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Variable sets method for urban flood vulnerability assessment

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On the basis of dialectics basic laws and mathematical theorems of variable sets, this paper proposes a variable sets method for urban flood vulnerability assessment. In this method, the comprehensive relative membership degree of multi-indices is represented by an index relative difference degree, which follows the characteristics of dialectical philosophy and mathematics. According to the quality-quantity exchange theorem, the relative difference degree of two adjacent levels (h and h+1), whose index standard interval values cross the boundaries, equals 0 in the urban flood vulnerability assessment. On the basis of the opposite unity theorem, the sum of relative membership degrees should be equal to 1 when indices lie in the adjacent degrees h and h+1. The variable sets method is proved to be theoretically rigorous and computationally simple. This paper takes 29 cities of Hunan province as an example to assess the urban flood vulnerability, and then compares the results from this newly developed method with the assessment results obtained from the fuzzy comprehensive evaluation and fuzzy set pair analysis methods.

variable sets, urban flood, vulnerability, assessment

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1 Introduction

Urbanization was one of the most impressive problems in the last 20th century. At the beginning of the 21st century, 50% of the world population lives in cities. Cities (especially the large cities), vulnerable to natural hazard, have become a focus for disaster defense in the world. The Geneva strategy, approved by the United Nations (U.K) in 1999, indicated that cities would be an emphasis of global disaster reduction in 21st century [1]. The comprehensive disaster prevention and reduction capability of cities will be an important indicator to measure the cities' macro-function and security defending ability in 21st century [2].

The vulnerability conception was first presented formally by Timmerman [3] in 1981; Kates et al. [4] analyzed disaster reduction in 1992 by combining social vulnerability to geology, hydrology, technology and other different types of disasters' influence. In the World Conference on Disaster Reduction of 1994, it was agreed that it is important to establish a more secure environment—Yokohama stratagem and plan of action, which attaches great importance to the urban flood vulnerability to meet the demands of urban sustainable development.

In the recent 20 years, disaster vulnerability has gained increasingly widespread international attention, and lots of cities were plagued by floods due to the urbanization tendency. By the end of July 2013 many domestic cities such as Beijing, Nanjing, Guangzhou, Dalian, Xi'an and Wuhan had suffered a serious urban water logging, leading to a lot of lives and property loss. Researchers have conducted a lot of studies on the urban flood problems from different aspects, including revealing the problems' mechanism and improving the urban flood defense software and hardware technologies and measures, in order to make an early forecast and warning before the floods and disaster assessments

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after the floods [5–10]. To assess urban flood disaster vulnerability, many social and economic factors, such as population, economy, environment, property, infrastructure and so on, are involved. Due to the complex relationships between vulnerability and hazard-affected components, selecting an appropriate assessment method is very important. The two methods, fuzzy comprehensive assessment (FCA) method and fuzzy set pair assessment (FSPA) method [11, 12] which were proposed in recent years and used for urban flood vulnerability assessment, have some problems. This paper proposes a variable sets (VS) method for urban flood vulnerability assessment by means of comparison and analysis, and points out the existing problems of FCA and FSPA, which is useful for further development of urban flood vulnerability assessment.

Chen [13] established the variable fuzzy sets theory in 2005. At the same time he proved the dialectics three basis laws of mathematical theorem-opposite unity, quantity-quality exchange and negation of negation theorem [14, 15], which were successfully applied to water resource field [16–18]. The paper proposes a new method for urban flood vulnerability assessment based on the VS theory. A brief introduction of the opposite unity, quantity-quality exchange and negation of negation comprehensive theorem are given firstly.

2 Dialectics three basis laws of mathematical theorem and the comprehensive theorem

2.1 Opposite unity theorem

Let U be a universe of discourse (UD), u be an element of U, $u \in U$. The contrasting properties of u is represented by $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The two endpoints P_l and P_r of a continuum are defined as 1, 0 or 0, 1. For u in U, a pair of measures are defined as $\mu_{\underset{\wedge}{A}}(u)$ and $\mu_{\underset{\wedge}{A}^c}(u)$ at any point of the continuum, which can be named the opposite relative membership degree (RMD) of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The mapping below is defined

$$\mu_{\underset{\wedge}{A}}, \mu_{\underset{\wedge}{A}^c} : U \to [0, 1], u| \to \mu_{\underset{\wedge}{A}}(u), \mu_{\underset{\wedge}{A}^c}(u) \in [0,\ 1]. \tag{1}$$

Eq. (1) is the opposite relative membership function (RMF) of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. No matter what kind of change the opposite measure makes, it always is 1 without change. That is

$$\mu_{\underset{\wedge}{A}}(u) + \mu_{\underset{\wedge}{A}^c}(u) = 1. \tag{2}$$

Let

$$D_{\underset{\wedge}{A}}(u) = \mu_{\underset{\wedge}{A}}(u) - \mu_{\underset{\wedge}{A}^c}(u). \tag{3}$$

Then $D_{\underset{\wedge}{A}}(u)$ is the opposite relative difference degree (RDD) of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$. The following mapping is defined as the opposite relative difference function (RDF) of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$:

$$D_{\underset{\wedge}{A}} : U \to [1, -1],\quad u| \to D_{\underset{\wedge}{A}}(u) \in [1, -1]. \tag{4}$$

Eq. (4) is expressed on the number-axis as Figure 1.

Adding up eqs. (2) and (3) together, the relationship of RMF and the RDF is

$$\mu_{\underset{\wedge}{A}}(u) = \left[1 + D_{\underset{\wedge}{A}}(u)\right] \big/ 2. \tag{5}$$

Let

$$D_{\underset{\wedge}{A}^c}(u) = \mu_{\underset{\wedge}{A}^c}(u) - \mu_{\underset{\wedge}{A}}(u). \tag{6}$$

Then the mapping below is the opposite RDF of u to $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$:

$$D_{\underset{\wedge}{A}^c} : U \to [-1, 1],\quad u| \to D_{\underset{\wedge}{A}^c}(u) \in [-1, 1]. \tag{7}$$

According to eqs. (3) and (6), an equation is obtained as follows

$$D_{\underset{\wedge}{A}}(u) = -D_{\underset{\wedge}{A}^c}(u). \tag{8}$$

Eq. (2) is called the opposite unity theorem of VS.

2.2 Quantity-quality exchange theorem

If $C(u)$ represents the change of u in UD, the three symbols $C_1(u)$, $C_2(u)$ and $C_3(u)$ respectively express the changes of u with time, space and condition's changes.

$$C(u) = \{C_1(u), C_2(u), C_3(u)\}. \tag{9}$$

Suppose $D_{\underset{\wedge}{A}}(u) \neq 0$, and let $D_{\underset{\wedge}{A}}(C(u))$ express the change of $D_{\underset{\wedge}{A}}(u)$.

If $D_{\underset{\wedge}{A}}(u)$ and $D_{\underset{\wedge}{A}}(C(u))$ satisfy

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) < 0,\ D_{\underset{\wedge}{A}}(C(u)) \neq 1, 0, -1 . \tag{10}$$

Then this change is named gradually qualitative change.

If $D_{\underset{\wedge}{A}}(u)$ and $D_{\underset{\wedge}{A}}(C(u))$ satisfy

$$D_{\underset{\wedge}{A}}(u) \cdot D_{\underset{\wedge}{A}}(C(u)) > 0,\ D_{\underset{\wedge}{A}}(C(u)) \neq 1, 0, -1. \tag{11}$$

Then this change is the quantity change.

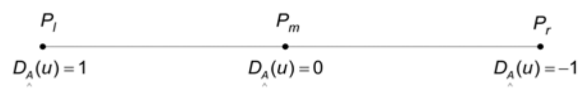

Figure 1 Chart of opposite relative difference function change.

Inequalities (10) and (11) are called the qualitative change and quantity change theorem of VS, which is named the quantity-quality exchange theorem uniformly.

2.3 Negation of negation theorem

The change of the value of $D_{\underset{\wedge}{A}}(u)$ from 1 to −1 can be called a period (see Figure 1). Suppose there are N periods (here N is a positive integer and $N \in [1, \infty]$). Before changing, the original state of $D_{\underset{\wedge}{A}}(u)$ is at the left endpoint P_l . After a whole periodic change (N=1), $D_{\underset{\wedge}{A}}(u)$ just is at the right endpoint P_r finally. That says the value of $D_{\underset{\wedge}{A}}(u)$ has changed from 1 to −1 and $D_{\underset{\wedge}{A}}(C(u)) = -1$. The negation of negation theorem can be expressed as $D_{\underset{\wedge}{A}}(u)\cdot D_{\underset{\wedge}{A}}(C(u)) = 1\cdot(-1) = (-1)^1$.

If there are several periodic changes ($N \succ 1$) of $D_{\underset{\wedge}{A}}(u)$ and the final state is at the right endpoint P_r (here N is an odd number) or the left endpoint P_r (here N is an even number), the process of change of $D_{\underset{\wedge}{A}}(u)$ is called N times negation($\underset{\wedge}{A}^{c^{\cdot^{c}}}$), there is

$$D_{\underset{\wedge}{A}}(u)\cdot D_{\underset{\wedge}{A}}(C(u)) = (-1)^N . \qquad (12)$$

When N is an odd number, $D_{\underset{\wedge}{A}}(u)\cdot D_{\underset{\wedge}{A}}(C(u)) = -1$ and $D_{\underset{\wedge}{A}}(u)\cdot D_{\underset{\wedge}{A}}(C(u)) = 1$ when N is an even number. When N=2, there is

$$D_{\underset{\wedge}{A}}(u)\cdot D_{\underset{\wedge}{A}}(C(u)) = 1. \qquad (13)$$

Eq. (13) is the mathematic express of the negation of negation theorem of VS.

2.4 Opposite unity, quantity-quality exchange and negation of negation comprehensive theorem

According to the opposite unity, quantity-quality exchange and negation of negation theorem, $D_{\underset{\wedge}{A}}(u)$, the N times evolution process of RDD can be expressed by a vector as follows.

$$D_{\underset{\wedge}{A}-\underset{\wedge}{A}^{c^{\cdot^{c}}}}(u) = (1,0,-1,0,1,0,-1,\cdots,1,0,-1). \qquad (14)$$

Eq. (14) shows the continuum's left endpoint $P_{l1}(\underset{\wedge}{A})$ (the original state of $D_{\underset{\wedge}{A}}(u)$ before the change). In vector (14), the first "−1" corresponds to $P_{r1}(\underset{\wedge}{A}^c)$, the right endpoint of $D_{\underset{\wedge}{A}}(C(u))$ after change (the final state of the change when N=1, and the original state P_{l2} of the change when N=2, and so on). Thinking the final state of every changing period as the original state of the next period, we can change every "−1" to "1". Then in N period' changing process eq. (14) can be

$$D_{\left(\underset{\wedge}{A}-\underset{\wedge}{A}^c\right)^N}(u) = (1,0,1,0,1,\cdots,1,0,1). \qquad (15)$$

Eq. (15) clearly expresses the dynamic process of things, phenomenon and its' opposite property RDD from quantitative change to qualitative change in the N period changing , which is called the opposite unity, quantity-quality exchange and negation of negation comprehensive theorem. And these are the theoretical basis of VS method for urban flood vulnerability assessment.

3 Variable sets method for urban flood vulnerability assessment

3.1 Index relative difference degree function

Let the level variable h, h=1,2,⋯,c (c is the sum of levels); index i (i=1,2,⋯,m, m is the sum of index) be classified by the protocol standard interval value [a_{ih}, b_{ih}] firstly, a_{ih} and b_{ih} be the upper and lower bounds of standard value of index i belonging to level h, respectively. For the-smaller-the-better indices, there is $a_{ih}<b_{ih}$ but $a_{ih}>b_{ih}$ for the-larger-the-better indices; b_{ih}, the cross point of adjacent levels index i standard interval value, amounts to the gradual qualitative change point when the level transforms from h to (h+1) in the opposite unity and quantity-quality exchange theorem, that means RMD of the cross point satisfies $\mu(b_{ih})=\mu(a_{i(h+1)})=0.5$.

Let the object u be recognized according to the standard value interval matrix of the multi-levels h and the multi-index i as follows

$$\boldsymbol{Y} = \left(\left[a_{ih}, b_{ih}\right]\right), \quad i = 1,2,\cdots,m;\ h = 1,2,\cdots,c. \qquad (16)$$

Owing to the gradual qualitative change point $\mu(b_{ih})=0.5$, there must be a condition that two-levels (or it can be called two poles) are opposed in both sides of the qualitative change point according to the opposite unity theorem. That means the levels h and h+1 of index i compose the opposite levels, $\underset{\wedge}{A}$ and $\underset{\wedge}{A}^c$ can be replaced by ih and $i(h+1)$, respectively. On the basis of opposite unity theorem, the total RMD of object u index i to levels h and h+1 is 1, there is

$$\mu_{ih}(u) + \mu_{i(h+1)}(u) = 1. \qquad (17)$$

It is sufficient just to calculate $\mu_{ih}(u)$ in eq. (17).

The corresponding point value matrix K where both RDD $D_{ih}(u)$ and $D_{i(h+1)}(u)$ are equal to 1 can be confirmed on the basis of standard interval matrix Y of level h and index i. According to ref. [19], there is

$$\begin{cases} k_{i1} = a_{i1}, \\ k_{ih} = \dfrac{a_{ih} + b_{ih}}{2}, \; h = 2,3,\cdots,c-1, \\ k_{ic} = b_{ic}. \end{cases} \tag{18}$$

According to the standard value interval matrix Y and eq. (18), the point value mapping matrix is

$$\boldsymbol{K} = (k_{ih}). \tag{19}$$

Let the index feature matrix of object u be

$$\boldsymbol{X} = (x_1, x_2, \cdots, x_m) = (x_i), \; i = 1,2,\cdots,m. \tag{20}$$

Let x_i, the index i's feature value of u, be at the point value interval $\left[k_{ih}, k_{i(h+1)}\right]$ of matrix K where the levels h and h+1 index i's feature value RDD $D_{ih}(u)$ and $D_{i(h+1)}(u)$ are both equal to 1; besides there must be a gradual qualitative change point b_{ih}, $D_{ih}(u)$=0 in the interval at the same time, so $D_{ih}(u)$, the RDD of x_i to levels h and h+1, is calculated by

$$D_{ih}(u) = \begin{cases} \dfrac{b_{ih} - x_i}{b_{ih} - k_{ih}}, & x_i \in [k_{ih}, b_{ih}], \\ -\dfrac{b_{ih} - x_i}{b_{ih} - k_{i(h+1)}}, & x_i \in [b_{ih}, k_{i(h+1)}]. \end{cases} \tag{21}$$

$D_{i(h+1)}(u)$ is ascertained by eq. (8). According to the physical conception, the RDD of index i to the levels smaller than h or larger than (h+1) should be always equal to −1, that is

$$\begin{cases} D_{i(<h)}(u) = -1, \\ D_{i(>(h+1))}(u) = -1. \end{cases} \tag{22}$$

Eqs. (21) and (22) are applied to single index condition, while urban flood vulnerability assessment is a multi-indices recognition problem, so the multiple indices comprehensive RMD nonlinear model, expressed by RDD of indices, is derived as follows.

3.2 Comprehensive relative membership degree model

According to Figure 1, the definition domain of $D_{ih}(u)$, RDF of index i to the opposite levels h and h+1, is [1, −1], if the index weight vector is w=($w_1,w_2,\cdots,w_m$), $D_{ih}(u)$, the RDD of the generalized weight range of the recognition project u to the left pole p_l and the right pole p_r (that is, the levels h and h+1) are shown respectively as follows

$$d_{ih}(u_j) = \left\{ \sum_{i=1}^{m} \left[w_i \left(1 - D_{ih}(u_j)\right) \right]^p \right\}^{\frac{1}{p}}, \tag{23}$$

$$d_{i(h+1)}(u_j) = \left\{ \sum_{i=1}^{m} \left[w_i \left(D_{ih}(u_j) - (-1)\right) \right]^p \right\}^{\frac{1}{p}}, \tag{24}$$

where p is a distance parameter, and when p=1, it is called Hamming distance and when p=2 it is called Euclidean distance.

Let $v_h(u)$ be a multi-indices comprehensive RMD of recognition object u to level h; the objective function is established as

$$\min\left\{ F\left(v_h(u)\right) = \left(v_h(u)\right)^2 \left(d_{ih}(u)\right)^\alpha + \left(v_{h+1}(u)\right)^2 \left(d_{i(h+1)}(u)\right)^\alpha \right\}, \tag{25}$$

where α is an optimization criterion parameter. When α=2, eq. (25) is called the least square optimization criterion; when α=1 it is called a least-absolute criterion.

On the basis of the opposite unity theorem

$$v_h(u) + v_{h+1}(u) = 1. \tag{26}$$

Taking derivative of variable $v_h(u)$ of eq. (25) and equating it to 0, we have

$$\frac{\mathrm{d}F(v_h(u))}{\mathrm{d}v_h(u)} = 2\left[d_{ih}(u)\right]^\alpha v_h(u) - 2\left[1 - v_h(u)\right]\left[d_{i(h+1)}(u)\right]^\alpha = 0.$$

Then we obtain

$$v_h(u) = \frac{1}{1 + \left[\dfrac{d_{ih}(u)}{d_{i(h+1)}(u)}\right]^\alpha} = \frac{1}{1 + \left[\dfrac{\sum_{i=1}^{m}\left[w_i(1 - D_{ih}(u))\right]^p}{\sum_{i=1}^{m}\left[w_i(1 + D_{ih}(u))\right]^p}\right]^{\frac{\alpha}{p}}}. \tag{27}$$

Eq. (27) is called the multi-indices comprehensive RMD nonlinear model of level h which is expressed by the index RDD.

When α=1, p=1, eq. (27) changes into a simple linear model

$$v_h(u) = \left[1 + \sum_{i=1}^{m}\left[w_i D_{ih}(u)\right]\right] \Big/ 2. \tag{28}$$

Using the rank feature value function in ref. [20] we have

$$H(u) = \sum_{h=1}^{c} v_h(u) \cdot h, \quad h=1,2,\cdots,c. \tag{29}$$

According to the rank feature value, we can obtain the urban flood vulnerability assessment result.

4 Case study

4.1 Assessment objects and indices

To compare the methods for urban flood vulnerability assessment in refs. [11] and [12], we take 29 cities of Hunan province as the assessment object. Hunan, located in the middle reach of the Yangtze River, with a continental humid subtropical monsoon climate, raining almost the whole year and the average annual precipitation between 1200 and

1700 mm, is one of the most rainfall provinces in China. In urban areas there concentrate a large population with high density and integrated function of industry, business, education, humanity and others. With the development of economy and urbanization, the security, especially the major natural disasters like earthquake and floods vulnerability, has been an increasingly prominent problem in the urban area. Based on that, we use the VS theorem to assess the urban flood vulnerability of Hunan province.

As for the index system establishment, we use 5 indices cited from ref. [11]: 1) Population density (x_1), 2) industrial output density (x_2), 3) road network density (x_3), 4) drainage pipeline density (x_4), 5) build-up area greening rate (x_5) as the assessment certainty indices of urban flood vulnerability firstly. According to Hunan province statistical yearbook of 1999, the certainty indices of 29 cities of Hunan province are shown in Table 1. At the same time 3 fuzziness indices are added to reflect the system response capacity and psychology: 6) information and communication security (x_6), 7) floods risk integrated management (x_7), 8) system calling efficiency (x_8). We establish the urban flood vulnerability assessment system considering the both sides of certainty and fuzziness.

Due to the limited space, just data of provincial capital—Changsha are listed in the table.

4.2 Assessment criterion and levels

According to previous studies, urban flood vulnerability is usually divided into 5 levels. Level 1 is the very high vulnerability. Level 2 is the high vulnerability. Level 3 is the medium vulnerability. Level 4 is the low vulnerability and level 5 is the very low vulnerability. We cite the level standard of the 5 certainty indices form ref. [12] and show it in Table 2.

4.3 Weight vector

To compare the methods FCA and FSPA and analyze the sensitivity of indices weights, we cite 5 certainty index weight vectors from refs. [11] and [12] separately and show them in Table 3.

4.4 Assessment procedure and counting process

On the basis of the proposed VS assessment method for urban flood vulnerability, the basic calculation procedure is shown as Figure 2.

According to the basic calculation procedure, we calculate the index RDD matrix, comprehensive RMD and the rank feature value through the certainty index subsystem 1) and the fuzzy uncertainty index subsystem 2). The detail process is as follows.

4.4.1 Subsystem 1)

According to the level index standard interval values in Table 2, the 5 levels index standard values interval matrix would be

Table 1 Hunan province urban flood vulnerability certainty indices

City	x_1 (person km^2)	x_2 (10 Thousand (a km^2)$^{-1}$)	x_3 (km km^{-2})	x_4 (km km^{-2})	x_5 (%)
Changsha	11749	16243	8.47	5.45	25.9

Table 2 Hunan province urban flood vulnerability certainty indices level standard

Levels	x_1 (person km^2)	x_2 (10 Thousand (a km^2)$^{-1}$)	x_3 (km km^{-2})	x_4 (km km^{-2})	x_5 (%)
Level 1	>11400	>19000	>8.3	<3.2	<12
Level 2	[10100, 11400]	[15700, 19000]	[7.3, 8.3]	[3.2, 3.7]	[12, 17]
Level 3	[7400, 10100]	[9100, 15700]	[5.4, 7.3]	[3.7, 5.9]	[17, 26]
Level 4	[6100, 7400]	[5800, 9100]	[4.5, 5.4]	[5.9, 7.0]	[26, 31]
Level 5	<6100	<5800	<4.5	>7.0	>31

Table 3 Hunan province urban flood vulnerability certainty indices weights

Certainty indices	x_1 (person km^2)	x_2 (10 Thousand (a km^2)$^{-1}$)	x_3 (km km^{-2})	x_4 (km km^{-2})	x_5 (%)
Ref. [11] (weight 1)	0.211	0.255	0.137	0.149	0.248
Ref. [12] (weight 2)	0.151	0.263	0.147	0.22	0.219

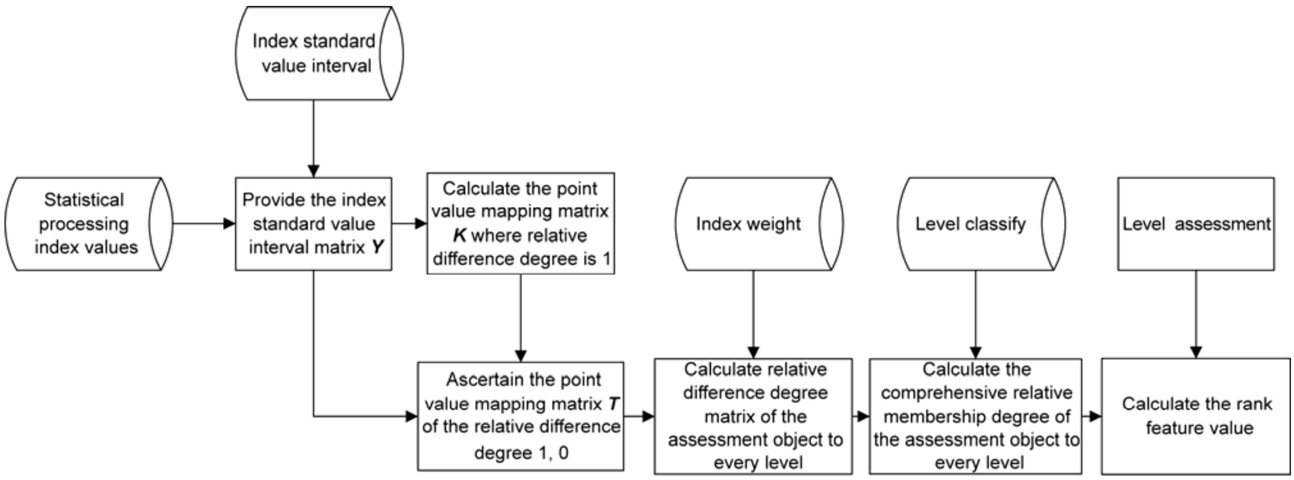

Figure 2 Urban flood vulnerability assessment variable sets basic procedure.

$$\boldsymbol{Y}=\begin{bmatrix} \geqslant 11400 & [11400,10100] & [10100,7400] & [7400,6100] & \leqslant 6100 \\ \geqslant 19000 & [19000,15700] & [15700,9100] & [9100,5800] & \leqslant 5800 \\ \geqslant 8.3 & [8.3,7.3] & [7.3,5.4] & [5.4,4.5] & \leqslant 4.5 \\ \leqslant 3.2 & [3.2,3.7] & [3.7,5.9] & [5.9,7.0] & \geqslant 7.0 \\ \leqslant 12 & [12,17] & [17,26] & [26,31] & \geqslant 31 \end{bmatrix}.$$

Using eq. (18) and matrix $\boldsymbol{Y}$, the point value mapping matrix where the RDD equals 1 is obtained as

$$\boldsymbol{K}=\begin{bmatrix} 11400 & 10750 & 8750 & 6750 & 6100 \\ 19000 & 17350 & 12400 & 7450 & 5800 \\ 8.3 & 7.8 & 6.35 & 4.95 & 4.5 \\ 3.2 & 3.45 & 4.8 & 6.45 & 7 \\ 12 & 14.5 & 21.5 & 28.5 & 31 \end{bmatrix}.$$

By the gradual quality change point b_{in} in matrix $\boldsymbol{Y}$, the point value mapping matrix where the RDD equals 1 and 0 is

$$\begin{matrix} & (k_{i1}) & (b_{i1}) & (k_{i2}) & (b_{i2}) & (k_{i3}) & (b_{i3}) & (k_{i4}) & (b_{i4}) & (k_{i5}) \end{matrix}$$
$$\boldsymbol{T}=\begin{bmatrix} 11400 & 11075 & 10750 & 10100 & 8750 & 7400 & 6750 & 6425 & 6100 \\ 19000 & 18175 & 17350 & 15700 & 12400 & 9100 & 7450 & 6625 & 5800 \\ 8.3 & 8.05 & 7.8 & 7.3 & 6.35 & 5.4 & 4.95 & 4.725 & 4.5 \\ 3.2 & 3.325 & 3.45 & 3.7 & 4.8 & 5.9 & 6.45 & 6.725 & 7 \\ 12 & 13.25 & 14.5 & 17 & 21.5 & 26 & 28.5 & 29.75 & 31 \end{bmatrix}=\left(t_{i(2c-1)}\right).$$

Now we take Changsha (u_1) as an example to analyze the detail process. It's known from Table 1 that the index feature value vector is x=(11749, 16243, 8.47, 5.45, 25.90). x_1=11749, the feature value of index 1), is larger than k_{11}=11400 in matrix $\boldsymbol{T}$, so the RMD is obtained to be $\mu_{11}(u_1)$=1. According to eq. (22), the RDD matrix of index 1) to every level is $D_1(u_1)$=(1,–1,–1,–1,–1). The feature value of index 2) x_2=16243 falls into the interval between k_{22} and b_{22} of matrix $\boldsymbol{T}$. Using eq. (21) the RDD $D_{22}(u_1)$= 0.33 is obtained and $D_{23}(u_1)$=–0.33 by eq. (8). Through calculating the RDD of indices 3)–5) in the same way, the RDD matrix of indices 1)–5) to every level of Changsha is obtained as

$$D(u_1)=\begin{bmatrix} 1 & -1 & -1 & -1 & -1 \\ -1 & 0.33 & -0.33 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 0.41 & -0.41 & -1 \\ -1 & -1 & 0.022 & -0.022 & -1 \end{bmatrix}.$$

By eq. (28) and the index weight vector w=(0.211, 0.255, 0.137, 0.149, 0.248) of ref. [11] in Table 3, the RMD vector of the object (u_1) to every level is $v(u_1)$=(0.348, 0.170, 0.317, 0.165, 0). By eq. (29), the rank feature value of Changsha about subsystem 1) can be finally obtained: $H_1(u_1)$=2.299. Similarly, by the index weight vector w=(0.151, 0.263, 0.147, 0.220, 0.219) of ref. [12] in Table 3, the results change to be $v(u_1)$=(0.298, 0.175, 0.355, 0.172, 0), $H_1(u_1)$=2.401. From the results obtained by the two different weight vectors, it can be seen that the index sensibility is inapparent.

4.4.2 Subsystem 2)

According to the index fuzziness of subsystem 2), weight vectors of indices 6)–8) can be obtained by the binary comparison method of engineering fuzzy set theory in ref. [20]. So we take the importance binary comparison of 3 fuzziness indices. Considering the relationships of system reaction capacity with people's psychology, index 7) is always thought more important than indices 6) and 8). Using the mood operator and quantity scale RMD relationship in ref. [20] (Table 5–1 in ref. [20]), the mood operator 'relatively' corresponds to the RMD of 0.538 in this table, then the non-normalized weight vector of fuzziness indices 6)–8) is (0.538, 1.0, 0.538), so the normalized indices weight vector is (0.259, 0.482, 0.259).

On the basis of Hunan province statistical yearbook for 1999, information and communication security, integrated floods risk management and the system efficiency of Changsha are always increasing but still not strengthened enough, so the fuzziness indices 6)–8) are assigned to be 0.75, 0.70 and 0.65, separately. Considering the traditional 5 levels assessment standard: excellent, good, medium, bad and worse and the value features of fuzziness indices in the interval [0,1], and using the fuzzy recognition method in ref. [21], 5 levels standard values of the fuzziness indices 6)–8) are ascertained to be [0,0.3], [0.3,0.6], [0.6,0.8], [0.8,0.9], [0.9,1]. According to the same way for the certainty index system—VS assessment method, the rank feature value of fuzziness index system 2) would be $H_2(u_1)$=3.0

Considering that there are 3 engineering indices that belong to the urban flood resistance infrastructure in the certainty index system 1), we assign 0.7 to the weight value of system 1) and 0.3 to system 2). When using the weight 1, the final rank feature value is $H(u_1)$=0.7×2.299+0.3×3.0= 2.509 and $H(u_1)$=2.581 when weight 2. Both the two rank feature values show that Changsha flood vulnerability is between level 2 and level 3. That means the flood vulnerability of the city is between the high level and the medium level. The assessment results are consistent with the reality. Through the historical statistical data, the whole Yangtze River basin suffered a heavy flood in 1998, and resulted in serious flood loss. Changsha locates in the middle reaches of Yangtze River, and Xiangjiang, which is a branch of Yangtze River, just goes through Changsha, so it suffered a bad flood and had serious loss. With a certain flood disaster resist capability of Changsha, as assessed in this paper, the real flood of Changsha wasn't up to the heavy level in 1999 (ref. [22]). That shows the assessment results of VS method are reasonable.

4.5 Results

We list the assessment results of VS, refs. [11] and [12] in Table 4 and do the comparison and analysis. Restricted by the limited space, just the data of Changsha are listed.

Ref. [11] used FCA method to assess the flood vulnerability of 29 cities of Hunan province, the results are much different from those of this paper. The main reason is that it improperly used the maximum membership principle (discussed in ref. [20]). For instance ref. [11] came to the conclusion that Changsha flood vulnerability belongs to level 1, just based on that the city's maximum RMD of flood vulnerability to level 1 is 0.251. It can be seen at page 4 in this ref. that the FCA value is $\tilde{B}=(0.251,\ 0.224,\ 0,\ 0.156,\ 0.125)$. ($\tilde{B}$ in ref. [11] was incorrect, it should be $\tilde{B}$=(0.462, 0.257, 0, 0.247, 0.034)). Although the RMD of flood vulnerability of Changsha to level 1 is the largest, but the sum of the RMD to levels 2 and 5 is larger than that to level 1. That's incorrect in physical conception to assess the city's flood vulnerability to be level 1 apparently. In the 5 certainty indices of Changsha, the feature value of indices 1) and 3) belong to level 1 and the two indices' weight sum is 0.397; the feature value of index 2) belongs to level 2 and the indices' weight sum is 0.255, feature value of the other two indices 4) and 5) belong to level 3 and the index's weight sum is 0.397. Through the above analysis it's reasonable that the rank feature value of the certainty system belongs to level 2. If we further consider the fuzziness index system 2), the final flood vulnerability assessment result of Changsha is between levels 2 and 3 will make sense.

The assessment results of this paper are different from the results of ref. [12], which were obtained by the FSPA method to assess the flood vulnerability of Hunan 29 cities. The specific reason has been expounded and analyzed by Chen in ref. [19]. Due to the value of index 5) x_5=25.6, index 5) has to be in the range of the third standard interval [17, 26] (see Table 2), but the connection degree of index 5) to level 3 is 0.371 but 0.629 to level 4 by the FSPA method in ref. [22] (see Table 4–28, ref. [12]). Apparently, this is discordant with the physical conception and we can confirm that the flood vulnerability assessment results of Hunan

Table 4 Hunan province urban flood vulnerability assessment results and comparison

City	VS		Ref. [11]	Ref. [12]
	Weight 1	Weight 2		
Changsha	2–3	2–3	1	2

province 29 cities in refs. [11] and [12] are not reasonable.

5 Conclusions

The development of urbanization, the increasing urban population and the concentration of urban properties have resulted in continuous increase of urban flood disaster and losses. Accordingly the urban flood vulnerability assessment, as the basis component of urban flood disaster prevention and reduction, becomes much more important. This paper, based on the VS and dialectics three-basic laws of mathematical theorem proposed in 2007 by Chen, firstly combines dialectics philosophy and mathematics thinking into urban flood vulnerability assessment, and proposes the VS assessment method, which has rigorous theory, clear structure, and simple calculation and makes a breakthrough in assessment methods for urban flood vulnerability. Through a case study of urban flood vulnerability assessment, this paper highlights how to use the dialectical thinking to solve multi-indices recognition problems.

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