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Springback equation of small curvature plane bending

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A new analytical method for springback of small curvature plane bending is addressed with unloading rule of classical elastic-plastic theory and principle of strain superposition. We start from strain analysis of plane bending which has initial curvature, and the theoretic derivation is on the widely applicable basic hypotheses. The results are unified to geometry constraint equations and springback equation of plane bending, which can be evolved to straight beam plane bending and pure bending. The expanding and setting round process is one of the situations of plane bending, which is a bend-stretching process of plane curved beam. In the present study, springback equation of plane bending is used to analyze the expanding and setting round process, and the results agree with the experimental data. With a reasonable prediction accuracy, this new analytical method for springback of plane bending can meet the needs of applications in engineering.

plane bending, springback, geometry constraint equations, springback equation, the expanding and setting round process

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1 Introduction

Plane bending is defined in ref. [1] as "when all the external forces act on the beam in the vertical symmetry plane, the beam axis after deformation will be located in the symmetrical plane." Actually, a non-symmetric beam, being imposed appropriate constraints, can also bend at the centroid plane, which is known as plane bending. Plane bending is one of the most fundamental and common bending problems. Many bending problems in engineering fall into the category of plane bending, including not only the straight beam and symmetry cross-section plane bending but also curved beam and non-symmetry cross-section plane bending According to the level of forcing bearing, plane bending can be categorized into pure bending, stretch-bending and compress-bending. Actually, pure bending is very rare in applications, but the latter two cases are common. For example, aluminum profiles stretch-bending process, large pipes JCOE/UOE forming process, large pipes expanding/compression and setting round processes, all fall into the categories of stretch-bending or compress-bending.

With the rapid development of science and technology and increasingly fierce competition in product manufacturing, the accurate prediction and precise control of bending springback has become the key issues to be solved. For the past several decades, a lot of research has been done about plane bending springback, particularly stretch-bending of aluminum profiles [2–4]. El-Megharbel et al. [5] estimated the springback after stretch-bending and residual stress within the section of 7075 aluminum plate. El-Domiaty et al. [6, 7] analyzed stretch-bending of U cross-section beam to study the influences of material properties and cross-section geometry on forming load and bending springback. El-sharkawy et al. [8] established the mathematical model of T cross-section profile. Zhang [9] studied the bending defor-

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mation under transverse compression and longitudinal stretch of plane frame profile by grid test. Guan [10, 11] analyzed the stress state of stretch-bending and proposed an energy method to calculate springback angle. The stress distributions of wide and narrow plates in stretch-bending were developed in refs. [12, 13] by Du et al. who analyzed the impact law of loading history with constant volume conditions by plastic deformation, incremental theory and differential equations. Qian et al. [14] studied the springback calculation formulas of rail profile, and during the process the springback angle after unloading was divided into two parts: the springback by bending moment unloading and the springback by tension unloading. With the rapid development of finite element analysis software, numerical simulation methods are widely used in profile stretch-bending and its springback law study. Jin and Zhou et al. [15, 16] used Pamstamp2000 to simulate the stretch-bending process with hollow rectangular aluminum profile and compared it with the experiment. Xie et al. [17] studied two different cross-section aluminum profiles at six different stretch-bending processes by Marc finite element software to explore the influences of forming process, bending radius and the cross-section geometry of profile on stretch-bending springback. The works above showed that to date there has not been an integrated theoretical system for analyzing the plane bending springback problems. Neither have the unified and well-recognized results been achieved.

Small curvature plane bending refers to the fact that the ratio of curvature radius to blank thickness is greater than 10. In this case, the strain generated by loading moment can be approximated as a linear distribution [18]. From the analysis of curved beam's small curvature deformation characteristics, the geometric constraint equations and the springback equation in different loading and deforming situations are derived, respectively in this study according to the classical elastic-plastic theory and principle of strain superposition. Furthermore, the unified expressions of the geometric constraint equations and the springback equation of plane bending are developed under the pre-defined symbol system. It is both theoretically and practically significant to express the complex geometric relationships before and after springback of straight beam and curved beam under different plane bending situations in a unified way. The results of using the springback equation of plane bending to analyze the expanding and setting round process are in good agreement with the theoretical results. Hence, it lays a theoretical foundation for engineering applications.

2 Definitions and basic hypotheses

The main research in this study includes plane bending springback of beam-plate, pipe, bar and various cross- section profiles. The corresponding definitions of these terms are given as follows.

1) Beam: in the bending plane, the length of the part is much larger than the height of the cross-section, for example, metal strip, tube, profile, and so on.

2) Micro-beam section: a section of infinitesimal length intercepted along the centroid line of the beam cross-section, as shown in Figure 1.

3) Plane-curved beam: The centroid line of the beam is in one space plane. This study focuses on the small curvature plane-curved beams [19], where the ratio of initial bend radius to blank thickness is greater than 5.

4) Curved beam plane bending: The combined effect of all external loads imposed on a plane-curved beam only causes the beam to deform in the space plane where the initial section centroid curve occurs.

5) Straight beam plane bending: The combined effect of all external loads causes the straight beam's section centroid curve to be in one space plane.

6) Straight beam plane bending and curved beam plane bending are referred to as plane bending.

7) Bending plane: The space plane where the section centroid curve goes.

Symmetrical cross-section beam is used as an example of equation derivation for the sake of clarity and convenience. As is shown in Figure 1, the centroid of micro-beam's cross-section o is the coordinate origin, and we use cross-section of micro-beam and bending plane as coordinate planes to establish $oxyz$ Cartesian coordinate system. The intersection points of inner and outer layers with z axis are points A and B. The height of cross-section is t. The distance of point A to y axis is e. $B_{(z)}$ is the width of cross-section which is a function of z. x axis coincides with the section centroid curve; ρ_0 is the curvature radius of the cross-section geometric center layer.

The basic hypotheses of this study are formulated as follows.

1) Plane section hypothesis: Any plane section remains plane after deforming and no aberrance occurs. That is, when plane bending takes place, the movements of strain neutral layer are caused by axial force and the cross-section geometric center layer does not move.

2) Uniaxial state of stress hypothesis: Any point on the beam is uniaxial stretched or compressed when deforming occurs regardless of the effect of uniaxial force on the section size.

3) Conventional elastic-plastic material model hypothesis:

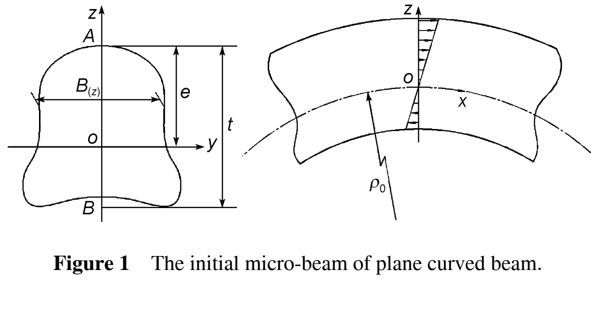

Figure 1 The initial micro-beam of plane curved beam.

The beam is homogeneous, linear-elastic material and consistent with Hooke's law and classical elastic-plastic unloading law.

Since the research object is the small curvature plane-curved beam, the initial curvature of micro-beam $1/\rho_0$ can be expressed in terms of initial equivalent strain ε_0, which satisfies the following relations:

$$\varepsilon_0 = \frac{z}{\rho_0}, \quad e-t \leqslant z \leqslant e. \tag{1}$$

Then the concept of micro-beam equivalent strain ε_{eq} is introduced, that is, the relative length change of micro-beam considering the initial beam deflection is taken to be equivalent strain ε_{eq}, whose value is the algebra sum of true strain ε_{tr} and initial equivalent strain ε_0 as follows:

$$\varepsilon_{\text{eq}} = \varepsilon_{\text{tr}} + \varepsilon_0. \tag{2}$$

The classification of plane bending is shown in Figure 2. Straight beam is a special case of curved beams, while stretch-bending and compress-bending are completely similar, so a detailed analysis of curved beam plane bending will be shown as an example to derive the geometric constraint equations and the springback equation of plane bending.

3 Springback analysis of curved beam stretch-bending

3.1 Springback analysis of equidirectional stretch-bending

3.1.1 The loading strain

Under the axial stretch force T and the bending moment M equidirectional with the initial curvature, the micro-curved-beam undergoes an elastic-plastic bending strain ε, whose equivalent strain distribution is shown in Figure 3, where the curvature radius of cross-section's center layer is ρ when loading, and the curvature radius of equivalent strain neutral layer is ρ_s when loading.

The equivalent strain neutral layer when loading is possibly outside the cross-section. Along the equivalent strain neutral layer when loading, in the $x_1o_1z_1$ coordinate system set up based on hypotheses (1), the equivalent strain when loading is given by

$$\frac{z_1}{\rho_\varepsilon} = \varepsilon + \varepsilon_0,$$

where

$$z_1 = z + \rho - \rho_\varepsilon,$$

so

$$\varepsilon = z\left(\frac{1}{\rho_\varepsilon} - \frac{1}{\rho_0}\right) + \frac{\rho - \rho_\varepsilon}{\rho_\varepsilon}, \quad e-t \leqslant z \leqslant e. \tag{3}$$

3.1.2 The residual strain after unloading springback

After unloading springback the residual strain is ε_p. The equivalent residual strain distribution is still linear as shown in Figure 4, where the curvature radius of cross-section's center layer after unloading is ρ_p, the curvature radius of equivalent strain neutral layer after unloading is ρ_{ap}.

Along the equivalent strain neutral layer after unloading, in the $x_2o_2z_2$ coordinate system set up based on basic hypotheses (1), the equivalent strain after unloading is given by

$$\frac{z_2}{\rho_{\varepsilon p}} = \varepsilon_p + \varepsilon_0,$$

where

$$z_2 = z + \rho_p - \rho_{\varepsilon p},$$

so

$$\varepsilon_p = z\left(\frac{1}{\rho_{\varepsilon p}} - \frac{1}{\rho_0}\right) + \frac{\rho_p - \rho_{\varepsilon p}}{\rho_{\varepsilon p}}, \quad e-t \leqslant z \leqslant e. \tag{4}$$

3.1.3 The elastic strain under reverse load

If the micro-beam shown in Figure 1 is considered to be a pure elastic body, the springback strain ε_e produced by force T_e and moment M_e, which are equal, opposite, and at the same action points of force T and moment M, is the pure elastic strain. The equivalent springback strain is shown in Figure 5, where the curvature radius of cross-section's center

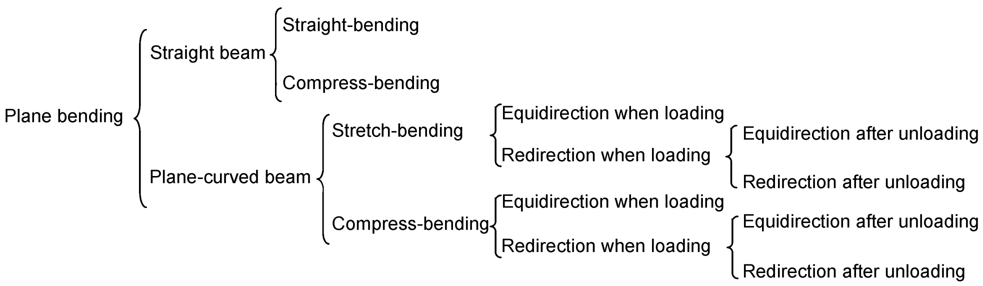

Figure 2 The classification of plane bending.

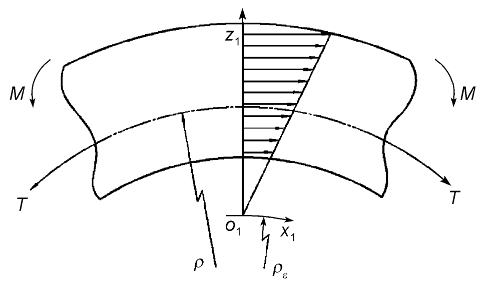

Figure 3 Equivalent strain when loading.

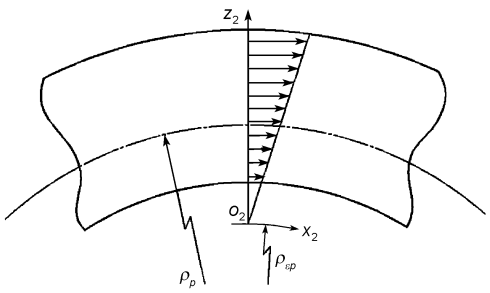

Figure 4 Equivalent strain after unloading.

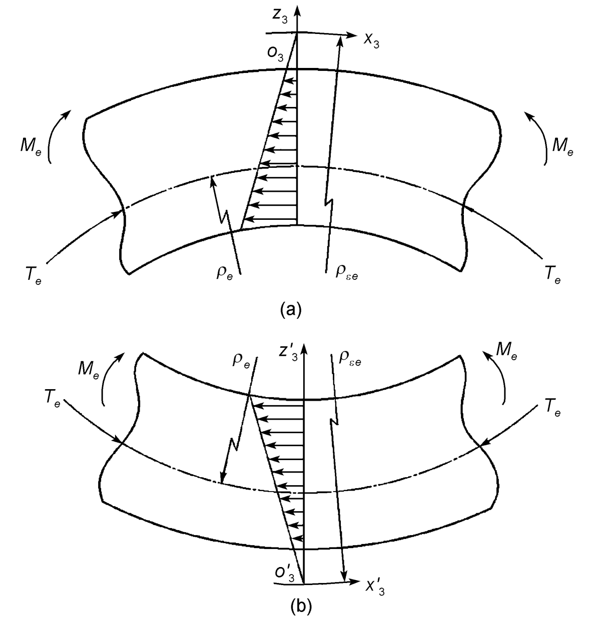

Figure 5 Elastic equivalent strains under reverse load: (a) Elastic equivalent strain under reverse load: bending direction unchanged; (b) Elastic equivalent strain under reverse load: bending direction reversed.

layer is ε_e, and the curvature radius of equivalent strain neutral layer is $\rho_{\varepsilon e}$. One case is that the micro-beam's deflection direction and the initial deflection are in the same direction when loaded by T_e and M_e, as shown in Figure 5(a). Along the equivalent strain neutral layer, we set the $x_3o_3z_3$ coordinate system based on basic hypotheses (1), then the equivalent strain is given by

$$\frac{z_3}{\rho_{\varepsilon e}} = \varepsilon_e + \varepsilon_0,$$

where

$$z_3 = z + \rho_e - \rho_{\varepsilon e},$$

so

$$\varepsilon_e = z\left(\frac{1}{\rho_{\varepsilon e}} - \frac{1}{\rho_0}\right) + \frac{\rho_e - \rho_{\varepsilon e}}{\rho_{\varepsilon e}}, \quad e - t \leqslant z \leqslant e. \tag{5}$$

Another case is that the micro-beam's deflection direction is with the initial deflection in the opposite direction when loaded by T_e and M_e, as shown in Figure 5(b). Along the equivalent strain neutral layer, the $x_3'o_3'z_3'$ coordinate system is set up based on basic hypotheses (1), and then the equivalent strain is given by

$$-\frac{z_3'}{\rho_{\varepsilon e}} = \varepsilon_e' + \varepsilon_0,$$

where

$$z_3' = z + \rho_{\varepsilon e} - \rho_e,$$

so

$$\varepsilon_e' = -z\left(\frac{1}{\rho_{\varepsilon e}} + \frac{1}{\rho_0}\right) + \frac{\rho_e - \rho_{\varepsilon e}}{\rho_{\varepsilon e}}, \quad e - t \leqslant z \leqslant e. \tag{6}$$

3.1.4 The geometric constraint equations and the springback equation

According to basic hypotheses 3, the elastic strain when micro-beam unload is equivalent to the pure elastic strain when the micro-beam as a pure elastic body is loaded by force and moment of equal, opposite and at the same action points. Based on small deformation theory, the strain can be superimposed. The residual strain after unloading springback is equivalent to the superposition of the loading strain and the springback strain.

For the case of Figure 5(a), the relationship of ε, ε_p and ε_e is given by

$$\varepsilon_p = \varepsilon + \varepsilon_e, \tag{7}$$

where ε, ε_p and ε_e have been given by eqs. (3)–(5). So, eq. (7) can be expressed by

$$z\left(\frac{1}{\rho_{\varepsilon p}} - \frac{1}{\rho_\varepsilon} - \frac{1}{\rho_{\varepsilon e}} + \frac{1}{\rho_0}\right) = \frac{\rho}{\rho_\varepsilon} + \frac{\rho_e}{\rho_{\varepsilon e}} - \frac{\rho_p}{\rho_{\varepsilon p}} - 1. \tag{8}$$

The necessary and sufficient conditions of eq. (8) permanently valid within the definition region are

$$\begin{cases} \dfrac{1}{\rho_{\varepsilon p}} - \dfrac{1}{\rho_\varepsilon} - \dfrac{1}{\rho_{\varepsilon e}} + \dfrac{1}{\rho_0} = 0, \\ \dfrac{\rho}{\rho_\varepsilon} + \dfrac{\rho_e}{\rho_{\varepsilon e}} - \dfrac{\rho_p}{\rho_{\varepsilon p}} = 1. \end{cases} \tag{9}$$

Based on basic hypotheses (2) and (3), when springback the equivalent stress σ_e is given by

$$\sigma_e = E\varepsilon_e = zE\left(\frac{1}{\rho_{\varepsilon e}} - \frac{1}{\rho_0}\right) + E\left(\frac{\rho_e}{\rho_{\varepsilon e}} - 1\right), \quad e - t \leqslant z \leqslant e, \tag{10}$$

where E is the Young's modulus of micro-beam.

Force T_e and moment M_e can be calculated by σ_e as follows:

$$\begin{cases} T_e = \displaystyle\int_A \sigma_e \mathrm{d}A = EA\left(\frac{\rho_e}{\rho_{\varepsilon e}} - 1\right), \\ M_e = \displaystyle\int_A \sigma_e \cdot z\mathrm{d}A = EI_y\left(\frac{1}{\rho_{\varepsilon e}} - \frac{1}{\rho_0}\right), \end{cases} \tag{11}$$

where the area of micro-beam cross-section is A, and the moment of inertia is I_y.

The balance equation of plane bending is given by

$$\begin{cases} T + T_e = 0, \\ M + M_e = 0. \end{cases} \tag{12}$$

By eqs. (9), (11) and (12), ρ_p is given by

$$\rho_p = \frac{\rho - \dfrac{T}{EA}\rho_\varepsilon}{1 - \dfrac{M}{EI_y}\rho_\varepsilon}. \tag{13}$$

In the case of Figure 5(b), the relationship of ε, ε_p and ε_e is given by

$$\varepsilon_p = \varepsilon + \varepsilon_e', \tag{14}$$

where ε, ε_p and ε_e are given by eqs. (3), (4) and (6). So eq. (14) can be expressed by

$$z\left(\frac{1}{\rho_{\varepsilon p}} - \frac{1}{\rho_\varepsilon} + \frac{1}{\rho_{\varepsilon e}} + \frac{1}{\rho_0}\right) = \frac{\rho}{\rho_\varepsilon} + \frac{\rho_e}{\rho_{\varepsilon e}} - \frac{\rho_p}{\rho_{\varepsilon p}} - 1. \tag{15}$$

The necessary and sufficient conditions of eq. (15) permanently valid within the definition are

$$\begin{cases} \dfrac{1}{\rho_{\varepsilon p}} - \dfrac{1}{\rho_\varepsilon} + \dfrac{1}{\rho_{\varepsilon e}} + \dfrac{1}{\rho_0} = 0, \\ \dfrac{\rho}{\rho_\varepsilon} + \dfrac{\rho_e}{\rho_{\varepsilon e}} - \dfrac{\rho_p}{\rho_{\varepsilon p}} = 1. \end{cases} \tag{16}$$

Similarly, we can obtain

$$\begin{cases} T_e' = \displaystyle\int_A \sigma_e \mathrm{d}A = EA\left(\frac{\rho_e}{\rho_{\varepsilon e}} - 1\right) = -T, \\ M_e' = \displaystyle\int_A \sigma_e z\mathrm{d}A = -EI_y\left(\frac{1}{\rho_{\varepsilon e}} + \frac{1}{\rho_0}\right) = -M. \end{cases} \tag{17}$$

By eqs. (16) and (17), ρ_p is given by

$$\rho_p = \frac{\rho - \dfrac{T}{EA}\rho_\varepsilon}{1 - \dfrac{M}{EI_y}\rho_\varepsilon}. \tag{18}$$

Eqs. (9) and (16) can be unified to

$$\begin{cases} \dfrac{1}{\rho_{\varepsilon p}} - \dfrac{1}{\rho_\varepsilon} \pm \dfrac{1}{\rho_{\varepsilon e}} + \dfrac{1}{\rho_0} = 0, \\ \dfrac{\rho}{\rho_\varepsilon} + \dfrac{\rho_e}{\rho_{\varepsilon e}} - \dfrac{\rho_p}{\rho_{\varepsilon p}} = 1. \end{cases} \tag{19}$$

Eq. (19) is called the geometric constraint equations of equidirectional stretch-bending. Eqs. (13) and (18) are identical, so they are called the springback equation of equidirectional stretch-bending.

3.2 The geometric constraint equations and the springback equation of redirectional stretch-bending

Similar to the analysis of equidirectional stretch-bending, redirectional stretch-bending is the micro-beam bended under the axial stretch force T and with initial curvature opposite direction moment M. Effected by the initial curvature, redirectional stretch-bending can be divided into three cases. The first case is the deflection after loading and the initial deflection occurs in the same direction. The second case is the deflection after unloading and the initial deflection occur in the opposite directions. The third case is the deflection after loading and the initial deflection occur in the opposite directions, but the deflection after unloading and the initial deflection occur in the same direction. The analysis details are similar to equidirectional stretch-bending, so here we only give the derivation of the results of the above three cases.

The geometric constraint equations and the springback equation of redirectional stretch-bending can be unified as follows. The symbols of the three cases are shown in Table 1.

$$\begin{cases} \dfrac{1}{\rho_{\varepsilon p}} \mp \dfrac{1}{\rho_\varepsilon} \mp \dfrac{1}{\rho_{\varepsilon e}} \pm \dfrac{1}{\rho_0} = 0, \\ \dfrac{\rho}{\rho_\varepsilon} + \dfrac{\rho_e}{\rho_{\varepsilon e}} - \dfrac{\rho_p}{\rho_{\varepsilon p}} = 1, \end{cases} \tag{20}$$

$$\pm\rho_p = \frac{\rho - \dfrac{T}{EA}\rho_\varepsilon}{1 \mp \dfrac{M}{EI_y}\rho_\varepsilon}. \tag{21}$$

Table 1 The symbols in the geometric constraint equations and the springback equation

The three cases of redivectional stretch-bending	$\frac{1}{\rho_\varepsilon}$	$\frac{1}{\rho_{\varepsilon e}}$	$\frac{1}{\rho_0}$	ρ_p	M
The first case	–	–	+	+	–
The second case	–	+	-	+	+
The third case	+	–	+	–	+

4 The unified presentation of the geometric constraint equations and the springback equation of plane bending and its corollary

4.1 Symbol system

The previous derivation is based on the strain and geometry relations of micro-beam bended before and after springback, so the geometric constraint equations and the springback equation are expressions of curvature radius. If a symbol system is defined, the geometric constraint equations and the springback equation can be uniformly expressed in curvature expressions. In this way, the expressions are further harmonized and simplified to reflect the physical meanings more conveniently and more clearly.

The positive direction of z axis is upward vertical, otherwise negative.

The moment when loading is expressed by M. Moment M is an algebraic number, whose value is calculated by integral of stress pointing on the cross-section's center layer. When M makes the straight beam protrude to the positive directive of z axis, M is defined as positive, otherwise negative.

The axial force when loading is expressed by T. Axial force T is an algebraic number, whose value is calculated by integral of stress in the area of cross-section.

Curvature is normally expressed by K, and the absolute value of K is equal to the reciprocal of the curvature radius. When K and M are in the same direction, K is defined as positive, otherwise negative.

4.2 The unified presentation

Based on the above symbols definition, the geometric constraint equations and the springback equation of different bend cases can be expressed as two unified formulas, namely

$$\begin{cases} K_{\varepsilon p} = K_\varepsilon + K_{\varepsilon e} - K_0, \\ \dfrac{K_\varepsilon}{K} + \dfrac{K_{\varepsilon e}}{K_e} - \dfrac{K_{\varepsilon p}}{K_p} = 1, \end{cases} \tag{22}$$

$$K_p = \frac{K_\varepsilon - \dfrac{M}{EI_y}}{\dfrac{K_\varepsilon}{K} - \dfrac{T}{EA}}. \tag{23}$$

The second equation in eq. (22) shows the proportional relationship between the equivalent strain neutral layer and the geometric center layer. It speaks volumes for that the strain neutral layer does not coincide with the geometric center layer when axial force effects in plane bending.

Based on the above symbols definition, the strain after loading ε can be expressed as

$$\varepsilon = \begin{cases} z\left(K_\varepsilon - K_0\right) + \left(\dfrac{K_\varepsilon}{K} - 1\right), \text{ equidirection bending,} \\ -z\left(K_\varepsilon - K_0\right) + \left(\dfrac{K_\varepsilon}{K} - 1\right), \text{ redirection bending.} \end{cases} \tag{24}$$

Let stress σ be a function of strain ε, that is,

$$\sigma = f(\varepsilon) = f\left(K_\varepsilon, z\right). \tag{25}$$

According to the balance condition of section load, axial force T and moment M are given by

$$T = \int_A \sigma \mathrm{d}A, \tag{26}$$

$$M = \int_A \sigma \cdot z \mathrm{d}A. \tag{27}$$

Usually, T or ε can be solved according to the deformation of the geometric center layer, so K_s can be obtained through eq. (26) or (24). T and M are functions of K_s, so the curvature of geometric center layer after unloading can be derived from the equations above. In the solution process, eq. (25) relates to the loading history, and therefore the solution of K_p reflects the impact of loading history of bending springback. Eq. (23) is a general solution to plane bending springback problems.

In fact, for a non-symmetric beam, when appropriate constraints are imposed on the beam to make it bended at the centroid plane, the analysis process and results are still applicable. Therefore, the springback equation of plane bending is the same for all the plane bending situations and objects that satisfy the basic hypotheses.

4.3 The deduction about curvature variations of equivalent strain neutral layers

The first formula of geometric constraint equations can be transformed into

$$K_{\varepsilon p} - K_0 = K_\varepsilon - K_0 + K_{\varepsilon e} - K_0, \tag{28}$$

$$\Delta K_{\varepsilon p} = \Delta K_\varepsilon + \Delta K_{\varepsilon e}. \tag{29}$$

Eq. (28) can be expressed as eq. (29), which is called curvature variations equation of equivalent strain neutral layers. It can be explained like this: the curvature increment of equivalent residual strain neutral layer is equal to the sum of the curvature increment of equivalent strain neutral layer when loading and the curvature increment of equivalent strain neutral layer when springback. Eq. (29) means that when axial force acts on plane bending process, there is not superposition between curvatures of geometry center layers, but there is superposition between curvature increments of equivalent strain neutral layers.

4.4 The conversion to straight beam plane bending

When $K_0 = \lim_{\rho_0\to\infty} 1/\rho_0 = 0$, the situation is straight beam plane bending, and eq. (22) can be conversed to eq. (30), which is called geometric constraint equations of straight beam plane bending. Eq. (23) does not contain K_0, so it does not change. That means eq. (23) contains the situation of straight beam plane bending. We can get the same results of eqs. (30) and (23) by directly deriving straight micro-beam section. Therefore, the conversion to straight beam plane bending further corroborates the results derived in this paper.

$$\begin{cases} K_{\varepsilon p} = K_{\varepsilon} + K_{\varepsilon e}, \\ \dfrac{K_{\varepsilon}}{K} + \dfrac{K_{\varepsilon e}}{K_e} - \dfrac{K_{\varepsilon p}}{K_p} = 1. \end{cases} \tag{30}$$

4.5 The conversion to pure bending

For pure bending, the strain neutral layer coincides with the geometric center layer. In this situation, the second formula of eq. (22) comes into existence permanently. It means in pure bending, only the first equation of eq. (22) can fully express the geometric relationship before and after springback. That is,

$$K_p = K + K_e - K_0. \tag{31}$$

In case of pure bending, eq. (13) can be expressed as follows:

$$K - K_p = \frac{M}{EI_y}. \tag{32}$$

It is the same as the pure bending results of classical elastic-plastic theory, as shown in ref. [20].

5 Expanding and setting round of line pipe

5.1 Analysis of expanding and setting round process

Expanding and setting round process is one of the important processes in the large-diameter longitudinal submerged-arc welded (LSAW) pipe molding process. With wide application of the large pipes in modern industry, to accurately predict and control the process parameters has drawn great attention in this field [21–22]. In this section, springback equation of plane bending is used to analyze the expanding and setting round process. The results will show the relationship of geometric parameters of pipe before and after springback and the main process parameters. Three experiments on the expanding and setting round process with different initial ellipticities are carried out to verify the theoretical analysis.

According to the temporal sequence, the expanding and setting round process can be divided into three phases, as shown in Figure 6. The first phase is the setting round phase. The centerline of pipe's cross-section is approximately an ellipse, with long half-axis a_0 and minor half-axis b_0, the curvature radius of any point at cross-section's centerline is ρ_0 as shown in detail in Figure 6(a).

The expanding mold consists of a number of petal molds. At the beginning, these petal molds form a full circle. When expanding, the force transmission system makes the petal molds move simultaneously at the same speed in its normal direction. Initially, the contact of mold contacts the pipe wall in a linear manner. With the continuous movement of the petal molds, the external surfaces of each petal mold join gradually the pipe's internal wall. Due to the stiffness of the pipe section after the mold join the pipe internal wall, the pipe's cross-section forms the shape of a circumcircle of the expanding mold. Hence, the curvature radius of cross-section's centerline is R_1. The circumferential force is small in this phase; the main deformation is the elastic bend deformation; the centerline of cross-section L_1 is approxi- mately unchanged; so this phase can be viewed as an elastic pure bending process as shown in details in Figure 6(b).

The second phase is the expanding phase. Under the effect of the mold, the pipe's diameter is expanded so that the circumferential cross-section engenders plastic elongation. In this phase, the pipe wall endures greater circumferential pull, so stretch is the main deformation. When the mold reaches the preset displacement, the pipe's cross-section is approximately round and its curvature radius of cross-section's centerline increases to R_2, as shown in details in Figure 6(c).

The third phase is springback. After keeping force, the petal molds move back and the pipe starts to be unloaded. After unloading, springback makes the curvature radius of any point at cross-section's centerline become ρ_p. If the centerline of pipe's cross-section is still considered as elliptic after unloading, we define its long half-axis as a, and minor half-axis as b, as shown in Figure 6(d).

Expanding and setting round for pipe is curved beam stretch-bending, which is one case of plane bending. Shown in Figure 6 is a Cartesian coordinate system *uov* established according to the center of the ellipse. It is easily known that segment $\widehat{\mathrm{BH}}$ and segment $\widehat{\mathrm{DF}}$ belong to the first case of equidirectional stretch-bending, and segment $\widehat{\mathrm{BD}}$ and segment $\widehat{\mathrm{FH}}$ belong to the first case of redirectional stretch-bending. The cutoff point in both cases is point $\rho_0=R_2$. Apparently, the cutoff point has been moved in the expanding

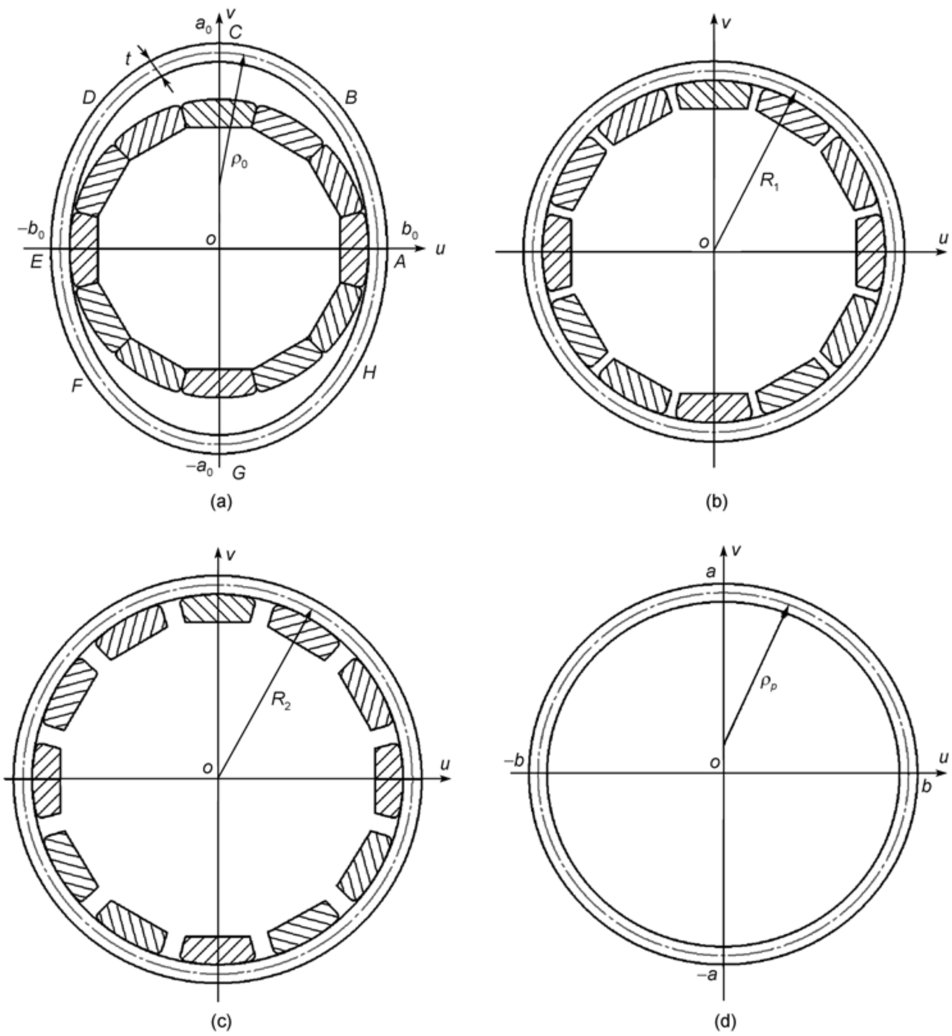

Figure 6 The process of expanding and setting round. (a) Initial state; (b) setting round; (c) expanding; (d) springback.

and setting round process. From the second section of this paper, we can reveal that the two cases have the same ε expression and the same springback equation; therefore we can discuss them together. To simplify the analysis process we assume that pipe's cross-section is always on the u axis and the v axis is symmetry. We only consider the $\widehat{ABC}$ segment of cross-section in the first quadrant as the object to be studied.

The ratio of diameter and wall thickness of a large pipe is much larger than 10, thus the expanding and setting round for pipe can be seen as plane bending of small curvature rectangular cross-section beam. The following basic hypotheses are introduced when springback equation is used to analyze the expanding and setting round process.

1) After springback arbitrary cross-section of the pipe remains in plane, no distortion occurs, and the change of pipe thickness is ignored.

2) The centerline of initial pipe's cross-section is an ellipse with long half-axis a_0 and minor half-axis b_0, and curvature radius of any point at cross-section's centerline is ρ_0.

3) The anisotropy is not considered. The Bauschinger effect is ignored, i.e., materials are consistent with the tension and compression. The laws of classical elastic-plastic theory of unloading are verified. Bilinear hardening material model is adopted, in which elastic modulus is E; plastic tangent modulus is D; intercept stress is σ_0; plastic strain at demarcation point is ε_E. The material model can be expressed as follows:

$$\sigma = \begin{cases} E\varepsilon, & \varepsilon \leqslant \varepsilon_E, \\ D\varepsilon + \sigma_0, & \varepsilon \geqslant \varepsilon_E. \end{cases} \tag{33}$$

4) During the forming process, the stress-strain state of any point on the pipe section is approximately uniaxial tension or compression.

5) The deformation of the first phase for setting round is pure-elastic bending.

6) The friction between pipe and mold is ignored.

When $\rho = R_2$, the strain of cross-section centerline is given by

$$\varepsilon\big|_{z=0} = \frac{\rho-\rho_\varepsilon}{\rho_\varepsilon} = \frac{R_2-\rho_\varepsilon}{\rho_\varepsilon} = \delta = \frac{R_2-R_1}{R_1}, \tag{34}$$

so

$$\rho_\varepsilon = R_1, \tag{35}$$

where δ is the expanding rate.

Set δ_1 to be the expanding rate cut-off point between pure-elastic deformation stage and elastic-plastic deformation stage, that is, when the expanding rate is less than δ_1, the pipe is in pure-elastic deformation stage; when the expanding rate is greater than δ_1, the pipe starts to come into plastic deformation stage. Set δ_2 to be the expanding rate cut-off point between elastic-plastic deformation stage and pure-plastic deformation stage, that is, when the expanding rate is less than δ_2, the pipe is in elastic-plastic deformation stage; when the expanding rate is greater than δ_2, the pipe is in pure-plastic deformation stage. According to the earlier results and the description of expanding and setting round, δ_1 and δ_2 are given by

$$\delta_1 = \varepsilon_E + \frac{t}{2}\left(\frac{1}{R_1}-\frac{1}{\rho_{0a}}\right), \tag{36}$$

$$\delta_2 = \varepsilon_E - \frac{t}{2}\left(\frac{1}{R_1}-\frac{1}{\rho_{0a}}\right). \tag{37}$$

When $\delta<\delta_1$, after unloading the elastic recovery makes the cross-section back to the state before. The situation is well known, and not repeated here.

When $\delta_1<\delta<\delta_2$, the pipe comes into the elastic-plastic deformation stage. By using springback equation and basic hypotheses (2) of expanding and setting round, the curvature radius ρ_{p1} of equidirectional stretch-bending segment $\widehat{\mathrm{AB}}$ after springback is given by

$$\rho_{p1} = \frac{t^2\left[R_2Et-f_1(z_E)\right]}{Et^3-12g_1(z_E)}, \tag{38}$$

where

$$\begin{aligned} f_1(z_E) = {} & \frac{1}{2}\left(1-\frac{R_1}{\rho_0}\right)\left(z_E^{\ 2}-\frac{t^2}{4}\right)(E-D) \\ & + z_E(R_2-R_1)(E-D) \\ & + \frac{t}{2}(R_2-R_1)(E+D)+\sigma_0R_1\left(\frac{t}{2}-z_E\right), \end{aligned} \tag{39}$$

$$\begin{aligned} g_1(z_E) = {} & \frac{t^3}{24}\left(1-\frac{R_1}{\rho_0}\right)(E+D)+\frac{z_E^{\ 3}}{3}\left(1-\frac{R_1}{\rho_0}\right)(E-D) \\ & + \frac{1}{2}(R_2-R_1)\left(z_E^{\ 2}-\frac{t^2}{4}\right)(E-D) \\ & + \frac{\sigma_0R_1}{2}\left(\frac{t^2}{4}-z_E^{\ 2}\right). \end{aligned} \tag{40}$$

Similarly, the curvature radius ρ_{p2} of redirectional stretch-bending segment $\widehat{\mathrm{AB}}$ after springback is given by

$$\rho_{p2} = \frac{t^2\left[R_2Et-f_2(z_E)\right]}{Et^3-12g_2(z_E)}, \tag{41}$$

where

$$\begin{aligned} f_2(z_E) = {} & \frac{1}{2}\left(1-\frac{R_1}{\rho_0}\right)\left(\frac{t^2}{4}-z_E^{\ 2}\right)(E-D) \\ & - z_E(R_2-R_1)(E-D) \\ & + \frac{t}{2}(R_2-R_1)(E+D)+\sigma_0R_1\left(\frac{t}{2}+z_E\right), \end{aligned} \tag{42}$$

$$\begin{aligned} g_2(z_E) = {} & \frac{t^3}{24}\left(1-\frac{R_1}{\rho_0}\right)(E+D)-\frac{z_E^{\ 3}}{3}\left(1-\frac{R_1}{\rho_0}\right)(E-D) \\ & - \frac{1}{2}(R_2-R_1)\left(z_E^{\ 2}-\frac{t^2}{4}\right)(E-D) \\ & + \frac{\sigma_0R_1}{2}\left(z_E^{\ 2}-\frac{t^2}{4}\right). \end{aligned} \tag{43}$$

In this case $\rho_{pa} = \rho_{p2}\big|_{\rho_0=\rho_{0a}}$, $\rho_{pb} = \rho_{p1}\big|_{\rho_0=\rho_{0b}}$, when $\delta>\delta_2$, the pipe cross-sections are all in the plastic deformation stage. Similarly, the curvature radius ρ_p of any point in cross-section centerline after unloading can be expressed as

$$\begin{aligned} \rho_p &= \frac{1-\dfrac{R_1}{R_2}\left[\dfrac{\sigma_0}{E}+\dfrac{D}{E}\left(\dfrac{R_2}{R_1}-1\right)\right]}{1-\dfrac{D}{E}\left(1-\dfrac{R_1}{\rho_0}\right)}R_2 \\ &= \frac{(E-\sigma_0)+\delta(E-D)}{E-D\left(1-\dfrac{R_1}{\rho_0}\right)}R_1. \end{aligned} \tag{44}$$

In this case $\rho_{pa} = \rho_p\big|_{\rho_0=\rho_{0a}}$, $\rho_{pb} = \rho_p\big|_{\rho_0=\rho_{0b}}$.

If the cross-section after springback is still considered as ellipstic, then the long half-axis a and minor half-axis b as well as ellipticity β can be calculated respectively by

$$a = \sqrt[3]{\rho_{pa}\cdot\rho_{pb}^{\ 2}}, \tag{45}$$

$$b = \sqrt[3]{\rho_{pa}^{\ 2}\cdot\rho_{pb}}, \tag{46}$$

$$\beta = \frac{2(a-b)}{a+b}. \tag{47}$$

5.2 Experiment of expanding and setting round

Based on the above theoretical results, three pipes of different initial ellipticities were chosen to do expanding and setting round experiment. The pipes' material is 20 steel, and its material properties are shown in Figure 7. It can be seen from the figure that when the strain is large, the power exponent hardening material model agrees well with the experimental data; when the strain is small, the bilinear hardening material model is in good agreement with the experimental data. Because the strain when expanding and setting round is not too large, usually about 0.01, to improve the accuracy of bilinear fit, we just took the strain less than 0.05 to fit the experimental data.

Table 2 shows the geometric parameters of initial pipes. Figure 8 shows the experimental results with different expanding ratios. It is clear that the agreement between the results is very good. It means the theoretical analysis model has high precision to meet the forecast need for engineering applications. Analytical analysis and experimental results demonstrate the process principle of expanding and setting round, that is, when stretch-bending to every point in the cross-section is elongation strain, the springback is small and initial ellipticity does not have large effect on springbake amount. That is why the same expanding ratio can be used to achieve $\sigma=815\varepsilon^{0.20375}$ different initial ellipticity pipes' expanding and setting round.

6 Conclusions

In summary, according to the basic hypotheses of small curvature plane bending, the geometric model and the

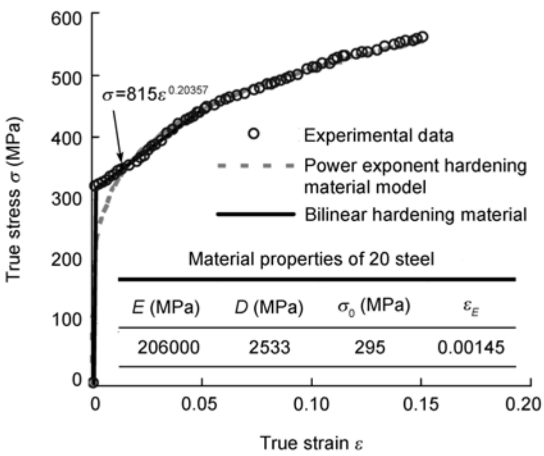

Figure 7 Material properties of experimental pipes.

Table 2 Geometric parameters of initial pipes

Pipe No.	R_1 (mm)	a_0 (mm)	b_0 (mm)	β_0 (%)	t (mm)
1	47.49	47.95	47.03	1.94	2.98
2	47.54	48.05	47.04	2.14	2.97
3	47.50	48.16	46.85	2.77	2.99

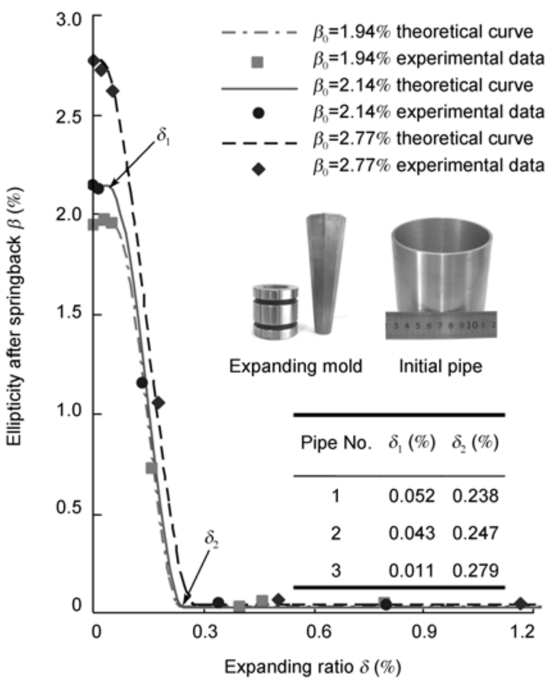

Figure 8 The law of effects of initial ellipticity and expanding ratio on ellipticity after springback.

mechanical model of curved beam under stretch-bending and compress-bending situations are founded. In the analysis process, the equivalent strain and equivalent strain neutral layer are firstly proposed. Through unloading law and strain superposition, the geometric constrain equations as well as springback equation of plane bending are derived. The results enrich the content of the classical theory of elasticplastic bending and are successfully applied to the expanding and setting round problem. Based on the above investigation, the following conclusions can be drawn:

1) Geometric constraint equations of plane bending describe the relationship between geometric center layer and equivalent strain neutral layer. Hereinto, the first equation not only reflects the initial impact of curvature of plane bending, but also shows that the curvatures of geometric center layer do not have superposition relationship but curvatures of equivalent strain neutral layer have when axial force effects plane bending. At the same time, the second equation reflects that curvature of strain neutral layer and curvature of geometric center layer have proportional relationship but they do not coincide with each other.

2) Springback equation of plane bending is the expression of the curvature of geometric center layer after unloading which correlates with the curvature of geometric center layer and the curvature of equivalent strain neutral layer after loading as well as material properties, axial force T, and moment M. The calculations of T and M are determined by loading history and strain state, so springback equation is the general solution of all plane bending problems which meet the basic hypotheses.

3) Straight beam plane bending and pure bending are two special cases of plane bending. The successful conversion to straight beam plane bending and pure bending further proves the correctness of the springback equation.

4) The springback equation is successfully used to analyze pipe's expanding and setting round process and excellent agreement is found between the theoretical prediction of springback and experimental results. Experiment verifies the correctness of analytic method and the accuracy for engineering application, while the principles of expanding and setting round process are also revealed.

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