the characteristics of bed load movement. However, some researchers believed that bedload movement in flows with vegetation was equivalent to the motion without vegetation, and semi-empirical equations for bed load transport rate were derived without considering the influence of the sediment incipient motion [7,11,12]. In this study, the bed resistance with vegetation was investigated using rigid circular cylinders to represent plant stems, and the differences in the bed resistance and critical shear stress of incipient motion of the sediment with and without the presence of vegetation were analyzed. The factors that affect the bed resistance in flows with emergent rigid vegetation, including the vegetation density, flow depth and grain sizes were also investigated.

2 Vegetation-related hydraulic radius r_v

The hydraulic diameter is a characteristic length commonly used in open channels. It is defined as the ratio of the cross-sectional area to the wetted perimeter:

$$r_0 = A_1/X\ , \tag{1}$$

where A_1 is the cross-sectional area and X is the wetted perimeter (Figure 1). For the channels or flumes with vegetation, this definition may be no more available. In this section, the vegetation-related hydraulic radius would be discussed.

First of all, we extend the definition of the hydraulic diameter from the section to volume. As shown in Figure 2, the unit control volume considered here has two cross sections and the length, L_1. Then the hydraulic diameter of the volume could be expressed as

$$r = V/A_2\ , \tag{2}$$

where V is the control volume and A_2 is the contact area between the flow and the bed surface in the control volume.

If these two cross sections have the identical area, A_1, and the identical wetted perimeter, X, eq. (2) could become

$$r = A_1L_1/XL_1, \tag{3}$$

which is exactly the same expression with eq. (1) when L_1 is removed.

Then, we tried to obtain the detailed expression of the vegetation-related hydraulic radius. The unit control volume was selected for the stress analysis of uniform flow with emergent rigid vegetation (Figure 2). Section 1, section 2 and

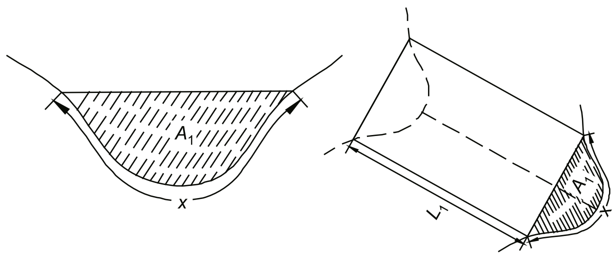

Figure 1 Sketch of hydraulic radius.

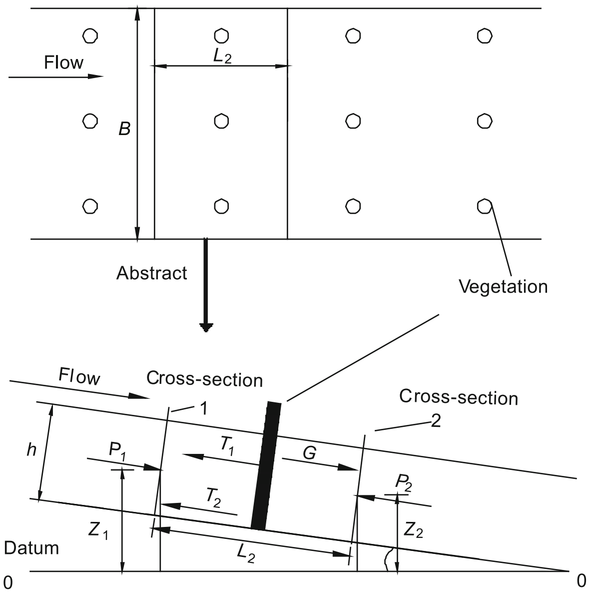

Figure 2 Sketch of the momentum balance.

the bed surface are three control surfaces of the control volume. L_2 is the distance between the centers of the adjacent model plants in the flow direction. B is the channel width. The water depth is represented by h; the vertical distances between the datum plane and the centroids of sections 1 are Z_1 and Z_2, respectively. The dynamic water pressures on the centroids are given by P_1 and P_2. The average shear stress of vegetation resistance is τ_1, and the average shear stress of the control surface is τ_2.

Along the flow direction, there are several external forces acting on the control volume.

1) The hydrodynamic pressures P_1hB and P_2hB act on sections 1 and 2, respectively.

2) The vegetation resistance $T_1 = \tau_1 A'$, where A' is the area that the shear stress of vegetation resistance acts on.

3) The shear force that acts on the surface of control volume is $T_2 = \tau_2 A''$, where A'' is the area of the bed surface.

4) The water gravity along the flow direction is G and can be specifically expressed as follows:

$$G = \rho g \sin\theta \left(BL_2 h - n\frac{\pi}{4}d^2 h \right), \tag{4}$$

where ρ is the water density; g is the gravity acceleration; d is the diameter of vegetation stem; θ is the angle between the flow direction and the datum edge; n is the number of vegetation stems in the control volume.

When the acceleration is zero during uniform flow at equilibrium, the forces can be expressed as follows:

$$P_1hB \quad P_2hB + \rho g \sin\theta \left(BL_2 h - n\frac{\pi}{4}d^2 h \right) \quad \tau_1 A' \quad \tau_2 A'' = 0, \tag{5}$$

where $\sin\theta = (Z_1 - Z_2)/L_2$.

The bed shear stress can be ignored for an even bed surface compared with the vegetation resistance [3,4,7,15,16] (e.g., the smooth side walls and bottom of experimental flume). When both sides of eq. (5) are divided by $\rho g BL_2 h$, the following expression is obtained:

$$\frac{Z_1 + \frac{P_1}{\rho g} - \left(Z_2 + \frac{P_2}{\rho g} \right)}{L_2} = \frac{Z_1 - Z_2}{L_2}\frac{n\pi d^2}{4BL_2} + \frac{\tau_1 A'}{\rho g BL_2 h}. \tag{6}$$

The energy equation from sections 1 and 2 is defined as follows:

$$Z_1 + \frac{P_1}{\rho g} + \frac{\partial_1 v_1^2}{2g} = Z_2 + \frac{P_2}{\rho g} + \frac{\partial_2 v_2^2}{2g} + h_f, \tag{7}$$

where v_1 and v_2 are the corresponding average velocities of sections 1 and 2, respectively. Furthermore, h_f is the head loss caused by the vegetation, which can be defined as the same expression of the frictional head loss, $h_f / L_2 = J$, where J is the energy slope.

For uniform flow, $v_1 = v_2$, and the dynamic pressures of sections 1 and 2 should be equal, which means that $P_1 = P_2$. Eq. (7) could be simplified as

$$Z_1 - Z_2 = JL_2. \tag{8}$$

Substituting this equation into eq. (6), the following expression holds true:

$$J = J\frac{n\pi d^2}{4BL_2} + \frac{\tau_1 A'}{\rho g BL_2 h}. \tag{9}$$

In the flume with rigid plants, the vegetation density, λ, can be expressed as follows [12,14,17]:

$$\lambda = \frac{n\pi d^2}{4BL_2}. \tag{10}$$

The following formula can be derived from eqs. (9) and (10):

$$(1-\lambda)J = \frac{\tau_1 A'}{\rho g BL_2 h}. \tag{11}$$

We assume that the shear stress of vegetation resistance τ_1 equals $\rho g J r_v$ when the bed shear stress is ignored. Then the vegetation-related hydraulic radius, r_v, can be expressed as

$$r_v = \frac{(1-\lambda)BL_2 h}{A'}. \tag{12}$$

The contact area A' is the only unknown quantity for the determination of the hydraulic radius. There are different expressions of the contact area, A', which should be compared to decide which is the best description for the contact area of the control volume with the presence of the rigid vegetation.

Definition 1.

The contact area could be defined as the contact area between the water and the rigid vegetation in the control volume according to the definition of the wetted perimeter. Thus, $A' = n\pi dh$. Substituting this expression into eq. (12), we obtain

$$r_v = \frac{1-\lambda}{4\lambda}d, \tag{13}$$

which yields the same results as eq. (2).

Definition 2.

Cheng and Nguyen [3] defined the contact area as the section of the vegetation area that blocks the water, that is, $A' = ndh$. Substituting this expression into eq. (12), we obtain

$$r_v = \frac{\pi}{4}\frac{1-\lambda}{\lambda}d. \tag{14}$$

eq. (14) agrees with the method proposed by Cheng et al. [3] and yields the same results as eq. (2).

Definitions 1 and 2 are reasonable if and only if the bed shear stress is small and negligible compared with the shear stress caused by the vegetation. Here we will give a definition of the contact area which is also suitable for the condition that both of these two shear stresses are considered.

Definition 3.

For uniform flow in a flume with an uneven bed surface, the bed shear stress cannot be ignored [10–12,18], an equation similar to eq. (6) which is obtained by the force balance is given:

$$\frac{Z_1+\dfrac{P_1}{\rho g}-\left(Z_2+\dfrac{P_2}{\rho g}\right)}{L_2}=\frac{Z_1-Z_2}{L_2}\lambda+\frac{\tau_1 A'}{\rho g B L_2 h}+\frac{\tau_2 A''}{\rho g B L_2 h}. \quad (15)$$

Both of A' and A'' represent the contact areas, but with different meanings. Here we assume that $A'=BL_2$, $A''=(1-\lambda)BL_2$ and the rationality of this assumption would be discussed later.

Substituting eq. (8) into eq. (15), we can obtain

$$J=J\lambda+\frac{\tau_1}{\rho g h}+\frac{(1-\lambda)\tau_2}{\rho g h}, \quad (16)$$

where J is the total energy slope according to the resistance caused by both of the bed surface and the vegetation. p_1 and p_2 are identical and removed in eq. (16).

The bed surface shear stress of the wide and shallow channel could be given as

$$\tau_2=\rho g J_b h, \quad (17)$$

where J_b is the energy slope according to the resistance caused by the bed surface.

Substituting eq. (17) into eq. (16), we can obtain the shear stress caused by the vegetation:

$$\tau_1=(1-\lambda)\rho g(J-J_b)h. \quad (18)$$

The flow energy slope can be expressed as follows with considering both of the resistances caused by the vegetation and bed surface [18]:

$$J=J_v+J_b, \quad (19)$$

where J_v is the energy slope according to the resistance caused by the vegetation. Then eq. (18) could be turned into

$$\tau_1=(1-\lambda)\rho g J_v h. \quad (20)$$

The hydraulic radius can express the vegetation-related shear as

$$\tau_1=\rho g J_v r_v. \quad (21)$$

Comparing eq. (20) with eq. (21), we can easily obtain the vegetation-related hydraulic radius:

$$r_v=(1-\lambda)h, \quad (22)$$

note that this hydraulic radius is available for the vegetated flume or channel if and only if $A'=BL_2$, $A''=(1-\lambda)BL_2$. In fact, the existence of vegetation results in the local head loss, compared with the processing head loss for bed resistance. Herein the local head loss caused by vegetation is divided by the bed surface (BL_2) equally in the flow direction [19,20], which is thought to be the processing head loss combined with bed resistance, because the processing head loss is equal for the uniform flow and vegetated flow is always treat as quasi-uniform flow [5,17,21]. The value of A'' is the contact area between the water and the bed surface.

Based on the eq. (17) and the value of A'', the actual value of bed resistance could be gotten. However, in order to get a constant value of the processing head loss, we also need to transform A'' into A' but the bed resistance is not change. So the bed-related radius is also equal to r_v and the same results are obtained as eq. (2) by using similar method of eq. (21). In fact, eq. (2) expresses the hydraulic radius for flow with vegetation and bedform. For the vegetation-related hydraulic radius, i.e. =bed-related radius which also yields the same results as eq. (2), the r_v is available for the flow with vegetation.

As for the case of the ignorance of bed resistance, substituting $A'=BL_2$ into eq. (12), we can get the hydraulic radius same as the formation of eq. (22).

In order to unite the formation of the hydraulic radius in the vegetated flow, herein we used eq. (22) to avoid the problem of the existence of the bed resistance. Definitions 1 and Definition 2 are not suitable for the case of vegetation with an uneven bed surface.

3 Experimental procedure

The laboratory experiment was carried out in a tilting, rectangular flume that was 12 m long, 0.42 m wide, and 0.7 m deep, with glass sidewalls and a marble bottom. The slope of the flume was adjustable to achieve different uniform flows. The flow discharges were defined by the average reading from an acoustic flow meter with a standard deviation of approximately 0.1%–0.5%. To accurately simulate the geometrical configuration of the vegetation, rigid cylinders with a circular cross-section were used in the laboratory studies and located in the middle of the flume [7,9,22]. The vegetation zone was 6 m long. All the tests were divided into two series: the tests in series A were conducted without a sediment bed; those in series B were conducted with a sediment bed. The *y*-axis was aligned with the main flow direction; the *x*-axis was perpendicular to the flow direction; the *z*-axis was vertical. The origin of the coordinate system was set at the center of the upstream end of the vegetation zone, as shown in Figure 3.

3.1 Series A experiments

Experimental series A was designed to investigate the vegetation drag coefficient, C_d, under different hydraulic conditions determined by the vegetal density, discharge and flow depth. The experiments were conducted in uniform flows and six types of vegetal density, λ, were adopted. The distances between the centers of the adjacent model plants were 4, 5 and 6 cm in the *x*-axis (L_1) and 5 cm and 10 cm along the *y*-axis (L_2). The diameter of one vegetation element was $d = 6$ mm. Three flow depths were tested: 6, 9 and 12 cm. A total of 112 tests were conducted in series A, and the average water temperature was 8°C, as shown in Table 1.

3.2 Series B experiments

Experimental series B was designed to establish the bed shear stress under different hydraulic conditions when the flume was covered by uniform sediment. Herein, the presence of net sediment transport through vegetation was defined as the criterion for the incipient motion condition of sediment flows with vegetation. The tests were performed using two sizes of quartz sand with a density of 2.65 g/cm^3. The two bed sediments showed a uniform in gradation with

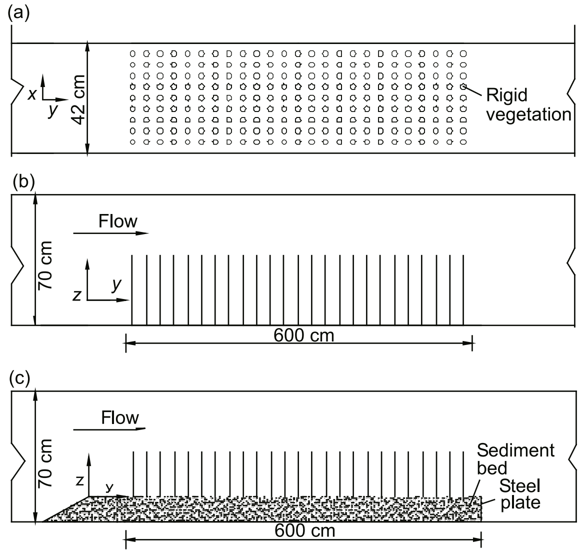

Figure 3 Experiment flume and rigid vegetation. (a) Top view; (b) lateral view in series A; (c) lateral view in series B.

a geometric standard deviation of sediment sizes that was less than 1.2. The sediment bed was saturated with seepage water prior to the start of the test. A total of 36 tests were conducted in series B, and the average water temperature was 16°C, as shown in Table 2. When the flow velocity increased to a certain value, noticeable sediment was notably transported from the scour holes to the outside of the vegetation zone. At this threshold condition, the stage was deemed as the incipient motion velocity for sediment transport in the channels covered with vegetations [14].

The aim of Series A experiments was to obtain the vegetation drag coefficient C_d and finally the function of C_d

Table 1 Experimental condition in series A

Test	$L_1 \times L_2$ (cm×cm)	λ	Flow depth h (cm)	Energy slope J (‰)	Discharge Q (L/S)
a	6×10	0.00471	6	3.1–8.6	3.97–7.85
	6×10	0.00471	9	2.6–7.8	5.76–12.12
	6×10	0.00471	12	2.1–5.7	7.2–13.87
b	5×10	0.00565	6	2.2–9.0	3.56–7.25
	5×10	0.00565	9	2.6–7.9	5.37–11.0
	5×10	0.00565	12	2.2–6.0	6.84–12.87
c	4×10	0.00707	6	3.6–11.0	3.53–7.22
	4×10	0.00707	9	3.2–9.1	5.50–10.56
	4×10	0.00707	12	2.8–7.8	7.0–13.35
d	6×5	0.00942	6	4.6–10.3	3.8–6.32
	6×5	0.00942	9	4.0–7.7	5.75–9.09
	6×5	0.00942	12	3.2–9.3	6.3–13.76
e	5×5	0.0113	6	4.5–10.9	3.12–5.76
	5×5	0.0113	9	4.3–10.4	5.55–9.2
	5×5	0.0113	12	3.9–8.4	6.70–11.28
f	4×5	0.01413	9	5.4–11.8	5.55–8.81
	4×5	0.01413	12	4.8–10.8	7.1–11.7

Table 2 Experimental condition in series B

Test	$L_1 \times L_2$ (cm×cm)	λ	D (cm)	J (‰)	Q (L/s)	h (cm)	u_c (cm/s)
1	4×5	0.01413	0.058	7.2	3.27	6	13.16
2	4×5	0.01413	0.058	7.2	5.03	9	13.50
3	4×5	0.01413	0.058	7.3	6.86	12	13.80
4	5×5	0.0113	0.058	6.6	3.33	6	13.38
5	5×5	0.0113	0.058	6.7	5.21	9	13.95
6	5×5	0.0113	0.058	6.6	6.99	12	14.02
7	6×5	0.00942	0.058	6.0	3.63	6	14.53
8	6×5	0.00942	0.058	5.6	5.51	9	14.71
9	6×5	0.00942	0.058	5.6	7.36	12	14.75
10	4×10	0.00706	0.058	5.4	3.76	6	15.03
11	4×10	0.00706	0.058	5.2	5.76	9	15.34
12	4×10	0.00706	0.058	5.5	7.91	12	15.80
13	5×10	0.00565	0.058	4.6	3.81	6	15.22
14	5×10	0.00565	0.058	4.8	5.96	9	15.85
15	5×10	0.00565	0.058	4.9	8.00	12	15.97
16	6×10	0.00471	0.058	4.6	4.13	6	16.47
17	6×10	0.00471	0.058	4.5	6.28	9	16.68
18	6×10	0.00471	0.058	4.4	8.37	12	16.69
19	4×5	0.01413	0.067	8.2	3.43	6	13.82
20	4×5	0.01413	0.067	7.9	5.31	9	14.25
21	4×5	0.01413	0.067	7.9	7.12	12	14.34
22	5×5	0.0113	0.067	7.0	3.55	6	14.27
23	5×5	0.0113	0.067	7.0	5.38	9	14.41
24	5×5	0.0113	0.067	7.3	7.45	12	14.95
25	6×5	0.00942	0.067	6.6	3.58	6	14.35
26	6×5	0.00942	0.067	6.7	5.56	9	14.86
27	6×5	0.00942	0.067	6.8	7.50	12	15.02
28	4×10	0.00706	0.067	5.4	3.92	6	15.65
29	4×10	0.00706	0.067	5.6	5.92	9	15.77
30	4×10	0.00706	0.067	5.9	8.09	12	16.17
31	5×10	0.00565	0.067	5.0	4.03	6	16.08
32	5×10	0.00565	0.067	5.1	6.09	9	16.21
33	5×10	0.00565	0.067	5.3	8.24	12	16.45
34	6×10	0.00471	0.067	5.0	4.19	6	16.70
35	6×10	0.00471	0.067	4.7	6.36	9	16.89
36	6×10	0.00471	0.067	4.9	8.65	12	17.24

under the condition of the bed shear stress was ignored and the total resistance was equal to the vegetal-related resistance. Based on the value of C_d, the aim of Series B experiments was to obtain bed shear stress and then analyze the composition of bed shear stress on the condition of the incipient motion of sediment.

4 Results and discussion

4.1 Vegetation drag coefficient C_d

Vegetation drag coefficient, C_d, is a useful parameter for quantifying the vegetation resistance. However, many scholars currently believe that the arrangement of plants has some influence on the water blocking of plants [6,7], and the arrangement of plants influences vegetation drag coefficient, C_d. Most papers have studied the C_d of plants in the staggering and random arrangements [4–6]. In this paper, the vegetation drag coefficient, C_d, was investigated for a regular vegetation pattern.

The vegetation drag coefficient, C_d, can be obtained in several ways, such as the direct measurement method [4,6,8,9] and energy slope method [3,15,16], etc. This study used the energy slope method to obtain experimental values of C_d. A corresponding empirical formula was then developed by data fitting. On the other hand, smooth plastic mold plates were used to fix the plants and create smooth bed surface, which reduced the bed surface shear stress, i.e., the value of the bed surface shear stress could be ignored compared with the plant shear stress (series A).

Hence, the "energy slope" method was adopted to obtain the parameter C_d and finally determine the vegetation resistance based on the experimental values [3,15,16].

Compared with the vegetation stress, the bed shear stress can be ignored in this study for flows with protruding vegetation not covered by sediment (series A). A control volume of the unit bed balance in the stream direction yields

$$\tau_w = \tau_v . \tag{23}$$

The parameter τ_w is the stream-wise component of the water weight per unit bed area, and can be expressed as follows [12]:

$$\tau_w = \rho g J h(1-\lambda) , \tag{24}$$

where ρ is the mass density of fluid, g is the acceleration of gravity, J is the energy slope, h is the flow depth and λ is the vegetation density.

The parameter τ_v represents the drag resistance around stems (stem resistance) and can be expressed as follows [12]:

$$\tau_v = \frac{1}{2}\rho C_d N d h u^2 , \tag{25}$$

where C_d is the vegetation drag coefficient, d is the stem diameter and N is the number of stems per unit plant area of the bed. Thus,

$$N = n / BL_2 = 4\lambda / \pi d^2 , \tag{26}$$

where u is the apparent vegetation layer velocity and can be expressed as follows (experiments in flume) [18]:

$$u = \frac{Q}{Bh(1-\lambda)}, \tag{27}$$

where Q is the discharge and B is the channel width.

By substituting eqs. (24), (25), and (26) into eq. (23), the following expression is obtained:

$$C_d = \frac{\pi g J (1-\lambda) d}{2\lambda u^2} . \tag{28}$$

Eq. (28) can be used to obtain the experimental values of C_d. In fact, the value of J in eq. (28) is equal to J_v (eq. (19)). However, the bed shear stress cannot be ignored or obtained directly for flows with vegetation covered in sediment [10–13]. In these flows, obtaining the value of J_v from eq. (28) is not necessary. The values calculated by eq. (28) were only obtained experimentally. Hence, the law of C_d was investigated based on the experimental values of C_d(Series A).

As discussed earlier, previous studies [4–6] proposed a specific form of C_d. We suggest that C_d could be expressed as follows:

$$C_d = M + N / R_v^{\ m} , \tag{29}$$

where R_v is a parameter similar to the Reynolds number, and the forms of M and N are different, as shown in Table 3.

In this study, the vegetation-related hydraulic radius, r_v, is used to express R_v. Thus,

$$R_v = \frac{u r_v}{v} = \frac{(1-\lambda)uh}{v} . \tag{30}$$

To empirically describe the relationship between C_d and R_v, a best-fit function was developed in this study:

$$C_d = \frac{90}{R_v^{0.5}} + 4.5\frac{d}{h} - 0.303\ln\lambda - 0.9 , \tag{31}$$

as shown in Figure 4. The change in the stem diameter of vegetation d was not considered due to the limitation of the experimental material.

Table 3 The form of R_v

Investigator	R_v	M	N	m
Ergun 1952 [16]	uh/v	constant	constant	constant
Tanino et al. 2008 [16]	uh/v	$f(\lambda)$	constant	constant
Cheng 2011 [3]	ur_1/v	$f(r_1)$	constant	constant
Kothyari et al. 2009 [6]	uh/v	$f(R_{ed})$	constant	constant

Note: The parameter v is the kinematic viscosity of fluid, and $r_1 = \frac{\pi}{4}\frac{1-\lambda}{\lambda}D$.

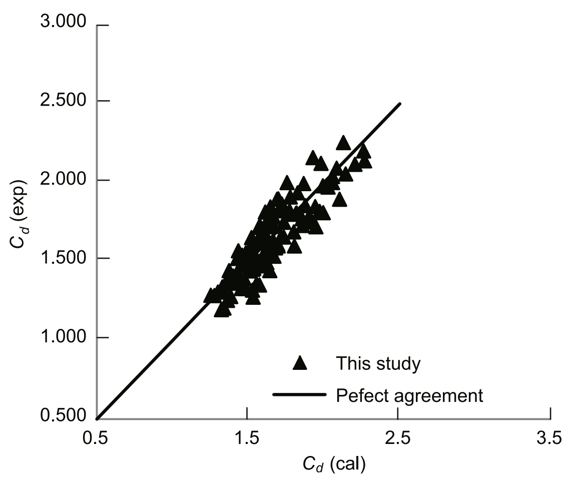

Figure 4 Comparison of experimental values of C_d and calculated values of C_d.

4.2 Bed shear stress

In fact, the bed shear stress cannot be ignored or obtained directly for flows with vegetation and movable beds [10–13]. A control volume of the unit bed balance in the stream direction yields [12,18]

$$\tau_w = \tau_v + \tau_b, \tag{32}$$

where τ_b is the bed resistance, also termed the bed shear stress.

For sediments in incipient motion, eqs. (24) and (25) can be substituted into eq. (32):

$$\tau_{bc} = \rho gJh(1-\lambda) - \frac{1}{2}\rho C_d Nhdu_c^2, \tag{33}$$

where the subscript c represents the situation of incipient motion. Eq. (31) was used herein to evaluate C_d [12], and the other parameters of eq. (33) could be measured in series B to determine τ_{bc}.

This study defined the incipient motion of the sediment with protruding vegetation as a continuous measurable sediment transport out of the vegetation zone when the equilibrium bedform had been reached [14]. The bedform was also found to be deformed when the sediment reached incipient motion. This phenomenon was attributed to the vegetation, as it was not observed in cases without vegetation.

Kothyari et al. [12] experimentally suggested that the ratio of the bed shear stress to vegetation resistance, τ_b/τ_v, was in the range of 0.1 to 0.7 during the bed load transport studies. They postulated that τ_b could not be negligible when compared with the vegetation resistance, τ_v. In this study, the value of τ_{bc}/τ_{vc} was computed using eqs. (25) and (33), as shown in Figure 5. The computed values for τ_{bc}/τ_{vc} ranged from 0.35 to 0.9, which indicates that τ_{bc} was 0.35–0.9 times the value of τ_{vc} in the vegetated flows. Thus, τ_{bc} was also not negligible in this study during incipient motion.

The grain shear stress, τ_{gc}, (also called the grain resistance) was considered equal to the critical shear stress τ_c without vegetation during the incipient motion of sediment with vegetation. The value of τ_c was computed using Shields' method, which is based on the Shields' Curve. The ratio of τ_{bc}/τ_{gc} was found to be in the range of 2–14, as shown in Figure 6, indicating that τ_{bc} was much larger than τ_{gc}.

Many studies have suggested that the bed shear stress is equal to the grain shear stress and that $\tau_{bc} = \tau_{gc} = \tau_c$ during the incipient motion of the sediment [7,11,12]. We assumed that τ_{bc} contained two parts, τ_{gc} and the shear stress of the sand dune τ_{Dc}. The latter is also called the resistance of the sand dune and arises from bed surface deformation. The following can be assumed in accordance with the principle of resistance superposition:

$$\tau_{bc} = \tau_{gc} + \tau_{Dc}. \tag{34}$$

The shear stress of the sand dune, τ_{Dc}, was not negligible based on the ratio of τ_{bc}/τ_{gc}.

4.3 Shear stress of the sand dune, τ_{Dc}

Compared with the incipient motion of sediments without vegetation, the bedform was deformed during incipient motion with protruding vegetation. The shear stress of sand dune, τ_{Dc} (10^{-5} N/cm^2), could be computed using eq. (34).

The relationship between τ_{Dc} and u_c^2 is shown in Figure 7 with different grain sizes. As expected, a linear relationship between τ_{Dc} and u_c^2 was generally obtained for rigid cylinders. The slope varied with the flow depth.

Figure 8 shows the relationship between τ_{Dc} and the vegetation density, λ, which indicated that τ_{Dc} increased

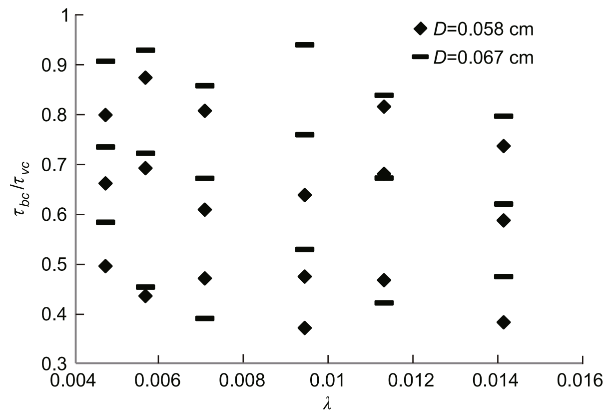

Figure 5 Ratio of τ_{bc}/τ_{vc}.

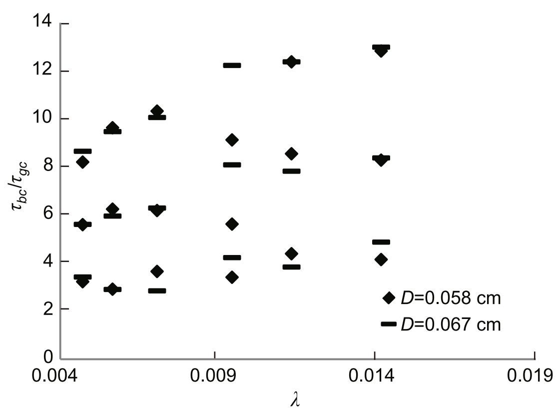

Figure 6 Ratio of τ_{bc}/τ_{gc}.

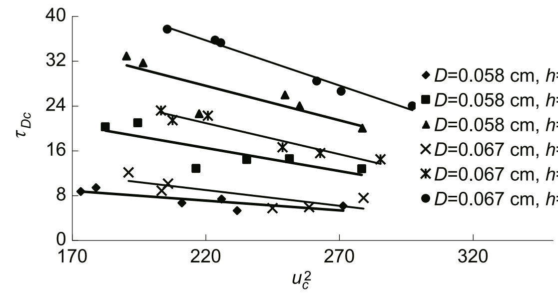

Figure 7 Variation of τ_{Dc} with u_c^2.

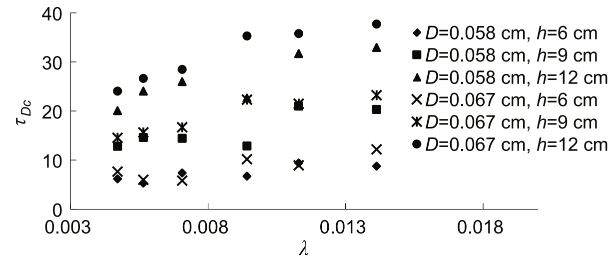

Figure 8 Variation of τ_{Dc} with λ.

with λ during the incipient motion of vegetation, albeit at a small rate.

Figure 9 shows how the Froude number, F_r $\left(F_r = \dfrac{u}{\sqrt{r_v g}}\right)$, affected the value of τ_{Dc}. Specifically, τ_{Dc} rapidly decreased with F_r.

In this study, τ_{Dc} was thought to be a function of the following equation:

$$\tau_{Dc} = f(u_c^2, h, \lambda, D, \rho). \tag{35}$$

This expression can be re-written by transforming all the parameters to their dimensionless form:

$$\tau_{Dc} / (\rho u_c^2) = f\left(\lambda, F_r, \frac{D}{d}, \frac{r_v}{d}\right), \tag{36}$$

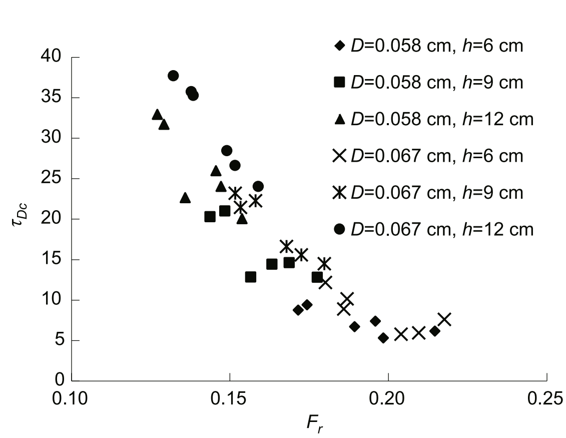

Figure 9 Variation of τ_{Dc} with F_r.

when the experimental data are substituted into eq. (16), the following relationship is obtained:

$$\frac{\tau_{Dc}}{\rho u_c^2} = 0.068 \frac{(\sqrt{r_v D} / d)^{2.6}}{F_r \sqrt{\dfrac{\pi/4 - \lambda}{\lambda}}}. \tag{37}$$

This equation could also be expressed as

$$\tau_{Dc} = 0.068 \rho u_c^2 \frac{(\sqrt{r_v D} / d)^{2.6}}{F_{rv} \sqrt{\dfrac{\pi/4 - \lambda}{\lambda}}}. \tag{38}$$

The maximum value of λ was $\pi/4$. Figure 10 shows the relationship between the observed τ_{Dc} values and those computed using eq. (38).

5 Conclusions

Experimental observations of the incipient motion of sediment in a uniform-flow flume with protruding rigid cylindrical stems were presented. The vegetation drag coefficient, C_d was computed using eq. (31) for a regular pattern of vegetation based on the derivation of the hydraulic radius, r_v. The bed shear stress, τ_{bc}, was obtained for the incipient motion of the sediment, which was not negligible compared with the vegetation resistance, τ_{vc}. This study revealed that τ_{bc} contained two parts: the grain shear stress, τ_{gc}, and the shear stress of the sand dune, τ_{Dc}. The latter arose from the deformation of the bedform during the incipient motion of sediment. Eq. (38) was found to appropriately describe the shear stress of the sand dune, τ_{Dc}, when vegetation was present. This stress was also not negligible compared with the grain shear stress. This study considered values smaller than 0.1, i.e., low-density conditions. Further research is needed to study the vegetation of higher density.

Notation

A_1= area of flow section
A_2= contact area between the control volume and bedform
A' = area that the shear stress of vegetation resistance acts on
A'' = area of the lateral wall and bedform
B = channel width
C_d= vegetation drag coefficient
d = stem diameter
D = sediment size
F_r = Froude number
g = acceleration of gravity
G = water gravity along the flow direction

h = flow depth
h_f = head loss
J = energy slope
J_b = portion of friction attributable to bed shear
J_v = portion of friction attributable to stem drag
L_1 = unit length
L_2 =the spacing along the flow direction
n = number of stems in the control volume
N = number of stems per unit plan area of the bed
P_1, P_2 =dynamic water pressure on centroids
Q = discharge
R_v = parameter which has the similar form of Reynolds number
r = hydraulic radius
r_v = vegetation-related hydraulic radius
T_1 = vegetation resistance
T_2 = shear force acting on the surface of control volume
u = apparent vegetation layer velocity
V = control volume
v_1, v_2 = average velocities of sections 1 and 2, respectively
X = wetted perimeter
Z_1, Z_2 = datum plane
τ_1 = average shear stress of vegetation resistance
τ_2 = average shear stress of control surface
ρ = mass density of fluid
θ = the angle between flow direction and datum edge
λ = vegetation density
τ_w = stream-wise component of the water weight per unit bed area
τ_v = drag resistance around stems (stem resistance)
ν = kinematic viscosity of fluid
τ_b = bed resistance
τ_{gc} = grain shear stress
τ_{Dc} = shear stress of sand dune

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