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Constructal optimization for H-shaped multi-scale heat exchanger based on entransy theory

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Analogizing with the definition of thermal efficiency of a heat exchanger, the entransy dissipation efficiency of a heat exchanger is defined as the ratio of dimensionless entransy dissipation rate to dimensionless pumping power of the heat exchanger. For the constraints of the total tube volume and total tube surface area of the heat exchanger, the constructal optimization of an H-shaped multi-scale heat exchanger is carried out by taking entransy dissipation efficiency maximization as optimization objective, and the optimal construct of the H-shaped multi-scale heat exchanger with maximum entransy dissipation efficiency is obtained. The results show that for the specified total tube volume of the heat exchanger, the optimal constructs of the first order T-shaped heat exchanger based on the maximizations of the thermal efficiency and entransy dissipation efficiency are obviously different with the lower mass flow rates of the cold and hot fluids. For the H-shaped multi-scale heat exchanger, the entransy dissipation efficiency decreases with the increase in mass flow rate when the heat exchanger order is fixed; for the specified dimensionless mass flow rate M ($M<32.9$), the entransy dissipation efficiency decreases with the increase in the heat exchanger order. The performance of the multi-scale heat exchanger is obviously improved compared with that of the single-scale heat exchanger. Moreover, the heat exchanger subjected to the total tube surface area constraint is also discussed in the paper. The optimization results obtained in this paper can provide a great compromise between the heat transfer and flow performances of the heat exchanger, provide some guidelines for the optimal designs of heat exchangers, and also enrich the connotation of entransy theory.

constructal theory, thermal efficiency, entransy dissipation efficiency, H-shaped multi-scale heat exchanger, generalized thermodynamic optimization

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1 Introduction

Heat exchanger, as an important thermodynamic device, is widely used in many fields, such as steel industry, chemical industry, energy industry, power industry and spaceflight. Therefore, it is significant to carry out optimal designs of the heat exchangers to improve the energy utilization of each industry. Some scholars analyzed and optimized the performances of the heat exchangers by taking minimum entropy generation as optimization objective [1–5]. However, the entropy generation paradox [1, 2], describing as the paradox between the minimum entropy generation and maximum effectiveness, appeared in the optimization designs. To solve the problem of entropy generation paradox, some scholars have defined various modified objectives based on the definition of entropy generation [6–11].

To reflect the essential property of heat transfer process, Guo et al. [12, 13] defined a new physical quantity, “entransy” (ever interpreted as heat transfer potential capacity in ref. [14]) and the extremum principle of entransy dissipation (a

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new theoretical guideline and criteria for heat transfer optimizations), and defined an equivalent thermal resistance for multi-dimensional heat conduction problems based on the entransy dissipation rate. The physical meaning of entransy was further expounded from the points of views of heat conduction physical mechanism and electro-thermal simulation experiment, etc. [15–17]. Henceforth, many scholars carried out a series of researches on the heat transfer optimizations based on entransy theory [18–56]. Specially, the research work in refs. [19–41] is the nonesuch of the combination of entransy theory and heat exchanger optimization, and refs. [42–56] are the representational work of the combination of entransy theory and constructal theory. In the analyses and optimizations of the heat exchangers, Liu et al. [19, 20] and Guo et al. [21] defined the equivalent thermal resistance based on the entransy dissipation, and analyzed the performance of the heat exchanger based on this thermal resistance. Song et al. [22] and Xia et al. [23] validated the uniformity principle of temperature difference field with the fixed heat transfer coefficient, and Guo et al. [24, 25] further discussed the relationship between the uniformity principles of entransy dissipation and temperature difference field with the fixed heat transfer coefficient and unfixed one, respectively. Liu et al. [26], Qian and Li [27] and Cheng et al. [28] analyzed and compared various heat exchanger performances based on entropy generation theory and entransy theory, and found that the design based on entransy theory was more suitable for the optimal design of heat exchangers. Guo et al. [29, 30] and Li et al. [31] studied the applications of the entransy dissipation number in the optimal designs of heat exchangers. The groups and networks of the heat exchangers were optimized in refs. [32–36], respectively. Xu et al. [37] deduced the entransy dissipation expression of the heat exchanger brought by the flow resistance, and Li et al. [38] considered the entransy dissipation brought by the finite heat transfer temperature difference and fluid viscosity simultaneously. Guo et al. [39, 40] analyzed the effect of viscous heating on the two-fluid heat exchangers with the fixed fluid viscosity and unfixed one, respectively. Guo et al. [41] found that the entransy dissipation number brought by the heat transfer was three magnitude orders larger than that brought by the flow resistance, and adopted the multi-objective optimization to solve the problem that the flow resistance was ignored in the optimization process.

The research work mentioned above chiefly focused on the performance analyses and optimizations of single scale heat exchangers. Based on constructal theory [57–64], some scholars carried out studies on multi-scale heat exchangers [65–67]. da Silva et al. [65] carried out constructal design of H-shaped multi-scale counter-flow heat exchanger with Murry law for the tube diameter ratio, analyzed its heat transfer and flow performances, and obtained the monotonic decreasing relationship between the thermal resistance and pumping power of the heat exchanger. Zimparov et al. [66] compared the performances of the multi-scale parallel- and counter-flow tree-shaped heat exchangers, and the results showed that the performance of the counter-flow heat exchanger was better than that of the parallel-flow heat exchanger for a higher value of pumping power. da Silva et al. [67] further carried out experimental study on counter-flow tree-shaped heat exchanger. Moreover, Chen et al. [68–71] analyzed the thermal efficiency (the ratio of heat transfer rate to the pumping power) of the fractal tree-shaped heat exchanger, made a compromise between its heat transfer rate and pumping power, and found that the thermal efficiency of the tree-shaped heat exchanger was obviously higher than that of the parallel-flow one.

In the performance optimizations of the heat exchangers, the factor of flow resistance can be easily annihilated when the sum of entransy dissipations brought by the heat transfer and flow resistance are considered. The annihilation of the flow resistance factor will lead to large pumping power consumption of the heat exchanger. Based on this consideration and analogizing with the definition of thermal efficiency in refs. [68–71], the entransy dissipation efficiency of a heat exchanger is defined as the ratio of dimensionless entransy dissipation rate to dimensionless pumping power in this paper. The objective of entransy dissipation efficiency could effectively overcome the shortcoming of the ignoring of flow resistance factor, gave attention to both the performances of heat transfer and fluid flow, and presented a compromise between the heat transfer rate and pumping power of the heat exchanger. Based on ref. [65], the constructal optimization of an H-shaped multi-scale heat exchanger will be carried out by taking entransy dissipation efficiency maximization as optimization objective, and the optimal construct of the H-shaped multi-scale heat exchanger considering both heat transfer and fluid flow performances will be obtained. The performance comparisons between the result obtained in this paper and those obtained based on H-shaped single-scale heat exchanger and maximum thermal efficiency objective will be performed.

2 Definition of entransy dissipation rate [12]

Entransy, which is a new physical quantity reflecting heat transfer ability of an object, was defined in ref. [12] as

$$E_{vh} = \frac{1}{2} Q_{vh} U_h = \frac{1}{2} Q_{vh} T, \tag{1}$$

where $Q_{vh}=Mc_vT$ is thermal capacity of an object with constant volume, U_h or T represents the thermal potential. The entransy dissipation function, which represents the entransy dissipation per unit time and per unit volume, is deduced as

$$\dot{E}_{h\phi} = -\dot{q}\nabla T = k(\nabla T)^2, \tag{2}$$

where $\dot{q}$ is thermal current density vector, and ∇T is the temperature gradient.

The entransy dissipation rate of the whole volume is [12]

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi} \mathrm{d}v = \int_v |\dot{q}\nabla T| \, \mathrm{d}v, \tag{3}$$

where v is the control volume.

For the performance optimization problem of the heat exchanger with a fixed boundary heat flux, the performance of the heat exchanger is optimized when its entransy dissipation rate is minimized; for the fixed boundary inlet temperature of the hot and cold fluids, the heat exchanger is optimized when its entransy dissipation rate is maximized.

3 Constructal optimization for first order T-shaped multi-scale heat exchanger

As shown in Figure 1 [65], the upper part of the T-shaped counter-flow heat exchanger in the rectangular area $A(=4L_0\times 2L_1)$ is composed of one tube with diameter D_1 and length L_1 and two tubes with diameter D_0 and length L_0. The hot fluid (mass flow rate $\dot{m}_1 = \dot{m}$ and inlet temperature $T_{h,\mathrm{in}}$) flows in from the inlet of D_1 tube, and finally flows out of the heat exchanger from the end of the D_0 tube located at the center of the elemental volume (mass flow rate $\dot{m}_0 = \dot{m}_1/2$). The structure of the lower part of the T-shaped heat exchanger is the same as that of the upper part one. The cold fluid (the same kind of fluid as hot fluid) flows in from the inlet of the D_0 tube (mass flow rate $\dot{m}_0$ and inlet temperature $T_{c,\mathrm{in}}$) located at the center of the elemental volume, and finally flows out of the heat exchanger from the end of

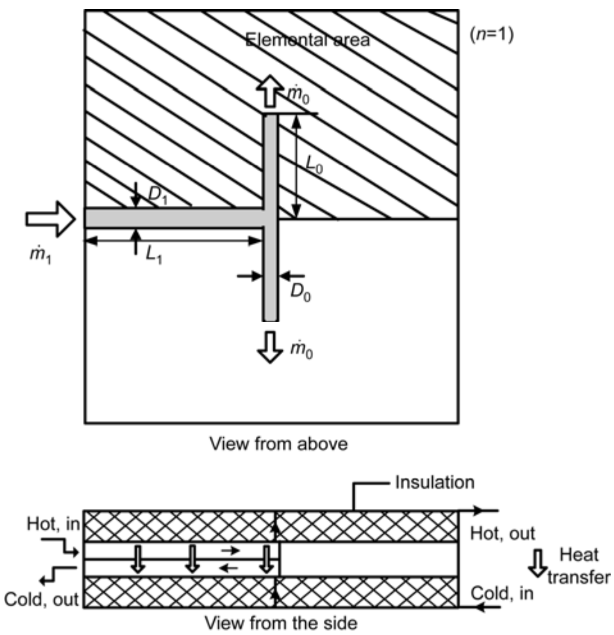

Figure 1 T-shaped multi-scale heat exchanger in a rectangular area [65].

the D_1 tube (mass flow rate $\dot{m}_1$). The hot and cold fluids exchange heat through the tube walls, and the tube walls of the two sides are adiabatic from the surroundings. The flows in cold and hot side tubes are both in Hagen-Poiseuille regime, and the fluid is the single-phase one with specific heat c_p, kinematic viscosity v and density ρ. Due to the different diameters of the D_1 and D_0 tubes, the heat exchanger as shown in Figure 1 can be named as multi-scale heat exchanger.

Both the total volume of the hot and cold fluid tubes and area occupied by the H-shaped heat exchanger network can be, respectively, given by

$$V = \frac{2\pi}{4}\left(2D_0^{\,2}L_0 + D_1^{\,2}L_1\right), \tag{4}$$

$$A = 8L_0L_1. \tag{5}$$

Assume that the tube diameter ratio of the heat exchanger obeys Murry law, i.e., $D_1=2^{1/3}D_0$. For the specified total volume V and area A occupied by the heat exchanger network and from eqs. (4) and (5), we yield

$$D_0 = \left[\frac{8V^2\tilde{L}}{\pi^2 A\left(1+2^{-1/3}\tilde{L}\right)^2}\right]^{1/4}, \tag{6}$$

$$L_0 = \left(\frac{A}{8\tilde{L}}\right)^{1/2}, \tag{7}$$

where $\tilde{L} = L_1/L_0$.

When the heat capacity rates of the cold and hot fluids are identical, the temperature difference ΔT along the length is a constant [22]. For the specified inlet temperatures $T_{h,\mathrm{in}}$ and $T_{c,\mathrm{in}}$ of the cold and hot fluids, the first law of thermodynamics requires

$$\left(U_1\pi D_1L_1 + 2U_0\pi D_0L_0\right)\Delta T = \dot{m}c_p\left(T_{h,\mathrm{in}} - T_{c,\mathrm{in}} - \Delta T\right), \tag{8}$$

where $U_i(i=0,1)$ is the total heat transfer coefficient between the two tubes when the heat conduction resistances of the dirt and tube wall are ignored,

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{1}{h_i}, \tag{9}$$

where $h_i(i=0,1)$ is the heat transfer coefficient of the fluid in single tube,

$$h_i = kNu_i/D_i, \tag{10}$$

where k is the thermal conductivity of the fluid, and $Nu_i(i=0,1)$ is the Nusselt number. When the flow in the tube is in laminar flow regime, the differences of the Nu_i in D_0 and D_1 tubes are small. To simplify the calculation, the value of the Nu_i in each tube is assumed to be identical [65, 72] (signed as Nu).

From eqs. (8)–(10), the temperature difference of the cold and hot fluids is

$$\Delta T = \frac{2MA^{1/2}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)}{2L_0 + L_1 + 2MA^{1/2}}, \tag{11}$$

where the dimensionless mass flow rate is $M = \dfrac{\dot{m}c_p}{\pi kNuA^{1/2}}$. From eqs. (3) and (8)–(11), the total heat transfer rate and entransy dissipation rate of the heat exchanger are, respectively, given by

$$\begin{aligned} q &= \left(U_1\pi D_1 L_1 + 2U_0\pi D_0 L_0\right)\Delta T \\ &= \frac{\pi kNuA^{1/2}M\left(2L_0 + L_1\right)\left(T_{h,\text{in}} - T_{c,\text{in}}\right)}{2L_0 + L_1 + 2A^{1/2}M}, \end{aligned} \tag{12}$$

$$\dot{E}_{vh\phi} = q\Delta T = \frac{2\pi kNuAM^2\left(L_1 + 2L_0\right)\left(T_{h,\text{in}} - T_{c,\text{in}}\right)^2}{\left(L_1 + 2L_0 + 2A^{1/2}M\right)^2}. \tag{13}$$

According to the formats of eqs. (12) and (13), these two equations can be nondimensionalized as

$$\tilde{q} = \frac{q}{\pi kNuA^{1/2}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{2^{1/2}M\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right)}{2^{1/2}\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right) + 8M}, \tag{14}$$

$$\begin{aligned} \tilde{\dot{E}}_{vh\phi} &= \frac{\dot{E}_{vh\phi}}{\pi kNuA^{1/2}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)^2} \\ &= \frac{8\cdot 2^{1/2}M^2\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right)}{\left[2^{1/2}\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right) + 8M\right]^2}. \end{aligned} \tag{15}$$

The dimensionless total pumping power consumed by the cold and hot fluids of the heat exchanger is

$$\begin{aligned} \tilde{W} &= W\frac{V^2}{4\left(kNu/c_p\right)^2\left(\nu/\rho\right)A^{5/2}} \\ &= 2\times\frac{128\dot{m}\nu}{\pi\rho}\times\left(\dot{m}_0\frac{L_0}{D_0^4} + \dot{m}\frac{L_1}{D_1^4}\right)\frac{V^2}{4\left(kNu/c_p\right)^2\left(\nu/\rho\right)A^{5/2}} \\ &= \frac{\pi^3M^2\left(2^{1/3} + \tilde{L}\right)^3}{2^{1/2}\tilde{L}^{3/2}}. \end{aligned} \tag{16}$$

Chen et al. [68–71] built a function of the thermal efficiency (the ratio of heat transfer quantity to pumping work) by considering heat transfer quantity and pumping work simultaneously. According to the definition of the heat exchanger thermal efficiency in refs. [68–71], the thermal efficiency η_{qW} of the multi-scale heat exchanger in this paper is defined as the ratio of dimensionless heat transfer rate to dimensionless pumping power

$$\eta_{qW} = \frac{\tilde{q}}{\tilde{W}}. \tag{17}$$

Analogizing with the definition of thermal efficiency of a heat exchanger, the entransy dissipation efficiency of the heat exchanger is defined as the ratio of dimensionless entransy dissipation rate to dimensionless total pumping power of the heat exchanger,

$$\eta_{EW} = \frac{\tilde{\dot{E}}_{vh\phi}}{\tilde{W}}, \tag{18}$$

where η_{EW} reflects the compromise of the heat transfer entransy dissipation rate and total pumping power of the heat exchanger. Substituting eqs. (14)–(16) into eqs. (17) and (18), the thermal efficiency η_{qW} and entransy dissipation efficiency η_{EW} of the first order T-shaped multi-scale heat exchanger are, respectively, given by

$$\eta_{qW} = \frac{\tilde{q}}{\tilde{W}} = \frac{2\tilde{L}\left(2 + \tilde{L}\right)}{\pi^2M\left(2^{1/3} + \tilde{L}\right)^3\left[2^{1/2}\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right) + 8M\right]}, \tag{19}$$

$$\eta_{EW} = \frac{\tilde{\dot{E}}_{vh\phi}}{\tilde{W}} = \frac{16\tilde{L}\left(2 + \tilde{L}\right)}{\pi^2\left(2^{1/3} + \tilde{L}\right)^3\left[2^{1/2}\left(2\tilde{L}^{-1/2} + \tilde{L}^{1/2}\right) + 8M\right]^2}. \tag{20}$$

From eqs. (19) and (20), for the fixed M, η_{qW} and η_{EW} are only the functions of $\tilde{L}$, and the constructal optimization of the first order T-shaped heat exchanger can be optimized by varying $\tilde{L}$.

Figure 2 shows the comparison of the optimal constructs of the first order T-shaped heat exchanger with different dimensionless mass flow rate M based on the maximum thermal efficiency and maximum entransy dissipation efficiency, respectively. From Figure 2, one can see that the optimal tube length ratios of the first and elemental tubes

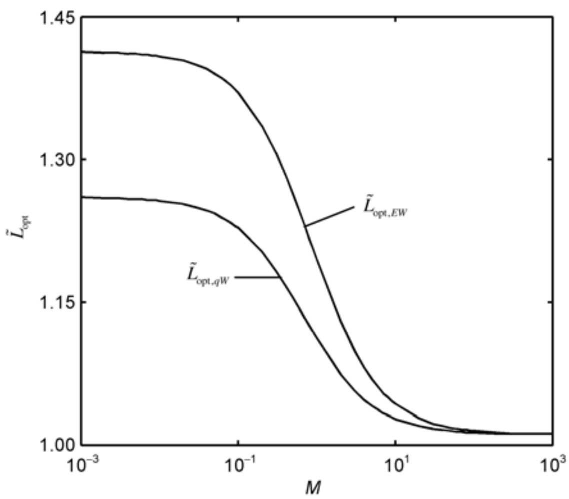

Figure 2 Characteristic of $\tilde{L}_{\text{opt}}$ versus M.

$\tilde{L}_{\text{opt},qW}$ and $\tilde{L}_{\text{opt},EW}$ based on the maximizations of the two objectives decrease with the increase in M. When $M << 1$, the two optimal tube length ratios of the first and elemental tubes are $\tilde{L}_{\text{opt},EW} = 1.413$ and $\tilde{L}_{\text{opt},qW} = 1.260$, respectively; with the increase in M, $\tilde{L}_{\text{opt},EW}$ is always larger than $\tilde{L}_{\text{opt},qW}$; when $M >> 1$, both $\tilde{L}_{\text{opt},qW}$ and $\tilde{L}_{\text{opt},EW}$ keep constant at 1.012. Therefore, for the lower mass flow rates of the cold and hot fluids, the optimal constructs of the first order T-shaped heat exchanger based on the maximizations of the thermal efficiency and entransy dissipation efficiency are obviously different.

4 Constructal optimization for the higher order T-shaped multi-scale heat exchanger

A higher order (n=4) of H-shaped counter-flow heat exchanger in the rectangular area A is shown in Figure 3 [65]. The hot fluid (mass flow rate $\dot{m}_n = \dot{m}$ and inlet temperature $T_{h,\text{in}}$) flows in from the inlet of D_n tube, exchanges heat with the cold fluid inside the two identical structures when flowing through the multi-scale tubes (diameter D_i, length L_i), and finally flows out of the heat exchanger from the end of the D_0 tube located at the center of the elemental volume (mass flow rate $\dot{m}_0$). The cold fluid, the same fluid as hot fluid, flows in from the inlet of the D_0 tube, and the mass flow rate and inlet temperature are $\dot{m}_0$ and $T_{c,\text{in}}$, respectively. The relationships for the size and mass flow rate of the H-shaped multi-scale heat exchanger can be, respectively, given by

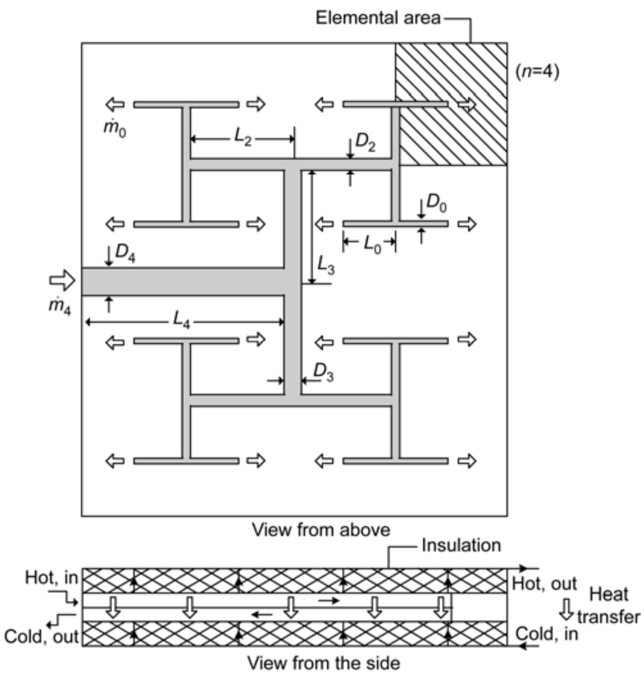

Figure 3 H-shaped multi-scale heat exchanger in a rectangular area [65].

$$L_i = 2^{i/2} L_0,\ D_i = 2^{i/3} D_0,\ \dot{m}_i = 2^{i-n}\dot{m},\quad n_i = 2^{n-i},\ (i = 0,\ 1,\ 2,\ \cdots,\ n), \tag{21}$$

$$V = 2\sum_{i=0}^{n} n_i L_i \frac{\pi D_i^2}{4}, \tag{22}$$

$$A = 2^n \left(2L_0 \times 2L_1\right). \tag{23}$$

From eqs. (21)–(23), one has

$$D_0 = \left[\frac{V\left(2^{1/6}-1\right)}{\pi A^{1/2}\left(2^{-\frac{25}{12}+\frac{2n}{3}} - 2^{-\frac{9}{4}+\frac{n}{2}}\right)}\right]^{1/2}, \tag{24}$$

$$L_0 = 2^{-\left(n+2+\frac{1}{2}\right)/2} A^{1/2}. \tag{25}$$

When the temperature difference ΔT of the cold and hot fluids is independent of its location, the first law of thermodynamics gives

$$\frac{\pi k Nu \Delta T}{2}\sum_{i=0}^{n} n_i L_i = \dot{m} c_p \left(T_{h,\text{in}} - T_{c,\text{in}} - \Delta T\right). \tag{26}$$

Substituting eqs. (21) and (25) into eq. (26) yields

$$\Delta T = \frac{4M\left(2-2^{1/2}\right)\left(T_{h,\text{in}} - T_{c,\text{in}}\right)}{2^{\frac{3}{4}+\frac{n}{2}} - 2^{1/4} + \left(8-2^{5/2}\right)M}. \tag{27}$$

From eqs. (3), (21), (24), (25) and (27), the dimensionless total heat transfer rate, the dimensionless entransy dissipation rate and the dimensionless total pumping power can be, respectively, given by

$$\tilde{q} = \frac{\pi k Nu}{2}\Delta T \sum_{i=0}^{n} n_i L_i \frac{1}{\pi k Nu A^{1/2}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{2^{1/4}\left[2^{(n+1)/2}-1\right]M}{2^{1/4}\left[2^{(n+1)/2}-1\right] + 4M\left(2-2^{1/2}\right)}, \tag{28}$$

$$\tilde{\dot{E}}_{vh\phi} = q\Delta T \frac{1}{\pi k Nu A^{1/2}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)^2} = \frac{2^{9/4}\left(2-2^{1/2}\right)\left[2^{(n+1)/2}-1\right]M^2}{\left[2^{1/4}\left(2^{n/2+1/2}-1\right) + 4M\left(2-2^{1/2}\right)\right]^2}, \tag{29}$$

$$\tilde{W} = 2\times\frac{128\dot{m}\nu}{\pi\rho}\sum_{i=0}^{n}\dot{m}_i\frac{L_i}{D_i^4}\frac{V^2}{4\left(kNu/c_p\right)^2\left(\nu/\rho\right)A^{5/2}} = \frac{\pi^3 M^2 2^{1/4-n/2}\left[2^{(n+1)/6}-1\right]^3}{\left(2^{1/6}-1\right)^3}. \tag{30}$$

From eqs. (28)–(30), the entransy dissipation efficiency η_{EW} of the H-shaped multi-scale heat exchanger is

$$\eta_{EW} = \tilde{\tilde{E}}_{vh\phi}\Big/\tilde{W} = \frac{2^{2+\frac{n}{2}}\left(2^{1/6}-1\right)^3\left(2-2^{1/2}\right)\left[2^{(n+1)/2}-1\right]}{\pi^2\left(2^{\frac{1+n}{6}}-1\right)^3\left\{2^{1/4}\left[2^{(n+1)/2}-1\right]+4M\left(2-2^{1/2}\right)\right\}^2}. \tag{31}$$

From Eq. (31), the entransy dissipation efficiency η_{EW} is relevant to M and n, and entransy dissipation efficiency performance of the H-shaped multi-scale heat exchanger can be analyzed by varying M and n.

Figure 4 shows the effect of the heat exchanger order number n on the characteristic of the entransy dissipation efficiency η_{qW} versus the dimensionless mass flow rate M. From Figure 4, one can see that for the specified n, η_{EW} decreases with the increase in M; the decrement gradually becomes large, and the entransy dissipation efficiency of the heat exchanger becomes small. When M<32.9, the entransy dissipation efficiency decreases with the increase in n; for a large mass flow rate (when n⩽20, M>508.1), the entransy dissipation efficiency of the heat exchanger reaches its minimum when n=4. This is because the entransy dissipation efficiency gives attention to both heat transfer and fluid flow performances, and the relationship between the entransy dissipation efficiency and heat exchanger order is no more than a monotonic one. When n>4, the entransy dissipation efficiency increases with the increase in n.

Figure 5 shows the effect of tube-diameter ratio on the entransy dissipation efficiency η_{EW} of the H-shaped multi-scale heat exchanger. When the tube-diameter ratio is $D_i=2^{i/3}D_0$ (i=1, 2, ···, n), the tube-diameter ratio obeys Murry law; when this ratio is $D_i=2^{i/2}D_0$ (i=1, 2, ···, n), the flow velocities in the tubes are equal to each other; when this ratio is $D_i=D_0$(i=1, 2, ···, n), all the tube cross section areas are equal to each other, and the heat exchanger belongs to the H-shaped single-scale one in this case. From Figure 5, one can see that when n=4, the entransy dissipation efficiency of the H-shaped multi-scale heat exchanger with Murry law is increased by 36.54% compared with that with equal flow velocity in the tubes, and by 198.94% compared with that with equal tube cross section area of the tubes. These increments will further increase when n increases. Therefore, the performance of the H-shaped multi-scale heat exchanger with Murry law for the tube-diameter ratio is greatly improved. Actually, the tube-diameter ratio obeying Murry law is the optimal tube-diameter ratio of the multi-scale heat exchanger in this paper, which can be further verified by the numerical calculations. Compared with the H-shaped multi-scale heat exchanger with the H-shaped single-scale heat exchanger, the performance of the multi-scale heat exchanger is obviously improved.

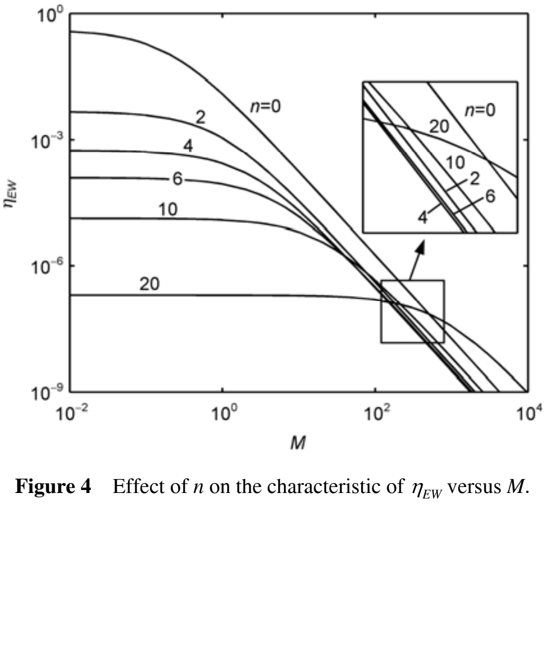

Figure 4 Effect of n on the characteristic of η_{EW} versus M.

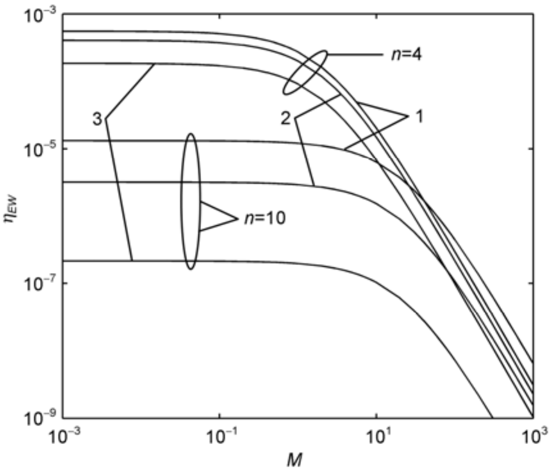

Figure 5 Effect of tube ratio on η_{EW}. 1, $D_i=2^{i/3}D_0$; 2, $D_i=2^{i/2}D_0$; 3, $D_i=D_0$.

5 Constructal optimization for H-shaped heat exchanger subjected to tube surface area constraint

In Sections 3 and 4, the constructal optimizations of the H-shaped multi-scale heat exchangers are carried out subjected to the total tube volume constraint. In practical designs of heat exchangers, the heat transfer area of the tubes is an important constraint. The constructal optimization of the H-shaped multi-scale heat exchanger will be carried out subjected to the tube surface area constraint in this section.

The total tube surface area on the cold and hot sides of the H-shaped multi-scale heat exchanger is

$$A_a = 2\sum_{i=0}^{n}\pi n_i L_i D_i. \tag{32}$$

For the fully-developed laminar flow in the tubes, the tube diameter ratio subjected to the tube surface area constraint can be given as [73]

$$D_i = 2^{2i/5} D_0 \left(i = 1,\ 2,\ \cdots,\ n \right). \tag{33}$$

From eqs. (21), (23), (32) and (33), one has

$$D_0 = \frac{2^{-13/20-2n/5}\left(2-2^{9/10}\right)A_a}{\pi A^{1/2}\left[2^{(n+1)/10}-1\right]}. \tag{34}$$

The dimensionless total pumping power consumed by cold and hot fluids of the H-shaped multi-scale heat exchanger is

$$\begin{aligned}\tilde{W} &= W\frac{{A_a}^4}{16\left(kNu/c_p\right)^2\left(\nu/\rho\right)A^{7/2}} \\ &= 2\times\frac{128\dot{m}\nu}{\pi\rho}\sum_{i=0}^{n}\dot{m}_i\frac{L_i}{D_i^4}\frac{{A_a}^4}{16\left(kNu/c_p\right)^2\left(\nu/\rho\right)A^{7/2}} \\ &= \frac{64\times2^{1/4}\left[2^{(n+1)/10}-1\right]^5\pi^5M^2}{\left(2-2^{9/10}\right)^5}.\end{aligned} \tag{35}$$

From eqs. (29) and (35), the entransy dissipation efficiency η_{EW} of the H-shaped multi-scale heat exchanger becomes

$$\begin{aligned}\eta_{EW} &= \tilde{\dot{E}}_{vh\phi}\Big/\tilde{W} \\ &= \frac{\left(2-2^{1/2}\right)\left(2-2^{9/10}\right)^5\left[2^{(n+1)/2}-1\right]}{16\pi^4\left[2^{(n+1)/10}-1\right]^5\left[2^{1/4}\left(2^{\frac{1+n}{2}}-1\right)+4M\left(2-2^{1/2}\right)\right]^2}.\end{aligned} \tag{36}$$

Figure 6 shows the effect of the heat exchanger order number n on the entransy dissipation efficiency η_{qW} subjected to the tube surface area constraint. From Figure 6, one can see that for the specified n, η_{EW} also decreases with the increase in M. For the specified M, the entransy dissipation efficiency always decreases with the increase in n, and the decrement tends to be small. This is obviously different from the characteristic of η_{EW} versus M with a large dimensionless mass flow rate.

Figure 7 shows the effect of tube-diameter ratio on the entransy dissipation efficiency η_{EW} subjected to the tube surface area constraint. From Figure 7, the entransy dissipation efficiency of the H-shaped multi-scale heat exchangers with equal flow velocity in the tubes only has a little difference from that with Murry law. When n=4, the entransy dissipation efficiency of the H-shaped multi-scale heat exchanger with Murry law is increased by 283.91% as compared with that with equal tube cross section area of the tubes. With the further increase in n, the increment of the entransy dissipation efficiency becomes more obvious. Therefore, for the tube surface area constraint, the performance of the multi-scale heat exchanger remains improved compared with that of the single-scale one.

6 Conclusions

By analogizing with the definition of thermal efficiency of a heat exchanger, the entransy dissipation efficiency of a heat exchanger is defined (the ratio of dimensionless entransy dissipation rate to dimensionless pumping power of the heat exchanger). For the specified total tube volume V and total tube surface area A_a, the constructal optimization of the H-shaped multi-scale heat exchanger is carried out, and its optimal construct with maximum entransy dissipation efficiency is obtained. The results show that:

(1) For the specified total tube volume V of the heat exchanger, the optimal constructs of the first order T-shaped heat exchanger based on the maximizations of the thermal efficiency and entransy dissipation efficiency are obviously different with the lower mass flow rates of the cold and hot fluids.

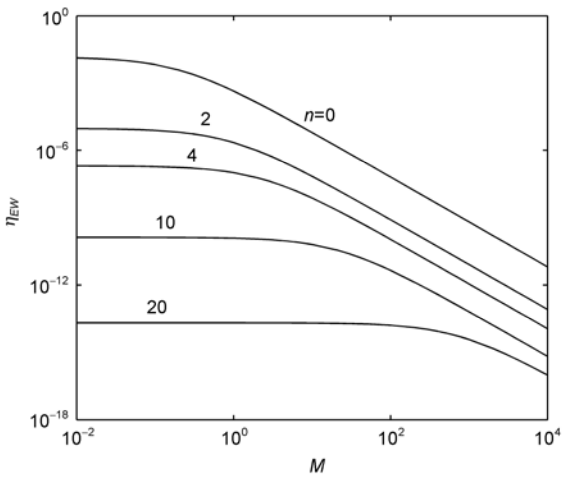

Figure 6 Effect of n on η_{EW} subjected to the tube surface area constraint.

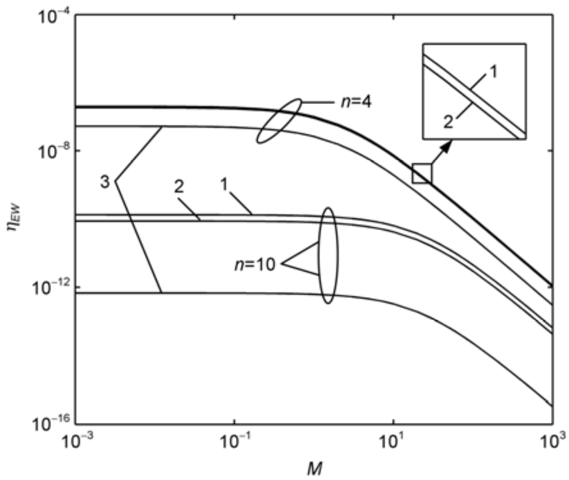

Figure 7 Effect of tube ratio on η_{EW} subjected to the tube surface area constraint. 1, D_i=$2^{i/3}D_0$; 2, D_i=$2^{i/2}D_0$; 3, D_i=D_0.

(2) For the H-shaped multi-scale heat exchanger with the fixed heat exchanger order number, the entransy dissipation efficiency decreases with the increase in dimensionless mass flow rate; this decrement gradually becomes large, and the corresponding entransy dissipation efficiency of the heat exchanger becomes small.

(3) When M<32.9, the entransy dissipation efficiency decreases with the increase in heat exchanger order number n; for a large dimensionless mass flow rate (when $n\leqslant 20$, M>508.1), the entransy dissipation efficiency of the heat exchanger reaches its minimum when n=4, and increases with the increase in n when n>4.

(4) When n=4, the entransy dissipation efficiency of the H-shaped multi-scale heat exchanger with Murry law is increased by 36.54% compared with that with equal flow velocity in the tubes, and by 198.94% compared with that with equal tube cross section area of the tubes. Therefore, compared with the H-shaped multi-scale heat exchanger with the H-shaped single-scale heat exchanger, the performance of the multi-scale heat exchanger is greatly improved; the same conclusion can be derived for the tube surface area constraint.

The entransy dissipation rate objective is the measurement of the heat transfer irreversibility. For the fixed inlet temperatures of the cold and hot fluids, the performance of the heat exchanger is optimized when its entransy dissipation rate is minimized. However, this optimization does not consider the flow performance of the heat exchanger. The objective of entransy dissipation efficiency could effectively overcome the shortcomings of the annihilation of flow resistance factor and a large total pumping power of the heat exchanger, gives attention to both heat transfer and fluid flow performances, and presents a compromise between the entransy dissipation rate brought by the heat transfer and total pumping power of the heat exchanger. Moreover, it is assumed that the cold and hot fluids in the heat exchanger are the same fluid; for the different cold and hot fluids, the method used in this paper can also be adopted to analyze the performance of the heat exchanger when the heat capacity rates of the cold and hot fluids are identical and the specific heat difference of the two fluids is little. The constructal optimization of the heat exchanger based on entransy theory can provide some important guidelines for the designs of heat exchangers. Therefore, one can adopt the optimal construct of the heat exchanger obtained based on maximum entransy dissipation efficiency when carrying out the optimal design of the heat exchangers in engineering.

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