SCIENCE CHINA
Technological Sciences

May 2013 Vol.56 No.5: 1307–1312
doi: 10.1007/s11431-013-5182-9

Vehicle's fuel consumption of car-following models

TANG TieQiao*, LI JinGang, WANG YunPeng & YU GuiZhen

School of Transportation Science and Engineering, Beijing Key Laboratory for Cooperative Vehicle Infrastructure Systems and Safety Control, Beihang University, Beijing 100191, China

Received October 26, 2012; accepted January 14, 2013; published online March 27, 2013

Many car-following models with their own merits have been proposed to explore various traffic problems, but researchers have rarely used vehicle's performance to evaluate them. In this paper, we study each vehicle's fuel consumption of the optimal velocity model, full velocity difference model, full velocity and acceleration difference model and the car-following model with consideration of the traffic interruption probability under two traffic situations, respectively. The numerical results show that the car-following model with consideration of traffic interruption probability can reduce vehicle's fuel consumption in the two studied traffic situations and thus improve the vehicle's fuel economy.

car-following model, fuel consumption, numerical test

Citation: Tang T Q, Li J G, Wang Y P, et al. Vehicle's fuel consumption of car-following models. Sci China Tech Sci, 2013, 56: 1307–1312, doi: 10.1007/s11431-013-5182-9

1 Introduction

To date, many traffic problems (e.g., jam, accident, fuel consumption, pollution, etc.) have influenced human's daily life and attracted many researchers to establish a number of traffic flow models (e.g., first-order model, high-order model, car-following model, cellular automaton model, etc.) to explore complex traffic phenomena [1–8]. Although the existing traffic flow models can describe and explain the formation mechanisms of many complex traffic phenomena and obtain many important results, they cannot be used to explore vehicle's fuel consumption which has never been considered.

Today, vehicle's fuel consumption accounts for a considerable proportion of the energy consumption. According to the International Energy Agency (IEA), the growth of the energy consumption in the transportation sector would increase by 1.5% from 2006 to 2030, while the average global growth would be only 1.4% [9]. Thus the vehicle's fuel consumption should be reduced, which has attracted researchers to improve the performance of engine [10–13] and propose models and methods to explore the vehicle's fuel consumption [14–19]. But these models and methods cannot be used to directly evaluate the existing traffic flow models. Ahn proposed a model for vehicle's fuel consumption and found that the vehicle's fuel consumption was related to its current speed and acceleration. Thus he defined the vehicle's fuel consumption approximately as a function of its current speed and acceleration by use of testing data [18]. Rakha et al. [20] incorporated Ahn's model into the simulation tool INTEGRATION to explore the effects of traffic light on the vehicle's fuel consumption and emissions. However, little effort has been made to study each vehicle's fuel consumption of the car-following models. In this paper, we use the theories of ref. [18] to explore the vehicle's fuel consumption of car-following models.

2 Related models

In this paper, we choose the optimal velocity (OV) model

---

*Corresponding author (email: tieqiaotang@buaa.edu.cn)

© Science China Press and Springer-Verlag Berlin Heidelberg 2013

[21], full velocity difference (FVD) model [22], full velocity and acceleration difference (FVAD) model [23] and the car-following model with consideration of the traffic interruption probability [24, 25] to study the vehicle's fuel consumption under two traffic situations. First, we introduce the four car-following models, respectively.

2.1 Optimal velocity model

The classical OV model was developed by Bando et al. [21] and the control equation can be reduced to

$$\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}=\alpha\left[V\left(s_i(t)\right)-v_i(t)\right], \tag{1}$$

where $v_i(t)$ and $s_i(t)=x_{i-1}(t)-x_i(t)$ are respectively the *i*th vehicle's speed and the headway at time t ($x_i(t)$ is the *i*th vehicle's position, $x_{i-1}(t)$ is the position of its preceding vehicle); α is the driver's reactive time; $V(\cdot)$ is the optimal velocity that the driver prefers.

Helbing and Tilch [26] used empirical data to calibrate the OVM and defined the optimal velocity function as

$$V(s)=V_1+V_2\tanh[C_1(s-l_c)-C_2], \tag{2}$$

where $l_c=5\ \mathrm{m}$ is the vehicle's length. And other parameters are defined as follows:

$$\begin{aligned}&\alpha=0.85\ \mathrm{s}^{-1},\ V_1=6.75\ \mathrm{m/s},\ V_2=7.91\ \mathrm{m/s},\\&C_1=0.13\ \mathrm{m}^{-1},\ C_2=1.57.\end{aligned} \tag{3}$$

2.2 Full velocity difference model

Under some specific conditions, too high an acceleration and unrealistic deceleration will occur if using eq. (1) to describe the vehicle's movement behavior. To overcome the drawback, Helbing and Tilch [26] developed a generalized force (GF) model. But Jiang et al. [22] pointed out that the GF model does not well describe the vehicle's delay time and kinematic wave speed at the jam density, thus they developed the FVD model [22], i.e.,

$$\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}=\alpha\left[V\left(s_i(t)\right)-v_i(t)\right]+\lambda\Delta v_i(t), \tag{4}$$

where $\Delta v_i=v_{i-1}-v_i$ is the i th vehicle's relative velocity. In the FVD model, $\alpha=0.41\,s^{-1}$ and the parameter λ is defined as follows:

$$\lambda=\begin{cases}0.5, & \text{if } s\leqslant s_{\mathrm{c}},\\0, & \text{otherwise,}\end{cases} \tag{5}$$

where $s_{\mathrm{c}}=100\ \mathrm{m}$ is a critical value.

2.3 Full velocity and acceleration difference model

Eq. (4) can overcome the drawbacks of the OV and GF models, but it will produce collision under some specific conditions. In order to conquer this drawback, Zhao and Gao [23] proposed the FVAD model, i.e.,

$$\begin{aligned}\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}&=\alpha\left[V\left(s_i(t)\right)-v_i(t)\right]+\lambda\Delta v_i(t)\\&\quad+kg\left(\Delta a_i(t-1),a_{i-1}(t)\right)\Delta a_i(t-1),\\\Delta a_i(t)&=a_{i-1}(t)-a_i(t)=\frac{\mathrm{d}v_{i-1}(t)}{\mathrm{d}t}-\frac{\mathrm{d}v_i(t)}{\mathrm{d}t},\\g&\left(\Delta a_i(t-1),a_{i-1}(t)\right)\\&=\begin{cases}-1, \Delta a_i(t-1)>0 \text{ and } a_{i-1}(t)\leqslant 0,\\1,\ \text{otherwise,}\end{cases}\end{aligned} \tag{6}$$

where $\Delta a_i(t)$ is the acceleration difference between the i th vehicle and its preceding vehicle. The parameter k is defined as follows:

$$k=\begin{cases}0.5, & \text{if } s\leqslant s_{\mathrm{c}},\\0, & \text{otherwise.}\end{cases} \tag{7}$$

The function $g(\cdot)$ is the sign of the acceleration difference term.

2.4 Car-following model with consideration of traffic interruption probability

Eq. (6) can conquer some drawbacks of the OV, GF and FVD models, but it cannot be used to study the impacts of interruption factors on the traffic flow. To study the impacts of interruption factor on the traffic flow, Tang et al. [24, 25] explored the driving behavior in the traffic system with interruption factor and found that each vehicle may be interrupted. Thus they proposed a new car-following model with consideration of the traffic interruption probability, i.e.,

$$\begin{aligned}\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}&=\alpha\left[V\left(s_i(t)\right)-v_i(t)\right]+\lambda_1P_{i-1}(-v_i(t))\\&\quad+\lambda_2(1-P_{i-1})\Delta v_i(t),\end{aligned} \tag{8}$$

where P_{i-1} is the probability that the (*i*-1)th vehicle is interrupted, λ_1 and λ_2 are two parameters. Tang et al. [25] proved that the interruption probability can improve the stability of the traffic flow under some specific conditions and that the traffic flow is the most stable when each vehicle's interruption probability is equal to 0.2, $\lambda_1=0.5$ and $\lambda_2=0.2$. So we define each vehicle's interruption probability as 0.2, $\lambda_1=0.5$ and $\lambda_2=0.2$ in the following numerical tests.

For simplicity, we here define the OV model, FVD model, FVAD model and the following model with consideration of the traffic interruption respectively as Model A, Model B, Model C and Model D. In addition, the results obtained in refs. [21–25] illustrate that the above four car-following models can reproduce many complex traffic phenomena from different perspectives, so we can use the four car-following models to explore the vehicle's fuel consumption.

2.5 Fuel consumption models

However, the above car-following models cannot be used to explore the vehicle's fuel consumption. To explore the vehicle's fuel consumption, Ahn developed a VT-micro model for the vehicle's fuel consumption [18]. In ref. [18], Ahn used 5 light-duty automobiles and 3 light-duty trucks to measure each vehicle's fuel consumption in a laboratory at the Oak Ridge National Laboratory. The testing results are perfectly consistent with the results of the VT-Micro model, so we can here use this model to study the vehicle's fuel consumption of the above four car-following models from the qualitative perspective. The VT-Micro model can be expressed as follows:

$$\ln\left(\mathrm{MOEe}\right)=\sum_{i=0}^{3}\sum_{j=0}^{3}\left(K_{i,j}^{\mathrm{e}}\times s^{i}\times a^{j}\right), \tag{9}$$

where MOEe is the vehicle's fuel consumption rate whose unit is $\mathrm{mL/s}$, $K_{i,j}^{\mathrm{e}}$ is the regression coefficient, s is the vehicle's instantaneous speed, and a is the vehicle's instantaneous acceleration.

3 Simulations

In this section, we focus on using the VT-Micro model to explore the vehicle's fuel consumptions of the above four car-following models under two different conditions. It is difficult to use the above four car-following models to study each vehicle's movement from an analytical perspective and the numerical schemes have no qualitative effects on the following numerical results, so we use the Euler difference scheme to discretize the above four models, i.e.,

$$v_i(t+\Delta t)=v_i(t)+\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}\cdot\Delta t, \tag{10}$$

$$x_i(t+\Delta t)=x_i(t)+v_i(t)\cdot\Delta t+\frac{1}{2}\cdot\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}\cdot(\Delta t)^2, \tag{11}$$

where $v_i(t+\Delta t)$ and $x_i(t+\Delta t)$ denote the velocity and position of the ith vehicle, respectively; $\Delta t=0.005\,\mathrm{s}$ is the time step length. In addition, we use MATLAB to carry out the following numerical tests.

3.1 Simulation in traffic situation Ⅰ

First, we study each vehicle's fuel consumption during the vehicle's starting and braking processes by means of the above four car-following models, where the initial conditions are set as follows. There are 11 vehicles that uniformly distribute on a road with a headway of 7.4 m[a)], where the 1st vehicle is the leading one and the 11th vehicle lies at the origin; there is a signal light and a barrier on the road, where the signal light is located at 74 m and the barrier is located at 500 m; all the vehicles are still when $t<0$; the signal light turns green and all the vehicles will immediately start at $t=0$; all the vehicles will eventually stop because of the barrier (see Figure 1).

Based on the above discussions, we can obtain the evolution of each vehicle's fuel consumption of the above four car-following models during the vehicle's starting and braking processes (see Figure 2)[b)].

From Figure 2, we can conclude the following results.

(1) The evolution of each vehicle's fuel consumption can be divided into three prominent stages during the starting process. In the first stage, each vehicle's fuel consumption increases sharply because its speed and acceleration increase sharply. In the second stage, each vehicle's speed still increases but its fuel consumption begins to decrease because its acceleration begins to decrease. In the third stage, each vehicle's fuel consumption does not change because its speed increases to its maximum value and its acceleration becomes zero.

(2) During the braking process, the evolution of each vehicle's fuel consumption may be irregular and very complex, but each vehicle's fuel consumption will gradually decrease. As for the exact phenomena, we should further study it with experiments in the future.

(3) In all the car-following models, each vehicle's fuel consumption during the starting process is larger than the

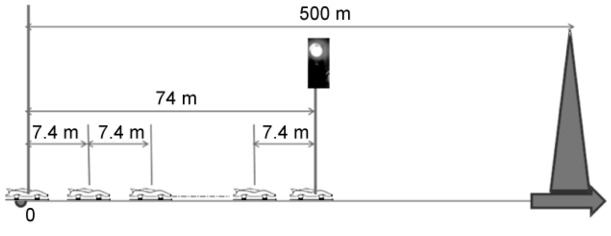

Figure 1 The car-following scheme during the starting and braking processes.

---

a) Each vehicle's headway has no effect on the numerical results from the qualitative perspective, so we here define the headway as 7.4 m based on ref. [22].

b) In this paper, the car-following model's related parameters are defined respectively as refs. [21–25] and the related parameters of the VT-Micro model is defined as ref. [18].

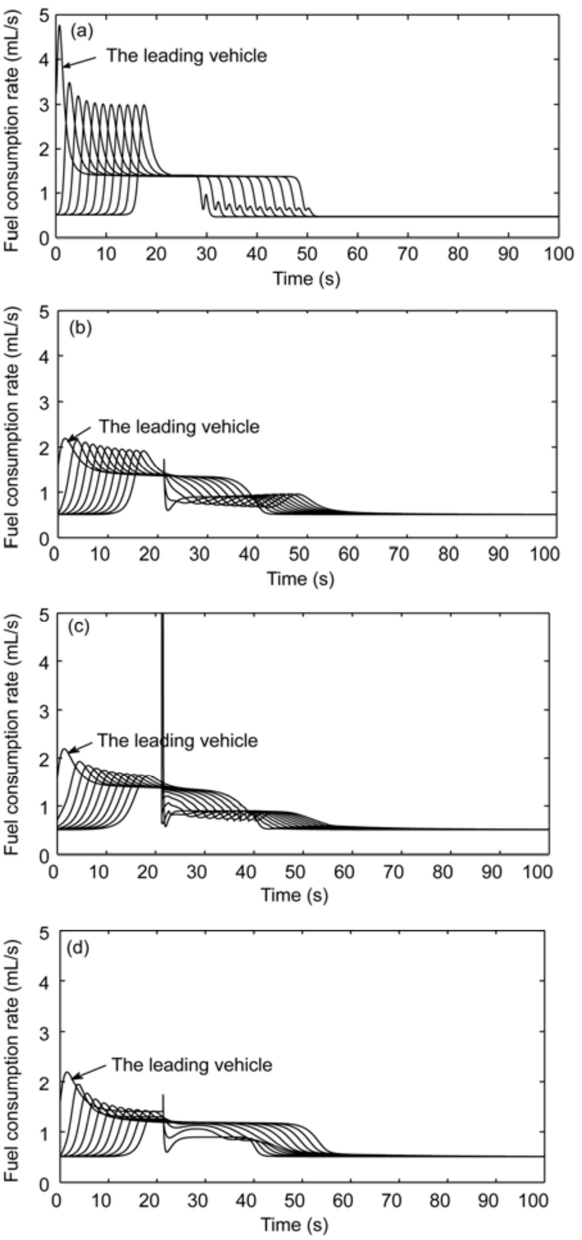

Figure 2 Evolution of each vehicle's fuel consumption. (a) Model A; (b) Model B; (c) Model C; (d) Model D.

one during the braking process. The reasons are as follows. When opening throttle to make the vehicle's engine accelerate, it needs more fuel to avoid leaning mixture and make engine work smoothly, which inevitably sacrifices the vehicle's fuel economy.

(4) The leading vehicle's fuel consumption has prominent difference. Since the leading vehicle's acceleration quickly increases in Model A, the maximum value of the leading vehicle's fuel consumption is much higher than the other vehicles'. The leading vehicle's fuel consumption has a prominent transient in Models B, C and D, but there exists an unrealistic fuel consumption value in Model C, i.e., the maximum value of the leading vehicle's fuel consumption increases to 5.19×10^4 mL/s due to the transient acceleration, which shows that Model C is unreasonable from the perspective of fuel consumption.

To further display the differences of the vehicle's fuel consumption of the four car-following models, we calculate the total fuel consumption of the 11 vehicles (see Figure 3).

From Figure 3, we can conclude the following results.

(1) The total fuel consumption of Model A quickly changes, i.e., it quickly increases and quickly decreases, which produces the impractual total fuel consumption. This is because Model A will encounter the problems of too high an acceleration in the starting process and unrealistic deceleration in the braking process.

(2) The total fuel consumptions of Model B and Model C have a trasient change; this is because all the vehicles' accelerations have a transient change in the two models. In addition, there is no prominent difference between the two models.

(3) The total fuel consumption of Model D is less than those of the other three models at first but higher than those of the other three models at last. This is because the drivers are more careful due to the consideration of the traffic interruption probability.

To validate which model consumes the least fuel during the starting and braking processes, we calculate the 11 vehicles' total fuel consumptions of the four models from 0 to 80 s (see Table 1)[c].

From Table 1, the total fuel consumption of Model D is less than those of the other three models, which shows that Model D is the best from the perspective of vehicle's fuel economy.

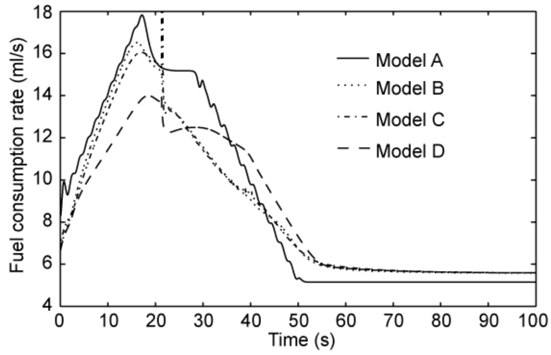

Figure 3 The total fuel consumptions of the four car-following models during the starting and braking processes.

---

c) From Figure 3, we can find that the vehicle's fuel consumptions are the same in Models B, C and D after 80 s. After 80 s, the total fuel consumption of Model A has an unrealistic value since Model A encounters the problem of unrealistic deceleration in braking process, so we only calculate the 11 vehicles' total fuel consumptions from 0 to 80 s.

Table 1 The total fuel consumption of the whole system during the starting and braking processes

Car-following model	The total fuel consumption (mL)
Model A	780.6963
Model B	747.3544
Model C	2299.3
Model D	737.8510

3.2 Simulation in traffic situation II

In this section we use the above four car-following models and VT-micro model to explore each vehicle's fuel consumption when a small perturbation occurs in a traffic system with periodic boundary condition. Here, the initial conditions are set as follows:

$$\begin{aligned} &x_1(0)=1\text{ m};\ x_n(0)=(n\text{-}1)L/N,\\ &\text{if }\ n\neq 1;\ v_n(0)=V(L/N), \end{aligned} \tag{12}$$

where $N=100$ is the total number of vehicles, $L=$ 1500 m is the road's length. Thus, we can obtain each vehicle's fuel consumption of the above four car-following models after a long time (see Figure 4).

From Figure 4, we can obtain the following results.

(1) Each vehicle's fuel consumption of Model D will be a constant after a long time, with the reasons as follows. The traffic interruption probability will make the drivers more careful, so they can response more quickly to a small perturbation and the small perturbation will eventually evolve into a uniform flow; each vehicle's fuel consumption of the other three car-following models will produce oscillating phenomena since the small perturbation cannot be dissipated after a long time and the stop-and-go traffic will occur.

(2) Each vehicle's fuel consumption of Model D is lower than that of the other three car-following models, which shows that Model D is the best of the car-following models from the perspective of vehicle's fuel consumption.

To display the differences of the vehicle' fuel consumptions of the above four car-following models, we calculate the evolution of the total fuel consumption after 3×10^5 time steps and the total fuel consumption value of the whole system during the period of 2000 time steps after 3×10^5 time steps (see Figure 5 and Table 2).

From Figure 5 and Table 2, we can conclude the following results.

(1) The changes of the total fuel consumptions of Model B, Model C and Model D are very slight, but the change of the total fuel consumption of Model A is relatively prominent (see Figure 4).

(2) The total fuel consumption of Model D is less than those of the other three car-following models (see Figure 4 and Table 2). The reasons are as follows. The vehicle fleet in Model D can travel at a nearly steady speed after a long time; the other three car-following models will produce traffic jams and the stop-and-go phenomenon, which makes the vehicles start and brake more frequently and the vehicle's fuel consumption increase. This result further proves that Model D is the best in the four car-following models from the perspective of fuel economy.

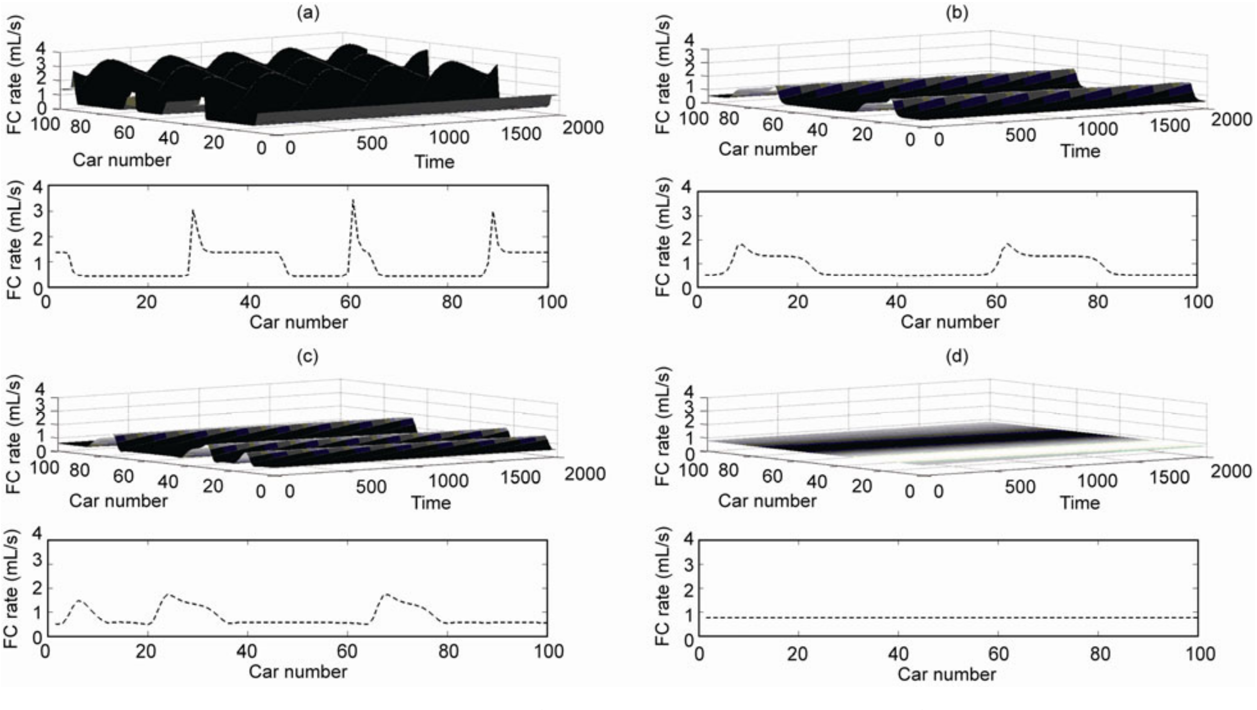

Figure 4 Evolution of each vehicle's fuel consumption after 3×10^5 time steps and the profile at time step 3×10^5. (a) Model A; (b) Model B; (c) Model C; (d) Model D.

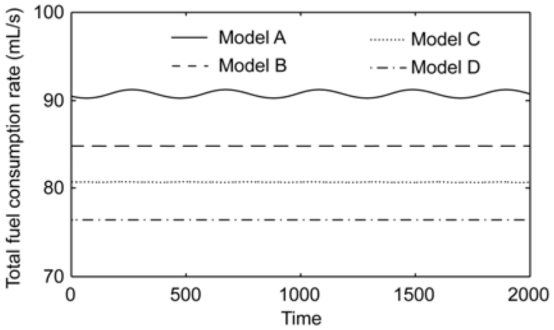

Figure 5 Total fuel consumption rate comparison.

Table 2 The total fuel consumption of the whole system during the period of 2000 time steps after 3×10^5 time steps

Car-following model	The total fuel consumption (mL)
Model A	907.699
Model B	848.231
Model C	806.553
Model D	763.753

4 Conclusions

In this paper, we introduced the vehicle's fuel economy into the car-following model and explored the relationship between the four car-following models and the vehicle's fuel consumption under two different traffic situations. The numerical results have shown that the vehicle's fuel consumption of Model D is the least in the four car-following models, showing that Model D can improve vehicle's fuel economy. Thus based on the characteristic of Model D, the driver of the following vehicle can consider traffic interruption before he/she makes a decision to accelerate or decelerate, even if traffic interruption will not happen. In addition, the drivers had better keep patience when meeting traffic jams, or else they will frequently start and brake vehicles, which can result in an increase in fuel consumption.

However, this paper has the following limitations.

Firstly, the results obtained in this paper depend on the assumed values of all parameters, which are from the original reference. Secondly, vehicles have emerged as one of the most critical sources of air pollution which is becoming worse, so it is necessary to study fuel consumption and emissions together. Thirdly, we do not incorporate vehicle's fuel consumption into the car-following model. In view of the aforementioned limitations, we should further study vehicle's fuel consumption experimentally and develop an exact model to study the vehicle's fuel consumption and emissions.

This work was supported by the National Natural Science Foundation of China (Grant Nos. 70971007 and 71271016) and the National Basic Research Program of China ("973" Project) (Grant No. 2012CB725404).

1 Pipes L A. An operational analysis of traffic dynamics. J Appl Phys, 1953, 24: 274–281

2 Chandler R E, Herman R, Montroll E W. Traffic dynamics: Studies in car-following. Oper Res, 1958, 6: 165–184

3 Gazis D C, Herman R, Rothery R W. Nonlinear follow-the-leader models of traffic flow. Oper Res, 1961, 9: 545–567

4 Chowdhury D, Santen L, Schreckenberg A. Statistical physics of vehicular traffic and some related systems. Phys Rep, 2000, 329: 199–329

5 Nagel K, Schreckenberg M. A cellular automaton model for freeway traffic. J Phys I France, 1992, 2: 2221–2229

6 Lenz H, Wagner C K, Sollacher R. Multi-anticipative car-following model. Eur Phys J B, 1999, 7: 331–335

7 Zhao B H, Hu M B, Jiang R, et al. A realistic cellular automaton model for synchronized traffic flow. Chin Phys Lett, 2009, 26: 118902

8 Helbing D. Traffic and related self-driven many-particle systems. Rev Mod Phys, 2001, 73: 1067–1141

9 Toshihiko N, Mikhail R, Diego S, et al. Shift to a low carbon society through energy systems design. Sci China Tech Sci, 2010, 53: 134–143

10 Ricardo M B, Apostolos P, Yang M Y. Overview of boosting options for future downsized engines. Sci China Tech Sci, 2011, 54: 318–331

11 Guzzella L, Wenger U, Martin R. IC-Engine downsizing and pressure-wave supercharging for fuel economy. SAE Paper, 2000-01-1019

12 Zhang Y J, Chen T, ZhuGe W L, et al. An integrated turbocharger design approach to improve engine performance. Sci China Tech Sci, 2010, 53: 69–74

13 Lin Y, Zheng X Q, Jin L, et al. A novel experimental method to evaluate the impact of volute's asymmetry on the performance of a high pressure ratio turbocharger compressor. Sci China Tech Sci, 2012, 55: 1695–1700

14 Ahn K, Rakha H, Trani A, et al. Estimating vehicle fuel consumption and emissions based on instantaneous speed and acceleration levels. J Trans Eng, 2002, 128: 182–190

15 Rakha H A, Ahn K, Moran K, et al. Viginia Tech comprehensive power-based fuel consumption model: Model development and testing. Trans Res D, 2011, 16: 492–503

16 Si B F, Zhang H Y, Zhong M, et al. Multi-criterion system optimization model for urban multimodal traffic network. Sci China Tech Sci, 2011, 54: 947–954

17 Wu C X, Zhao G Z, Ou B. A fuel economy optimization system with applications in vehicles with human drivers and autonomous vehicles. Trans Res D, 2011, 16: 515–524

18 Ahn K. Microscopic fuel consumption and emission modeling. Dissertation for Master Degree. Blacksburg: Virginia Polytechnic Institute and State University, 1998

19 Ferreira L. Modelling urban fuel consumption: Some empirical evidence. Trans Res A, 1985, 19: 253–268

20 Rakha H, Van Aerde M, Ahn K, et al. Requirements for evaluating traffic signal control impacts on energy and emissions based on instantaneous speed and acceleration measurements. Trans Res Rec, 2000, 1738: 56–67

21 Bando M, Hasebe K, Nakayama A, et al. Dynamical model of traffic congestion and numerical simulation. Phys Rev E, 1995, 51: 1035–1042

22 Jiang R, Wu Q S, Zhu Z J. Full velocity difference model for a car-following theory. Phys Rev E, 2001, 64: 017101

23 Zhao X M, Gao Z Y. A new car-following model: Full velocity and acceleration difference model. Eur Phys J B, 2005, 47: 145–150

24 Tang T Q, Huang H J, Wong S C, et al. A new car-following model with consideration of the traffic interruption probability. Chin Phys B, 2009, 18: 0975–0983

25 Tang T Q, Huang H J, Xu G. A new macro model with consideration of the traffic interruption probability. Physica A, 2008, 387: 6845–6856

26 Helbing D, Tilch B. Generalized force model of traffic dynamics. Phys Rev E, 1998, 58: 133–138