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Constructal entransy dissipation rate minimization for leaf-like fins

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Based on constructal theory, the constructs of the leaf-like fins are optimized by taking minimum entransy dissipation rate (for the fixed total thermal current, i.e., the equivalent thermal resistance) as optimization objective. The optimal constructs of the leaf-like fins with minimum dimensionless equivalent thermal resistance are obtained. The results show that there exists an optimal elemental leaf-like fin number, which leads to an optimal global heat conduction performance of the first order leaf-like fin. The Biot number has little effects on the optimal elemental fin number, optimal ratios of length and width of the elemental and first order leaf-like fins; with the increase of the thermal conductivity ratio of the vein and blade, the optimal elemental fin number and optimal ratio of the length and width of the elemental leaf-like fin increase, and the optimal shape of the first order leaf-like fin becomes tubbier. The optimal construct based on entransy dissipation rate minimization is obviously different from that based on maximum temperature difference minimization. The dimensionless equivalent thermal resistance based on entransy dissipation rate minimization is reduced by 11.54% compared to that based on maximum temperature difference minimization, and the global heat conduction performance of the leaf-like fin is effectively improved. For the same volumes of the elemental and first order leaf-like fins, the minimum dimensionless equivalent thermal resistance of the first order of the leaf-like fin is reduced by 30.10% compared to that of the elemental leaf-like fin, and the global heat conduction performance of the first order leaf-like fin is obviously better than that of the elemental leaf-like fin. Essentially, this is because the temperature gradient field of the first order leaf-like fin based on entransy dissipation rate minimization is more homogenous than that of the elemental leaf-like fin. The dimensionless equivalent thermal resistance defined based on entransy dissipation rate reflects the average heat transfer performance of the leaf-like fin, and can provide some guidelines for the thermal design of the fins from the viewpoint of heat transfer optimization.

constructal theory, entransy dissipation rate, leaf-like fin, heat transfer optimization, generalized thermodynamic optimization

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1 Introduction

With the development of the micro-electronic technology, the miniaturization and high integration of the electronic devices are strong trends, and thermal current densities of the electronic devices become higher and higher. Therefore, how to dissipate the heats from the interior of the electronic devices quickly and efficiently is a key question that needs to be solved in the development way of the super micro-electronic devices. Fins, as a common technology of enhancement heat transfer, are widely applied in the field of cooling electronic devices.

Bejan firstly applied the constructal theory [1–7] in the

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"volume-point" heat conduction problem of cooling electronic devices [8]. Henceforth, many scholars had done plenty of researches on the fin heat transfer problems [9–27] based on constructal theory. Bejan and Dan [10] used the analytical method and finite element method to carry out constructal optimization of a tree-shaped fin, and the results showed that the results obtained based on these two methods agreed with each other. Almogbel and Bejan [11] carried out constructal optimizations of the cylindrical tree-shaped assemblies composed of pin fin and taper fin elements, respectively, and the result showed that the more efficient construct of the tree-shaped assemblies looked more natural. Bejan and Almogbel [12] carried out the constructal optimizations of T-, τ- and umbrella-shaped fins by taking maximum heat transfer rate as optimization objective. Moreover, some scholars further optimized the single- and multi-stage T-, Y-, T-Y- and two-stage Y-shaped fins [13–19] by using analytical and numerical methods, respectively. Combelles [20] optimized a single leaf-like fin by taking maximum heat transfer rate as optimization objective and using analytical and numerical methods, respectively, and obtained the optimal construct of the leaf-like fin. The effects of the Biot number and thermal conductivity ratio of the vein and blade on the optimal construct of the leaf-like fin were analyzed. Khaled [21] studied the heat transfer performances of five kinds of fins with different heat convection coefficients and free stream temperatures at their upper and lower surfaces, and derived the analytical solutions of the heat transfer rates of four kinds of fins. The results showed that the heat transfer performances of the fins could be improved when the effective thermal conductivity, cross-sectional area and some other parameters of the fins increased. Khaled [22] further studied the heat transfer performance of a permeable plate fin, and the result showed that the permeable fin had a better heat transfer performance than the conventional fins. Ref. [23] studied the heat transfer performance of a plate fin for the fixed volume of the fin, and derived the maximum heat transfer rate performance of the plate fin with nature and forced convection boundaries by optimizing a single variable. Zhang and Liu [24] optimized the vertical rectangular fin arrays by taking maximum heat transfer rate as optimization objective and using analytical and finite volume methods, respectively, obtained the optimal spacing of the rectangular fin arrays, and found that the optimization results obtained by these two methods were consistent. Bello-Ochende et al. [25] carried out the constructal optimization of the pin fin arrays with two columns, the optimal geometries obtained were the multi-scale pin-fin arrays with different diameters and heights, and the result obtained by magnitude analysis agreed with that obtained by the numerical calculation. Kundu and Bhanja [26] investigated the heat transfer enhancement problem of porous fins by using analytical method, and compared the heat transfer performances of the porous fins with simple, approximate and exact models by using analytical and numerical methods, respectively. Sharqawy and Zubair [27] studied the circle fin with simultaneous heat and mass transfer, and obtained the analytical solution of fin temperature distribution under fully wet condition, which included the special solution for a dry-fin case.

The researches of the pin heat transfer performances above are all based on the optimization objectives of heat transfer rate and efficiency, and cannot reflect the global heat transfer performances of the fins. To illustrate the essential characteristics of heat transfer processes, Guo et al. [28, 29] put forward a new physical quantity, "entransy" (ever interpreted as heat transfer potential capacity in ref. [30]) and the extremum principle of entransy dissipation (a new theoretical guideline and criteria for heat transfer optimizations), and defined an equivalent thermal resistance for multidimensional heat conduction problems based on the entransy dissipation rate. The physical meaning of entransy was further expounded from the angles of heat conduction physical mechanism and electrothermal simulation experiment, etc. [31–33]. Henceforth, many scholars carried out a series of deep researches on the heat transfer optimization based on minimum entransy dissipation rate [34–67], and their work further illustrated that the optimization objective of the entransy dissipation rate had great advantages in fields of heat transfer optimizations. Specially, the research work in refs. [55–67] was the major research progress by the combination of the extremum principle of entransy dissipation and constructal theory, and refs. [58, 61] were the representational work of the heat transfer optimization of fins based on the extremum principle of entransy dissipation and constructal theory.

Based on ref. [20], the constructs of the leaf-like fins will be optimized by taking minimum entransy dissipation rate as optimization objective and using analytical method. The elemental leaf-like fin was studied in ref. [20], and the model of a first order leaf-like fin will be established by assembling many elemental leaf-like fins. The global heat transfer performance comparisons of the elemental leaf-like fin based on the minimizations of entransy dissipation rate and maximum temperature difference, as well as elemental and first order leaf-like fins based on the minimization of entransy dissipation rate will be carried out.

2 Definition of entransy dissipation rate [28]

Entransy, which is a new physical quantity reflecting heat transfer ability of an object, was defined in ref. [28] as

$$E_{vh}=\frac{1}{2}Q_{vh}U_h=\frac{1}{2}Q_{vh}T, \tag{1}$$

where $Q_{vh}=Mc_vT$ is thermal capacity of an object with constant volume, U_h or T represents the thermal potential. The entransy dissipation function, which represents the entransy dissipation per unit time and per unit volume, is deduced as [28]

$$\dot{E}_{h\phi} = -\dot{q} \cdot \nabla T = k(\nabla T)^2, \tag{2}$$

where $\dot{q}$ is thermal current density vector, and ∇T is the temperature gradient. In steady-state heat conduction, $\dot{E}_{h\phi}$ can be calculated as the difference between the entransy input and the entransy output of the object, i.e.

$$\dot{E}_{h\phi} = E_{h,\text{in}} - E_{h,\text{out}}. \tag{3}$$

The entransy dissipation rate of the whole volume in the "volume-to-point" conduction is

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi} \mathrm{d}v = \int_v k(\nabla T)^2 \mathrm{d}v. \tag{4}$$

where v is the control volume.

The equivalent thermal resistance for multidimensional heat conduction problems with specified heat flux boundary condition is given as follows [28]:

$$R_h = \dot{E}_{vh\phi} \big/ \dot{Q}_h^2, \tag{5}$$

where $\dot{Q}_h$ is the thermal current. From eq. (5), for the fixed $\dot{Q}_h$, the entransy dissipation rate $\dot{E}_{vh\phi}$ is proportional to the equivalent thermal resistance R_h, the heat transfer optimization based on the minimization of entransy dissipation rate is equal to that based on the minimization of equivalent thermal resistance. Therefore, for the fixed total thermal current of the leaf-like fin, the constructal optimization of the leaf-like fin can be carried out by taking the minimization of equivalent thermal resistance as optimization objective.

3 Constructal optimization of the elemental leaf-like fin

An elemental leaf-like fin with a blade ($2H_0 \times L_0 \times t, k_0$) and a vein ($A_0 \times L_0$, k_p) attached to the symmetry axis of the blade is shown in Figure 1 [20]. The heat current (q_0) flows along the vein from the root (x=0) to the top ($x=L_0$) of the vein, and then flows into the blade ($H_0 \times t \times \Delta x$) located on the both sides of the vein along the direction perpendicular to the vein. Finally, the heat is dispersed from the surface of the blade by heat convection. The materials of leaf-like fin are isotropic, and their thermal conductivities k_0 and k_p are constants ($k_p > k_0$). The heat transfer coefficient h of the blade is of homogenization over all the external surfaces, and the top of the vein and the perimeter of the blade are considered to be adiabatic. It is assumed that the thermal conductivity of the vein (k_p) is much higher than that of the blade (k_0), the cross-sectional area of the vein (A_0) is small, both the vein and the blade are slender enough ($H_0 \ll L_0$).

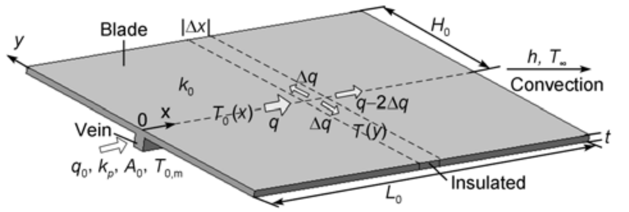


Figure 1 Elemental leaf-like fin [20].

With these assumptions, the flows in the vein and blade are considered to be one-dimensional, i.e. the heat transfer direction is approximately parallel to x-direction in the vein, and is parallel to y-direction in the blade. Heat exchanging does not exist in the adjacent blades, which will be verified later in this section. For the fixed heat transfer rate of the elemental leaf-like fin q_0, ambient temperature T_∞ and total volume of the elemental leaf-like fin $V_0=(A_0+2H_0t)L_0$, the elemental vein cross-sectional area A_0, the width H_0, length L_0 as well as thickness t of the elemental leaf-like fin are free to vary.

The temperature distribution of the blade $H_0 \times t \times \Delta x$ (y >0) in the elemental leaf-like fin is [68]

$$T(y) - T_\infty = [T_0(x) - T_\infty] \frac{\cosh[a_0(H_0 - y)]}{\cosh(a_0 H_0)}, \tag{6}$$

where $a_0 = (2h/k_0 t)^{1/2}$. For the case y<0, the temperature distribution can be obtained by replacing H_0 by $-H_0$ in eq. (6).

The temperature distribution along x axis of the vein and the heat current entering the root of the vein are, respectively, given by [20]

$$T_0(x) - T_\infty = (T_{0,\text{m}} - T_\infty) \frac{\cosh[a_1^{1/2}(L_0 - x)]}{\cosh(a_1^{1/2} L_0)}, \tag{7}$$

$$q_0 = k_p A_0 \left(\frac{\mathrm{d}T_0}{\mathrm{d}x} \right)_{x=0} = k_p A_0 (T_{0,\text{m}} - T_\infty) \tanh(a_1 L_0), \tag{8}$$

where $a_1 = \dfrac{4h}{k_p A_0 a_0} \tanh(a_0 H_0)$.

From eqs. (4) and (6)–(8), the entransy dissipation rate of the k_0 blade is

$$\begin{aligned} \dot{E}_{vh\phi 01} &= 2t \int_0^{L_0} \int_0^{H_0} k_0 \left[\frac{\mathrm{d}T(y)}{\mathrm{d}y} \right]^2 \mathrm{d}y\mathrm{d}x \\ &= k_0 a_0 t q_0^2 \operatorname{csch}^2(a_1^{1/2} L_0) \operatorname{sech}^2(a_0 H_0) \\ &\quad \times [\sinh(2a_0 H_0) - 2a_0 H_0][2a_1^{1/2} L_0 \\ &\quad + \sinh(2a_1^{1/2} L_0)]/(8A_0^2 k_p^2 a_1^{3/2}). \end{aligned} \tag{9}$$

From eqs. (4), (7) and (8), the entransy dissipation rate of the k_p vein is

$$\dot{E}_{vh\phi 02}=\int_0^{L_0}k_pA_0\left[\frac{\mathrm{d}T_0(x)}{\mathrm{d}x}\right]^2\mathrm{d}x$$
$$=\frac{q_0{}^2\mathrm{csch}^2(a_1^{1/2}L_0)[a_1^{-1/2}\sinh(2a_1^{1/2}L_0)-2L_0]}{4A_0k_p}. \quad (10)$$

According to eqs. (9) and (10), the entransy dissipation rate of the whole elemental leaf-like fin is

$$\dot{E}_{vh\phi 0}=\dot{E}_{vh\phi 01}+\dot{E}_{vh\phi 02}. \quad (11)$$

From eqs. (5) and (11), the dimensionless equivalent thermal resistance of the whole elemental leaf-like fin becomes

$$\begin{aligned}\tilde{R}_{h0}&=\frac{\dot{E}_{vh\phi 0}}{q_0{}^2\cdot k_0{}^{-1}\cdot V_0{}^{-1/3}}\\&=\mathrm{csch}^2(a_1^{1/2}\tilde{L}_0V_0^{1/3})\{-a_0{}^2\tilde{H}_0\tilde{t}V_0^{1/3}\mathrm{sech}^2(a_0\tilde{H}_0V_0^{1/3})\\&\quad\times[2a_1^{1/2}\tilde{L}_0V_0^{1/3}+\sinh(2a_1^{1/2}\tilde{L}_0V_0^{1/3})]+2a_1^{1/2}\tilde{L}_0V_0^{1/3}\\&\quad\times[\tilde{t}a_0\tanh(a_0\tilde{H}_0V_0^{1/3})-a_1\tilde{k}\tilde{A}_0V_0^{1/3}]+\sinh(2a_1^{1/2}\tilde{L}_0V_0^{1/3})\\&\quad\times[a_1\tilde{k}\tilde{A}_0V_0^{1/3}+\tilde{t}a_0\tanh(a_0\tilde{H}_0V_0^{1/3})]\}/(4a_1^{3/2}\tilde{k}^2\tilde{A}_0{}^2V_0^{2/3}),\end{aligned} \quad (12)$$

where $\tilde{A}_0=A_0/V_0^{2/3}$, $(\tilde{H}_0,\tilde{L}_0,\tilde{t})=(H_0,L_0,t)/V_0^{1/3}$, $\tilde{k}=k_p/k_0$. The dimensionless equivalent thermal resistance of the elemental leaf-like fin $\tilde{R}_{h0}$ is the function of $\tilde{A}_0$, $\tilde{H}_0$, $\tilde{L}_0$ and $\tilde{t}$.

According to refs. [20, 68], for the fixed heat current entered from the root of the blade $H_0\times t\times\Delta x$, the optimal construct of the blade $H_0\times t\times\Delta x$ based on minimum maximum temperature difference is

$$\tilde{t}^{1/2}=Bi^{1/2}\tilde{H}_0, \quad (13)$$

where the Biot number is $Bi=0.996^2hV_0^{1/3}/k_0$ [20]. To compare the result obtained in this paper with that obtained in ref. [20] conveniently, the ratio parameter r is set as

$$r=\frac{Bi^{1/2}\tilde{H}_0}{\tilde{t}^{1/2}}. \quad (14)$$

When r =1, eq. (14) is simplified into eq. (13). The total volume constraints of the elemental leaf-like fin can be nondimensionalized as

$$(\tilde{A}_0+2\tilde{H}_0\tilde{t})\tilde{L}_0=1. \quad (15)$$

Substituting eq. (12) into eqs. (14) and (15) to eliminate $\tilde{H}_0$ and $\tilde{L}_0$, the function $\tilde{R}_{h0}$ in eq. (12) is relative to Bi, $\tilde{k}$, $\tilde{A}_0$, $\tilde{t}$ and r. For the fixed Bi in eq. (14) and $\tilde{k}$ in eq. (12), the function $\tilde{R}_{h0}$ in eq. (12) after eliminating $\tilde{H}_0$ and $\tilde{L}_0$ has three degrees of freedom $\tilde{A}_0$, $\tilde{t}$ and r, and one can use $\tilde{A}_0$, $\tilde{t}$ and r as optimization variables to carry out constructal optimization for the elemental leaf-like fin.

Figure 2 shows the dimensionless equivalent thermal resistance $\tilde{R}_{h0}$ versus $\tilde{A}_0$ characteristic with Bi=0.01, $\tilde{k}$ = 100, r =2 and different $\tilde{t}$. From Figure 2, for the fixed $\tilde{t}$, there exists an optimal $\tilde{A}_0$ ($\tilde{A}_{0,\mathrm{opt}}$) which leads to minimum $\tilde{R}_{h0}$ ($\tilde{R}_{h0,\mathrm{m}}$); with the increase in $\tilde{t}$, $\tilde{R}_{h0,\mathrm{m}}$ decreases first, and then increases.

Figure 3 shows $\tilde{R}_{h0,\mathrm{m}}$ and $\tilde{A}_{0,\mathrm{opt}}$ versus $\tilde{t}$ characteristics with Bi=0.01, $\tilde{k}=100$ and r=2. The subscripts "m" and "mm" denote $\tilde{R}_{h0}$ minimized once and twice, respectively, and "oo" denotes the corresponding twice optimization. From Figure 3, $\tilde{A}_{0,\mathrm{opt}}$ increases first and then decreases with the increase in $\tilde{t}$; there exist the optimal $\tilde{t}$ ($\tilde{t}_{\mathrm{opt}}=0.0119$) and twice optimal $\tilde{A}_0$ ($\tilde{A}_{0,\mathrm{oo}}=0.0522$) which lead to double minimum $\tilde{R}_{h0}$ ($\tilde{R}_{h0,\mathrm{mm}}=2.2082$).

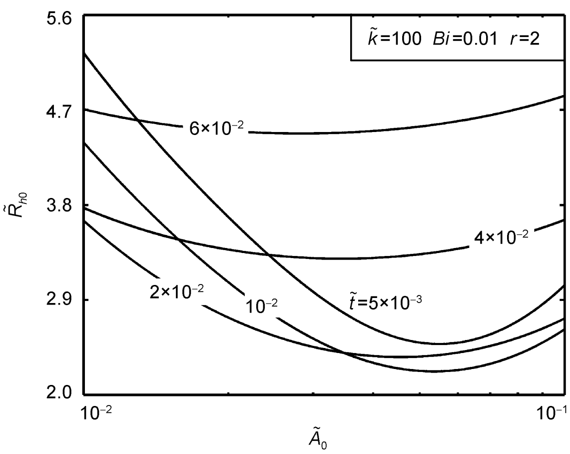


Figure 2 Effect $\tilde{t}$ on the characteristic of $\tilde{R}_{h0}$ versus $\tilde{A}_0$.

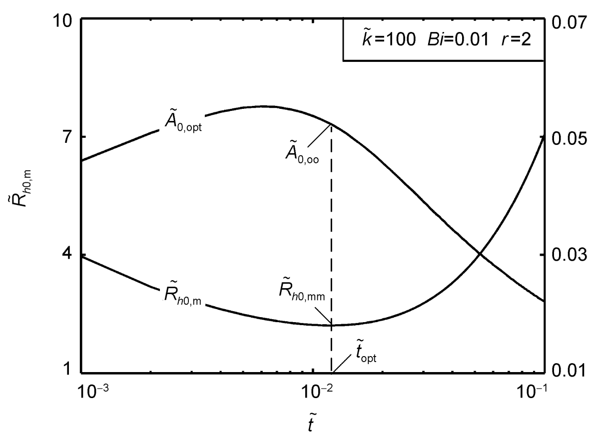


Figure 3 $\tilde{R}_{h0,\mathrm{m}}$ and $\tilde{A}_{0,\mathrm{opt}}$ versus $\tilde{t}$ characteristics.

Figure 4 shows $\tilde{R}_{h0,\mathrm{mm}}$, $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ versus r characteristics with Bi =0.01 and $\tilde{k}=100$. From Figure 4, the twice optimal elemental vein cross-sectional area in the elemental leaf-like fin and optimal thickness of the blade decrease with the increase in r; $\tilde{R}_{h0,\mathrm{mm}}$ decreases with the decrease in r, and there does not exist an optimal r (r_{opt}) which leads to minimum $\tilde{R}_{h0,\mathrm{mm}}$. Actually, the elemental vein cross-sectional area and the thickness of the blade are small, therefore, it is impossible to get an infinitesimal r which leads to minimum $\tilde{R}_{h0,\mathrm{mm}}$.

Figure 5 shows the effect of the Biot number Bi on the optimal construct of the elemental leaf-like fin with $\tilde{k}=100$ and r=1. From Figure 5, when $\tilde{k}=100$ ($\tilde{k} \gg 1$), with the increase of Bi, the length and width of the elemental leaf-like fin decrease, and the vein cross-sectional area and the thickness of the blade increase, however, the ratio of the length and width as well as the cross-sectional area ratio of the vein and blade of the elemental leaf-like fin almost keep constant. The optimal shape of the blade keeps slender, and the cross-sectional area of the vein is equal to that of the blade. When $Bi \ll 1$, the heat transfer thermal resistance of the blade surface is much larger than the heat conductance thermal resistance, the shape of the blade keeps slender, and the assumption that the heat conduction in the blade is one-dimensional is valid. With the increase in Bi, the heat convection at the surface of the elemental leaf-like fin becomes stronger, the minimum dimensionless equivalent thermal resistance $\tilde{R}_{h0,\mathrm{mm}}$ decreases, and the global heat conduction performance of the elemental leaf-like fin improves.

Figure 6 shows the effect of the thermal conductivity ratio of the vein and blade $\tilde{k}$ on the optimal construct of the elemental leaf-like fin with Bi=0.01 and r=1. From Figure 6, with the increase of $\tilde{k}$, the width, thickness and vein cross-

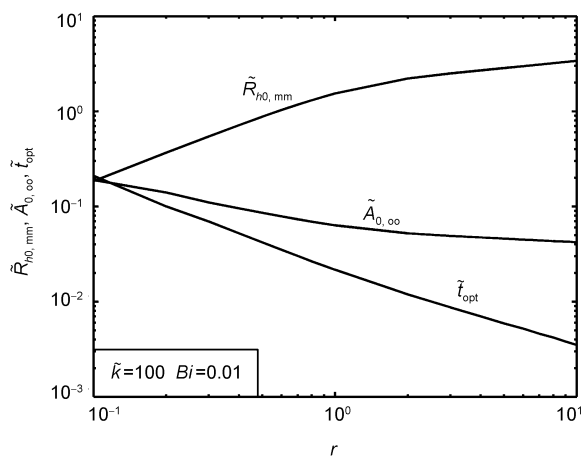


Figure 4 $\tilde{R}_{h0,\mathrm{mm}}$, $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ versus r characteristics.

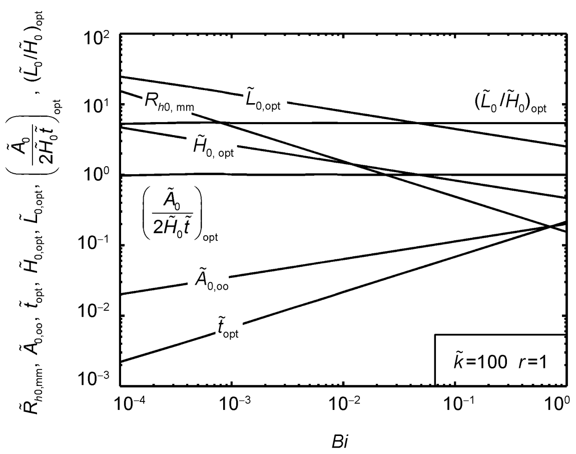


Figure 5 Effect of Bi on the optimal construct of the elemental leaf-like fin.

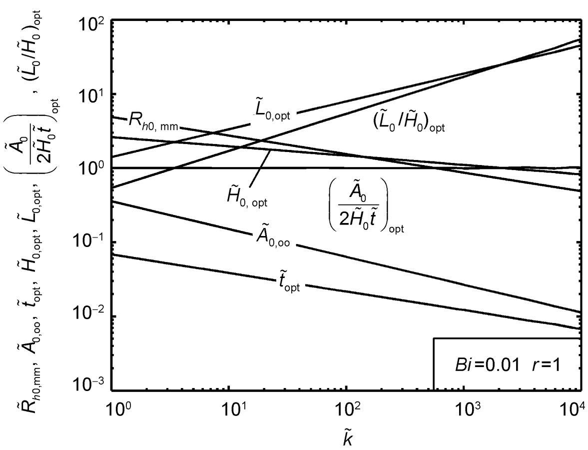


Figure 6 Effect of $\tilde{k}$ on the optimal construct of the elemental leaf-like fin.

sectional area of the elemental leaf-like fin decrease, the length and ratio of length and width increase, and the global heat conduction performance of the elemental leaf-like fin improves. From the change law of the geometrical parameter of the elemental leaf-like fin, the cross-sectional area ratio of the vein and blade of the elemental leaf-like fin almost remains 1, and to be pointed out that, the physical mechanisms herein need to be further investigated. When $\tilde{k}\to 1$, the shape of the blade is no longer slender, and the vein cross-sectional area for longitudinal conduction is larger than the blade cross section for transversal conduction ($(\tilde{A}_0/(2\tilde{L}_0\tilde{t}))_{\mathrm{opt}}>1$). In this case, the heat exchange between the adjacent blades ($H_0\times t\times\Delta x$) can be ignored, and the assumption that the heat conduction in the blade ($H_0\times t\times\Delta x$) is one-dimensional remains valid.

To compare the optimal constructs of the elemental leaf-like fin based on the minimizations of entransy dissipation rate and maximum temperature difference, the ratio of the length and thickness of the blade $\tilde{H}_0/\tilde{t}^{1/2}$ is the same as that in ref. [20], i.e., r =1. Table 1 and Figure 7 show the

Table 1 Optimal constructs of the elemental leaf-like fin based on the minimizations of entransy dissipation rate and maximum temperature difference

Optimization objective	$\tilde{A}_{0,\text{opt}}$	$\tilde{t}_{\text{opt}}$	$\tilde{H}_{0,\text{opt}}$	$\tilde{L}_{0,\text{opt}}$	$\tilde{R}_{t0}$	$\tilde{R}_{h0}$
Entransy dissipation rate minimization	0.0633	0.0216	1.4697	7.8870	3.8180	1.5381
Maximum temperature difference minimization[a)]	0.0407	0.0161	1.2689	12.2613	3.4753	1.7387

a) This row is the result of ref. [20].

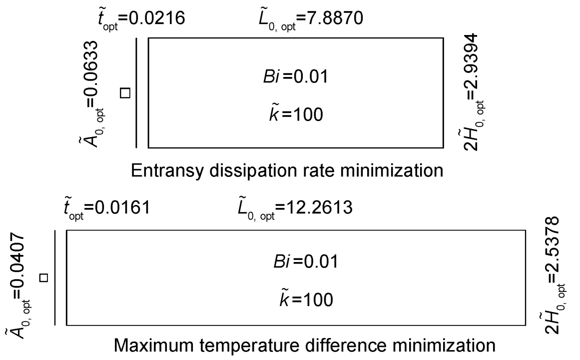


Figure 7 Optimal constructs of the elemental leaf-like fin based on the minimizations of entransy dissipation rate and maximum temperature difference.

optimal constructs of the elemental leaf-like fin based on the minimizations of these two objectives with Bi =0.01, $\tilde{k}$ =100 and r =1. The small squares and column bars are the cross-sectional areas of the vein and blade viewed from the side of the elemental leaf-like fin. From Table 1 and Figure 7, the optimal constructs of the elemental leaf-like fin based on the minimizations of entransy dissipation rate and maximum temperature difference are obviously different. Comparing the optimal construct of the elemental leaf-like fin based on the minimization of entransy dissipation rate with that based on the minimization of maximum temperature difference, the vein cross-sectional area is bigger, the blade is thicker and the shape of the leaf-like fin is tubbier based on the minimization of entransy dissipation rate. The dimensionless equivalent thermal resistance based on the minimization of entransy dissipation rate is reduced by 11.54% compared with that based on maximum temperature difference, and the dimensionless maximum thermal resistance ($\tilde{R}_{t0}=T_{0,\mathrm{m}}/(q_0\cdot k_0^{-1}V_1^{-1/3})$) based on the minimization of entransy dissipation rate is increased by 8.97% compared with that based on maximum temperature difference. The equivalent thermal resistance of the leaf-like fin based on the minimization of entransy dissipation rate presents the local entransy dissipation, but the maximum thermal resistance of the leaf-like fin based on the minimization of maximum temperature difference only considers the minimization of maximum temperature difference of the leaf-like fin. The minimization of maximum temperature difference can reduce the entransy dissipation where the temperature difference is small; however, the heat transfer rate of the leaf-like fin is reduced simultaneously, i.e., the balance between the entransy dissipation and heat transfer rate at different positions and temperature differences in the leaf-like fin is not considered. Therefore, there exists certain limitation in the optimal construct of the leaf-like fin based on the minimization of maximum temperature difference, and the optimal construct of the leaf-like fin based on the minimization of entransy dissipation rate is more scientific. On the premise that the temperature of the leaf-like fin is lower than the limiting temperature, the temperature gradient field of the elemental leaf-like fin based on entransy dissipation rate minimization is more homogenous than that based on maximum temperature difference minimization essentially. Therefore, the optimal construct of the leaf-like fin based on entransy dissipation rate minimization can effectively reduce the average heat transfer performance of the whole leaf-like fin, and its global heat transfer performance improves simultaneously.

4 Constructal optimization of the first order leaf-like fin

One way to assemble a number of n_1 of the elemental leaf-like fins to form the first order leaf-like fin, which is composed of the first order blade ($2H_1\times L_1\times t$, k_0) and the first order vein ($A_1\times L_1$, k_p) attached to the symmetry axis of the blade, is shown in Figure 8. The dashed lines in Figure 8 represent the assembling boundaries of the elemental leaf-like fins. The heat current q_1 flows along the first order vein from its root to the top, and part of it q_{0i} (i=1, 2, …, $n_1/2$, n_1 is even) enters the root of the elemental vein perpendicular to the first order vein. The heat current q_{0i} flows along the elemental vein, and, the heat is finally dispersed from the surface of the elemental blade by heat convection. The top of the first order leaf-like fin is considered to be adiabatic, and the adjacent elemental leaf-like fins (the dashed lines in Figure 8) are also adiabatic. For simplification, it is assumed that the first order vein only exchanges heat with the elemental leaf-like fins at the boot of the elemental vein, and the cross-sectional area of the first order vein is larger than that of the elemental vein ($A_1>A_0$). For the fixed total volume of the first order leaf-like fin $V_1=n_1(A_0+2H_0t)L_0+A_1H_0(n_1-1)$, n_1, A_0, A_1, H_0, L_0 and t of the first order leaf-like fin are free to vary.

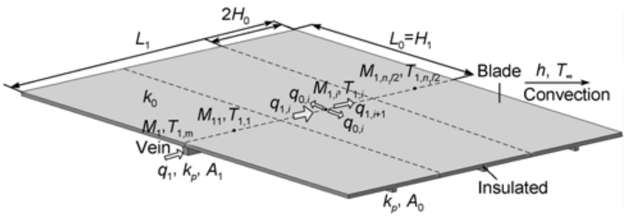


Figure 8 First order leaf-like fin.

From eqs. (8) and (11), the inward heat current $q_{0,i}$ and the entransy dissipation rate $\dot{E}_{vh\phi 0,i}$ of each elemental leaf-like fin (i) are, respectively, given by

$$q_{0,i}=k_pA_0a_1^{1/2}(T_{1,i}-T_\infty)\tanh(a_1^{1/2}L_0)\quad(1\leqslant i\leqslant n_1/2), \tag{16}$$

$$\begin{aligned}\dot{E}_{vh\phi 0,i}&=q_{0,i}^{\ 2}\operatorname{csc h}^2(a_1^{1/2}L_0)\{-2a_0^{\ 2}tH_0k_0\operatorname{sec h}^2(a_0H_0)\\&\times[2a_1^{1/2}L_0+\sinh(2a_1^{1/2}L_0)]+4a_1^{1/2}L_0[a_0tk_0\\&\times\tanh(a_0H_0)-a_1k_pA_0]+2\sinh(2a_1^{1/2}L_0)\\&\times[a_1k_pA_0+a_0tk_0\tanh(a_0H_0)]\}/(8a_1^{3/2}A_0^{\ 2}k_p^{\ 2}).\end{aligned} \tag{17}$$

The heat currents between the adjacent nodes ($M_{1,i}$, $M_{1,i+1}$) of the first order vein are

$$q_{1,i}=2q_{0,i}+q_{1,i+1},\quad(1\leqslant i\leqslant n_1/2-1), \tag{18}$$

$$q_{1,n_1/2}=2q_{0,n_1/2}, \tag{19}$$

$$q_{1,i}=k_pA_1\frac{(T_{1,i-1}-T_{1,i})}{2H_0},\quad(2\leqslant i\leqslant n_1/2), \tag{20}$$

$$q_1=k_pA_1\frac{(T_{1,\mathrm{m}}-T_{1,1})}{H_0}, \tag{21}$$

where $q_{1,1}=q_1$, $T_{1,i}$ is the temperature of the first order vein at node $M_{1,i}$. According to eq. (2), the entransy dissipation rate between the adjacent node along the first order vein is the product of heat current and temperature difference

$$\dot{E}_{h\phi,M_1M_{11}}=\dot{E}_{h,M_1}-\dot{E}_{h,M_{11}}=q_1(T_{1,\mathrm{m}}-T_{1,1})=\frac{q_1^{\ 2}H_0}{k_pA_1}, \tag{22}$$

$$\begin{aligned}\dot{E}_{h\phi,M_{1,i-1}M_{1,i}}&=\dot{E}_{h\phi,M_{1,i-1}}-\dot{E}_{h\phi,M_{1,i}}\\&=q_{1,i}(T_{1,i-1}-T_{1,i})=\frac{2q_{1,i}^{\ 2}H_0}{k_pA_1},\quad 2\leqslant i\leqslant n_1/2,\end{aligned} \tag{23}$$

where $\dot{E}_{h\phi,M_{1,j}}$ ($2\leqslant i\leqslant n_1/2$) represents the entransy at point $M_{1,j}$. According to eqs. (22) and (23), the entransy dissipation rate along the first order vein is

$$\begin{aligned}\dot{E}_{h\phi,M_1M_{1,n_1/2}}&=\dot{E}_{h\phi,M_1M_{11}}+\sum_{i=2}^{n_1/2}\dot{E}_{h\phi,M_{1,i-1}M_{1,i}}\\&=\frac{H_0}{k_pA_1}\left(q_1^{\ 2}+2\sum_{i=2}^{n_1/2}q_{1,i}^2\right),\quad n_1\geqslant 2.\end{aligned} \tag{24}$$

According to eqs. (17) and (24), the entransy dissipation rate of the first order leaf-like fin can be derived

$$\dot{E}_{vh\phi 1}=\dot{E}_{h\phi,M_1M_{1,n_1/2}}+2\sum_{i=1}^{n_1/2}\dot{E}_{vh\phi 0,i}. \tag{25}$$

From eqs. (5) and (25), the dimensionless equivalent thermal resistance of the whole first order leaf-like fin becomes

$$\begin{aligned}\tilde{R}_{h1}&=\frac{\dot{E}_{vh\phi 1}}{q_1^{\ 2}\cdot k_0^{\ -1}\cdot V_1^{-1/3}}\\&=\frac{\tilde{H}_0}{\tilde{k}\tilde{A}_1}[1+2\sum_{i=2}^{n_1/2}\left(\frac{q_{1,i}}{q_1}\right)^2]+2B\sum_{i=1}^{n_1/2}\left(\frac{q_{0,i}}{q_1}\right)^2,\end{aligned} \tag{26}$$

where

$$\begin{aligned}B=&\operatorname{csc h}^2(a_1^{1/2}\tilde{L}_0V_1^{1/3})\{-\tilde{H}_0a_0^{\ 2}\tilde{t}V_1^{1/3}\operatorname{sec h}^2(a_0\tilde{H}_0V_1^{1/3})\\&\times[2a_1^{1/2}\tilde{L}_0V_1^{1/3}+\sinh(2a_1^{1/2}\tilde{L}_0V_1^{1/3})]\\&+2a_1^{1/2}\tilde{L}_0V_1^{1/3}[-a_1\tilde{k}\tilde{A}_0V_1^{1/3}+\tilde{t}a_0\tanh(a_0\tilde{H}_0V_1^{1/3})]\\&+\sinh(2a_1^{1/2}\tilde{L}_0V_1^{1/3})[\tilde{t}a_0\tanh(a_0\tilde{H}_0V_1^{1/3})\\&+a_1\tilde{k}\tilde{A}_0V_1^{1/3}]\}/(4a_1^{3/2}\tilde{k}^2\tilde{A}_0^{\ 2}V_1^{2/3}),\end{aligned}$$

$$(\tilde{A}_0,\tilde{A}_1)=(A_0,A_1)/V_1^{2/3},$$
$$(\tilde{H}_0,\tilde{L}_0,\tilde{t})=(H_0,L_0,t)/V_1^{1/3}.$$

For the fixed inward heat current of the first order leaf-like fin q_1, eqs. (16) and (18)–(21) include $3\cdot(n_1/2)$ unknown variables ($q_{0,i}$ $(i=1,2,\ldots,n_1/2)$, $q_{1,i}$ $(i=2,\ldots,n_1/2)$, $T_{1,i}$ (i=1, $2,\ldots,n_1/2$) and $T_{1,\mathrm{m}}$) and $3\cdot(n_1/2)$ equations, and these $3\cdot(n_1/2)$ unknown variables can be solved successfully. Substituting $q_{0,i}$ $(i=1,2,\ldots,n_1/2)$ and $q_{1,i}$ $(i=2,\ldots,n_1/2)$ into eq. (26), the dimensionless equivalent thermal resistance of the first order leaf-like fin $\tilde{R}_{h1}$ is only the function of $\tilde{A}_0$, $\tilde{A}_1$, $\tilde{H}_0$, $\tilde{L}_0$ and $\tilde{t}$.

The total volume constraint of the first order leaf-like fin and the relationship of the length and thickness of the elemental blade can be nondimensionalized as follows:

$$n_1(\tilde{A}_0+2\tilde{H}_0\tilde{t})\tilde{L}_0+\tilde{A}_1\tilde{H}_0(n_1-1)=1, \tag{27}$$

$$r=\frac{Bi^{1/2}\tilde{H}_0}{\tilde{t}^{1/2}}, \tag{28}$$

where Biot number is Bi=$0.996^2hV_1^{1/3}/k_0$, and r is the ratio parameter. Substituting eqs. (27) and (28) into eq. (26) to eliminate $\tilde{H}_0$ and $\tilde{L}_0$, the function $\tilde{R}_{h1}$ in eq. (26) is relative to Bi, $\tilde{k}$, $\tilde{A}_0$, $\tilde{t}$, r, n_1 and $\tilde{A}_1$. For the fixed Bi in eq. (28) and $\tilde{k}$ in eq. (26), the function $\tilde{R}_{h1}$ in eq. (26) after eliminating $\tilde{H}_0$ and $\tilde{L}_0$ is only the function of five independent variables $\tilde{A}_0$, $\tilde{t}$, r, n_1 and $\tilde{A}_1$, and one can use $\tilde{A}_0$, $\tilde{t}$, r, n_1 and $\tilde{A}_1/\tilde{A}_0$ as optimization variables to carry out constructal optimization for the first order leaf-like fin.

Figure 9 shows the dimensionless equivalent thermal resistance $\tilde{R}_{h1}$ versus $\tilde{A}_0$ characteristic with Bi=0.01, $\tilde{k}$ = 100, r=2, n_1=8, $\tilde{A}_1/\tilde{A}_0=2$ and different $\tilde{t}$. From Figure 9, for the fixed $\tilde{t}$, there exists an optimal $\tilde{A}_0$ ($\tilde{A}_{0,\mathrm{opt}}$) which leads to minimum $\tilde{R}_{h1}$ ($\tilde{R}_{h1,\mathrm{m}}$); with the increase in $\tilde{t}$, $\tilde{R}_{h1,\mathrm{m}}$ decreases first, and then increases.

Figure 10 shows $\tilde{R}_{h1,\mathrm{m}}$ and $\tilde{A}_{0,\mathrm{opt}}$ versus $\tilde{t}$ characteristics with Bi=0.01, $\tilde{k}=100$, r=2, n_1=8 and $\tilde{A}_1/\tilde{A}_0=2$. From Figure 10, $\tilde{A}_{0,\mathrm{opt}}$ increases first and then decreases slowly with the increase in $\tilde{t}$; there exist the optimal $\tilde{t}$ ($\tilde{t}_{\mathrm{opt}}=0.0012$) and twice optimal $\tilde{A}_0$ ($\tilde{A}_{0,\mathrm{oo}}=0.0141$) which lead to double minimum $\tilde{R}_{h1}$ ($\tilde{R}_{h1,\mathrm{mm}}=1.7198$).

Figure 11 shows $\tilde{R}_{h1,\mathrm{mm}}$, $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ versus r characteristics with Bi=0.01, $\tilde{k}$=100, n_1=8 and $\tilde{A}_1/\tilde{A}_0=2$. From Figure 11, the twice optimal elemental vein cross- sectional area and optimal thickness of the first order leaf-like fin decrease with the increase in r; $\tilde{R}_{h1,\mathrm{mm}}$ decreases with the increase in r, but r cannot tend to be infinitesimal, and this is because there does not exist an optimal r (r_{opt}) which leads to minimum $\tilde{R}_{h1,\mathrm{mm}}$. Compared with Figure 4, for the identical r, the twice optimal elemental vein cross-sectional area and optimal thickness of the first order leaf-like fin are smaller than those of the elemental leaf-like fin.

Figure 12 shows the minimum dimensionless equivalent thermal resistance $\tilde{R}_{h1,\mathrm{mm}}$ and the corresponding optimal construct ($\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$) versus the number of elemental leaf-like fins n_1 characteristics with Bi=0.01, $\tilde{k}=100$, r=1 and $\tilde{A}_1/\tilde{A}_0=2$. The subscript "mmm" and "ooo" denote $\tilde{R}_{h1}$ minimized triple and the corresponding triple optimization, respectively. From Figure 12, with the increase in n_1, both $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ of the first order leaf-like fin decrease;

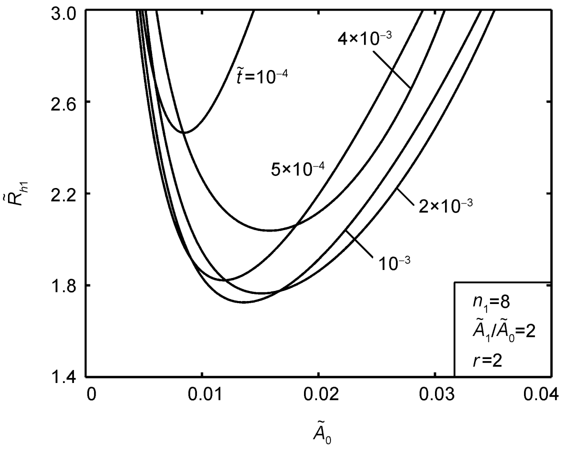


Figure 9 Effect $\tilde{t}$ on the characteristic of $\tilde{R}_{h1}$ versus $\tilde{A}_0$.

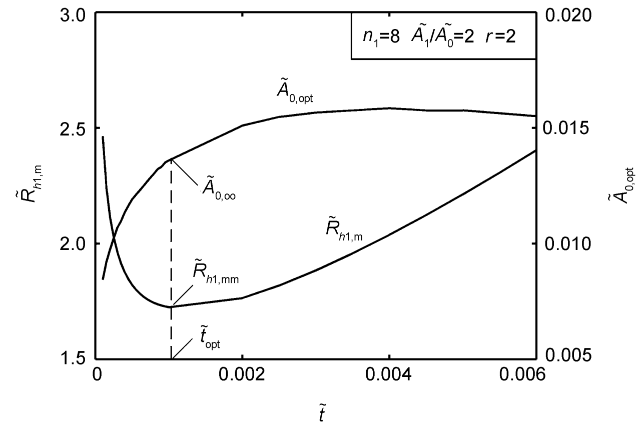


Figure 10 $\tilde{R}_{h1,\mathrm{m}}$ and $\tilde{A}_{0,\mathrm{opt}}$ versus $\tilde{t}$ characteristics.

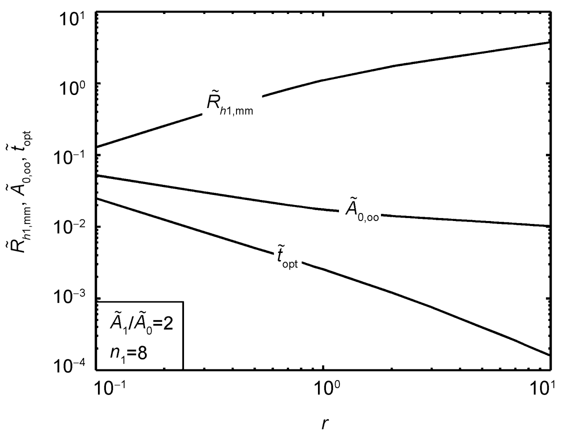


Figure 11 $\tilde{R}_{h1,\mathrm{mm}}$, $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ versus r characteristics.

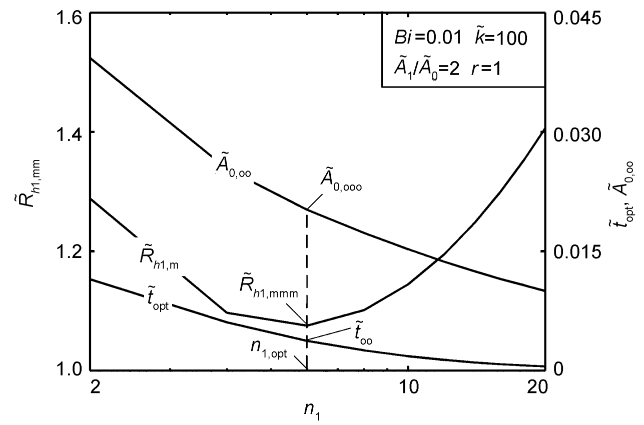


Figure 12 $\tilde{R}_{h1,\mathrm{mm}}$, $\tilde{A}_{0,\mathrm{oo}}$ and $\tilde{t}_{\mathrm{opt}}$ versus n_1 characteristics.

$\tilde{R}_{h1,\mathrm{mm}}$ decreases first, and then increases. There exists an optimal n_1 ($n_{1,\mathrm{opt}}$=6) which leads to triple minimum $\tilde{R}_{h1}$ ($\tilde{R}_{h1,\mathrm{mmm}}$=1.0752).

Figure 13 shows the triple minimum dimensionless equivalent thermal resistance of the first order leaf-like fin $\tilde{R}_{h1,\mathrm{mmm}}$ and the corresponding optimal construct ($\tilde{A}_{0,\mathrm{ooo}}$, $\tilde{t}_{\mathrm{oo}}$ and $n_{1,\mathrm{opt}}$) versus the vein cross-sectional area ratio of the first order and elemental leaf-like fins $\tilde{A}_1/\tilde{A}_0$ characteristics with Bi=0.01, $\tilde{k}=100$ and r=1. From Figure 13, with the increase in $\tilde{A}_1/\tilde{A}_0$, the triple optimal elemental vein cross-sectional area and the twice optimal thickness of the first order leaf-like fin decrease, and the optimal number of elemental leaf-like fins increases. When $\tilde{A}_1/\tilde{A}_0=1$, $\tilde{R}_{h1,\mathrm{mmm}}$ reaches its maximum; $\tilde{R}_{h1,\mathrm{mmm}}$ decreases with the increase in $\tilde{A}_1/\tilde{A}_0$, but the decrement is small, and there does not exist an optimal $\tilde{A}_1/\tilde{A}_0$ which leads to minimum $\tilde{R}_{h1,\mathrm{mmm}}$.

Figure 14 shows the effect of the Biot number Bi on the optimal construct of the first order leaf-like fin with $\tilde{k}=100$, r=1 and $\tilde{A}_1/\tilde{A}_0=2$. From Figure 14, with the increase in Bi, the optimal elemental vein cross-sectional area $\tilde{A}_{0,\mathrm{ooo}}$ and the optimal thickness of the first order leaf-like fin $\tilde{t}_{\mathrm{oo}}$ increase; the optimal number of elemental leaf-like fins $n_{1,\mathrm{opt}}$ and the optimal ratios of the length and width of the elemental and first order leaf-like fins $(L_0/H_0)_{\mathrm{opt}}$ and $(L_1/H_1)_{\mathrm{opt}}$ almost keep constant. The triple minimum dimensionless equivalent thermal resistance of the first order leaf-like fin $\tilde{R}_{h1,\mathrm{mmm}}$ decreases with the increase in Bi, and the global heat conduction performance of the first order leaf-like fin improves in this case.

Figure 15 shows the effect of the thermal conductivity

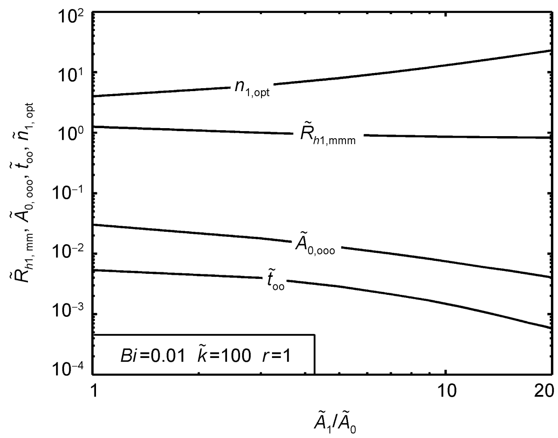


Figure 13 $\tilde{R}_{h1,\mathrm{mmm}}$, $\tilde{A}_{0,\mathrm{ooo}}$, $\tilde{t}_{\mathrm{oo}}$ and $n_{1,\mathrm{opt}}$ versus $\tilde{A}_1/\tilde{A}_0$ characteristics.

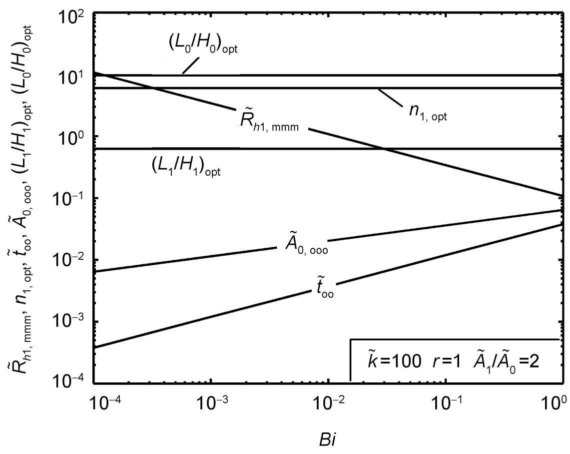


Figure 14 Effect of Bi on the optimal construct of the first order leaf-like fin.

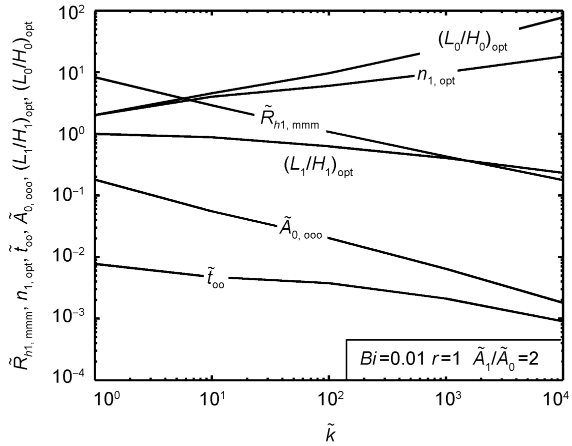
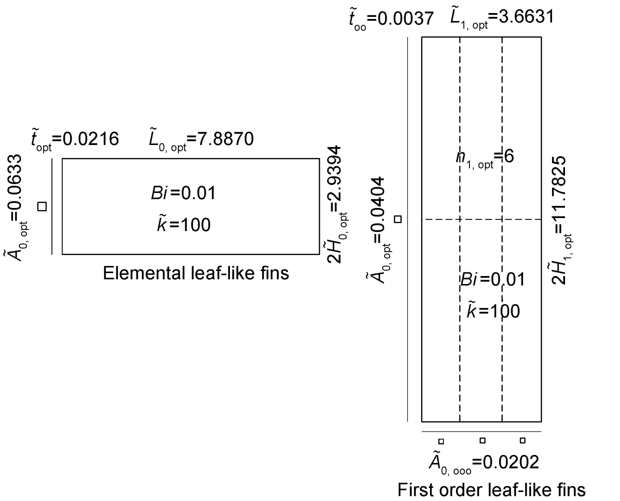


Figure 15 Effect of $\tilde{k}$ on the optimal construct of the first order leaf-like fin.

ratio of the vein and blade $\tilde{k}$ on the optimal construct of the first order leaf-like fin with Bi=0.01, r=1 and $\tilde{A}_1/\tilde{A}_0$=2. From Figure 15, with the increase in $\tilde{k}$, the optimal ratio of the length and width of the elemental leaf-like fin $(L_0/H_0)_{\mathrm{opt}}$ and the optimal number of elemental leaf-like fins $n_{1,\mathrm{opt}}$ in the first order leaf-like fin increase, and the optimal elemental vein cross-sectional area $\tilde{A}_{0,\mathrm{ooo}}$, the optimal thickness of the first order leaf-like fin $\tilde{t}_{\mathrm{oo}}$, the optimal ratio of the length and width of the first order leaf-like fin $(L_1/H_1)_{\mathrm{opt}}$ as well as minimum dimensionless equivalent thermal resistance $\tilde{R}_{h1,\mathrm{mmm}}$ decrease. Therefore, by increasing $\tilde{k}$, the global heat conduction performance of the first order leaf-like fin improves simultaneously.

To compare the heat conduction performance of the elemental and first order leaf-like fins, the comparison of the optimal constructs of the elemental and first order leaf-like fins with Bi=0.01, $\tilde{k}=100$, r=1 and $\tilde{A}_1/\tilde{A}_0=2$ is shown in Figure 16. The small squares and column bars are the



-	$n_{1,\mathrm{opt}}=6,$
$(\tilde{L}_0/\tilde{H}_0)_{\mathrm{opt}}=5.3664,$	$(\tilde{L}_0/\tilde{H}_0)_{\mathrm{opt}}=9.6495,$
$\tilde{A}_{0,\mathrm{oo}}=0.0633,$	$\tilde{A}_{0,\mathrm{ooo}}=0.0202,$
$\tilde{t}_{\mathrm{opt}}=0.0216,$	$\tilde{t}_{\mathrm{oo}}=0.0037,$
$\tilde{R}_{h0,\mathrm{mm}}=1.5381,$	$\tilde{R}_{h1,\mathrm{mmm}}=1.0752.$

Figure 16 Comparison of the optimal constructs of the elemental and first order leaf-like fins.

cross-sectional areas of the vein and blade viewed from the side of the first order leaf-like fin. The optimal number of elemental leaf-like fins is $n_{1,\mathrm{opt}}$=6. For the same volumes of elemental and first order leaf-like fins, comparing the optimal construct of the first order leaf-like fin with that of the elemental leaf-like fin, the optimal shape of the first order leaf-like body becomes much tubbier, the shape of each elemental leaf-like fin in the first order leaf-like fin becomes slenderer, the first order leaf-like fin becomes thinner, and the elemental vein cross-sectional area in the first order leaf-like thin becomes smaller. The dimensionless equivalent thermal resistance of the first order leaf-like fin is reduced by 30.10% compared with that of the elemental leaf-like fin. The equivalent thermal resistance of the first order leaf-like fin is greatly reduced based on the minimization of entransy dissipation rate, this is essentially because that the temperature gradient field of the first order leaf-like fin based on entransy dissipation rate minimization is more homogenous than that of the elemental leaf-like fin. Meanwhile, the global heat conduction performance of the first order leaf-like fin can be obviously improved.

5 Conclusions

For the fixed total volumes of the fins, the constructs of the elemental and first order leaf-like fins are optimized respectively by taking minimum entransy dissipation rate (for the fixed total thermal current, i.e. the equivalent thermal resistance) as optimization objective. The optimal constructs of the elemental and first order leaf-like fins with minimum dimensionless equivalent thermal resistance are obtained. The results show that there does not exist an optimal ratio of the length and thickness of the elemental blade r_{opt} which makes the twice minimum dimensionless equivalent thermal resistance $\tilde{R}_{h1,\mathrm{mm}}$ lead to its minimum, but there exists an optimal number of elemental leaf-like fins $n_{1,\mathrm{opt}}$ which makes $\tilde{R}_{h1,\mathrm{mm}}$ lead to its minimum. Meanwhile, there does not exist an optimal vein cross-sectional area ratio of the first order and elemental leaf-like fins $(\tilde{A}_1/\tilde{A}_0)_{\mathrm{opt}}$ which makes the triple minimum dimensionless equivalent thermal resistance $\tilde{R}_{h1,\mathrm{mmm}}$ lead to its minimum. With the increase in Bi, the optimal number of elemental leaf-like fins and the optimal ratios of the length and width of the elemental and first order leaf-like fins almost keep constant. With the increase in $\tilde{k}$, the optimal number of elemental leaf-like fins and the optimal ratio of the length and width of the elemental leaf-like fin increase, and the optimal ratio of the length and width of the first order leaf-like fin decreases. Moreover, by increasing Bi and $\tilde{k}$, the global heat conduction performance of the first order leaf-like fin improves. The optimal construct based on entransy dissipation rate minimization is obviously different from that based on maximum temperature difference minimization. When Bi=0.01, $\tilde{k}$= 100 and r=1, the dimensionless equivalent thermal resistance based on entransy dissipation rate minimization is reduced by 11.54% compared with that based on maximum temperature difference minimization. Therefore, the optimal construct of the leaf-like fin based on entransy dissipation rate minimization can effectively reduce the average temperature difference of the leaf-like fin compared with that based on maximum temperature difference minimization, and its heat transfer performance improves simultaneously. For the same volumes of the elemental and first order leaf-like fins, when Bi=0.01, $\tilde{k}$=100, r=1 and $\tilde{A}_1/\tilde{A}_0$=2, comparing the optimal construct of the first order leaf-like fin with that of the elemental leaf-like fin, the dimensionless equivalent thermal resistance of the first order of the leaf-like fin is reduced by 30.10% compared with that of the elemental leaf-like fin. Essentially, this is because the temperature gradient field of the first order leaf-like fin based on entransy dissipation rate minimization is more homogenous than that of the elemental leaf-like fin. It is shown that the global heat conduction performance of the first order leaf-like fin is obviously improved by using the constructal optimization method. The equivalent thermal resistance defined based on entransy dissipation rate minimization can reflect the global heat transfer performance of the system, i.e. the smaller the equivalent thermal resistance, the better the heat transfer performance, the lower the average temperature difference as well as the higher heat transfer efficiency of the construct. Therefore, on the premise that the maximum temperature difference and other performance indicators are taken into account, the constructal design scheme with minimum equivalent thermal resistance should be adopted when making thermal design in engineering.

Actually, the elemental vein is not always perpendicular to the first order vein, and there may exist multistage vein networks in the whole blade. Therefore, one can release the constraint that the elemental vein is perpendicular to the first order vein, and further investigate the constructal optimization problem of the leaf-like fins with multistage vein networks. The assumption that the heat currents in the vein and blade of the leaf-like fins are one-dimensional is made in this paper, and therefore, one can build the two- and three-dimensional flow models in the leaf-like fins to further carry out constructal optimizations of the leaf-like fins.

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1 Bejan A. Shape and Structure, from Engineering to Nature. Cambridge: Cambridge University Press, 2000
2 Bejan A, Lorente S. Thermodynamic optimization of flow geometry in mechanical and civil engineering. J Non-Equilib Thermodyn, 2001, 26(4): 305–354
3 Bejan A, Lorente S. The Constructal Law (La Loi Constructale). Paris: L'Harmatan, 2005
4 Bejan A, Marden J H. Unifying constructal theory for scale effects in running, swimming and flying. J Exp Biol, 2006, 209(2): 238–248
5 Bejan A, Merkx G W. Constructal Theory of Social Dynamics. New York: Springer, 2007
6 Bejan A, Lorente S. Design with Constructal Theory. New Jersey: Wiley, 2008
7 Bejan A, Lorente S, Miguel A F, et al. Constructal Human Dynamics, Security & Sustainability. Amsterdam: IOS Press, 2009
8 Bejan A. Constructal-theory network of conducting paths for cooling a heat generating volume. Trans ASME, J Heat Transfer, 1997, 40(4): 799–816
9 Siddique M, Khaled A R A, Abdulhafiz N I, et al. Recent advances in heat transfer enhancements: a review report. Int J Chem Eng, doi: 1155/2010/106461
10 Bejan A, Dan N. Constructal trees of convective fins. Trans ASME J Heat Transfer, 1999, 121(3): 675–682
11 Almogbel M, Bejan A. Cylindrical of pin fins. Int J Heat Mass Transfer, 2000, 43(23): 4285–4297
12 Bejan A, Almogbel M. Constructal T-shaped fins. Int J Heat Mass Transfer, 2000, 43(12-15): 2101–2115
13 Almogbel M. Constructal tree-shaped fins. Int J Therm Sci, 2005, 44(4): 342–348
14 Lorenzini G, Rocha L A O. Constructal design of Y-shaped assembly of fins. Int J Heat Mass Transfer, 2006, 49(23-24): 4552–4557
15 Lorenzini G, Moretti S. A CFD application to optimize T-shaped fins: comparisons to the constructal theory's results. Trans ASME J Electron Packag, 2007, 129(3): 324–327
16 Lorenzini G, Moretti S. Numerical performance analysis of constructal I and Y finned heat exchanging modules. Trans ASME J Electron Packag, 2009, 131(3): 031012
17 Lorenzini G, Rocha L A O. Constructal design of T–Y assembly of fins for an optimized heat removal. Int J Heat Mass Transfer, 2009, 52(5-6): 1458–1463
18 Xie Z, Chen L, Sun F. Constructal optimization of twice level Y-shaped assemblies of fins by taking maximum thermal resistance minimization as objective. Sci China Tech Sci, 2010, 53(10): 2756–2764
19 Kundu B, Bhanja D. Performance and optimization analysis of a constructal T-shaped fin subject to variable thermal conductivity and convective heat transfer coefficient. Int J Heat Mass Transfer, 2010, 53(1-3): 254–267
20 Combelles L, Lorente S, Bejan A. Leaflike architecture for cooling a flat body. J Appl Phys, 2009, 106(4): 044906
21 Khaled A R A. Analysis of heat transfer through Bi-convection fins. Int J Thermal Sci, 2009, 48(1): 122–132
22 Khaled A R A. Investigation of heat transfer enhancement through permeable fins. J Heat Transfer, 2010, 132(3), 034503
23 Karvinen R, Karvinen T. Optimum geometry of fixed volume plate fin for maximizing heat transfer. Int J Heat Mass Transfer, 2010, 53(23-24): 5380–5385
24 Zhang X, Liu D. Optimum geometric arrangement of vertical rectangular fin arrays in natural convection. Energy Convers Mgmt, 2010, 51(12): 2449–2456
25 Bello-Ochende T, Meyer J P, Bejan A. Constructal multi-scale pin-fins. Int J Heat Mass Transfer, 2010, 53(13-14): 2773–2779
26 Kundu B, Bhanja D. An analytical prediction for performance and optimum design analysis of porous fins. Int J Refrigeration, 2011, 34(1): 337–352
27 Sharqawy M H, Zubair S M. Efficiency and optimization of an annular fin with combined heat and mass transfer — An analytical solution. Int J Refrigeration, 2007, 30(5): 751–757
28 Guo Z, Zhu H, Liang X. Entransy — A physical quantity describing heat transfer ability. Int J Heat Mass Transfer, 2007, 50(13/14): 2545–2556
29 Li Z, Guo Z. Field Synergy Principle of Heat Convection Optimization. Beijing: Science Press, 2010
30 Guo Z, Cheng X, Xia Z. Least dissipation principle of heat transport potential capacity and its application in heat conduction optimization. Chin Sci Bull, 2003, 48(4): 406–410
31 Han G, Zhu H, Cheng X, et al. Transfer similarity among heat conduction, elastic motion and electric conduction (in Chinese). J Eng Thermophys, 2005, 26(6): 1022–1024
32 Han G, Guo Z. Physical mechanism of heat conduction ability dissipation and its analytical expression (in Chinese). Proc CSEE, 2007, 27(17): 98–102
33 Zhu H, Chen J, Guo Z. Electricity and thermal analogous experimental study for entransy dissipation extreme principle (in Chinese). Prog Natural Sci, 2007, 17(12): 1692–1698
34 Chen Q, Ren J. Generalized thermal resistance for convective heat transfer and its relation to entransy dissipation. Chin Sci Bull, 2008, 53(23): 3753–3761
35 Chen Q, Ren J, Guo Z. The extremum principle of mass entransy dissipation and its application to decontamination ventilation designs in space station cabins. Chin Sci Bull, 2009, 54(16): 2862–2870
36 Xia S, Chen L, Sun F. Optimization for entransy dissipation minimization in heat exchanger. Chin Sci Bull, 2009, 54(19): 3587–3595
37 Wang S, Chen Q, Zhang B. An equation of entransy and its application. Chin Sci Bull, 2009, 54(19): 3572–3578
38 Guo J, Cheng L, Xu M. Entransy dissipation number and its application to heat exchanger performance evaluation. Chin Sci Bull, 2009, 54(15): 2708–2713
39 Chen Q, Wang M, Pan N, et al. Optimization principles for convective heat transfer. Energy, 2009, 34(9): 1199–1206
40 Guo Z, Liu X B, Tao W Q, et al. Effectiveness-thermal resistance method for heat exchanger design and analysis, Int J Heat Mass Transfer, 2010, 53(13/14): 2877–2884
41 Chen Q, Yang K, Wang M, et al. A new approach to analysis and optimization of evaporative cooling system I: Theory. Energy, 2010, 35(6): 2448–2454
42 Xia S, Chen L, Sun F. Entransy dissipation minimization for liq-

uid-solid phase processes. Sci China Tech Sci, 2010, 53(4): 960–968
43 Guo J, Xu M, Cheng L. Principle of equipartition of entransy dissipation for heat exchanger design. Sci China Tech Sci, 2010, 53(5): 1309–1314
44 Guo J, Xu M, Cheng L. The entransy dissipation minimization principle under given heat duty and heat transfer area conditions. Chin Sci Bull, 2011, 56(19): 2071–2076
45 Guo J, Xu M, Chen L. The influence of viscous heating on the entransy in two-fluid heat exchangers. Sci China Tech Sci, 2011, 54(5): 1267–1274
46 Cheng X, Xu X, Liang X. Application of entransy to optimization design of parallel thermal network of thermal control system in spacecraft. Sci China Tech Sci, 2011, 54(4): 964–971
47 Cheng X, Liang X, Guo Z. Entransy decrease principle of heat transfer in an isolated system. Chin Sci Bull, 2011, 56(9): 847–854
48 Li X, Guo J, Xu M, et al. Entransy dissipation minimization for optimization of heat exchanger design. Chin Sci Bull, 2011, 56(20), 2174–2178
49 Hu G, Cao B, Guo Z. Entransy and entropy revisited. Chin Sci Bull, 2011, 56(27): 2974–2977
50 Xia S, Chen L, Sun F. Entransy dissipation minimization for a class of one-way isothermal mass transfer processes. Sci China Tech Sci, 2011, 54(2): 352–361
51 Qian X, Li Z. Analysis of entransy dissipation in heat exchangers. Int J Thermal Sci, 2011, 50(4): 608–614
52 Liu W, Liu Z, Jia H, et al. Entransy expression of the second law of thermodynamics and its application to optimization in heat transfer process. Int J Heat Transfer, 2011, 54(13-14): 3049–3059
53 Chen Q, Pan N, Guo Z. A new approach to analysis and optimization of evaporative cooling system II: Applications. Energy, 2011, 36(5): 2890–2898
54 Xu M. The thermodynamic basis of entransy and entransy dissipation. Energy, 2011, 36(7): 4272–4277
55 Wei S, Chen L, Sun F. “Volume-point” heat conduction constructal optimization with entransy dissipation minimization objective based on rectangular element. Sci China Ser E-Tech Sci, 2008, 51(8): 1283–1295
56 Wei S, Chen L, Sun F. Constructal multidisciplinary optimization of electromagnet based on entransy dissipation minimization. Sci China Ser E-Tech Sci, 2009, 52(10): 2981–2989
57 Xie Z, Chen L, Sun F. Constructal optimization for geometry of cavity by taking entransy dissipation minimization as objective. Sci China Ser E-Tech Sci, 2009, 52(12): 3504–3513
58 Xie Z, Chen L, Sun F. Constructal optimization on T-shaped cavity based on entransy dissipation minimization. Chin Sci Bull, 2009, 54(23): 4418–4427
59 Wei S, Chen L, Sun F. Constructal optimization of discrete and continuous-variable cross-section conducting path based on entransy dissipation rate minimization. Sci China Tech Sci, 2010, 53(6): 1666–1677
60 Xiao Q, Chen L, Sun F. Constructal entransy dissipation rate minimization for “disc-to-point” heat conduction. Chin Sci Bull, 2011, 56(1): 102–112
61 Xiao Q, Chen L, Sun F. Constructal entransy dissipation rate minimization for umbrella-shaped assembly of cylindrical fins. Sci China Tech Sci, 2011, 54(1): 211–219
62 Xiao Q, Chen L, Sun F. Constructal entransy dissipation rate and flow-resistance minimizations for cooling channels. Sci China Tech Sci, 2010, 53(9): 2458–2468
63 Wei S, Chen L, Sun F. Constructal entransy dissipation rate minimization of round tube heat exchanger cross-section. Int J Therm Sci, 2011, 50(7): 1285–1292
64 Chen L, Wei S, Sun F. Constructal entransy dissipation rate minimization of a disc. Int J Heat Mass Transfer, 2011, 54(1-3): 210– 216
65 Xie Z, Chen L, Sun F. Comparative study on constructal optimizations of T-shaped fin based on entransy dissipation rate minimization and maximum thermal resistance minimization. Sci China Tech Sci, 2011, 41(7): 962–970
66 Xiao Q, Chen L, Sun F. Constructal design for a steam generator based on entransy dissipation extremum principle. Sci China Tech Sci, 2011, 54(6): 1462–1468
67 Xiao Q, Chen L, Sun F. Constructal entransy dissipation rate minimization for a heat generating volume cooled by forced convection. Chin Sci Bull, 2011, 56(27): 2966–2973
68 Bejan A. Heat Transfer. New York: Wiley, 1993