SCIENCE CHINA
Technological Sciences

December 2011 Vol.54 No.12: 3275–3280
doi: 10.1007/s11431-011-4573-z

Ill-conditioned problems of dam safety monitoring models and their processing methods

GU ChongShi[1,2,3*], WANG YaChao[1,2,3], PENG Yan[1,2,3] & XU BaoSong[1,2,3]

[1] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing 210098, China;
[2] College of Water-conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, China;
[3] National Engineering Research Center of Water Resources Efficient Utilization and Engineering Safety, Hohai University, Nanjing 210098, China

Received October 13, 2010; accepted June 29, 2011; published online November 5, 2011

The focus of this paper is the ill-conditioned problems in the dam safety monitoring model. The reasons to give rise to the ill-conditioned problems in statistical models, deterministic models and hybrid models are analyzed in detail, and the criterions for ill-conditioned models are investigated. It is shown that safety monitoring models are not easy to be ill-conditioned if the number of influence factors is less than seven. Moreover, the models have a high accuracy and can meet the engineering requirements. Another frequently encountered problem in establishing a safety monitoring model is the existence of inflection points, which are often present in the mathematical model for the hydraulic components in deterministic models and hybrid models. The conditions for inflection points are studied and their treatments are suggested. Numerical example indicates that the treatments proposed in this paper are effective in removing the ill-conditioned problems.

dam safety, monitoring model, ill-conditioned, processing method

Citation: Gu C S, Wang Y C, Peng Y, et al. Ill-conditioned problems of dam safety monitoring models and their processing methods. Sci China Tech Sci, 2011, 54: 3275–3280, doi: 10.1007/s11431-011-4573-z

1 Introduction

Dam safety monitoring models are generally established to monitor the dam behaviors using prototype observations. The different components in the monitoring models can be physically interpreted and used to monitor the working state of the dams. At present, dam safety monitoring models can be classified into two kinds as follows [1].

The first kind of monitoring models is the so-called statistical model, which can be mathematically expressed as follows:

$$\delta = \delta_H + \delta_T + \delta_\theta, \tag{1}$$

where δ is one of the monitored response variables, such as displacement, stress, uplift pressure, and seepage quantity. δ_H is the hydraulic component and is often expressed as a polynomial function of the hydraulic head H (the difference between the upstream level and the downstream level), i.e., the influence factors are denoted as H, H^2, $\cdots$, H^n (n denotes the highest order). δ_T is the thermal component and can be expressed as a linear function of the variations of corresponding variables. If the thermal field achieves a quasi-stationary state, the influence factors in δ_T can be expressed by periodic terms (e.g. the sine and cosine functions of accumulative days). δ_θ is the time effect component, which can reflect the mechanical behaviors of dam body, foundation and the geological structure of bedrock comprehensively. Generally, the time effect component changes

*Corresponding author (email: csgu@hhu.edu.cn)

rapidly during the initial stage, and it tends to be stable gradually during the later stage in normal operation. Therefore, δ_θ is often composed of a linear part and a logarithmic part, both of which are functions of time.

Based on the foregoing analysis, it can be concluded that there are many influence factors related to the same response variable in the statistical model. For instance, the number of influence factors in engineering practice is usually greater than seven.

The second kind of models for dam safety monitoring is the so-called deterministic model and the hybrid model, both of which are established with the aid of finite element calculations and data fitting procedures. The expressions can be presented as follows:

Deterministic model:

$$\delta = \delta_H + \delta_T + \delta_\theta = Xf(H) + Yf(T) + \delta_\theta, \quad (2)$$

Hybrid model:

$$\delta = \delta_H + \delta_T + \delta_\theta = Xf(H) + \delta_T + \delta_\theta, \quad (3)$$

where $f(H)$ is the hydraulic component obtained from finite element calculations with a certain number of model parameters, e.g., the elastic modulus and the Poisson ratio of dam materials and foundation materials. $f(T)$ is the thermal component, which is also obtained from FEM calculations with thermal parameters such as the linear expansion coefficients of the dam materials and foundation materials. X and Y are the weighting parameters of $f(H)$ and $f(T)$, respectively.

It is shown from eq. (2) that the response variables in deterministic model are affected only by $f(H)$, $f(T)$ and the influence factors in δ_θ. Therefore, the number of influence factors in deterministic models is generally less than that in a statistical model. For most of dam engineering models, the total number of influence factors in a deterministic model is less than seven. Differently, for a hybrid model only the influence factors of the hydraulic component are calculated and the other influence factors are the same as those in a statistical model. Therefore, the total number of influence factors in a hybrid model is generally greater than seven.

It can be seen that different dam safety monitoring models have different numbers of influence factors. Therefore, the dam safety monitoring models established with the same observations are usually different. It is a common experience that the accuracy of dam safety monitoring models is influenced by many factors, among which the number of influence factors and the incomplete information are the dominant ones. A poorly established model is prone to suffer the problem of ill condition and often has an unacceptable accuracy. In this study, the ill-conditioned problems in the statistical, deterministic and hybrid models are studied in depth. The criterions for the judgment of ill condition are proposed and the corresponding treatments are suggested.

2 Ill-conditioned problems of dam safety monitoring models

Based on the prototype observations, a dam safety monitoring model is established by the optimization method to reflect the relationship between the response variable δ and the influence quantities (e.g., the influence factors of the hydraulic, thermal and time effect component). The most important task is to define the effect coefficients of the influence quantities in the model, i.e., n-dimensional effect coefficients $b_i (i = 1,2,\cdots,n)$ of influence quantities are determined by m-dimensional monitoring data of the response variables $\delta_t (t = 1,2,\cdots,m)$ and the influence quantities x_{ti}. Since the number of equations, m, is usually greater than the number of unknowns, n, the above equation is a contradictory one and is often solved by deriving the normal equations using the least square principle. However, two numerical problems are often encountered in solving the normal equations [2–4]. Firstly, the operation load increases because of the rounding errors introduced in forming the coefficient matrix of normal equations. Special features of the coefficient matrix may be destroyed in some cases. Secondly, the ill-conditioned degree of the coefficient matrix in the normal equations is considerably exacerbated. The first problem is obvious while the second one is not so evident. Herein, a simple example is used to demonstrate the second problem.

Let us suppose that the contradictory equations of a dam safety monitoring model are

$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} + \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{Bmatrix} = 0, \quad (m \gg n). \quad (4)$$

Eq. (4) can be rewritten in a compacted form, i.e.,

$$\boldsymbol{CX} + \boldsymbol{\delta} = 0, \quad (5)$$

where $\boldsymbol{C}$ is an $m\times n$ matrix and its element is C_{ij}; n is the rank of $\boldsymbol{C}$; $\boldsymbol{X}$ is an n-dimensional solution vector, i.e. $(x_1, x_2, \cdots, x_n)^{\mathrm{T}}$; $\boldsymbol{\delta}$ is an m-dimensional known vector, i.e. $(\delta_1, \delta_2, \cdots, \delta_m)^{\mathrm{T}}$.

The ill-conditioned degree of the contradictory equation is defined as the change of the solution induced by a perturbation of the coefficient matrix or the free term. Under certain condition, it can be measured approximately by the so-called condition number $f(\boldsymbol{C})$, which is defined as follows:

$$f(\boldsymbol{C}) = \frac{\max\limits_{\|x\|_2=1} \|\boldsymbol{C}_x\|_2}{\min\limits_{\|x\|_2=1} \|\boldsymbol{C}_x\|_2} = \frac{\mu_1}{\mu_m}, \quad (6)$$

where $\mu_1 = \sqrt{\max(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})}$ and $\mu_m = \sqrt{\min(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})}$.

If a perturbation $\Delta\boldsymbol{C}$ and a perturbation $\Delta\boldsymbol{\delta}$ are made to the coefficient matrix $\boldsymbol{C}$ and the free term $\boldsymbol{\delta}$, respectively, the change of the solution $\Delta\boldsymbol{X}$ should satisfy the following inequality:

$$\frac{\|\Delta\boldsymbol{X}\|_2}{\|\boldsymbol{X}\|_2} \leqslant f(\boldsymbol{C})\left(k_1\frac{\|\Delta\boldsymbol{C}\|_2}{\|\boldsymbol{C}\|_2} + k_2\frac{\|\Delta\boldsymbol{\delta}\|_2}{\|\boldsymbol{\delta}\|_2}\right), \tag{7}$$

where k_1 and k_2 are coefficients.

As mentioned above, the ill-conditioned degree of equations can be measured by the condition number $f(\boldsymbol{C})$ of coefficient matrix $\boldsymbol{C}$. For the case of the normal eq. (5), the condition number of coefficient matrix $\boldsymbol{C}$ reads

$$P(\boldsymbol{C}) = P(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}) = (\mu_1/\mu_m)^2 = (f(\boldsymbol{C}))^2. \tag{8}$$

It can be seen from eq. (8) that the condition number of the coefficient matrix is scaled up to the square of its original value when eq. (5) is transferred into normal equations [5, 6], which means the ill-conditioned degree is greatly increased. For this reason, a problem that is not so ill conditioned may become a seriously ill-conditioned one after the normal equations are derived. Generally, equation is prone to be ill-conditioned when its rank n is greater than or equal to seven. This conclusion can be proved in the following example.

Suppose that $x_1 < x_2 \cdots < x_n$ compose a set of observations of independent variables and $\delta_1, \delta_2, \cdots, \delta_n$ are the related observations of response variables. The dam safety monitoring model established by them can be expressed as

$$\delta = b_0 + \sum_{i=1}^{n} b_i x_i .$$

Denoting

$$R = \sum_{j=1}^{m}(\delta(x_{ij}) - \delta_j)^2 = \sum_{j=1}^{m}\left(b_0 + \sum_{i=1}^{n} b_i x_{ij} - \delta_j\right)^2,$$

and based on the least square principle, one has

$$\partial R/\partial b_i = 0, \quad (i = 0, 1, \cdots, n),$$

or

$$\partial R/\partial b_i = 2\sum_{j=1}^{m}\left(b_0 + \sum_{i=1}^{n} b_i x_{ij} - \delta_j\right) = 0.$$

Thus, $x_1, x_2, \cdots, x_n$ fit the linear normal equation as follows:

$$\sum_{j=1}^{n} c_{ij} b_j = g_i, \quad (i = 0, 1, \cdots, n), \tag{9}$$

where c_{ij} and g_i are the elements of the coefficient matrix and the load matrix in normal eq. (5), respectively. Among them, $c_{ij}=\sum_{t=1}^{m}(x_{it} - \overline{x}_i)(x_{jt} - \overline{x}_j)$, $g_i=\sum_{t=1}^{m}(x_{it} - \overline{x}_i)(\delta_t - \overline{\delta})$, $\overline{x}_i = \frac{1}{m}\sum_{t=1}^{m} x_{it}$, and $\overline{\delta} = \frac{1}{m}\sum_{t=1}^{m}\delta_t$.

If $x_1, x_2, \cdots, x_m$ are equidistant nodes in the interval of $[0,1]$, namely $x_1 = 0, \cdots, x_k = (k-1)/(m-1)$, $\cdots$, $x_m = 1$, then one can obtain $c_{ij} = \sum_{k=1}^{m} x_k^{i+j-2} \approx (m-1)/(i+j-1)$, $i, j = 1, 2, \cdots, n$. Therefore, $\boldsymbol{C}$ approaches to $(m-1)\boldsymbol{A}_n$ when m tends to be infinite, and the concrete form of matrix $\boldsymbol{A}_n$ is

$$\boldsymbol{A}_n = \begin{bmatrix} 1 & 1/2 & \cdots & 1/n \\ 1/2 & 1/3 & \cdots & 1/(n+1) \\ \vdots & \vdots & \ddots & \vdots \\ 1/n & 1/(n+1) & \cdots & 1/(2n-1) \end{bmatrix},$$

$\boldsymbol{A}_n$ is a symmetric positive definite matrix and its inverse matrix is $\boldsymbol{A}_n^{-1}$, so the ratio of maximum and minimum elements of modulus in $\boldsymbol{A}_n^{-1}$ is

$$\left[\left(\frac{3n-1}{2}\right)!\right]^2 \Big/ n^3\left[\left(\frac{n-1}{2}\right)!\right]^6 .$$

The magnitude of the ratio is approximated to 10^n when n is in the interval of $(7,15)$, i.e., the error of the solution will reach 10^n when the error of the right-hand member g_i in eq. (9) is 1. Obviously, the error is not caused by the specific solving method itself, it should be attributed to the extreme sensitivity of the solution to the disturbance of the right-hand member. It can be seen from the simple example that the calculation error of equation will be greatly amplified when n is greater than seven. As a result, the solution of normal equations in eq. (5) is rather prone to be ill-conditioned.

As summarized in section 1, there are three kinds of dam safety monitoring models. For the statistical models, the number of influence factors is often greater than seven and even reaches dozens in some special cases. Therefore, statistical models are prone to be ill-conditioned when solving the relative effect coefficients of the influence factors. For the deterministic model, the number of influence factors is often less than seven, and the normal equation often has a better numerical performance. For the hybrid models, the number of influence factors is greater than that of deterministic models and smaller than that of statistical models. Therefore, the possibility of being ill-conditioned for hybrid models lies between statistical models and deterministic models.

It is shown in the foregoing analysis that the fewer the influence factors are, the higher the model accuracy will be. Not only the monitoring workload can be decreased but also the accuracy of the model is improved considerably. The following research is conducted to demonstrate the improvement of the model accuracy.

Suppose that S and R are the residual mean square error and the residual squares sum of the response variable, respectively. The relationship between them reads

$$S = \sqrt{R/(m-n-1)}. \tag{10}$$

It is evident that a smaller n results in a smaller S and a higher accuracy of the model when R changes slightly. By carefully selecting factors composed of representative influence quantity, the model with fewer influence factors not only can have a higher accuracy but also will not easily be ill-conditioned.

3 Ill-conditioned model caused by information incomplete and its processing method

3.1 Processing method towards the ill-conditioned model based on the method of dynamic iteration

As pointed out previously that the hydraulic component $f(H)$ in eqs. (2) and (3) should be calculated using FEM so as to establish a deterministic model or a hybrid model for the safety monitoring. The displacement of dam (especially concrete dams) caused by hydrostatic pressure is generally a smooth curve without inflection points. The $f(H)$ is usually expressed by a polynomial function of the hydraulic head H, i.e.,

$$f(H) = \sum_{i=0}^{m} a_i H^i, \tag{11}$$

where a_i are the fitting coefficients; m is the highest order of the polynomial function.

According to the theory about hydraulic structures and engineering mechanics, the displacement of certain point in dams shall be a smooth curve without any inflection points under the hydrostatic pressure. However, the ill-conditioned phenomenon of inflection point can often appear when determining the curve, which means the expression of $f(H)$ will not be able to reflect objectively the dam deformation regulation under the hydrostatic pressure, so it causes great difficulty to the analysis of dam deformation regulation and very probably leads to a false conclusion. The causes of such a phenomenon can be classified into two categories. The first one is the small number of the load cases selected to analyze the influence of water pressure on dam deformation, i.e., the considered load cases are too few to include all the cases completely so that unreasonable phenomenon of inflection points appears. The other reason is that the fitting coefficients a_i determined directly by the conventional method are not always the most suitable, and then the inflection point in the expression of $f(H)$ will appear.

The treatment to remove the inflection points in an ill-conditioned problem is studied. Without loss of generality, let us assume that m=4. Eq. (11) can now be expanded as follows:

$$f(H) = a_0 + a_1 H + a_2 H^2 + a_3 H^3 + a_4 H^4. \tag{12}$$

Obviously, $f(H)$ is possible to have inflection points within the range of the hydraulic head H. The necessary condition for the existence of inflection points is that the first and second derivatives of $f(H)$ exist. Furthermore, from the viewpoint of mathematical theory, there should be one real root at least for equation $f''(H)=0$ if the inflection points exist. This condition can be expressed as follows:

$$\Delta = 36a_3^2 - 96a_2a_4 \geqslant 0. \tag{13}$$

In order to avoid the inflection points existing in $f(H)$, $a_i (i = 2,3,4)$ have to meet the following condition:

$$\Delta = 36a_3^2 - 96a_2a_4 < 0. \tag{14}$$

As indicated by eq. (14), the inflection points can be effectively avoided by reasonably adjusting the values of a_2, a_3 and a_4.

As to eq. (12), the displacement $f(H_i)$ of any point in the dam under the hydraulic head $H_i (i = 1,2,\cdots,n)$ can be obtained by FEM. If $f(H)$ is the optimal equation, then it has to fit the following equation according to the least square principle:

$$[c_{ij}]_{m\times n}\{a_i\}_{n\times 1} = \{c_i\}_{m\times 1}, \tag{15}$$

where $[c_{ij}]$ and $\{c_i\}$ are the coefficient matrix and the load matrix of the normal equation in eq. (12), respectively, both of which are related to H_i, $f(H_i)$ and the number of load cases n selected in the FEM calculations.

The matrices $\{c_{ij}\}$ and $\{c_i\}$ are usually different for different combinations of H_i and $f(H_i)$, so a_i obtained using eq. (15) are also different. Under the premise of meeting the engineering accuracy, function $f(H)$ can avoid the inflection points only if the related coefficients fit the condition of eq. (14). There are two methods to solve the problem mentioned above. Firstly, the load cases of different hydraulic head H_i are selected within the possible range as much as possible so that the displacement $f(H_i)$ obtained by FEM is used to establish the model without inflection points. The second one is the so-called dynamic iteration method [7], in which the cost of FEM calculations can be considerably reduced.

3.2 Processing method towards the ill-conditioned model based on the entropy theory

In this part, the method to establish a model for $f(H)$ without inflection points is emphatically studied based on the entropy theory.

The entropy theory was first proposed by Clausius in thermodynamics to express the irreversibility of heat conduction. Later it was applied to information science to measure the average information in information theory [8]. The information entropy of discrete and continuous random variables x are defined respectively as follows:

Discrete variables: $$S(x) = -\sum_{i=1}^{n} p_i \ln p_i, \quad (16)$$

Continuous variables: $$S(x) = -\int_R f(x) \ln f(x) \mathrm{d}x, \quad (17)$$

where $S(x)$ is the measurement of system uncertainty, which is used to characterize the magnitude of information. p_i is the occurrence probability of x_i in discrete random variable x. $f(x)$ is the probability density function (PDF) of continuous random variable x.

There are two meanings in eqs. (16) and (17): One is that the information entropy $S(x)$ can be calculated once the occurrence probability of information is known; the other one is that the information entropy $S(x)$ can be considered as a function of probability distribution or PDF. According to the statistical inference rule of probability distribution proposed by Jaynes [9], the probability distribution that has the maximum entropy should be selected when inferred from partial information. This is because the maximum entropy means the least human assumptions on the condition of data inadequacy, which ensures that the results obtained conform to the reality the best.

It is well known that the relationship between the hydraulic head and the displacement of a dam is certain, i.e., the ratio of the area of the shaded parts in Figure 1 to the area of the curved triangle $H_{\max}AH_{\min}$ is also certain when ΔH_i tends to zero. Therefore, the PDF of displacement distribution can be defined by the dataset $(H_i, f(H_i))$, which includes different hydraulic heads and the corresponding displacement calculated by FEM, i.e.,

$$F(H_i) = f(H_i)\Delta H_i \Bigg/ \left(\sum_{i=1}^{n} f(H_i)\Delta H_i \right), \quad i = 1,2,\cdots,n, \quad (18)$$

where n is the number of FEM calculations.

The principle of the maximum entropy asserts that the PDF $F(H)$ with minimum deviation will ensure that the corresponding entropy value reaches its maximum value under the known constraint conditions of the sample data information. i.e.,

$$\begin{cases} \max & S(H) = -\int_{\Omega} F(H)\ln(F(H))\mathrm{d}H, \\ \text{s.t.} & \begin{cases} \int_{\Omega} F(H)\mathrm{d}H = 1, \\ \int_{\Omega} H^j F(H)\mathrm{d}H = \mu_j, \quad (j = 1,2,\cdots,N), \end{cases} \end{cases} \quad (19)$$

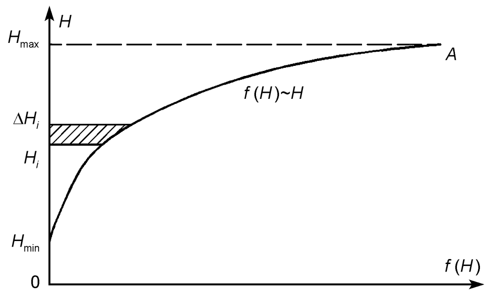

Figure 1 The relationship curve of hydraulic head and displacement.

where Ω is the integral space; $\mu_j (j = 1,2,\cdots,N)$ is the jth-order origin moment of the known sample; N denotes the maximum order. According to the experience, the calculation precision is satisfactory when N reaches 4.

The optimization problem expressed in eq. (19) can be solved by the method of Lagrange multiplier. i.e.,

$$F(H) = \exp\left(\sum_{i=0}^{N} \lambda_i H^i \right). \quad (20)$$

Then, the expression of $f(H)$ in eq.(12) can be obtained by associating eq. (18) with eq. (20).

Because the maximum entropy means the least information added by human, the expression of $f(H)$ obtained by the principle of maximum information entropy is the closest to the real relationship between the hydraulic head and the corresponding displacement that satisfies the requirement of eq. (14). Therefore, the expression of $f(H)$ obtained by this method can be considered as the best expression for the hydraulic component without inflection points.

4 Example analysis

To verify the effectiveness of the algorithm proposed in the study, the displacements monitored along 9# section of a gravity arch dam in Northwest China are chosen for analysis. The height of the selected section is 178 m, and the displacements of the dam body and the foundation are monitored by direct plumb line and inverted plumb line installed.

Because a long sequence of monitoring data is available, we attempt to establish a deterministic monitoring model so as to avoid the ill-conditioned problem. The dam is discretized with 23568 elements and the displacements of the dam body and the foundation are calculated by FEM, considering eight different hydraulic heads $H_i (i = 1-8)$, i.e., 68, 98, 108, 118, 128, 143, 148 and 168 m. The radial horizontal displacements $f(H_i)$ of the measuring point in $\nabla 2600$ obtained by FEM calculations are 0.01, 2.88, 5.60, 9.90, 14.03, 20.74, 23.50 and 45.60 mm, respectively.

Based on the least square principle, the eight groups of results obtained are fitted, and the fitting coefficients $a_i (i = 0-4)$, the discriminant Δ distinguishing the existence of inflection points and the multiple correlation coefficient R characterizing the accuracy of $f(H)$ are listed in Table 1. As can be inferred from the sign of the discriminant, there are two inflection points (H_1=115.71 m, H_2=112.14 m) in the model $f(H)$ established directly by the least square method, indicating the inadequacy of the method and the necessity for employing other approaches.

The coefficents $a_i (i = 0-4)$, Δ and R of $f(H)$ obtained by the method of dynamic iteration and maximum information entropy are also listed in Table 1 for comparison. In particular, the fourth iterative results obtained by

Table 1 The result statistic table of $f(H)$ obtained by different methods

Methods		Least square	Dynamic iteration	Maximum information entropy
Fitting coefficient	a_0	3.0243×10^{2}	3.0244×10^{2}	3.1089×10^{2}
	a_1	-1.1454×10^{1}	-1.1454×10^{1}	-1.1719×10^{1}
	a_2	1.5581×10^{-1}	1.5579×10^{-1}	1.5874×10^{-1}
	a_3	-9.1197×10^{-4}	-9.1110×10^{-4}	-9.2566×10^{-4}
	a_4	2.0012×10^{-6}	2.0005×10^{-6}	2.0242×10^{-6}
Δ		7.3448×10^{-9}	-3.5442×10^{-8}	-3.9309×10^{-10}
R		0.9998	0.9925	0.9998

dynamic iteration method are used to approximate the results of the least square method.

Some meaningful conclusions can be deawn from Table 1.

1) The discriminant Δ of $f(H)$ obtained by the method of least square is greater than zero, indicating the existence of inflection points ill-condition of$f(H)$.

2) The expression of $f(H)$ without inflection points can be obtained by the method of dynamic iteration and maximum information entropy. For the method of dynamic iteration, there are many solutions meeting the condition of eq. (14). The final solution should be selected so as to preserve a higher accuracy. Since iterations are involved in the dynamic iteration method, the computational expense is generally high. On the contrary, the method of maximum information entropy can provide a function of $f(H)$ without inflection points directly, so it is economic to the cost of computation.

Based on the comparison of the mentioned methods, the maximum information entropy seems to be the most cost-effective way to establish the expression of $f(H)$ without inflection point.

5 Conclusions

In this paper, we concentrate our attention on the ill-conditioned problems of dam safety monitoring models. The causes leading to the ill-conditioned problems are first analyzed and the criterions for the judgment are then studied, finally the corresponding treatments are proposed. The main conclusions are summarized as follows.

1) The reason for the ill-conditioned problems of dam safety monitoring models lies in the fact that the number of influence factors exceeds seven. Since the number of influence factors in deterministic models is six at most, the degree of ill condition is much weaker than those in statistical models and hybrid models. The number of influence factors in dam safety monitoring models should be as less as possible for the sake of accuracy.

2) The ill-conditioned problems caused by incomplete information are explored. After analyzing the causes of inflection points of the hydraulic component in dam safety monitoring models, the criterion of ill-conditioned problems is proposed and the corresponding processing methods are subsequently suggested to remove the inflection points.

3) During the analysis of engineering problems, massive in-situ monitoring data are processed and the corresponding analysis models are of paramount importance in monitoring the safety of the engineering. Due to the generality of theory proposed in the study, it could also be used in evaluating the performance of structures in other fields for higher accuracy and validity.

This work was supported by the National Natural Science Foundation of China (Grant Nos. 51079046, 50909041, 50809025, 50879024, 51139001), the National Science and Technology Support Plan (Grant Nos. 2008BAB29B03, 2008BAB29B06), the Special Fund of State Key Laboratory of China (Grant Nos. 2009586012, 2009586912, 2010585212), the Fundamental Research Funds for the Central Universities (Grant Nos. 2009B08514, 2010B20414, 2010B01414, 2010B14114), China Hydropower Engineering Consulting Group Co. Science and Technology Support Project (Grant No. CHC-KJ-2007-02), Jiangsu Province "333 High-Level Personnel Training Project" (Grant No. 2017-B08037), the Graduate Innovation Program of Universities in Jiangsu Province (Grant No. CX09B_163Z), and the Science Foundation for the Excellent Youth Scholars of Ministry of Education of China (Grant No. 20070294023).

1 Gu C S, Wu Z R. Safety Monitoring of Dams and Dam Foundations—Theories & Methods and Their Applications. Nanjing: Hohai University Press, 2006. 8–20, 129–144
2 Feng K. Numerical Computation Method. Beijing: National Defence Press, 1978. 37–39
3 Cao W P, Ma J P. The generalized solution of ill-posed boundary problem. Sci China Ser A-Math Sci, 2006, 49: 902–911
4 Liu Z H, Li J, Li Z W. Regularization method with two parameters for nonlinear ill-posed problems. Sci China Ser A-Math Sci, 2008, 51: 70–78
5 Huang J Q. Practical Modeling Method of Static and Dynamic Mathematical Model. Beijing: China Machine Press, 1988. 136–138
6 Eric W, Luc P. Identification of Parametric Models from Experimental Data. Masson: Springer Verlag, 1997. 74–82
7 Wu Z R, Gu C S. Back Analysis and Application of Dam. Nanjing: Jiangsu Science and Technology Press, 2000. 51–55
8 Su H Z, Wen Z P, Hu J, et al. Evaluation model for service life of dam based on time-varying risk probability. Sci China Ser E-Tech Sci, 2009, 52(7): 1966–1973
9 Jaynes E T. Information theory and statistical mechanics. Phys Rev, 1957, 106(4): 620–630