

## Comparative study on constructal optimizations of T-shaped fin based on entransy dissipation rate minimization and maximum thermal resistance minimization

XIE ZhiHui, CHEN LinGen\* & SUN FengRui

*College of Naval Architecture and Power, Naval University of Engineering, Wuhan 430033, China*

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The constructal optimizations of T-shaped fin with two-dimensional heat transfer model are carried out by finite element method and taking the minimization of equivalent thermal resistance based on entransy dissipation and the minimization of maximum thermal resistance as optimization objectives, respectively. The effects of the global parameter  $a$  (integrating the coefficient of convective heat transfer, the overall area occupied by fin and its thermal conductivity) and the volume fraction  $\Phi$  of fin on the minimums of equivalent thermal resistance and maximum thermal resistance as well as their corresponding optimal configurations are analyzed. The comparison of the results based on the above two optimization objectives is conducted. The results show that the optimal structures based on the two optimization objectives are obviously different from each other. Compared with the optimization result by taking the minimization of maximum thermal resistance as the objective, the optimization result by taking the equivalent thermal resistance minimization as the objective can reduce the average temperature difference in the fin obviously. The increases of  $a$  and  $\Phi$  can all improve the working status of local hot spot and the global heat transfer performance of the system. But the improvement effects of the increases of  $a$  and  $\Phi$  on the minimization of equivalent thermal resistance are different from those on the minimization of maximum thermal resistance. For either objective, the effect of  $a$  is different from that of  $\Phi$ . The T-shaped fin with minimum equivalent thermal resistance is much taller than that with minimum maximum thermal resistance; for either optimization objective, the stem of fin is thicker than the branches of fin, and the stem thickness is relatively close to branch thickness when the minimization of equivalent thermal resistance is taken as the optimization objective. The T-shaped fin with flat stem and slender branches can benefit the reduction of the maximum thermal resistance.

**constructal theory, entransy dissipation extremum principle, fin, multi-scale, generalized thermodynamic optimization**

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### 1 Introduction

With the quick developments of industry fields such as energy, materials, electric information, etc., the importance of heat transfer problem is more prominent than ever. In the study of heat transfer, heat transfer enhancement and heat

transfer optimization have obviously different physical meanings. Heat transfer enhancement always pursues the objectives such as maximization of heat transfer rate or minimization of thermal resistance. For convective heat transfer, conventional technologies of heat transfer enhancement always result in the increase of flow resistance, and the global effect of energy saving is not surely good [1–4]. The concept of heat transfer optimization includes more categories. The field synergy principle [1–4] and the

\*Corresponding author (email: lgchenna@yahoo.com; lingchen@hotmail.com)

entransy dissipation extremum principle [4–6] put forward by Guo are the new theories developed presently for heat transfer optimization. And the theories not only present the unified theoretical explanation and knowledge for conventional technologies of heat transfer enhancement, but also supply new theoretical bases for developing a series of optimization technologies of heat transfer for effective energy saving. Fin and near fin structures are the effective cells and modules for controlling temperature, which are employed widely in the fields of energy, chemical industry, space-flight, electronic industry, etc. The optimization studies for them are the important contents of heat transfer optimization all along [4, 7–26].

In the field of engineering heat transfer optimization, for different application situations, one should employ different optimization criterions and single or multiple objective optimizations. It is principal for the minimization of loss of useful energy to employ entropy generation minimization theory for heat transfer study. The result of entropy generation minimization corresponds to the highest exergy efficiency. To reflect the essential property of heat transfer, Guo et al. [5, 6] defined a new physical quantity, entransy, which represents the global heat transfer ability of an object based on the classical analogy method between thermal and electrical systems and put forward the new theoretical basis and criterion for heat transfer optimization, i.e., the entransy dissipation extremum principle. This attracts many scholars to conduct a series of in-depth corresponding investigations in various directions such as heat conduction [24, 25, 27–38], convective heat transfer and heat exchanger [4, 39–50], convective mass transfer [51, 52], phase change process [53], radiative heat transfer [54, 55], multiple transfer process [56], etc. When the objective is to reduce average temperature difference of heat transfer and enhance the global heat transfer efficiency, the entransy dissipation extremum principle should be employed. Refs. [24, 25, 29–31, 36–38, 48–50] combined the entransy dissipation extremum principle with constructal theory and obtained better system constructs for heat transfer. Refs. [42, 52, 54] combined the entransy dissipation extremum principle with finite time thermodynamics and obtained better heat exchanger configurations.

Since the constructal theory was put forward by Bejan in 1996 [56], it has been applied to optimizations of spatial geometry structure and rhythm in time for various flow problems (fluid flow, energy flow, species, etc.) [57–62], especially the enhancement and optimization of heat transfer and mass transfer [7–26, 29–31, 36–38, 63–76]. The optimizations of fin and near fin structures have become a class of important issues [7–26, 49]. The corresponding study methods mainly include mathematical analytical method [7–9, 21], finite element method [10–13, 15, 22–26] and computational fluid dynamics (CFD) method [14, 16–20]. The structure types mainly include plate fins [7, 21], circular fins [8], pin fins [21], T-shaped fins [9, 16],

z-shaped fins [9], umbrellas of cylindrical fins [9, 49], Y-shaped fins [13, 17], two level assembly of Y-shaped fins [26], T-Y assembly of fins [22], modular systems of Y-shaped fins [18] and modular systems of I-shaped fins [18, 19], etc. The performance indexes mainly include heat transfer rate [9], maximum thermal resistance [13, 26], entransy dissipation rate [24, 25, 49], fin efficiency [14, 17, 18] and a weighted sum of heat flux and loss of pressure [19, 20], etc. The purpose of optimizations is to enhance heat transfer and give attention to work consumption. Ref. [9], in which the constructal optimization of T-shaped fin was conducted, is a magnum opus.

Because homogenization of temperature field of heat transfer component has important effect on stability of system and stress distribution, the design for the homogenization of temperature field has become an important task in many industry fields. Based on simulated annealing method and bionic optimization method, Cheng et al. [32] investigated a two-dimensional heat conduction problem numerically and optimized distribution of material with high conductivity by taking homogenization of temperature field (minimization of quadratic mean difference of temperature) and homogenization of temperature gradient field (minimization of entransy dissipation rate) as optimization objectives, respectively. The results showed that the two objectives have consistency. For a thermal radiator in space, Cheng et al. [34] optimized distribution of internal finite material with high conductivity by taking homogenization of temperature field as optimization objective and using bionic optimization method based on entransy dissipation.

For a fin structure with external boundary conditions of convective heat transfer, this paper takes the minimization of maximum thermal resistance reflecting local ultimate performance of safety and the minimization of equivalent thermal resistance based on entransy dissipation reflecting global average performance of heat transfer as optimization objectives, respectively, considers inhomogenous of temperature distribution of practical heat transfer in fin, and employs two-dimensional heat transfer model and finite element method to conduct constructal optimization of T-shaped fin. For providing some theoretical supports for heat transfer optimizations of fin and its near structures, the results with the two optimization objectives are compared with each other to investigate similarities and differences between the characteristic of fin structure and optimal local ultimate performance and characteristic of fin structure and the optimal global average performance.

## 2 Problem description

### 2.1 Definition of entransy dissipation rate [6]

Ref. [6] defined the overall heat transfer ability of an object: physical quantity entransy ( $E_{vh}$ )

$$E_{vh} = Q_{vh} T / 2, \quad (1)$$

where  $Q_{vh} = Mc_v T$  is heat stored in an object with constant volume, and  $T$  is the object temperature. The entransy dissipation per unit time and per unit volume, which is called the entransy dissipation function  $\phi_h$ , is [6]

$$\phi_h = -\dot{q} \cdot \nabla T = k(\nabla T)^2, \quad (2)$$

where  $\dot{q}$  is the thermal current density vector, and  $\nabla T$  is the temperature gradient.

The entransy dissipation rate of the whole volume is

$$\dot{E}_{vh\phi} = \int_V \phi_h dV. \quad (3)$$

Based on the above, the equivalent thermal resistance for multi-dimensional heat conduction problems with specified heat flux boundary condition is [6]

$$R_h = \dot{E}_{vh\phi} / \dot{Q}_h^2, \quad (4)$$

where  $\dot{Q}_h$  is the heat flow (thermal current) across boundaries. The mean temperature difference for multi-dimensional heat conduction can be expressed as

$$\Delta \bar{T} = R_h \dot{Q}_h. \quad (5)$$

### 2.2 Model

Consider T-shaped assembly of fins as shown in Figure 1 [9], two elemental plate fins ( $t_0 \times L_0$ ) form branches of fin stem ( $t_1 \times L_1$ ).  $W$  represents the third dimension which is perpendicular to the plane of the paper,  $W \gg L_0, L_1$ , and the changes of all parameters along the  $W$  dimension are negligible. The model is simplified into a two-dimensional case and the thickness of the body is fixed at 1. The fin material is isotropic, the thermal conductivity  $k$  is a constant. For giving prominence to optimization method, the physical model is simplified and the heat transfer coefficient  $h$  of external fin surface is consumed as homogenization. The

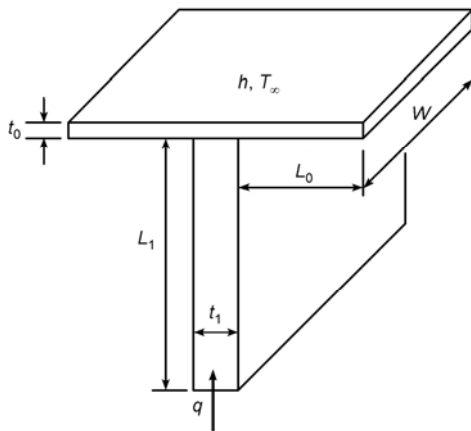


Figure 1 T-shaped assembly of fins [9].

heat transfer rate  $q$  from the fin root and the ambient temperature  $T_\infty$  are fixed.

The constraints for optimization herein are that the total volume of space occupied by the fin and the mass of fin material are fixed [9], i.e., the envelope area  $A$  and the straight-cut area  $A_f$  of fin are fixed, i.e.,

$$A = (2L_0 + t_1)(L_1 + t_0), \quad (6)$$

$$A_f = t_1 L_1 + t_0(t_1 + 2L_0). \quad (7)$$

Eq. (7) can be transformed to  $\Phi = A_f / A \ll 1$ , i.e., the occupancy ratio of fin is fixed. Let

$$\tilde{T} = \frac{T - T_\infty}{q l (k \cdot W)}, \quad (8)$$

$$\tilde{x}, \tilde{y}, \tilde{t}_0, \tilde{t}_1, \tilde{L}_0, \tilde{L}_1 = \frac{x, y, t_0, t_1, L_0, L_1}{A^{1/2}}.$$

The constraints are transformed to the dimensionless equations as follows

$$\tilde{A} = (2\tilde{L}_0 + \tilde{t}_1)(\tilde{L}_1 + \tilde{t}_0) = 1, \quad (9)$$

$$\Phi = \tilde{t}_1 \tilde{L}_1 + \tilde{t}_0(\tilde{t}_1 + 2\tilde{L}_0). \quad (10)$$

The dimensionless two-dimensional differential equation of heat conduction in the fin is

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} = 0. \quad (11)$$

The boundary condition of heat conduction at the fin root is

$$-\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{\tilde{t}_1}. \quad (12)$$

The boundary condition of convective heat transfer between fin and ambient fluid is

$$-\frac{\partial \tilde{T}}{\partial \tilde{x}} = \frac{a^2}{2} \tilde{T}, -\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{a^2}{2} \tilde{T}, \quad (13)$$

where  $a = \left( \frac{2hA^{1/2}}{k} \right)^{1/2}$  [9].

Because of symmetry, the adiabatic boundary condition of the straight-cut plane at  $t_2/2$  of fin stem ( $t_1 \times L_1$ ) is

$$\frac{\partial \tilde{T}}{\partial \tilde{x}} = 0. \quad (14)$$

From eq. (8), the maximum thermal resistance of fin is

$$\tilde{R}_t = \frac{T_{\max} - T_\infty}{q l (k \cdot W)}, \quad (15)$$

$T_{\max}$  represents the highest temperature of fin (local hot

spot), and appears at the fin root inevitably. It is the limit of safety for the fin. The higher  $T_{max}$  is, the lower the safety of equipment is. For special fin structure, maximum thermal resistance  $\tilde{R}_t$  characterizes its local ultimate performance of safety, and the lower its value is, the better it is.

The entransy dissipation extremum principle provides new warranty and criterion for optimization of heat transfer. From eq. (4), the equivalent thermal resistance of the T-shaped assembly of fins based on entransy dissipation is

$$R_h = \frac{\int_{A_f} k(\nabla T)^2 d\Omega}{q^2} = \frac{\int_{A_f} k[(\partial T / \partial x)^2 + (\partial T / \partial y)^2] d\Omega}{q^2} = \frac{\int_{\tilde{t}} \int_{\tilde{L}} [(\partial \tilde{T} / \partial \tilde{x})^2 + (\partial \tilde{T} / \partial \tilde{y})^2] d\tilde{x}d\tilde{y}}{k} \quad (16)$$

So, the dimensionless equivalent thermal resistance is

$$\tilde{R}_h = R_h k = \int_{\tilde{t}} \int_{\tilde{L}} [(\partial \tilde{T} / \partial \tilde{x})^2 + (\partial \tilde{T} / \partial \tilde{y})^2] d\tilde{x}d\tilde{y} \quad (17)$$

From eq. (5), the average temperature difference  $\Delta \bar{T}$  is

$$\Delta \bar{T} = R_h \dot{Q}_h = \frac{\int_{\tilde{t}} \int_{\tilde{L}} [(\partial \tilde{T} / \partial \tilde{x})^2 + (\partial \tilde{T} / \partial \tilde{y})^2] d\tilde{x}d\tilde{y}}{k} q \quad (18)$$

So, the dimensionless average temperature difference  $\tilde{\Delta \bar{T}}$  is

$$\tilde{\Delta \bar{T}} = \frac{\Delta \bar{T}}{q/k} = \int_{\tilde{t}} \int_{\tilde{L}} [(\partial \tilde{T} / \partial \tilde{x})^2 + (\partial \tilde{T} / \partial \tilde{y})^2] d\tilde{x}d\tilde{y} = \tilde{R}_h \quad (19)$$

From the comparison of eq. (15) and eq. (17), one can see that the dimensionless equivalent thermal resistance is the integral of the square of module of dimensionless temperature gradient in the whole region, and can reflect the global characteristics of field much more than the dimensionless maximum thermal resistance does. The entransy dissipation extremum principle is equivalent to the minimum thermal resistance principle, and the smaller the equivalent thermal resistance, the more homogenous the temperature gradient field in heat transfer structure, the lower the mean temperature difference, the better the global heat transfer performance, and the higher the heat transfer efficiency. Specially, for the problems with complex struc-

ture and complex boundary conditions of heat transfer, the equivalent thermal resistance based on entransy dissipation also has clear physical meanings.

### 3 Numerical method and result discussions

#### 3.1 Numerical method

The two-dimensional model shown in Figure 1 just can be solved numerically. The optimization process is to search the structures of fins with the minimum  $\tilde{R}_h$  and with the minimum  $\tilde{R}_t$ , respectively, under the global constraint eqs. (9) and (10). Because of the symmetry of model, only the part of  $x > t_1/2$  needs to be calculated. Generally, the order of heat transfer coefficient  $h$  on surface with forced air convective heat transfer is  $10^2 \text{ W/m}^2 \text{ K}$ , the orders of thermal conductivities of metals such as aluminum and copper are  $10^2 \text{ W/m}^2 \text{ K}$ ,  $A^{1/2} \sim 1 \text{ cm}$ , so  $a \sim 0.1$  [9]. For microminaturization,  $\Phi \ll 1$  holds in practical engineering. In general, the degrees of freedom for optimization are geometric characteristic parameters, i.e., fin thickness ratio  $\tilde{t}_1 / \tilde{t}_0$  and fin height ratio  $\tilde{L}_1 / \tilde{L}_0$ . In the optimization process, the degrees of freedom will be released one by one and the change laws of equivalent thermal resistance and maximum thermal resistance as well as the relations between the objectives and geometry characteristic parameters will be analyzed.

In MATLAB environment, one employs finite element method to determine the distribution of dimensionless temperature  $\tilde{T}$  under the corresponding boundary conditions for the different structures derived from the model shown in Figure 1, and then obtains the dimensionless equivalent thermal resistance  $\tilde{R}_h$ . The computation accuracy is controlled by the criterion  $|(\tilde{R}_h^{j+1} - \tilde{R}_h^j) / \tilde{R}_h^j| < 0.002$ , where  $j$  represents the numerical result corresponding to the former number of grids and  $j+1$  represents the numerical result corresponding to the number of grids refined once. If the computation accuracy is not satisfied, the element size will be refined and the computation will be re-conducted. One refinement increases the number of elements by four times. By searching numerically, one obtains the minimum  $\tilde{R}_h$  and the corresponding optimal geometry parameters. The constructal optimization based on maximum thermal resistance minimization is conducted by employing the same method. Table 1 shows a test for element independence.

**Table 1** A test for element independence ( $a = 0.1$ ,  $\Phi = 0.1$ ,  $\tilde{t}_1 / \tilde{t}_0 = 1$ ,  $\tilde{L}_1 / \tilde{L}_0 = 0.4$ )

Computation number	Element size	$\tilde{R}_h$	$ (\tilde{R}_h^{j+1} - \tilde{R}_h^j) / \tilde{R}_h^j  < 0.002$
1	63	11.8950	-
2	252	11.9427	0.0040
3	1008	11.9613	0.0016

To further validate the arithmetic herein, the preliminary optimization of T-shaped fin is carried out by taking the minimization of maximum thermal resistance  $\tilde{R}_t$  as optimization objective with  $a = 0.2$  and  $\Phi = 0.1$ . The results show that the arithmetic herein can find the minimum of maximum thermal resistance  $(\tilde{R}_t)_{\min}$ , and that the rules herein corresponding to two-dimensional optimization with the degrees of freedom  $\tilde{t}_1/\tilde{t}_0$  and  $\tilde{L}_1/\tilde{L}_0$  are consistent with those from analytical results in ref. [9]. So, the arithmetic herein is correct.

### 3.2 Result analyses

#### 3.2.1 Optimization when $a=0.1$ and $\Phi = 0.1$

Figure 2 shows the characteristics of  $\tilde{L}_1/\tilde{L}_0 - \tilde{R}_h$  and  $\tilde{L}_1/\tilde{L}_0 - \tilde{R}_t$  when  $\tilde{t}_1/\tilde{t}_0 = 1$  is set and the degree of freedom  $\tilde{L}_1/\tilde{L}_0$  is released. From the figure one can see that the rules of equivalent thermal resistance  $\tilde{R}_h$  reflecting global average performance of heat transfer and maximum thermal resistance  $\tilde{R}_t$  reflecting local ultimate performance are absolutely different. With the increase of  $\tilde{L}_1/\tilde{L}_0$ ,  $\tilde{R}_h$  decreases firstly and then increases. When  $\tilde{L}_1/\tilde{L}_0$  is at the optimum  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}} = 1.2$ ,  $\tilde{R}_h$  reaches to the minimum  $(\tilde{R}_h)_{\min} = 10.401$ , and the global heat transfer performance of T-shaped fin is the best. While with the increase of  $\tilde{L}_1/\tilde{L}_0$ ,  $\tilde{R}_t$  decreases firstly and then increases and decreases finally, it reaches to its minimum at  $\tilde{L}_1/\tilde{L}_0 = 0.1$ , and  $(\tilde{R}_t)_{\min} = 43.632$ . It should be noted that the maximum thermal resistance reaches to a bigger value ( $\tilde{R}_t = 61.691$ ,  $\tilde{L}_1/\tilde{L}_0 = 2.4$ ) when the global performance of heat transfer of T-shaped fin is the best, i.e. the safety of T-shaped fin is challenged.

Further releasing the second degree of freedom  $\tilde{t}_1/\tilde{t}_0$ , the work shown in Figure 2 is repeated for every  $\tilde{t}_1/\tilde{t}_0$ , and  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$  are obtained by comparison.

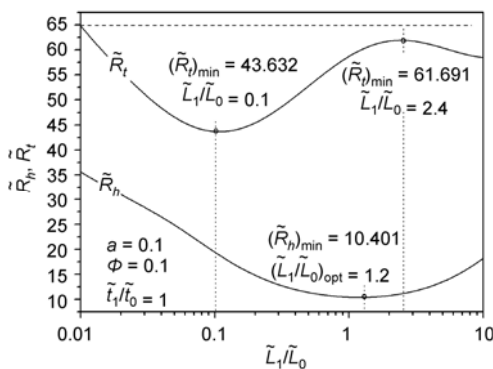


Figure 2 Optimization with a single degree of freedom.

Figure 3 shows the optimization results with two degrees of freedom  $\tilde{L}_1/\tilde{L}_0$  and  $\tilde{t}_1/\tilde{t}_0$ . From the figure one can see that with the increase of  $\tilde{t}_1/\tilde{t}_0$ ,  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$  all decrease firstly and then increase, and their minimums all appear, but their corresponding fin structures are different obviously. When  $(\tilde{R}_h)_{\min} = 6.623$ ,  $\tilde{t}_1/\tilde{t}_0 = 4.9$  and  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}} = 1$ . When  $(\tilde{R}_t)_{\min} = 38.614$ ,  $\tilde{t}_1/\tilde{t}_0 = 4.4$  and  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}} = 0.1$ . Moreover, the characteristics of  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}} - \tilde{t}_1/\tilde{t}_0$  and the optimums of  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}}$  by taking  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$  as objectives, respectively, are different obviously. When the objective is  $(\tilde{R}_h)_{\min}$ ,  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}}$  decreases with the increase of  $\tilde{t}_1/\tilde{t}_0$  and then increases a little (the fluctuation of wave mainly results from the step length of computation). When the objective is  $(\tilde{R}_t)_{\min}$ ,  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}}$  decreases quickly to 0.1 with the increase of  $\tilde{t}_1/\tilde{t}_0$  and then keeps stable. It should be noted that 0.1 is the result of optimization and not the low-limit value set for computation. For either  $\tilde{t}_1/\tilde{t}_0$ ,  $(\tilde{L}_1/\tilde{L}_0)_{\text{opt}}$  corresponding to  $(\tilde{R}_h)_{\min}$  is more 10 times larger than that corresponding to  $(\tilde{R}_t)_{\min}$  under general conditions. That is, the T-shaped fin with minimum equivalent thermal resistance is much taller than that with minimum maximum thermal resistance, and the T-shaped fin with flat stem and slender branches benefits reduction of the maximum thermal resistance.

#### 3.2.2 Effects of $a$ and $\Phi$ on optimization results

Figure 4 shows the effects of  $a$  on the characteristics of  $(\tilde{R}_h)_{\min} - \Phi$  and  $(\tilde{R}_t)_{\min} - \Phi$ . From the figure, one can see that for any  $a$ ,  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$  all decrease with the increase of  $\Phi$ , but the decreasing amplitude of  $(\tilde{R}_h)_{\min}$  is bigger than that of  $(\tilde{R}_t)_{\min}$ ; and for any  $\Phi$ ,  $(\tilde{R}_h)_{\min}$  and

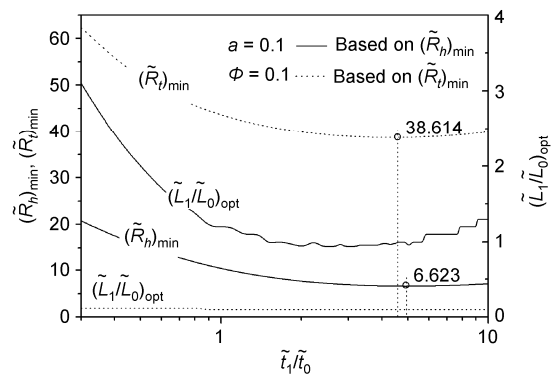
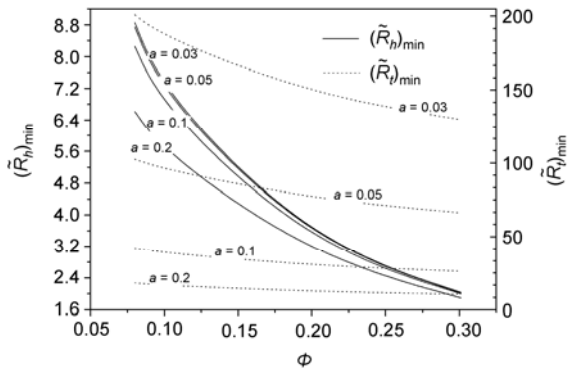


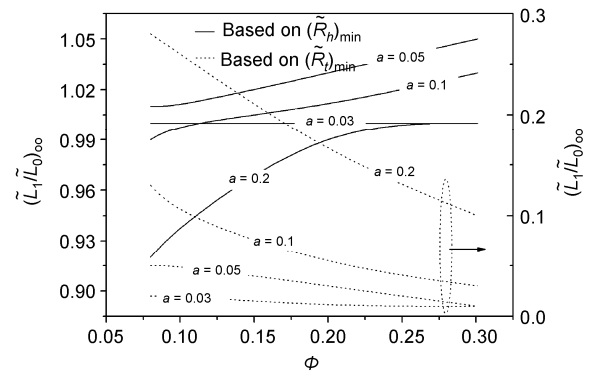
Figure 3 Optimization with double degrees of freedom.



**Figure 4** Effects of  $a$  on the characteristics of  $(\tilde{R}_h)_{\min} - \Phi$  and  $(\tilde{R}_t)_{\min} - \Phi$ .

$(\tilde{R}_t)_{\min}$  all decrease with the increase of  $a$ , but the decreasing amplitude of  $(\tilde{R}_h)_{\min}$  is much smaller than that of  $(\tilde{R}_t)_{\min}$ . It should be noted that the effect of  $a$  on  $(\tilde{R}_h)_{\min}$  is different from that of  $\Phi$  on  $(\tilde{R}_h)_{\min}$ , and the effect of  $a$  on  $(\tilde{R}_t)_{\min}$  is also different from that of  $\Phi$  on  $(\tilde{R}_t)_{\min}$ ; the effect of  $\Phi$  on  $(\tilde{R}_h)_{\min}$  is bigger than that of  $a$  on  $(\tilde{R}_h)_{\min}$ ; while the effect of  $\Phi$  on  $(\tilde{R}_t)_{\min}$  is smaller than that of  $a$  on  $(\tilde{R}_t)_{\min}$ , and the effect of  $\Phi$  decreases obviously with the increase of  $a$ . So, the increases of  $a$  and  $\Phi$  all benefit reduction of equivalent thermal resistance and maximum thermal resistance, i.e., they can improve the local working condition of hot spot and global average performance of heat transfer simultaneously. But the improvement effects of  $a$  and  $\Phi$  on  $(\tilde{R}_h)_{\min}$  are different from those on  $(\tilde{R}_t)_{\min}$ ; for any objective ( $(\tilde{R}_h)_{\min}$  or  $(\tilde{R}_t)_{\min}$ ), the effect of  $a$  is different from that of  $\Phi$ .

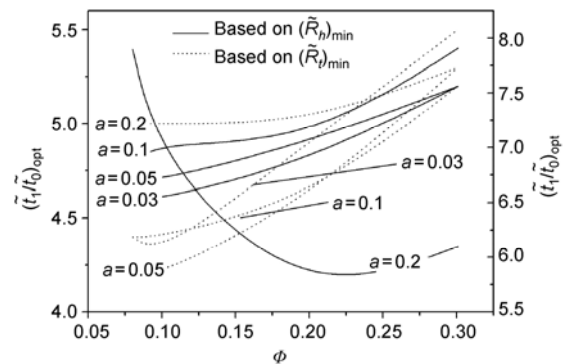
Figures 5 and 6 show the effects of  $a$  on characteristics of  $(\tilde{L}_1/\tilde{L}_0)_{oo} - \Phi$  and  $(\tilde{t}_1/\tilde{t}_0)_{opt} - \Phi$  corresponding to Figure 4. From Figure 5, one can find that when the objective is  $(\tilde{R}_h)_{\min}$ ,  $0.92 \leq (\tilde{L}_1/\tilde{L}_0)_{oo} \leq 1.05$ , and that when the objective is  $(\tilde{R}_t)_{\min}$ ,  $0.01 \leq (\tilde{L}_1/\tilde{L}_0)_{oo} \leq 0.28$ . That is, for the T-shaped fin based on  $(\tilde{R}_t)_{\min}$ , the branches are much longer than the stem, which favors enhancement of heat transfer and heat transfer rate is higher; while for the T-shaped fin based on  $(\tilde{R}_h)_{\min}$ , the length of branches is relatively near to that of stem, which favors uniform temperature gradient, heat transfer efficiency is higher. Moreover, the characteristic of  $(\tilde{L}_1/\tilde{L}_0)_{oo} - \Phi$  based on  $(\tilde{R}_h)_{\min}$  is more complex than that based on  $(\tilde{R}_t)_{\min}$ . When the objective is  $(\tilde{R}_h)_{\min}$  and  $a=0.03$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  is insensitive to  $\Phi$  and keeps stable; when  $a=0.05$  and  $0.1$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  in-



**Figure 5** Effect of  $a$  on characteristic of  $(\tilde{L}_1/\tilde{L}_0)_{oo} - \Phi$ .

creases monotonically with the increase of  $\Phi$ ; when  $a$  increases to  $0.2$ , with the increase of  $\Phi$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  increases firstly and then tends to stability. When the objective is  $(\tilde{R}_t)_{\min}$ , for any  $a$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  decreases monotonically with the increase of  $\Phi$ , and the smaller  $a$ , the smaller the decreasing amplitude of  $(\tilde{L}_1/\tilde{L}_0)_{oo}$ ; for any  $\Phi$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  increases monotonically with the increase of  $a$ . Besides, the results calculated show that with the increase of  $\tilde{t}_1/\tilde{t}_0$ ,  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  decreases firstly and then tends to stability in the same way for a different  $a$  and  $\Phi$ .

From Figure 6, one can see that  $(\tilde{t}_1/\tilde{t}_0)_{opt}$  and  $(\tilde{t}_1/\tilde{t}_0)_{opt} - \Phi$  characteristics based on  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$ , respectively are different obviously. The fundamental reasons for the differences are that the minimization of equivalent thermal resistance (the minimization of entransy dissipation rate) demands the good homogenization of temperature gradient, so the structure parameters and their change rules based on  $(\tilde{R}_h)_{\min}$  match with homogenization of temperature gradient; while the minimization of maximum thermal resistance pursues that the smaller maximum temperature difference, the better it is, so the structure parameters and their change rules based on  $(\tilde{R}_t)_{\min}$  match with the



**Figure 6** Effect of  $a$  on characteristic of  $(\tilde{t}_1/\tilde{t}_0)_{opt} - \Phi$ .

minimization of maximum temperature difference. When the objective is  $(\tilde{R}_t)_{\min}$  and  $a = 0.03, 0.05$  and  $0.1$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  increases monotonically with the increase of  $\Phi$ ; and for any  $\Phi$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  increases approximately with the increase of  $a$ ; when  $a=0.1$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  behaves fluctuation a little; and when  $a$  increases to  $0.2$ , the change rule of  $(\tilde{t}_1/\tilde{t}_0)_o$  with the increase of  $\Phi$  changes absolutely. With the increase of  $\Phi$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  decreases firstly and then increases and its minimum appears. When the objective is  $(\tilde{R}_t)_{\min}$  and  $a=0.03$ , the minimum of  $(\tilde{t}_1/\tilde{t}_0)_o$  also appears in the smaller region of  $\Phi$ , and then  $(\tilde{t}_1/\tilde{t}_0)_o$  increases monotonically with the increase of  $\Phi$ ; for other  $a$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  all increases monotonically with the increase of  $\Phi$ , but for any  $\Phi$ , the change rules of  $(\tilde{t}_1/\tilde{t}_0)_o$  with the increase of  $a$  do not show a fixed order. For a smaller  $\Phi$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  corresponding to the biggest  $a$  ( $0.2$ ) is the biggest; while for a bigger  $\Phi$ ,  $(\tilde{t}_1/\tilde{t}_0)_o$  corresponding to the smallest  $a$  ( $0.03$ ) is the biggest.

From Figures 5 and 6 one can conclude that the difference of  $(\tilde{t}_1/\tilde{t}_0)_o$  based on the two optimization objectives is obviously smaller than that of  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  based on the two optimization objectives, that when the objective is  $(\tilde{R}_h)_{\min}$ , the change amplitude of  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  is smaller than that of  $(\tilde{t}_1/\tilde{t}_0)_o$ , i.e., the effect of fin thickness ratio on the distribution of temperature gradient is bigger than that of fin height ratio on the distribution of temperature gradient, and that when the objective is  $(\tilde{R}_t)_{\min}$ , the change amplitude of  $(\tilde{L}_1/\tilde{L}_0)_{oo}$  is bigger than that of  $(\tilde{t}_1/\tilde{t}_0)_o$ . When one adjusts the performance of T-shaped fin between local ultimate performance and global average performance, adjusting  $\tilde{L}_1/\tilde{L}_0$  is more effective than adjusting  $\tilde{t}_1/\tilde{t}_0$ ; when one just only adjusts the global average performance, adjusting  $\tilde{t}_1/\tilde{t}_0$  is more effective than adjusting  $\tilde{L}_1/\tilde{L}_0$ ; when one just only adjusts the local ultimate performance, adjusting  $\tilde{L}_1/\tilde{L}_0$  is more effective than adjusting  $\tilde{t}_1/\tilde{t}_0$ . When one increases  $\Phi$  to reduce maximum thermal resistance, the material of branches should be added much more relatively; when one increases  $\Phi$  to reduce equivalent thermal resistance and  $a \leq 0.1$ , the material of stem should be added much more relatively. In general, the T-shaped fin with the minimum equivalent thermal resistance is much taller than that with the minimum maximum thermal resistance; for the two objectives, stems are all thicker than branches, but the thicknesses of stem and branch are relatively near to each other; the T-shaped fin with flat stem and slender branches benefits reduction of maximum thermal

resistance.

### 3.2.3 Comparison of average temperature difference between optimal constructs based on two objectives

Figure 7 shows the effects of  $a$  on characteristic of  $\tilde{\Delta T} - \Phi$  based on  $(\tilde{R}_h)_{\min}$  and  $(\tilde{R}_t)_{\min}$ , respectively. From the figure, one can see that the optimization based on  $(\tilde{R}_h)_{\min}$  reduces the average temperature difference in the fin more notably than the optimization based on  $(\tilde{R}_t)_{\min}$  does. When  $a=0.2$ , the difference of average temperature differences based on the two objectives is higher than 14.5%; the smaller  $a$ , the bigger the decreasing amplitude; it is beyond 88.2% when  $a = 0.03$ . When the objective is  $(\tilde{R}_h)_{\min}$ , because of  $\tilde{\Delta T} = \tilde{R}_h$ , the effect of  $a$  on  $\tilde{\Delta T} - \Phi$  characteristic is the effect of  $a$  on  $(\tilde{R}_h)_{\min} - \Phi$  characteristic shown in Figure 4. When the objective is  $(\tilde{R}_t)_{\min}$  and  $a=0.03, 0.1$  and  $0.2$ , the average temperature differences  $\tilde{\Delta T}$  of heat transfer are all decrease with the increase of  $\Phi$ ; but when  $a=0.05$ ,  $\tilde{\Delta T}$  decreases firstly and then increases with the increase of  $\Phi$ , and its minimum appears. For a specified  $\Phi$ ,  $\tilde{\Delta T}$  decreases with the increase of  $a$ .

## 4 Conclusions

The entransy dissipation extremum principle provides new warranty and criterion for optimization of heat transfer. Serving as the effective cells and modules for controlling temperature, fin and near fin structures are employed widely in the fields of energy, chemical industry, spaceflight, electronic industry, etc. In this paper, by combining finite element method and numerical searching method, the structural optimizations of two-dimensional T-shaped fin are carried out, in which the minimization of equivalent thermal resistance based on entransy dissipation and the minimization of maximum thermal resistance are taken as the objectives, respectively. The effects of the global parameter  $a$

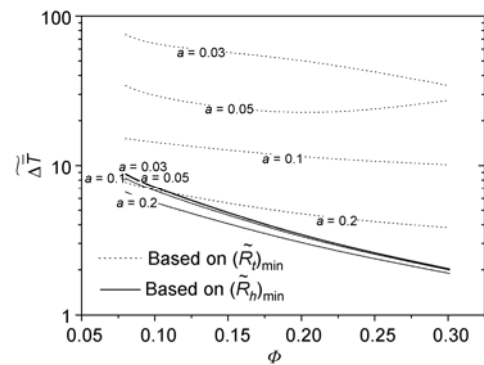


Figure 7 Effects of  $a$  on characteristics of  $\tilde{\Delta T} - \Phi$ .

( $a = (2hA^{1/2}\lambda^{-1})^{1/2}$ ) and the volume fraction  $\Phi$  of fin on the minimums of equivalent thermal resistance and maximum thermal resistance and their corresponding optimal configurations are analyzed. The comparison of the results based on the above two objectives is conducted.

The results show that the optimal geometry structure of T-shaped fin with minimum equivalent thermal resistance (minimum entransy dissipation rate) can be obtained by constructal optimization, and the optimal geometry structures based on the two optimization objectives are obviously different from each other. For any fin thickness ratio, the optimal fin height ratio corresponding to the minimum equivalent thermal resistance is more 10 times bigger than that corresponding to the minimum maximum thermal resistance when  $a=0.1$  and  $\Phi=0.1$ . Compared with the optimization by taking the minimization of maximum thermal resistance as the objective, the optimization by taking the minimization of equivalent thermal resistance as the objective can reduce the average temperature difference in the fin obviously, and the decreasing amplitude is beyond 88.2% in the numerical example herein. The increases of  $a$  and  $\Phi$  can improve the local working condition of hot spot and global average performance of heat transfer simultaneously. But the improvement effects of  $a$  and  $\Phi$  on the two objectives, i.e. the minimization of equivalent thermal resistance and the minimization of maximum thermal resistance, are different; for either objective, the effect of  $a$  is different from that of  $\Phi$ . The optimal fin height ratios based on two optimization objectives are different from each other obviously, and the characteristic of the optimal fin height ratio and volume ratio of fin based on equivalent thermal resistance minimization is more complex than that based on the maximum thermal resistance minimization. The difference of optimal fin thickness ratio based on the two different objectives is obviously smaller than that of optimal fin height ratio; when the objective is the minimization of equivalent thermal resistance, the change amplitude of optimal fin height ratio is smaller than that of the optimal fin thickness ratio, i.e., the effect of fin thickness ratio on the distribution of temperature gradient is bigger than that of fin height ratio on the distribution of temperature gradient; when the objective is the minimization of maximum thermal resistance, the change amplitude of optimal fin height ratio is bigger than that of optimal fin thickness ratio. So, when one adjusts the performance of T-shaped fin between local ultimate performance and global average performance, adjusting fin height ratio is more effective than adjusting fin thickness ratio; when one just only adjusts the global average performance, adjusting fin thickness ratio is more effective than adjusting fin height ratio; when one just only adjusts the local ultimate performance, adjusting fin height ratio is more effective than adjusting fin thickness ratio. When one increases  $\Phi$  to reduce the maximum thermal resistance, the material of branches should be added much more relatively;

when one increases  $\Phi$  to reduce the equivalent thermal resistance and  $a \ll 0.1$ , the material of stem should be added much more relatively.

Otherwise, the local heat transfer performance and global heat transfer performance of the heat transfer structures can not necessarily reach to their optimums simultaneously. In order to meet the needs of practical engineering, the optimization compromising local heat transfer performance with global heat transfer performance should be further considered in the optimization of heat transfer structures. And for practical structures with heat transfer, it can be considered to add the safety constraint of heat transfer for optimization [77].

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