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F-N curved surface method for establishing the integrated risk criteria of dam failure

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It is of great importance to establish the dam failure risk criteria for dam risk assessment and management. Presently, the *F-N* curve method is widely applied in practice, in which the *F-N* curves are used for establishing the criteria for separate risks caused by the dam failure in life, economy, environment, and society respectively. In this paper, in consideration of the overlying effect by two or more types of risk losses resulting from the dam failure at the same time, the *F-N* curved surface method is presented for establishing the integrated dam failure risk criteria. In this method, the named *F-N* curved surfaces are established, and by these curved surfaces, the acceptable, the allowable, and the unallowable integrated risk zones can be defined.

dam failure risk assessment, separate risk criteria, *F-N* curve method, integrated risk criteria, *F-N* curved surface method

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1 Introduction

In dam failure risk assessment, the risk grade depends on the annual dam failure probability and the dam failure consequences. Some approaches to determine the annual dam failure probability and the dam failure consequences have been proposed, and the named *F-N* curve method is widely used for establishing the risk criteria [1–5]. In these studies, the dam failure risk is usually separated into several types, such as the life risk, the economic risk, the environmental risk, and the social risk. In the *F-N* curve method, for each of these types of risks, the corresponding *F-N* curves can be established for establishing the risk criteria as shown in Figures 1 to 4, respectively on the basis of the characteristics of these risk losses. In fact each *F-N* curve expresses the relationship between the annual dam failure probability F and the risk loss N, i.e. $F=f(N)$, on which for all the points, $R=F\cdot N=\text{const}$, where R represents the risk level. In other words, all the points on each *F-N* curve are of the same risk level. It should be noted that in these figures the axes are formed by conversion of the original coordinates, i.e., the logarithm of the product of annual failure probability F multiplied by 1×10^7 is the vertical axis, the logarithm of the risk loss N is the horizontal axis. Then in this converted coordinate system, the *F-N* curves are in the form of straight lines.

By these *F-N* curves, three risk zones, i.e., the acceptable risk zone, the unallowable risk zone, and the allowable risk zone, are formed, which are respectively under the lower *F-N* curve, above the upper *F-N* curve, and between these two *F-N* curves. Then, the risk level of the separate risk loss can be easily determined, according to the location of the specific risk point corresponding to the risk event of dam failure.

In particular, the life risk criteria include the individual life risk criteria and the social life risk criteria. The individual life risk is the probability at which for a certain location

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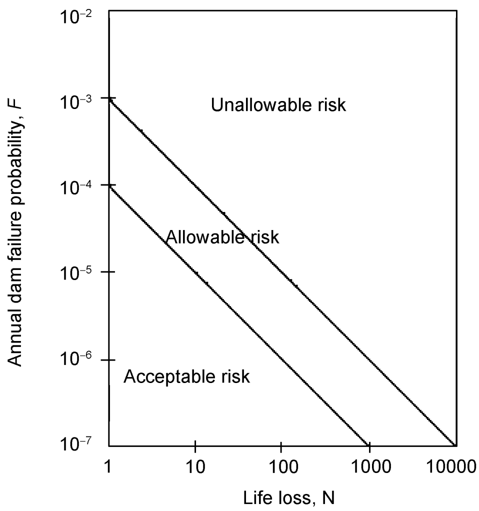

Figure 1 *F-N* curves for life risk criteria.

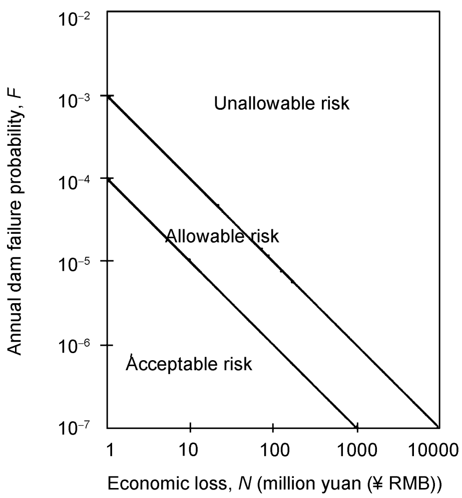

Figure 2 *F-N* curves for economic risk criteria.

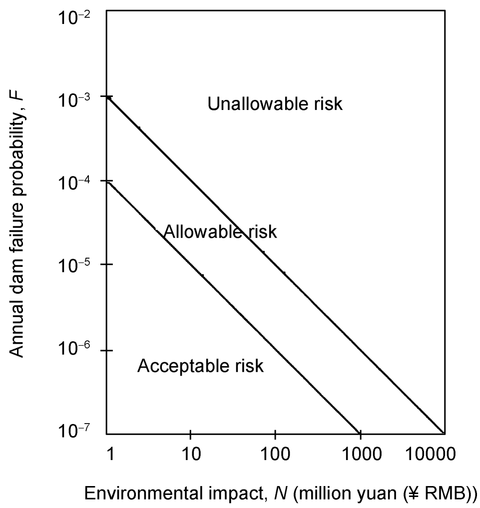

Figure 3 *F-N* curves for environmental risk criteria.

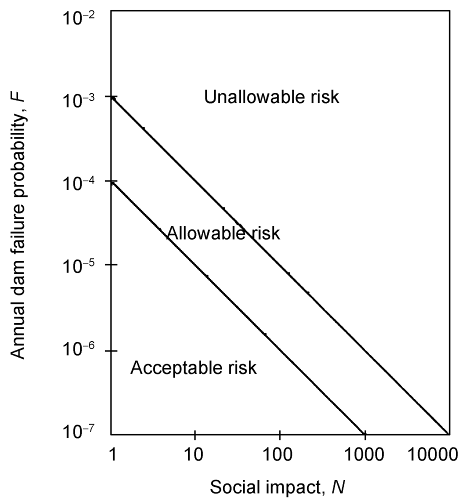

Figure 4 *F-N* curves for social risk criteria.

in the impacted area the individual life loss will be caused by dam failure. The social life risk is the cumulative probability at which the loss of life equal to or grater than *n* will be caused in the impacted area. For the individual life risk caused by dam failure, the risk criteria should be established in consideration of the life risk caused by some other accidents such as traffic accidents and production accidents in daily life. According to the relevant statistics of China [6], for the traffic accidents, the death rate in recent years is about 7.0×10^{-5}, and about 8.0×10^{-5} for the production accidents. Nowadays, such a safety situation in traffic and production is more and more complained by Chinese people. It seems that at present the upper limit of the allowable individual life risk for people in daily life should not be higher than 1.0×10^{-4}. On the basis of this, at present, the probability of 1.0×10^{-4} can be taken as the allowable criterion for individual life risk caused by dam failure. As for the acceptable individual life risk, in general, the lower the life risk probability, the more acceptable, and if the annual life risk probability were at 1.0×10^{-6} or lower, people would not worry about it. In practice, by increasing this probability of 1.0×10^{-6} by one order of magnitude, the probability of 1.0×10^{-5} can be taken as the acceptable criterion of the individual life risk caused by dam failure, which is the same as that suggested by ANCOLD (Australian National Committee on Large Dams) [7] and corresponding to the life risk caused by nuclear power station.

For the social life risk criteria, both the dam failure probability *F* and the number of life losses *N* caused by the dam failure should be taken into consideration. According to the relevant data of the years 1982 to 2000, in China the annual dam failure probability *F* took the value 0.88×10^{-4} for the large and medium scale dams, which comes close to the guidelines to evaluate the need and urgency to implement risk reduction activities of the United States [8], while the corresponding number of life losses N was from a few to hundreds or even more. If the number of life losses is in the range of 10–100 people in a dam failure, the social life risk $R=F\cdot N=0.88\times10^{-3}$–$0.88\times10^{-2}$ persons/year for the large and medium scale dams, while the lower limit of the range can be taken as the boundary of the allowable and unallowable social life risks. Based on this analysis, for the large and medium scale dams, the risk of 1.0×10^{-3} person/year approximate to the lower limit of the range can be defined as the allowable social life risk criterion which is also corresponding to that suggested by ANCOLD. As regards the acceptable social life risk criterion, in general, the lower the dam failure probability, the more acceptable, and in practice, it might be suitable that by decreasing the annual dam failure probability by one order of magnitude, the risk of 1.0×10^{-4} person/year can be taken as the acceptable social life risk criterion.

From the analysis above, in Figure 1, the upper *F-N* curve, the boundary of the allowable and the unallowable risk zones, goes down from the point of probability 1.0×10^{-3} on the vertical axis, which corresponds to the life loss of one person, and the lower curve, the boundary of the allowable and the acceptable risk zones, goes down from the point of probability 1.0×10^{-4}. These two curves are in the form of the straight lines with the same slope −1, which means that with the increase of the life loss, the probability will be decreased at the same speed while all the points on these curves are of the same product of the horizontal coordinate N, the life loss caused by the dam failure, multiplied by the vertical coordinate F, the corresponding probability, and then all the points on each curve are of the same the risk level. It should be pointed out that the *F-N* curve is supposed to go down with a slope less than −1, and even it takes the form of nonlinear curve, because in reality, with the increase of life loss, the unwillingness of the people may usually increase dramatically, and even in an acceleration. However, presently it is very difficult to exactly define the slope of the *F-N* curve and the nonlinear expression of the curve in detail. Thus in this paper the *F-N* curve in the linear form is adopted.

Similarly, the *F-N* curves for the economic risk criteria can also be established as shown in Figure 2. In general, for a country or a region, its ability to withstand the economic loss risk caused by the dam failure mainly depends on its level of the economic development. For instance, in Australia, it is suggested by ANCOLD that if the dam failure causes the economic loss of 100 million dollars ($AUD) while the dam failure probability is higher than 1.0×10^{-5}, the annual economic risk of 1.0×10^{-3} million dollars ($AUD)/year or higher should be considered unallowable, while the dam failure probability is lower than 1.0×10^{-6}, the economic risk of 1.0×10^{-4} million dollars ($AUD)/year or lower should be considered acceptable. But in China, from the data of the World Bank, its per capita GDP at purchasing power parity (PPP) in 2008 was 5962 dollars ($USD), only around one-sixth of the per capita GDP of Australia in that year, 35677 dollars ($USD) [9]. Then, by taking account of the difference in economic development between China and Australia and the exchange rate (around 6 from $AUD to ¥RMB), it is suitable that in China if the dam failure causes the economic loss of 100 million yuan (¥RMB) while the dam failure probability is higher than 1.0×10^{-5}, the annual economic risk of 1.0×10^{-3} million yuan (¥RMB)/year or higher should be considered unallowable, while the dam failure probability is lower than 1.0×10^{-6}, the economic risk of 1.0×10^{-4} million yuan (¥RMB)/year or lower should be considered acceptable.

On the basis of the analysis above, in Figure 2, the upper curve, the boundary of the allowable and the unallowable risk zones, goes down from the point of dam failure probability 1.0×10^{-3} on the vertical axis, which corresponds to the economic loss of one million yuan (¥RMB), and the lower curve, the boundary of the allowable and the acceptable risk zones, goes down from the point of dam failure probability 1.0×10^{-4}. It should be pointed out that in China because there is a great difference in the level of economic development between different regions, and the relatively developed eastern provinces are of the higher ability to withstand the economic loss risk caused by the dam failure than the underdeveloped western provinces, for different regions, the *F-N* curves for the economic risk criteria should not be the same. For the underdeveloped western provinces, the starting points of the *F-N* curves on the vertical axis are supposed to be properly lower, while for the relatively developed eastern provinces, the starting points of the *F-N* curves are supposed to be properly higher.

With respect to the environmental risk criteria, due to the complexity of the impact of dam failure on the environment, the environmental risk loss can be indicated in the environmental restoration cost in monetary unit. Then the way the *F-N* curves for the economic risk criteria are established is fully applicable to establishing the *F-N* curves for the environmental risk criteria as shown in Figure 3. The upper curve, the boundary of the allowable and the unallowable risk zones, goes down from the point of probability 1.0×10^{-3} on the vertical axis, which corresponds to the environmental restoration cost of one million yuan (¥RMB), and the lower curve, the boundary of the allowable and the acceptable risk zones, goes down from the point of probability 1.0×10^{-4}.

As for the social risk criteria, due to its complexity, it is quite difficult to define the social risk loss quantitatively. However, it is believed that in general the impact of dam failure on society is mainly related to such factors as the quantity of the flash flood, the importance of the impacted cities, and the features of the downstream area in the aspects of infrastructures, population, etc. Then it might be suitable to use a comprehensive index *CI* reflecting all the main factors associated with the social impact of dam failure, given by eq. (1), for indicating the magnitude of the social risk impact.

$$CI = k_1 k_2 VHSP, \quad (1)$$

where k_1 is the coefficient associated with the importance of the impacted cities, and normally k_1=1–10; k_2 is the coefficient related to the feature of infrastructures of the impacted cities, and normally k_2=1–3; *V* stands for the storage volume of the reservoir with 10^8 m^3 as the unit; *H* represents the dam height with 10 m as the unit; *S* is the slope of the river downstream; *P* indicates the number of people at risk downstream, in magnitude of 10^4.

Then the *F-N* curves can be established as shown in Figure 4. From the definition of the comprehensive social impact index *CI* above and by analysis of its composing factors, it could be supposed that *CI* takes the value of 10–100, while in China the probability of the dam failure takes the value 0.88×10^{-4} for the large and medium scale dams, the social risk is in the range of 0.88×10^{-3}–0.88×10^{-2}/year. Then it may be considered that for the large and medium scale dams, the risk of 1.0×10^{-3}/year approximate to the lower limit of the range can be defined as the allowable social risk criterion. On the other hand, it might be suitable that by decreasing the annual dam failure probability by one order of magnitude, the risk of 1.0×10^{-4}/year can be taken as the acceptable social risk criterion. Based on this, in Figure 4, the upper curve, the boundary of the allowable and the unallowable risk zones, goes down from the point of probability 1.0×10^{-3} on the vertical axis, which corresponds to the comprehensive social impact index *CI* of 1.0×10^8, and the lower curve, the boundary of the allowable and the acceptable risk zones, goes down from the point of probability 1.0×10^{-4}.

It should be pointed out that the *F-N* curves for all the separate risk criteria above can be applied to dam failure risk assessment and management when only one type of risk may be caused by dam failure. But in reality, by dam failure, two or more types of risks, including those in life, economy, environment, and society, may be brought about at the same time. So the integrated risk criteria should be established for dam failure risk assessment and management.

2 *F-N* curved surfaces for integrated risk criteria

For the sake of simplicity, suppose that only two types of risks, e.g. the life and the economic risks may be caused simultaneously by a risk event of the dam failure, and that the dam failure probability, the associated risk losses both in life and in economy have been properly calculated respectively. Then for these two types of losses, the separate risk level can be easily defined by use of the *F-N* curves for these two risk criteria above. For the life loss, the risk point corresponding to the risk event, with the loss of life and the dam failure probability as the horizontal and vertical coordinate values, respectively, can be easily defined in Figure 1. If the risk point is located in the zone above the upper *F-N* curve, the life risk is unallowable, if it is located in the zone under the lower *F-N* curve, the risk is acceptable, and if it is located in the zone between the upper and the lower *F-N* curves, the risk is allowable. In the same way, the economic risk level can also be easily defined by use of Figure 2.

However, it is not enough to define the level of these two types of risks separately for the dam failure risk assessment and management. In fact, for these two types of risks, there is an overlying effect, which means that because these two types of losses resulting from the dam failure at the same time will pile up on each other, the real total risk, or the integrated risk, may usually be higher than these two separate risks. Then the integrated risk criteria should be established to define the integrated risk level.

For this reason, by combining Figure 1 with Figure 2, a three-dimension coordinate system may be established as shown in Figure 5, with the axis *Z*, corresponding to the annual loss probability *F* caused by the dam failure, the axis *X*, corresponding to the life loss N_1, and the axis *Y*, corresponding to the economic loss N_2. For integrated risk assessment, the relationship between life loss and economic loss can be established in the loss plane *XOY*. For the sake of simplicity, by connecting points *B* and *F* in the plane *XOY*, the straight line *BF* expresses the linear relationship between life loss and economic loss. As can be seen, at point *B*, x_b=4 represents the upper limit of the allowable life risk loss N_b=10,000, y_b=0 means that no economic loss needs to be considered, and z_b=0 indicates the corresponding dam failure probability F=1.0×10^{-7}, while at point *F*,

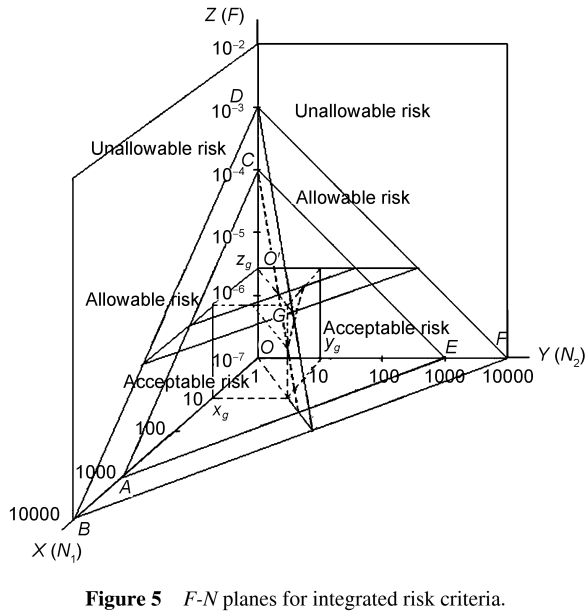

Figure 5 *F-N* planes for integrated risk criteria.

x_f=0 means that no life loss needs to be considered, y_f=4 represents the upper limit of the allowable economic risk loss N_f=10,000, and z_f=0 indicates the dam failure probability F=1.0×10^{-7}. Then, from the integrated risk viewpoint, both points B and F are of the same value of the risk level, $R=F\cdot N_b=F\cdot N_f=1.0\times10^{-3}$. On the other hand, on the straight line BF, with the life risk loss decreasing by an order of magnitude in comparison with point B, the economic risk loss will be increased by the same order of magnitude up to point F, thus, all the points on line BF have the same integrated risk level as points B and F. Then by this straight line BF, together with the *F-N* curves BD in plane XOZ and FD in plane YOZ, the plane BDF in which all the points are of the same integrated risk level can be established, while in fact the plane BDF, called the *F-N* plane, represents the upper limit of the allowable integrated risk.

Similarly, the other *F-N* plane ACE, formed by the line AE in plane XOY, together with the *F-N* curves AC in plane XOZ, and EC in plane YOZ, represents the upper limit of the acceptable integrated risk.

By these two *F-N* planes, the three-dimension risk space is divided into three zones, i.e. the zone above the *F-N* plane BDF in which the integrated risk is unallowable, the zone under the *F-N* plane ACE in which the integrated risk is acceptable, and the zone between the *F-N* planes BDF and ACE in which the integrated risk is allowable.

So far, when a risk event of the dam failure is given, the integrated risk can be easily defined according to the specific location of the corresponding risk point, in relation to the *F-N* planes in Figure 5, while in mathematics, the specific location of the risk point $G(x_g, y_g, z_g)$ in Figure 5 may be defined as follows.

If the *F-N* curve AC intersects the axes X and Z at points $A(a, 0, 0)$ and $C(0, 0, c)$, respectively, and EC intersects the axes Y and Z at points $E(0, e, 0)$ and $C(0, 0, c)$, respectively, then the *F-N* plane ACE may be expressed as follows:

$$\frac{x}{a}+\frac{y}{e}+\frac{z}{c}=1, \tag{2}$$

or,

$$A_1x+B_1y+C_1z-D_1=0, \tag{3}$$

where $A_1=e\cdot c$, $B_1=a\cdot c$, $C_1=a\cdot e$, and $D_1=a\cdot e\cdot c$.

Similarly, the *F-N* plane BDF may be expressed as below:

$$\frac{x}{b}+\frac{y}{f}+\frac{z}{d}=1, \tag{4}$$

or,

$$A_2x+B_2y+C_2z-D_2=0, \tag{5}$$

where $A_2=f\cdot d$, $B_2=b\cdot d$, $C_2=b\cdot f$, and $D_2=b\cdot f\cdot d$.

Let

$$I_1=A_1x_g+B_1y_g+C_1z_g-D_1, \tag{6}$$

and

$$I_2=A_2x_g+B_2y_g+C_2z_g-D_2. \tag{7}$$

From the basic principle of geometry, the location of the risk point $G(x_g, y_g, z_g)$ in the three-dimension coordinate system as shown in Figure 5 can be defined as follows [10]:

If $I_1<0$, then the risk point G is in the acceptable integrated risk zone under the *F-N* plane ACE;

If $I_2>0$, then the risk point G is in the unallowable integrated risk zone above the *F-N* plane BDF;

If $I_1>0$, and $I_2<0$, then the risk point G is in the allowable integrated risk zone between the *F-N* planes ACE and BDF.

It should be pointed out that in the method for establishing the integrated risk criteria above, i.e., the *F-N* plane method, it is supposed that there is a linear relationship between the risk loss in life and in economy, and the life loss and the economic loss are of the same weight on the integrated risk. However, for some cases, it is reasonable that these two risks are different from each other in the weight on the integrated risk. In this case, two curves AmE and BnF may be established. In correspondence, two curved surfaces, called *F-N* curved surfaces, in each of which all the points are of the same integrated risk level can be established for the integrated risk criteria as shown in Figure 6, while the formulation of the *F-N* curved surfaces is dependent on the function of the curves AmE and BnF. On the basis of this, when a risk event of the dam failure is given, the specific location of the corresponding risk point, in relation to the *F-N* curved surfaces, can be defined from the principle of geometry, and then the integrated risk level can be determined without any difficulties. Then, it can be seen that the

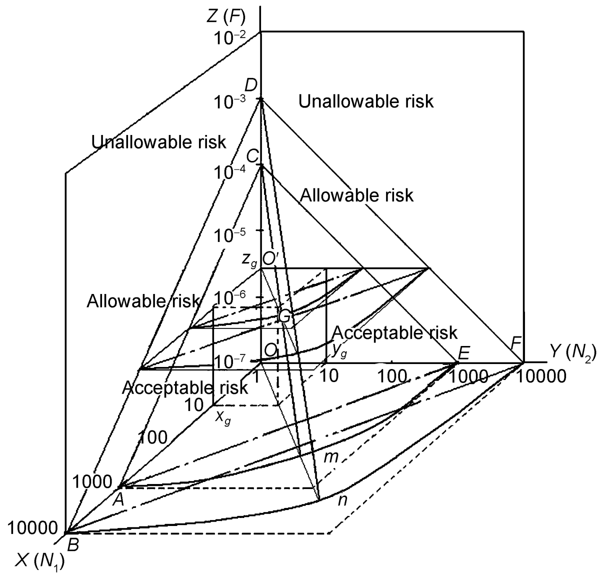

Figure 6 *F-N* curved-surfaces for integrated risk criteria.

F-N curved surface method for establishing the integrated risk criteria is, in fact, the natural extension of the *F-N* plane method. In other words, the *F-N* plane method, in which the relationship between life loss and economic loss is considered to be in linear form, is the special case of the *F-N* curved surface method for establishing the integrated risk criteria.

Theoretically, if the dam failure causes more than two types of risks simultaneously, the concept of the *F-N* curved surface and the analysis method above can be extended without any difficulties for establishing the integrated risk criteria.

3 Case study

The dam of a reservoir in the southern of China was built in 1970s, and the main task of the reservoir is to supply water for an irrigation district, and flood control, power generation, and so on are also taken into account. Because of some problems in the design and construction of this dam due to the historic reasons, the emergency situations have appeared several times since the dam was put into operation. In order to improve the conditions in dam safety, a specific project for removal of the ill-condition and the reinforcement of this dam was conducted in the years 2002 to 2004 [11]. The dam failure risk assessment was studied by use of the *F-N* curved surface method and the corresponding integrated risk criteria presented in this paper to demonstrate the effect of this specific project for removal of the ill-condition and the reinforcement of this dam.

By detailed calculation, it has been shown that at present the annual failure probability is 1.5×10^{-6} for this dam, with the estimated loss of life 8 people, and the economic loss 50.46 million yuan (¥RMB). From the separate risk criteria as shown in Figures 1 and 2, it can be seen that both the life risk and the economic risk of this dam are in the acceptable risk zones. However, because of the overlying effect of these two types of risks, it doesn't mean that the integrated risk is certainly acceptable. By analysis of the local conditions, it is believed that the life loss and the economic loss are of the same weight on the integrated risk, and the *F-N* plane method can be applied. By use of formulas (6) and (7), the location of the associated risk point G can be defined according to the values of I_1 and I_2 below:

$$I_1 = A_1x_g + B_1y_g + C_1z_g - D_1 = 3\times3\times\lg8 + 3\times3\times\lg50.46 + 3\times3\times\lg15 - 3\times3\times3 = 7.04 > 0,$$

$$I_2 = A_2x_g + B_2y_g + C_2z_g - D_2 = 4\times4\times\lg8 + 4\times4\times\lg50.46 + 4\times4\times\lg15 - 4\times4\times4 = -3.49 < 0.$$

It is clear that the risk point G is in the allowable integrated risk zone between the *F-N* planes *ACE* and *BDF*, so the integrated risk is allowable, which is higher than the separate risks in life and in economy. From this result, it can be concluded that the conditions of the reservoir in dam safety have been improved to certain degree by accomplishment of the specific project for removal of the ill- condition and the reinforcement of this dam.

4 Conclusions

This study is focused on the *F-N* curved surface method for establishing the integrated dam failure risk criteria. In this method, the *F-N* curved surfaces are established by combining the *F-N* curves for the separate risk criteria with one another. The integrated dam failure risk can be easily determined according to the specific location of the risk point with respect to the *F-N* curved surfaces. In consideration of the overlying effect of the different types of risks caused by dam failure, the integrated dam failure risk is, in general, higher than any of the separate risks. Through a case study, it is demonstrated that this *F-N* curved surface method is applicable to the dam failure risk assessment and management.

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