SCIENCE CHINA
Technological Sciences

October 2011 Vol.54 No.10: 2674–2686
doi: 10.1007/s11431-011-4533-7

Study on interference suppression based on joint fractional Fourier domain and time domain

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Received January 21, 2011; accepted July 11, 2011; published online August 7, 2011

An interference suppression algorithm is proposed to meet challenges of the traditional technique in dealing with the linear frequency modulation (LFM) interference, such as high loss of signal-to-noise ratio (SNR), the output signal-to-interference-plus-noise ratio (SINR) sensitive to input interference-to-signal ratio (ISR) that results in an unstable synchronization, and the spectrum leakage serious in strong ISR situation. This approach firstly makes use of the windowed and lapped technique to the fractional Fourier transform (FRFT) to enhance the ISR improvement and lower the SNR loss. Then by weakening the interference and a secondary threshold process, interference energy can be suppressed as much as possible and the output SINR is less sensitive to the ISR. Finally, a joint fractional Fourier domain and time domain technique is proposed to overcome the residual interference energy caused by the strong interference or the discontinuous-phase interference. Theoretical analysis and simulation results show that the proposed algorithm can achieve better performance than the conventional methods in suppressing both the multi period LFM interference and the multi chirp-rate LFM interference, especially in the strong interference environment.

fractional Fourier transform, linear frequency modulation interference, interference suppression, direct sequence spread spectrum

Citation: Huang K W, Tao R, Wu K, et al. Study on interference suppression based on joint fractional Fourier domain and time domain. Sci China Tech Sci, 2011, 54: 2674–2686, doi: 10.1007/s11431-011-4533-7

1 Introduction

Direct Sequence Spread Spectrum (DSSS) system, which has advantages including anti-jamming, anti-multipath interference and multiple access communication, has been widely used in fields of communication, navigation and so on. Due to its encoding gain and spreading gain, DSSS system has the ability of anti-jamming. However, strong interference exceeding its interference tolerance may deteriorate the performance of DSSS system because of the limitation by complexity and available bandwidth of communication system. Take the receivers of spread spectrum station, global positioning system (GPS), BeiDou navigation satellite system (BD), constellation satellite system for example, none of them can work normally in strong interference situation. Although the method of suppressing strong narrowband interference is mature at present [1–3], it is not effective for suppressing unstable broadband interference in practice.

Linear frequency modulation (LFM) signal, namely sweeping frequency signal or chirp signal, is one of the common unstable interferences with wide bandwidth and does harm to DSSS system. The typical form of LFM signal has one chirp rate in one or more cycles or has multi chirp rates within processing data. Therefore, the domestic and

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international scholars have carried on incessant studies on suppressing LFM interferences by using digital signal processing and have made achievements so far [4–11]. Fourier transform has been applied to suppress narrowband stable interferences, but it is not ideal to deal with quick time-varying broadband signals like LFM interference [4]. Time-frequency analysis methods proposed by Amin [5], Barbarossa [6], Chaparro [7] can recognize and eliminate broadband LFM interference to a certain extent. However, when there exists more than one LFM interference, it can hardly identify them separately due to the existence of cross term from Wigner-Ville Distribution (WVD). In addition, only when one interference exists in every moment can the method of tracing instantaneous frequency be useful. For example, the suppression method based on WVD-Hough transform proposed by Barbarossa can be used to identify several interferences, but the cross term of WVD by Hough transform will form the pseudo-peak which may cause false identification. Both short-time Fourier transform [8] and wavelet transform [9] are suitable for suppressing the pulse interference. The algorithm of suppressing LFM interferences based on fractional Fourier transform (FRFT) proposed by Olcay [10] and Qi [11] eliminates the effects of cross term so that it can be adopted to suppress the interference owning one chirp rate in one cycle within processing data. However, in practice, due to the existence of one or more frequency modulation rates in multi-cycles, this algorithm may result in high signal-to-noise ratio (SNR) loss. Besides, under the condition of strong interference, the residual interference energy in transform domain after suppression may affect the output SNR and results in instability of synchronization. Meanwhile, due to the phase discontinuity of strong interference, its residual interference resulting from the serious leakage of side lobes in time domain may deteriorate the bit error rate (BER) performance of DSSS system.

Section 2 in this paper analyzes the existing problems and reasons in traditional suppression method based on FRFT. As to the existent problems, a joint interference suppression method in fractional Fourier domain (FRFD) and time domain based on lapped technique is put forward in Section 3. This approach firstly makes use of the combination of lapped technique [12] and FRFT by windowing to enhance the interference-to-signal ratio (ISR) improvement and lower the SNR loss. Then by weakening the interference and doing a secondary threshold process in FRFD, the residual interference energy can be suppressed as much as possible in transform domain. Finally, a joint FRFD and time domain method is proposed to overcome the residual interference problem caused by the discontinuous-phase interference. Simulation and contrastive analysis presented in Section 4 analyzes the detection and suppression performance of the proposed method in DSSS system. The effects of phase skipping on conventional methods in the strong interference environment are also analyzed. The diagrams of both the output signal-to-interference-plus-noise ratio (SINR) and BER are presented to show the performance of suppressing both the multi period interference and the multi chirp-rate interference by comparing the proposed method and traditional methods. In the end, a summary is put forward.

2 Problems of traditional FRFT in interference suppression

2.1 Transform domain interference suppression principle

Interference suppression technology for DSSS system mainly includes time domain suppression, transform domain suppression and spatial domain suppression technologies. Transform domain suppression technology, mainly based on Fast Fourier Transform (FFT), is easy to implement in engineering, effective in narrowband interference and has been widely used [2, 3, 13]. Transform domain suppression technology is mainly composed of transformation basis selection, interference detection threshold criteria and notch suppression. The main principle of interference suppression in transform domain is shown in Figure 1.

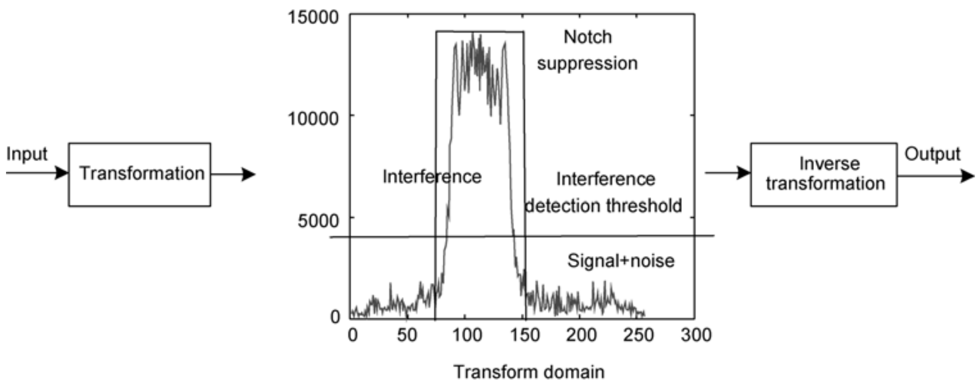

Figure 1 Principle of interference suppression in transform domain.

Based on the difference of transformation basis, transform domain interference suppression technique mainly includes interference suppression based on discrete Fourier transform (DFT), suppression technique based on lapped transform and sub-band transform and suppression technique based on time-frequency domain. Various transform domain interference suppression methods have different effects on different interferences, and the transformation basis, which can make interference energy gather, is the optimal one. For LFM interference, FRFT, which has the characteristic of gathering its energy, is an optimal transformation basis [4]. In addition, the interference threshold detection method mainly has weight leakage method, K spectral lines, N-sigma method and so on. Moreover, key performances include the ability of interference detection to adapt to ISR's change and implementation complexity. The notch process includes zero method and clamp method.

2.2 Definition of FRFT

Recently, as a new time-frequency analysis tool [14], the FRFT has been widely used in signal processing [15]. FRFT is a generalized form of Fourier transform. Signal's FRFT can be explained as its rotation around the axis and around the origin in the time-frequency plane.

The FRFT of the signal $x(t)$ is defined as

$$X_p(u)=\left\{F_p\left[x(t)\right]\right\}(u)=\int_{-\infty}^{+\infty}x(t)K_p(t,u)\,\mathrm{d}t, \tag{1}$$

in which $p=2\alpha/\pi$ is the FRFT's order, α is the rotation angle, $F_p[\cdot]$ is the FRFT's operator and $K_p(t,u)$ is the FRFT's transform kernel.

$$K_p(t,u)=\begin{cases}\sqrt{\dfrac{1-\mathrm{j}\cot\alpha}{2\pi}}\cdot\exp\left(\mathrm{j}\dfrac{t^2+u^2}{2}\cot\alpha-\mathrm{j}ut\csc\alpha\right), \\ \qquad\qquad \alpha\neq n\pi, \\ \delta(t-u), \qquad \alpha=2n\pi, \\ \delta(t+u), \qquad \alpha=(2n\pm1)\pi. \end{cases} \tag{2}$$

The inverse-FRFT (IFRFT) is

$$x(t)=\int_{-\infty}^{+\infty}X_p(u)\,K_{-p}(t,u)\,\mathrm{d}u\,. \tag{3}$$

Currently, there are several different types of discrete FRFT (DFRFT) fast algorithm [16–18], such as decomposition fast algorithm and discrete sampling algorithm, with different accuracy and computational complexity. This paper selects discrete sampling DFRFT algorithm [17], and the computational complexity is $O(\log_2 N)$, where N is sampling points. This algorithm has orthogonality, so transform sequences can be restored through the inverse transform and it is suitable for processing in the transform domain. Discrete sampling DFRFT algorithms can be expressed as

$$\begin{cases} X_\alpha(m)=A_\alpha\,\mathrm{e}^{\frac{\mathrm{j}}{2}\cot\alpha\, m^2\Delta u^2}\sum_{n=0}^{N-1}\mathrm{e}^{\frac{\mathrm{j}}{2}\cot\alpha\, n^2\Delta t^2}\,\mathrm{e}^{-\mathrm{j}\frac{2\pi nm}{M}}\,x(n), \\ \qquad\qquad \alpha\neq D\pi, \\ X_\alpha(m)=x(m), \qquad \alpha=2D\pi, \\ X_\alpha(m)=x(-m), \qquad \alpha=(2D+1)\pi, \end{cases} \tag{4}$$

where $A_\alpha=\sqrt{(\sin\alpha-\mathrm{j}\cos\alpha)/N}$ and D is integer.

2.3 Problems of traditional FRFT in interference suppression

The basic function of FRFT is a set of orthogonal chirp basis. In the appropriate FRFD, a LFM signal will be an impulse function. When the order $p\in(0,2)$, the chirp rate and focus order of the LFM signal have the following correspondence $\mu=-\cot(p\,\pi/2)$.

The principle of the FRFT interference suppression would be analyzed bellow. Let the LFM signal be

$$g(t)=A\exp\left(\mathrm{j}2\pi f_\mathrm{m}t+\mathrm{j}\pi\mu t^2+\mathrm{j}\varphi\right),\quad t\in[-T/2,T/2]. \tag{5}$$

Substituting eq. (5) into eq. (2), FRFT with order p of $g(t)$ is

$$\begin{aligned} G_\alpha(u)&=A\times\sqrt{(1-\mathrm{j}\cot\alpha)} \\ &\quad\times\int_{-T/2}^{T/2}\mathrm{e}^{\mathrm{j}\pi\left(t^2\cot\alpha-2ut\csc\alpha+u^2\cot\alpha\right)}\mathrm{e}^{\mathrm{j}\pi\left(\mu t^2+2f_\mathrm{m}t\right)+\mathrm{j}\varphi}\,\mathrm{d}t \\ &=\frac{A}{\sqrt{\sin\alpha}}\,\mathrm{e}^{\mathrm{j}\frac{\alpha}{2}+\mathrm{j}\frac{3\pi}{4}+\mathrm{j}\pi\left(u^2\cot\alpha\right)+\mathrm{j}\varphi} \\ &\quad\times\int_{-T/2}^{T/2}\mathrm{e}^{\mathrm{j}\pi t^2(\cot\alpha+\mu)}\mathrm{e}^{\mathrm{j}2\pi t\left(f_\mathrm{m}-u\csc\alpha\right)}\,\mathrm{d}t, \end{aligned} \tag{6}$$

where $p=\alpha\,2/\pi$. When $\mu=-\cot\alpha$, $f_\mathrm{m}=u\csc\alpha$, the fractional Fourier amplitude spectrum peak of $g(t)$ is

$$\left|G_\alpha(u)\right|^2_{\max}=\left|G_\alpha(u)\right|^2\Big|_{\mu=-\cot\alpha,f_\mathrm{m}=u\csc\alpha}=\frac{A^2\cdot T^2}{|\sin\alpha|}. \tag{7}$$

According to the LFM signal chirp rate μ, the main energy of the LFM signal can be extracted through p-order ($p=-2\,\mathrm{arc}\cot(\mu)/\pi$) FRFT. The statistical properties of the DS signal is similar to the Gaussian white noise distribution and in any FRFT domain, it always manifests itself as noise, so we can use FRFT's focusing property to chirp signal to separate the LFM interference and the spread spectrum signal.

Based on FRFT, traditional interference suppression methods have some problems such as the high loss of SNR, the difficulty in suppressing the interference energy completely in the FRFD through interference detection method and the residual interference energy in time domain caused by the discontinuous phase interference after inverse transform. Firstly, during FRFT, due to the limited interception to LFM signal, interference signal produces spectrum leakage and has a low side-lobe suppression ratio. If suppressing side lobe, it is likely to cause greater damage, and if not, the output SINR will be reduced by the residual side-lobe energy, especially there are multiple LFM interferences. Due to the need for multiple FRFT and IFRFT, the accumulation of the loss of SNR is significant. Secondly, the traditional interference detection method is sensitive to ISR, which leads to the great changes of the output SINR. In strong ISR situation, there may likely appear residual interference energy in FRFD after suppression, which influences the output SINR and leads to an unstable synchronization. Finally, there have been some literatures about LFM interference suppression [10, 11]. These literatures mainly deal with interference models with a single chirp frequency and a single cycle in the data section. However, in practical applications, LFM interference suppression is often manifested in the form that the processing data section includes a single chirp frequency and multiple cycles or multiple chirp frequency, and it is prone to cause discontinuous phase between interference cycles. When the interference energy is large, side-lobe energy leakage is serious which is difficult to suppress completely in FRFD. After the inverse transform, there is residual interference energy in the time domain and it is difficult to suppress interference completely only through conventional methods. Moreover, the performance of suppressing is poor under large ISR.

3 An interference suppression algorithm of joint FRFD and time domain based on lapped transform

An interference suppression algorithm of joint FRFD and time domain based on lapped transform is proposed in the paper for the aspects of choosing reasonable transform methods, suppressing the interference energy sufficiently in transform domain and removing the residual energy in time domain. This algorithm can achieve better performance in suppressing both the multi period LFM interference and the multi chirp-rate LFM interference. According to the problem of high loss of SNR, this approach firstly reduces the impact caused by the discontinuous edges when counting signals with the windowing, this operation can also improve the side-lobe suppression ratio of interfering signals, so the energy of the LFM interference will gather in the main lobe and reduce the damage to the signal. Then with the lapped transform principle, it solves the problem of reduction of the fringe signal energy caused by adding windows, so the LFM interference energy can be suppressed as much as possible. According to the problem of suppressing interference energy difficultly with traditional interference detection processing method in transform domain, an interference detection approach by weakening the interference and doing a secondary threshold process is proposed, which ensures to keep stable output SINR with different ISR. Finally, according to the problem of residual interference energy in time domain after interference suppression in FRFT domain, a joint interference suppression algorithm based on both FRFD and time domain is introduced to solve the problem of discontinuous phase in strong ISR situation. In the following, several aspects will be analyzed, such as adding windows processing and its effect, segmentation overlay processing, interference detection and suppression approach, eliminating impulse processing in time domain.

3.1 Adding windows processing and its effect

After sampling with LFM signals, certain length data are intercepted and then FRFT is implemented. Signal truncation process is equal to the process that the signal multiplies rectangular window function. Windowing is inevitable. Once signal is multiplied by window function, it means that the total transform is the convolution with the expected transform and window function transform. If the real power of signal is concentrated in the same frequency band, the convolution operation will expand the narrowband signals' power to adjacent areas, which causes serious spectrum leakage in FRFD. Leakage effect is connatural with DFRFT. It can be suppressed by using window functions weighted technique, a proper window function will be chosen to make the weighted signal smoother on the edge than rectangular window function, so the side-lobe energy is reduced which is caused by the sharp edges.

Take rectangular window for example, the first side lobe is 13 dB lower than its main lobe, that is the side lobe suppression degree is only 13 dB, compared with the interference which is higher than the useful signal, its side lobe is much higher than useful signal. The interference is suppressed incompletely, or the suppressed band range is expanded when interference suppression is processed, it increases the damage to the useful signal. In order to decrease the interference's spectrum leakage, window function which has lower side lobe is necessary. But when the side lobe is lower, the main lobe will grow wider rapidly, so we should make a compromise when choosing window functions.

In essence, windowing is weighting with the input data, the window function has an attenuation from center to the two ends, ensuring the smoothness of the two ends of the data segment, so spectrum leakage can be decreased. But the input signal will be distorted because of the window function, it brings extra SNR loss. The SNR loss, which is caused by windowing, is discussed below.

Suppose $r(k)$ is the sample sequence of receiving signal

$$r(k) = As(k) + n(k), \tag{8}$$

where $s(k)$ is the sending sequence with length N; $n(k)$ is the

Gaussian white noise sequence whose mean value is zero and variance is σ^2.

The windowed signal of $r(k)$ is

$$r_w(k) = As(k)w(k) + n(k)w(k), \tag{9}$$

where $w(k)$ is the window function.

Then by de-spreading N length $r_w(k)$ with relevant algorithm and carrying out integral operation, the output can be expressed as

$$y = \sum_{k=1}^{N} r_w(k)r(k) = \sum_{k=1}^{N} Aw(k) + \sum_{k=1}^{N} n(k)w(k)r(k). \tag{10}$$

The correlation output's mean value and variance are obtained as follows:

$$E(y) = A\sum_{k=1}^{N} w(k), \quad D(y) = \sigma_n^2 \sum_{k=1}^{N} w^2(k). \tag{11}$$

The relevant output SNR can be expressed as

$$SNR_w = \left[A\sum_{k=1}^{N} w(k) \right]^2 \Big/ \sigma_n^2 \sum_{k=1}^{N} w^2(k). \tag{12}$$

Similarly, the relevant output SNR with no windowing is

$$SNR = NA^2 / \sigma_n^2. \tag{13}$$

So the extra SNR loss caused by windowing can be expressed as

$$SNR_{\text{loss}} = \left[\sum_{k=1}^{N} w(k) \right]^2 \Big/ \left[N\sum_{k=1}^{N} w^2(k) \right]. \tag{14}$$

3.2 Segmentation overlay and de-overlay processing

Adding window to the receiving signal will reduce the influence because of edge data's discontinuity, but the signal will twist to some extent after windowing and truncating. Figure 2(a) shows signal with strong LFM interference, Figure 2(b) is the result of windowing. From Figure 2, we can see that the signal edge has obvious twist, which causes the SNR loss.

In order to reduce the twist, an overlap-processing technique is adopted. There are two signal-processing channels. Firstly, the first channel $a(x)$ has $M/4$ points zero-padding before the original signal, the input of the second channel $b(x)$ is the $M/4$ points delay signal compared with the original signal, after exchange processing of the first and second channels, the first and last $M/4$ points are discarded, but the $M/4$ points in between are saved, then the two channels become one channel. The aim of this processing is to remove signal components that have larger twist caused by windowing, and save signal components that have lower loss. When the two channels become one channel, the whole signals have smaller twist. Figure 2(d) shows signal wave with overlap processing, compared with Figure 2(b), the wave of signal with processing is more close to the original signal. Figure 3 gives the overlap processing flow chart.

The following analyses the SNR loss after overlap processing. The output sequence after overlap processing is as follows:

$$r_{w1}(k) = As(k)w_1(k) + n(k)w_1(k), \tag{15}$$

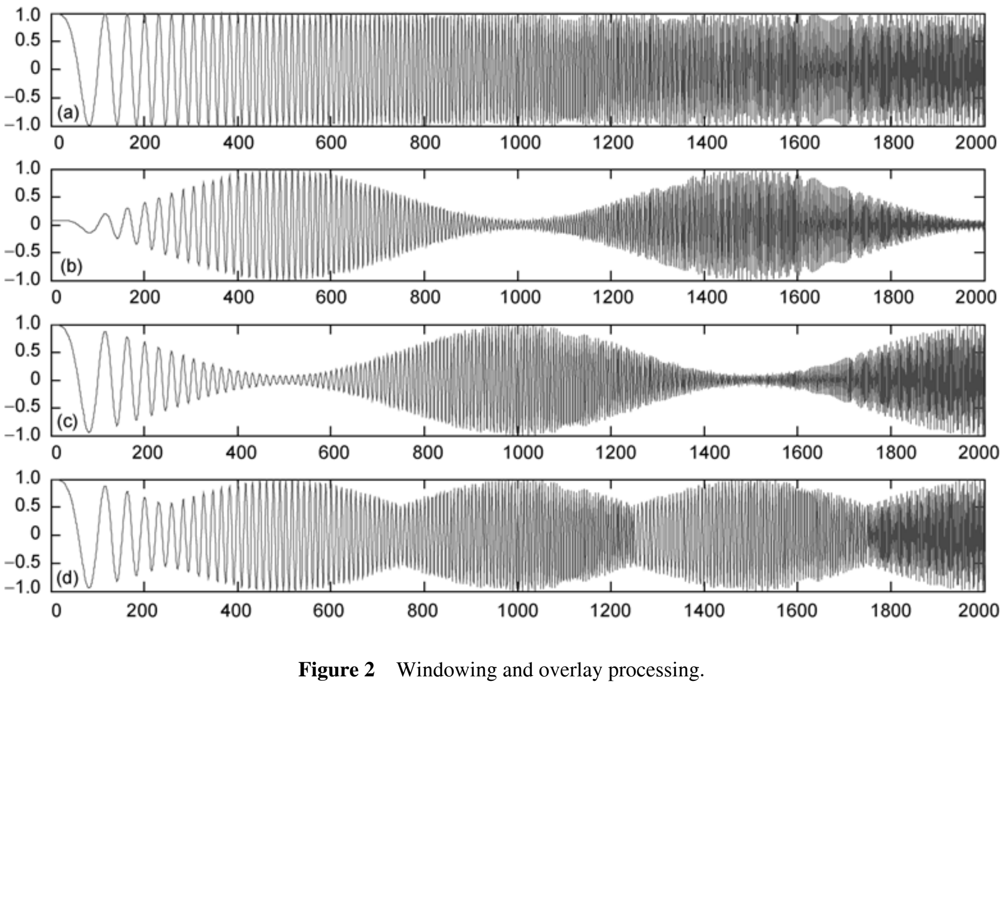

Figure 2 Windowing and overlay processing.

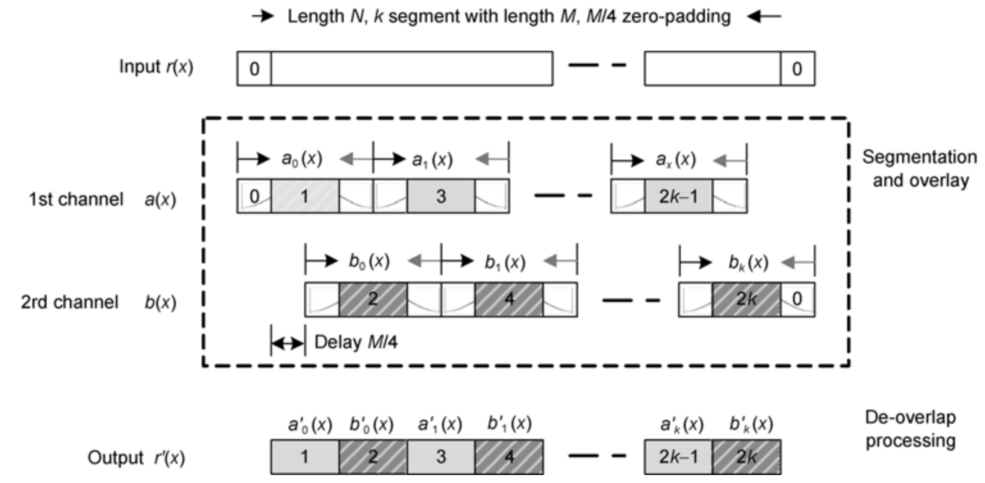

Figure 3 Overlay processing flow chart.

where

$$w_1(k)=\begin{cases} w(k+N/4), & 1\leqslant k\leqslant N/2,\\ w(k-N/4), & N/2<k\leqslant N.\end{cases} \tag{16}$$

So the SNR loss after overlap processing can be expressed as

$$SNR_{w1\text{loss}}=2\left[\sum_{k=N/4+1}^{3N/4} w(k)\right]^2 \Bigg/ \left[N\sum_{k=N/4+1}^{3N/4} w^2(k)\right]. \tag{17}$$

Table 1 analyses the side-lobe suppression of different window functions and the SNR loss with different processing modes, the data length N is 256. It calculates the SNR loss in both cases shown in eqs. (14) and (17). By comparison, we know that windowing can improve the side-lobe suppression, it is helpful to gather the main lobe's energy, but it will bring higher SNR loss. The approach that lapped transform combined with FRFT has better side-lobe suppression, it can also improve the performance of SINR and have lower SNR loss. What is more, the approach performs better than the traditional methods in the subsequent interference suppression. The Blackman window is usually chosen in practical application.

3.3 FRFD interference suppression based on overlap transform

The FRFD interference suppression processing consists of three steps: first, interference energy concentration by choosing the order corresponding to the LFM interference frequency modulation ratio, then the interference detection and suppression processing, and finally, inverse transformation to time domain. Time domain signal $r'(n)$ consists of damaged signal and residual interference.

$$\begin{aligned} r'(t) &= \text{IFRFT}\left\{R(u)\left[1-\sum_{i=1}^{K}\text{Rect}_i(u_1,u_2)\right]\right\}\\ &= \text{IFRFT}\left\{\left[S(u)+I(u)+N(u)\right]\left[1-\sum_{i=1}^{K}\text{Rect}_i(u_1,u_2)\right]\right\}\\ &= \left[s(t)+i(t)+n(t)\right]\tilde{\otimes}\left[1-\sum_{i=1}^{K}\text{sinc}(t)\right]\\ &= \left[s(t)+n(t)\right]\tilde{\otimes}\left[1-\sum_{i=1}^{K}\text{sinc}(t)\right]\\ &\quad + i(t)\tilde{\otimes}\left[1-\sum_{i=1}^{K}\text{sinc}(t)\right], \end{aligned} \tag{18}$$

where $\tilde{\otimes}$ represents fractional convolution [19], K represents the number of the interferences; $[S(u)+N(u)]$ represents the white noise envelop characteristic, and the energy is uniformly distributed in FRFD, suppressed by local rectangular window $\text{Rect}_i(u_1, u_2)$ and inverse transform to time domain, the loss of SNR is $-10\log\left(1-\sum_{i=1}^{K}\eta_i\right)\text{dB}$. η_i is the ratio of i^{th} rectangular width to the whole FRFD width. The energy of $i(t)$ in FRFD is mainly concentrated in the local rectangular window, but because of spectrum leakage caused by windowed operation and discontinuous phase, there are still partially extended components and side-lobe components out of the local rectangular window, namely

Table 1 SNR loss with different processing

Window function	Main lobe beam width	Side lobe peak attenuation (dB)	Windowing SNR loss (dB)	Overlay processing loss (dB)
Rectangle	$4\pi/N$	–13	0	0
Hanning	$8\pi/N$	–31	1.76	0.152
Hamming	$8\pi/N$	–41	1.34	0.125
Blackman	$12\pi/N$	–57	2.37	0.338

$i(t)\tilde{\otimes}\left[1-\sum_{i=1}^{K}\text{sinc}(t)\right]$. When ISR is larger, residual interference can still significantly worsen the system performance.

Threshold setting of interference detection in FRFD is a crucial issue, and whether the threshold setting is suitable will decide the anti-jamming effect. There are many methods to set threshold, and most of them are based on statistical distribution, such as N-sigma method, K spectrum method, weight-value leakage method. This paper, starting from the signal distribution, fully considering the engineering complexity and real-time suppression effect, proposes a interference detection method by weakening interference and doing secondary threshold processing, thereby improving interference suppression performance. The principle of the proposed interference detection method by weakening interference and doing a secondary threshold process is derived from this paper below. Spread spectrum signal is subject to Gaussian distribution. As FRFT is linear transformation, the FRFD signal is still Gaussian distribution in FRFD, and fractional Fourier spectrum of the signal is subject to Rayleigh distribution:

$$f(\alpha)=\frac{\alpha}{\sigma^2}\mathrm{e}^{-\frac{\alpha^2}{2\sigma^2}}, \tag{19}$$

where $\alpha>0$, the mean value of Rayleigh distribution is $\sigma\sqrt{\pi/2}$, the distribution function is $F(x)=1-\exp(-x^2/2\sigma^2)$, so when threshold is set as x, the probability of α greater than x is $\exp(-x^2/2\sigma^2)$. Therefore, if the product of amplitude mean multiplying coefficient P is used to represent the threshold G, the probability of x greater than G is shown in Table 2.

The selection of threshold not only can fully guarantee to suppress the interference energy but also can minimize the damage to signal. From Table 2, in weak interference situation, making a statistical mean, the condition of $P\in(2,3)$ can suppress interference effectively, without losing large SNR.

In strong interference situation, first make a statistical mean, choosing $P_1\in(3,4)$ to complete the process of weakening strong interference, then make a statistic mean of processed signal, choosing $P_2\in(2,3)$ to suppress interference and remove residual interference energy. Specific procedures are as follows:

$$\begin{cases}G_1=P_1\cdot\text{Mean}(\text{abs}(R(u))):\text{abs}(R(u))>G_1,\ R(u)=0,\\ G_2=P_2\cdot\text{Mean}(\text{abs}(R'(u))):\text{abs}(R'(u))>G_2,\ R'(u)=0,\end{cases} \tag{20}$$

where Mean represents the mean value, abs represents the amplitude. G_1 represents the threshold of weakening interference, which can be obtained by calculating fractional Fourier spectrum $R(u)$ statistical mean. G_1 is used to remove the power of main interference which is much more powerful than the signal in main lobe. G_2 represents the threshold of doing secondary threshold processing, which can be obtained by calculating the statistic mean of the mixed signal and residual interference energy $R'(u)$ after weakening interference. G_2 is used to remove the residual energy which is smaller than main interference energy but also significantly greater than the signal. In practice, the selection of the specific values of P_1 and P_2 can be fine tuned according to receiver loss and noise floor. This method can not only adapt to different ISR interferences, but also to multiple interferences.

Table 3 shows the performance comparison of interference detection under different threshold methods. Hereinto, N-sigma method is widely used in engineering because of its simple algorithm, but this algorithm is sensitive to the ISR, it is difficult for threshold setting to take account of the stability in both high ISR and low ISR situations. Therefore, it will have great influence on the following synchronization. K spectrum line method increases computational complexity due to the needs of sorting. Weight leakage method also increases the computational complexity, and it may encounter the problem of convergence rate for rapid change interference. The method proposed in this paper, first, weakening interference, then doing the secondary threshold processing, could better suppress residually extended main-

Table 2 Analysis of threshold probability

Threshold G	One time of mean: $\sigma\sqrt{\pi/2}$	Two times of mean: $2\sigma\sqrt{\pi/2}$	Three times of mean: $3\sigma\sqrt{\pi/2}$	Four times of mean: $4\sigma\sqrt{\pi/2}$
Probability of over threshold	$\exp\left(\frac{-\pi}{4}\right)=0.4559$	$\exp(-\pi)=0.0432$	$\exp\left(-\frac{9\pi}{4}\right)=8.5\times10^{-4}$	$\exp(-4\pi)=3.5\times10^{-6}$

Table 3 Performance comparison of interference detection under different threshold methods

	N-sigma method	K spectrum line method	Weight leakage method	Method of this paper
Response time	fast	fast	convergence slow	fast
Calculation complexity	low	high	high	low
Influence by ISR	sensitive	sensitive	insensitive	insensitive

lobe component in different ISR situations, making the SNR output stable after the interference suppression and benefit to the following synchronization. Besides, conventional methods are needed to calculate the mean and variance, or sort, etc. While the proposed method in this paper, despite doing the secondary threshold detection, only calculates twice statistic mean value of N points, low complexity and easy-to-work pipeline implementation.

The procedure of interference suppression based on lapped transform in FRFD is shown in Figure 4. First, through the windowing process, the combination of lapped transform principle and FRFT is used to improve the side-lobe suppression ratio and aggregate interference energy. Then the technique of weakening interference and secondary threshold processing is used to fully suppress interference energy in FRFD. After inverse transform, removing overlap processing is done, which enhances the ISR improvement ability and reduces the SNR loss.

3.4 Removing pulse processing in time domain

Phase-discontinuous signal is a kind of common signal in digital communication, such as BPSK, 2ASK, QAM signals of digital modulation and multi-cycle LFM signal. There are two kinds of phase jump in LFM interference, one is phase discontinuous between the beginning and end of LFM, and the other is that LFM signal has multiple cycles and phase is discontinuous between each cycle. The problem of phase discontinuity is difficult to avoid. Phase jump can easily result in high frequency component and interference signal extension in FRFD, and it is difficult to suppress thoroughly in FRFD. Especially in high ISR situation, the problem is particularly prominent. Figure 5 illustrates as ISR=50 dB, the comparison of LFM interference fractional spectrum in continuous phase situation and phase jump $\pi/2$ situation. After phase jump, main-lobe energy of interference lowers, spectrum widens and the energy spreads to the entire FRFD. The greater the phase jump, the more serious the situation is; the bigger the ISR is, the greater the influence is.

Due to phase discontinuity, the spectrum in FRFD is expanded. After removing main-lobe energy of interfering signal, certain expanding component and side-lobe energy still exist. Therefore, it is difficult to suppress the interference thoroughly and reserve the useful signal at the same time. Changing back into time domain, residual interfering signal $i(t)\tilde{\otimes}\left[1-\sum_{i=1}^{K}\mathrm{sinc}(t)\right]$ performs in pulse form when phase jumps. Through pulse handling in the time

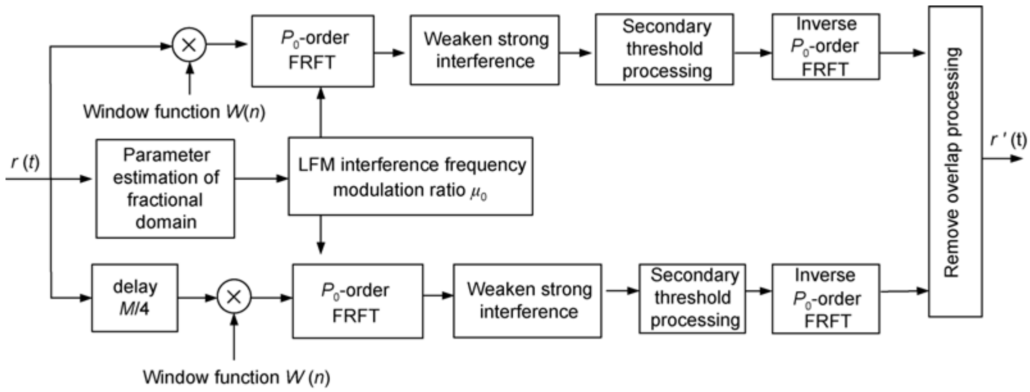

Figure 4 Interference suppression based on lapped transform in FRFD.

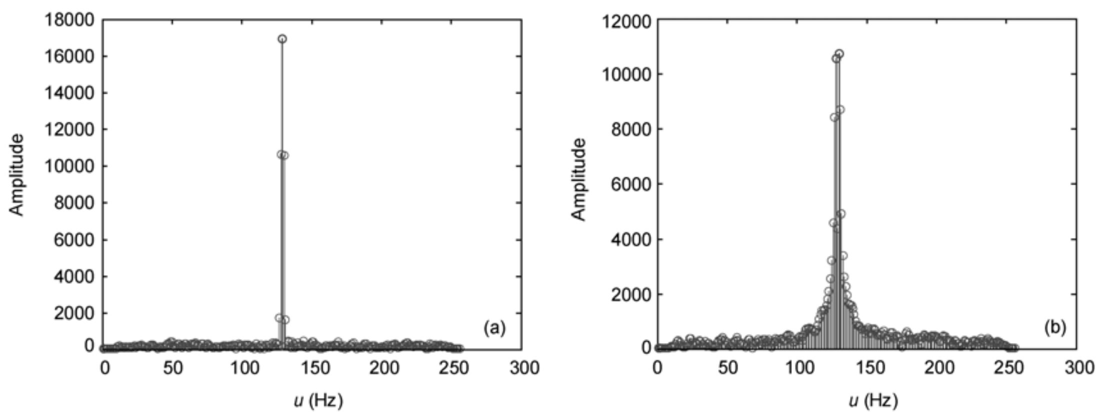

Figure 5 Comparison of fractional Fourier spectrum of continuous phase and phase jump. (a) Fractional spectrum of no phase jump; (b) fractional spectrum of phase jump.

domain, we further remove interference residual energy. Figure 6 gives an interference suppression technique of combining FRFD with time domain. First, we obtain the LFM interference adjustable frequency parameters through parameter estimation in FRFD. Then we make use of interference suppression in FRFD based on the overlap transform. If there are multiple adjustable frequency LFM interferences, we suppress interferences according to their corresponding order. At last, we complete suppression of residual interference energy by means of combining the method of removing pulse handling in time domain.

4 Analysis of simulation

The performance of the interference suppression technique in transform domain mainly includes: the aggregation level of interference energy depending on the transform base, the suppression level of interference energy, the SNR loss and so on, and it can be estimated throughout the output SNR and the BER curve after interference suppression. First a comparison is made below on the detection and suppression performance of strong interference between the weakening interference and second threshold processing and the traditional N-sigma method, then the influence on the traditional suppression method is simulated by the phase transition interference under the strong interference environment. Finally, the performance of the proposed method in this paper is analyzed through the output SNR and BER curve.

The main parameters in the system are as follows. The simulation system prefers to the CDMA IS-95 with 2048 spreading code length. Therefore, the gain of the system is approximately 33 dB. The speed of the baseband is 1.2288 Mbit s^{-1}, the spread spectrum signal through the shaping filter and digital analog converter (DAC) is modulated to intermediate frequency (IF) 70 MHz. The signal on the IF superposes noise and interference through the combiner, then analog digital converter (ADC) is used to complete the 40 MHz band-pass sampling, the data are changed to the baseband data by digital down conversion (DDC), and then the joint interference suppression is performed in FRFD and time domain. At last, the code acquisition, carrier recovery, data demodulation, bit error rate statistics and so forth are carried out.

4.1 Analysis of weakening interference and second threshold processing

The N-sigma method has been applied extensively to engineering [13] because of its low implementation complexity and having a certain degree of performance of suppression. In this paper a comparison is made with the typical N-sigma method. Suppose E_b/N_0=10 dB, *ISR*=50 dB, and superimpose the one-sweeping-cycle interference or the many-sweeping-cycles interference respectively in a spreading code chip period for analysis.

Figures 7, 8 and 9 show the simulation results respectively, which have demonstrated that the expanding components

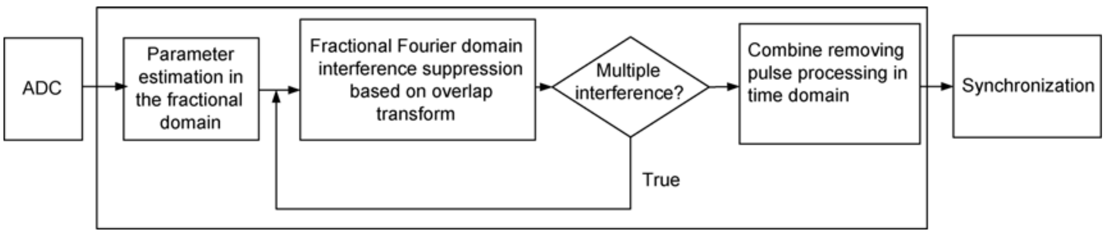

Figure 6 Joint interference suppression algorithm based on both FRFD and time domain.

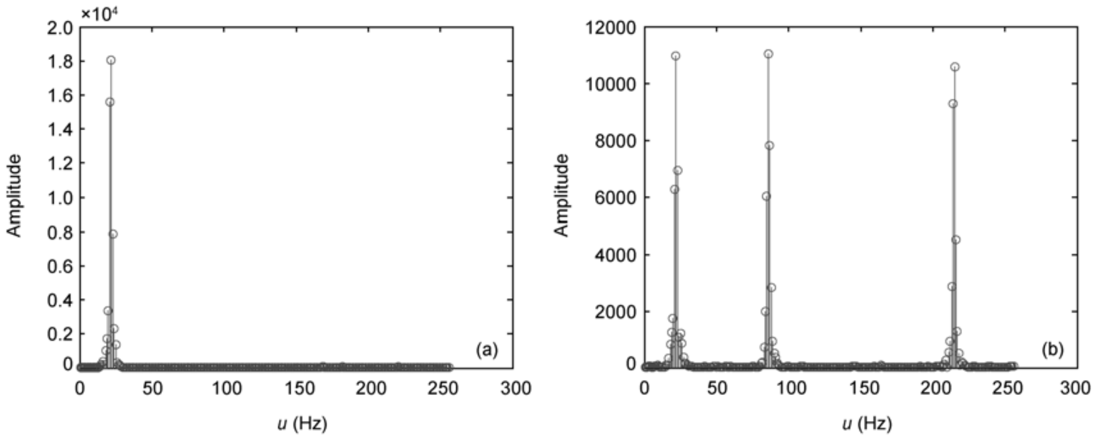

Figure 7 Fractional spectrum of strong interference. (a) One-sweeping-cycle interference; (b) many-sweeping-cycles interference.

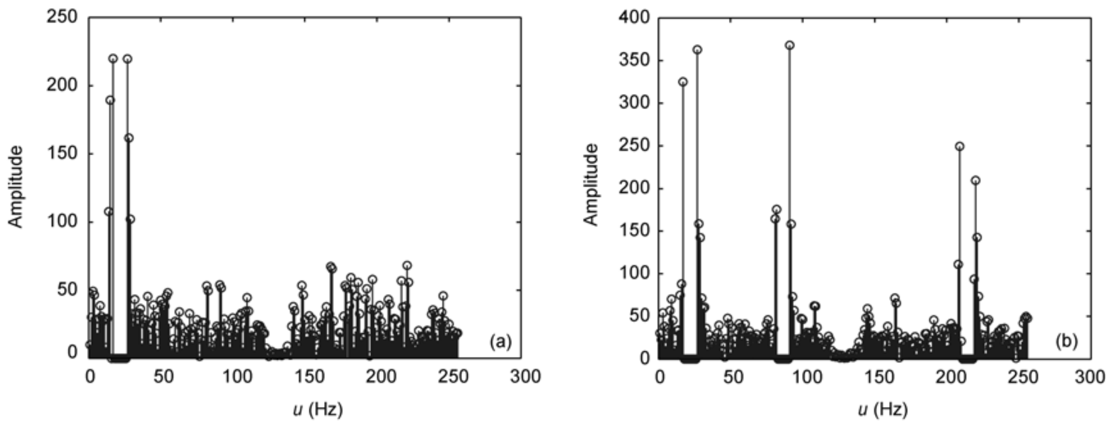

Figure 8 Performance of detection and suppression of N-sigma method. (a) One-sweeping-cycle interference; (b) many-sweeping-cycles interference.

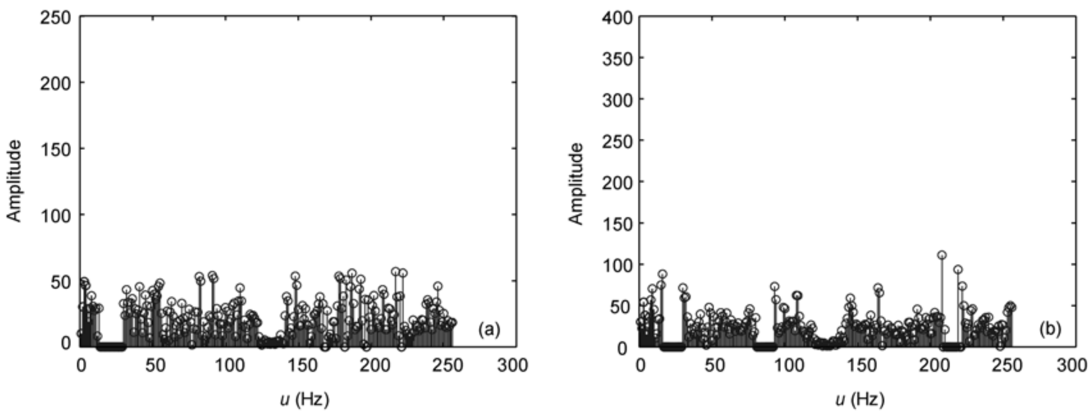

Figure 9 Performance of detection and suppression of weakening interference and second threshold processing. (a) One-sweeping-cycle interference; (b) many-sweeping-cycles interference.

of the main lobe of interference are smaller than those of the main lobe under the high SIR environment, however the energy of the interference is higher than the energy of signal, therefore N-sigma method cannot suppress the interference to the ground. Using the method proposed in this paper can suppress the interferences better.

4.2 Influence of discontinuous phase on interference suppression

Supposing E_b/N_0=10 dB, *ISR*=50 dB, in a spreading code chip period superimposing the 8-sweeping cycles interference, $\pi/2$ phase transition will appear at the sweeping signal during every period. The simulation result demonstrates the waveform of the signal with interference, the waveform of the signal that has been throughout the fractional domain interference suppression and then has completed the inverse transformation to time domain, and the waveform of signal after the pulse has been removed in the joint time domain.

The simulation result in Figure 10 demonstrates that the operations of suppression in FRFD and inverse transformation may produce the remnant interference energy in terms of pulse nearby the phase transition of the signal. The pulse process by joint time domain method can remove the interference energy to the ground completed.

4.3 Output SNR and analysis of BER

Throughout the interference suppression, the output SINR is defined as ratio of signal to residual interference and noise energy. The output SINR will directly influence the subsequent performance of synchronization. The conditions of simulation are: the spreading gain of the system is 33 dB, the sweeping interference is single tone frequency in Figure 11, every spreading cycle includes 8 sweeping cycles, however there are 2 interferences in Figure 12, one of them includes 2 sweeping cycles, another one includes 8 sweeping cycles in a spreading cycle.

Figures 11(a) and 11(b) show that the curve of output SINR varies with ISR and output SNR varies with input SNR under the conditions of fixed input SNR and fixed ISR respectively. The simulation result demonstrates that the interference suppression in FRFD based on lapped transform is better than the traditional method in the performance

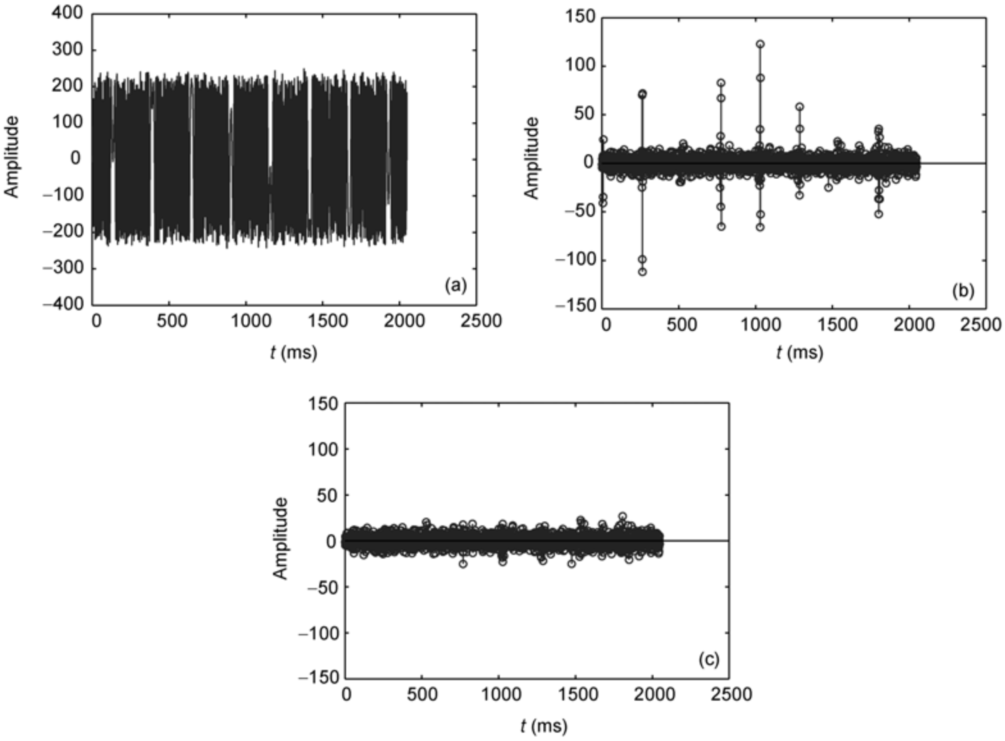

Figure 10 Influence of phase transition on interference suppression. (a) E_b/N_0=10 dB, *ISR*=50 dB, appearance of $\pi/2$ phase transition during the period of the sweeping interference; (b) interference suppressed signal in FRFD; (c) removed remnant pulse signal in time domain.

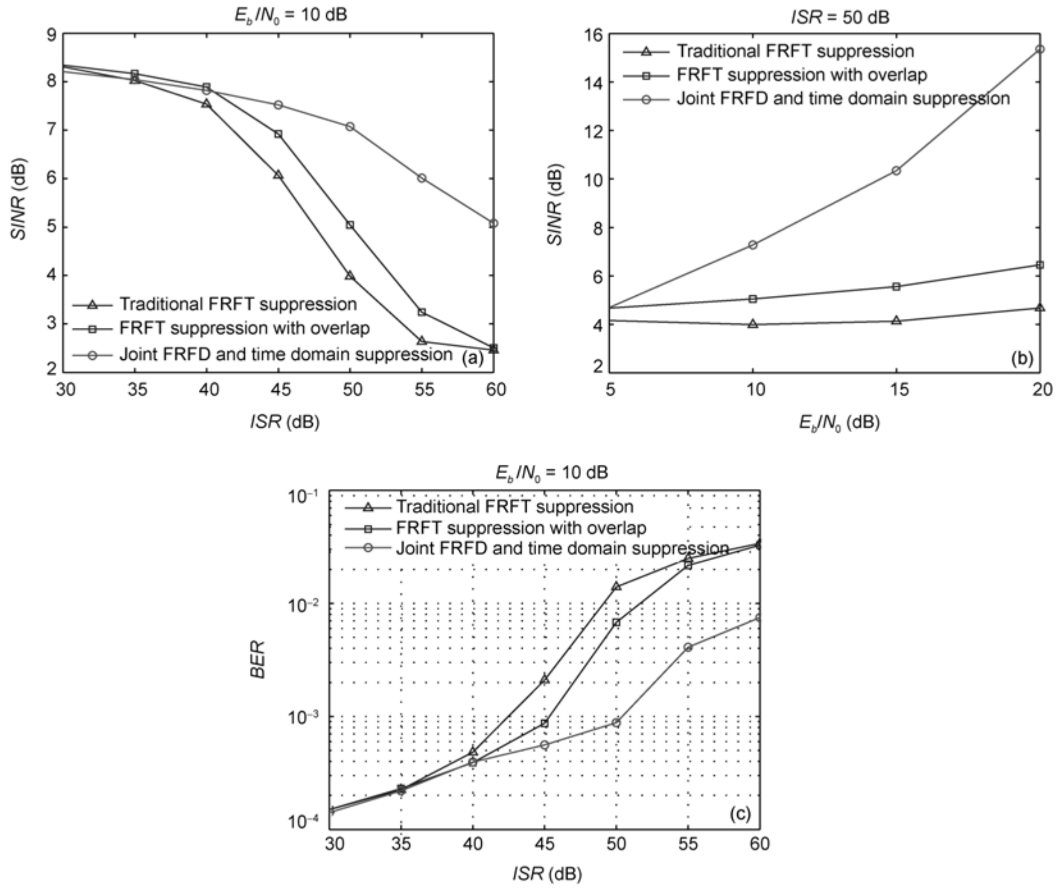

Figure 11 Performance comparison of many-sweeping-cycles interference with single tone frequency. (a) Output SINR under different ISR; (b) output SINR under different input SNR; (c) BER under different ISR.

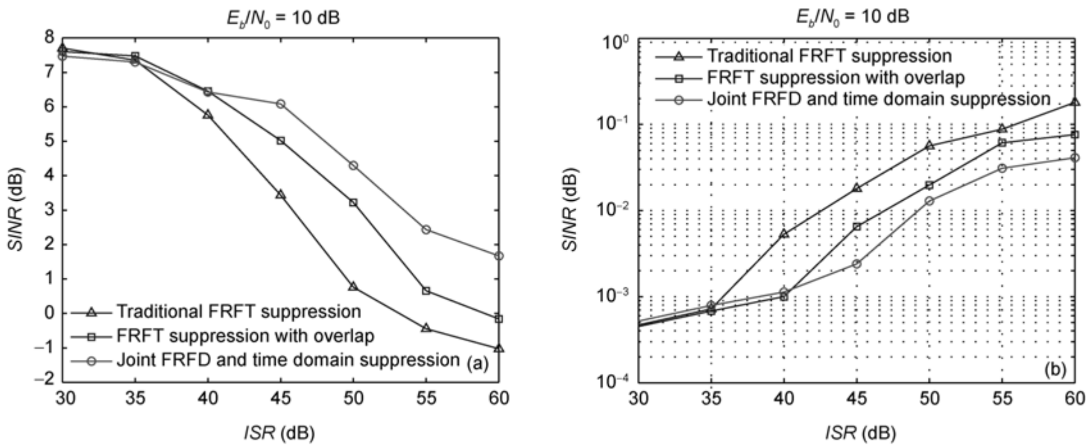

Figure 12 Performance comparison of sweeping interference with many tone frequencies. (a) output SINR under different ISR; (b) BER under different ISR.

of output SINR, however under the high ISR, removing pulse in joint time domain processing can further improve output SINR to ensure the stability of subsequent synchronization. The results of Figures 11(a) and 12(a) have shown that the method proposed in this paper has the advantage of 3 dB in terms of output SNR compared with the traditional method when input SNR is 10 dB and ISR is greater than 45 dB. Figures 11(c) and 12(b) have respectively demonstrated the performance comparison between traditional FRFT suppression and the method proposed in this paper under the condition of the many-sweeping-cycles interference with one tone frequency and the condition of many-tone-frequencies sweeping interference. The simulation result has shown that since the system itself has approximately 33 dB spreading gain, the interference performance is not very different from each other within 40 dB, however with the growth of ISR, the suppression technology in FRFD and joint time domain has an obvious improvement in performance compared with the traditional method.

5 Conclusion

A joint interference suppression algorithm based on both FRFD and time domain is proposed to meet the challenges of the traditional technique in dealing with the LFM interference, such as the high loss of SNR, the unstable synchronization because of the sensitiveness of the output SINR to input ISR, and the serious spectrum leakage in strong ISR situation. This approach firstly applies the windowed and lapped technique to the FRFT to enhance the ISR improvement and lower the SNR loss. Then by weakening the interference and doing a secondary threshold processing, interference energy can be suppressed as much as possible and the output SINR is less sensitive to the ISR. Finally, a joint FRFD and time domain technique is introduced to overcome the residual interference energy caused by the strong interference or the discontinuous-phase interference. Theoretical analysis and simulation results show that the proposed algorithm can achieve better performance than the conventional methods in suppressing both the multi-period LFM interference and the multi-chirp-rate LFM interference, especially in the strong interference environment. In addition, the proposed method can be used in other transform domains to improve the performance in strong interference environment. This method can provide reliable theoretic and practice basis for designing and upgrading communication systems such as the receiver of spread spectrum station, GPS, BD and constellation satellite system.

This work was partially supported by the National Natural Science Foundation of China for Distinguished Young Scholars (Grant No. 60625104) and the Foundation for Beijing Excellent Ph.D. Thesis (Grant No. 1320037010901).

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