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Finite difference numerical simulation of guided wave propagation in the full grouted rock bolt

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Based on guided wave theory and considering the grouted rock bolt as waveguide medium, we have constructed a three-dimensional model of grouted rock bolt with the dynamics of finite difference numerical simulation software FLAC^{3D}4.0, and simulated the propagation behavior of the guided wave in the full grouted rock bolt. The simulated waveform and wave velocity matched well with the experimental results. We have made a more in-depth and comprehensive study of the wave velocity, wave component and attenuation characteristics of the guided wave propagating in rock bolt, and found some new characteristics and phenomena. In addition, some phenomena that haven't been explained in the previous researches have also been discussed in this paper. The result showed that when guided wave propagates in grouted rock bolt, after the body wave decays, there is still the interface wave-Stoneley wave that does not decay in the axial direction of the bolt. The findings can provide some reference for rock bolt testing and the selection of the optimal excitation wave of testing.

guided wave, full grouted rock bolt, finite difference, numerical simulation

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1 Introduction

Rock bolts have been widely used in mining, slope, underground, tunnel engineering for its significant technical and economic advantages and ease of construction. Rock bolt is usually installed in the rock mass grouting with cement or resin, and its stability must be evaluated in order to ensure the safety of various types of engineering. In practical engineering, pull-out test and the stress wave reflection methods[1] are mainly used in inspecting the quality. In recent years, Beard and SLowe [2, 3] have applied guided wave as a new type of non-destructive detection technique to the detection of rock bolts.

The stress wave does not propagate in rock bolt in a simple form of longitudinal wave, but in the form of guided wave [4]. Sound waves are reflected repeatedly back and forth between the discontinuous interface (anchor interface) in the medium, and further produce a complex interference and the geometric dispersion, then comes into being the guided wave, whose propagation characteristics are directly related to the bolt border. One of the most important features of guided wave is dispersion, which means that its velocity and attenuation depend on the wave frequency. The domestic and foreign experts [5–11] have conducted some researches on evaluating the stability of grouted rock bolts with finite element numerically and experimentally based on guided wave theory. However, previous numerical sim-

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ulations were based on finite element analysis software. There exist two flaws: First, the artificial boundary conditions in finite element dynamic analysis may cause the reflection of outward propagating waves back into the model [6, 8, 11, 12], and thus bring a great impact on the simulation results; but the viscous boundary conditions of FLAC^{3D} software can be very good in absorbing the energy which should have been transmitted out of boundary without causing the reflection of waves. Second, for the grouted rock bolt, which is such a kind of model requiring very fine meshing (for the excitation wave length is very small, typically millimeters), the implicit algorithm commonly used in finite element software (Ansys) will cause the computation time so long that people find it difficult to accept; whereas the display algorithm used in finite difference will greatly reduce computation time. For this reason, we have established a three-dimensional structure model of rock bolt with finite difference numerical simulation software FLAC^{3D}4.0 to simulate the propagation characteristics of guided waves in the full grouted rock bolt, and meanwhile testify the accuracy of the numerical simulation with model experiment. By drawing lessons from previous studies, we have made a more in-depth and comprehensive study of the wave velocity, wave component and attenuation characteristic of the guided wave propagating in the grouted rock bolt, and found some new characteristics and phenomena. In addition, some phenomena that haven't been explained in the previous researches have also been discussed. The result showed that when the guided wave propagates in the grouted rock bolt, after the body wave decays, there is still the interface wave-Stonely wave that does not decay in the axial direction of the bolt. The findings can provide some references for rock bolt testing and the selection of the optimal excitation wave of testing.

2 Model and parameter selection

2.1 The basic principles of FLAC^{3D} dynamic analysis

FLAC^{3D}4.0 follows fully nonlinear analysis method, and its basic principle is the Lagrangian differential display method. The studied region is divided into a space grid whose mass is concentrated on grid nodes, and then all equations of motion should be solved. When the dynamic load is exerted, the equation of motion of the element node can be expressed as

$$\sigma_{ij,j} + \rho b_i = \rho \frac{\mathrm{d}v_i}{\mathrm{d}t}, \tag{1}$$

where ρ is the medium density, b_i is physical power, v_i is node speed, and $\frac{\mathrm{d}v_i}{\mathrm{d}t}$ is node acceleration.

The constitutive equation is

$$\left[\hat{\sigma}_{ij}\right] = H_{ij}\left(\sigma_{ij}, \zeta_{ij}, k\right), \tag{2}$$

where σ_{ij} indicates the stress tensor, $\left[\hat{\sigma}_{ij}\right]$ rate of stress tensor, H given function expression, k the parameter taking the loading history into account, and $\zeta_{ij} = \frac{\left(v_{ij} + v_{ji}\right)}{2}$ is strain rate tensor.

Through abovementioned simultaneous equations the stress rate tensor, strain rate tensor and velocity vector can be obtained. In solving these equations FLAC^{3D} uses the following three methods. ① discrete model approach: continuum is discretized into a number of interconnected hexahedral elements, and forces are concentrated on the nodes; ② finite difference method: the first derivative of variables on the space and time use finite difference to approximate, and the equations of motion and dynamic equations are solved by using explicit methods; ③ dynamic relaxation method: particle motion equations is applied to solving, and the system motion is made attenuation to a state of equilibrium by damping.

2.2 Finite difference model

The dynamics of finite difference software FLAC^{3D}4.0 developed by Itasca Consulting Group in the United States is used to establish a three-dimensional model of grouted rock bolts system, as is illustrated in Figure 1. In this model there are two layers of media with the outer layer being concrete for grouting media and inner layer being the rock bolt, and two kinds of media are simulated with solid elements. This ensures that the numerical simulation can be consistent with model experiments. Table 1 is for the concrete and rock bolt mechanical parameters. Rock bolt is fully grouted with a diameter of 20 mm and a length of 0.5 m, while the grouting medium is a cube whose length and breadth are all 0.5 m. In practical engineering, the surrounding rock around the bolt is infinitely large, and thus the guided waves propagating in

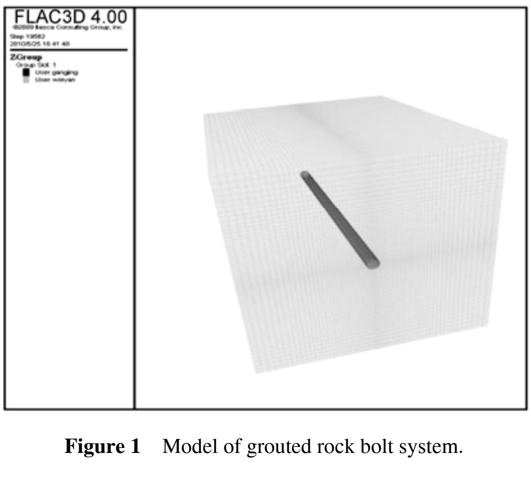

Figure 1 Model of grouted rock bolt system.

Table 1 Parameters of model material

Materials	Density ρ(kg /m^3)	Modulus E (GPa)	Poisson ratio ν
Rock bolt	7800	210	0.3
Concrete	2400	20	0.2

the grouted rock bolts would be transmitted to the surrounding rock. Although the grouting medium in this numerical model have been taken within the range of 0.5 m, the viscous boundary of the dynamic model of FLAC^{3D}4.0 would absorb the energy, which should have transmitted to the surrounding rock, and the reflection of wave will not be formed at the boundary. Such is the significant advantage of the simulation in this paper compared with those simulations using the finite element software.

2.3 Mesh, boundary conditions and constitutive model selection

According to the researches of Kuhlemeyer and Lysmer (1973), the size of meshing stands under control of the shortest wavelength of input waves. Suppose the maximum size of the grid is Δl , and the shortest wavelength of input waves is λ (the wavelength when the frequency amounts to maximum). Then Δl must be smaller than 1/10–1/8 of the shortest wavelength. Therefore, the size of radial grid is 2 mm, and the axial size 10 mm; then there would generate about 321000 elements in the model. On the top of the model is free boundary, and the displacements in the directions of X, Y, Z of other sections would be limited and viscous boundary conditions would also be imposed. When the viscous boundary conditions are set in FLAC^{3D}4.0, there is no need for user to set parameters, for the program would automatically set free dampers related to the node speed in the normal and tangential model boundaries to achieve the purpose of absorbing the wave. This damping in this model is Rayleigh damping. Since the excitation wave energy in testing is very small, the surrounding rock, mortar, and bolt are all in the elastic state in the course of bolt testing. Consequently, there is not any relative slip between the interfaces, so in the numerical simulation the elastic constitutive model is used in all the elements, and the relative slip between the interfaces is not taken into consideration.

2.4 Input of transient excitations

When viscous boundary condition is applied in model, the dynamic load can only be applied in the form of stress or force in FLAC^{3D}4.0. A transient excitation force load is imposed on the top of bolt according to the testing situation of the bolt and the characteristics of FLAC^{3D}4.0. In order to make the frequency dispersion of the excited signal in the propagation as low as possible, a single frequency signal is selected to excite testing signals. Beard [2] suggested that the more cycles signal should be selected so as to make the signal frequency domain much narrower and energy more focused. For this reason, the 7 period sine wave signal controlled by time and corresponding force is selected. Not only can this narrow-band excitation enhance the signal strength, but also increase the propagation energy of guided waves. Illustrated in Figure 2 is the time domain waveform of an excitation signal with the number of cycles as 7 and a center frequency of 40 kHz.

3 Verification of the correctness of numerical simulation

3.1 Verification of propagation of guided wave in free rock bolt

In the laboratory the bolt acoustic detector RSM-RBT developed by Wuhan COG was used to conduct an experimental research on the speed dynamic response of the top of a free rock bolt with a length of 5.14 m under the transient exciting force, and the speed dynamic response curve was shown in Figure 3(a); meanwhile, a numerical simulation (the excitation wave frequency is 20 kHz) was made to study the propagation of guided wave in the free bolt with a length of 5.14 m, and the dynamic response curve of the top of the bolt was shown in Figure 3(b), and the material parameters were listed in Table 1. It can be seen from Figure 3 that the speed dynamic response curve of the top bolt in numerical simulation matched well with that one in the test. First of all, for both of them there was a very small difference between the reflected waves and the first wave, which verifies the fact that the extent to which the propagation of the guided wave in free bolt attenuates is quite small. Secondly, the arrival times of both reflected waves stayed the same, which is 2.01 ms, whereby the velocity of wave can be worked out as 5100 m/s based on the length of the bolt, which is also consistent with the actual situation.

3.2 Verification of propagation of guided wave in grouted rock bolt

In this paper, the numerical simulations were conducted to

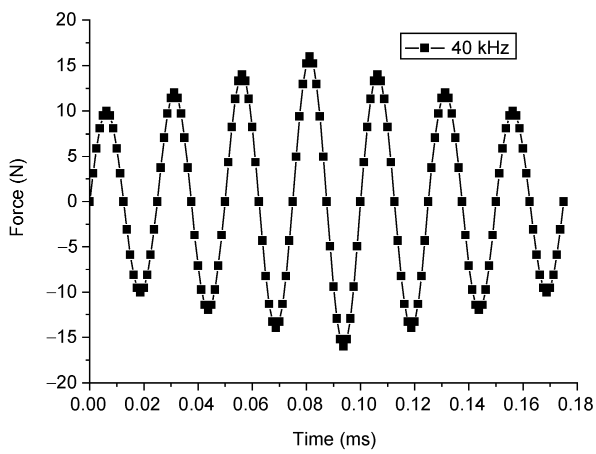

Figure 2 Excitation signal.

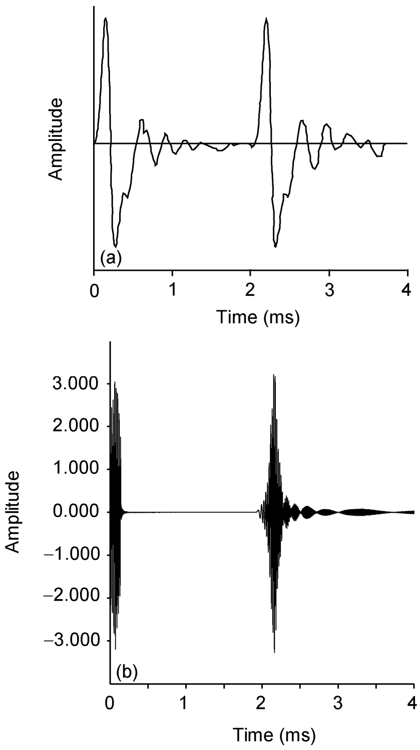

Figure 3 Acceleration-time curve of free bolt. (a) Test result; (b) numerical simulation result.

simulate the propagation of guided waves with excitation frequencies of 25, 40, 50, 75, 100 kHz, respectively in the fully grouted rock bolt, and the input excitation wave was single frequency signal with cycles of 7, and in Table 1 were the material parameters. The speed vibration waveforms of the top of the bolt monitored during the simulation were illustrated in Figure 4. When the numerical calculation was completed, first, the data of the vibration velocity recorded during the simulation were extracted by formulating Fish language, then by inputting these data to excel, the corresponding amplitudes and times when the amplitude of the first wave and the reflected wave reached their biggest points were precisely extracted, and accordingly the speed of guided waves was calculated. The comparison results between the velocities of each guided wave worked out according to the data in the numerical simulation under different frequencies of the excitation wave and those obtained through model test [13] were shown in Figure 5. It can be seen from the figure that there were certain differences between the numerical simulation results and the experimental results, but comparatively speaking, the differences were within 3%, and such differences could be caused because of inaccurate grouting medium parameters. Anyway, the variation of wave velocity was consistent with the change of frequency, which exerts no influence on the analysis of the propagation characteristics of guided wave in grouted rock bolt. Therefore, it can be proved that the numerical results agreed well with the experimental results, and the results of this paper are reliable.

4 Numerical simulation results analysis

4.1 Analysis of propagation velocity of guided wave in grouted rock bolt system

The speed vibration waveform of the top of the bolt monitored during the simulation was illustrated as in Figure 4. First, the data of the vibration velocity recorded during the simulation should be extracted by formulating Fish language, then by inputting these data to excel, the corresponding amplitude and time when the amplitude of the first wave and the reflected wave reache the biggest points should be precisely extracted, and finally the speed of guided waves can be calculated. The calculation results were shown in Figure 5 and Table 2. First of all, it can be seen from Figure 4 that the speed waveform at the top of the bolt is very different under the exciting waves of different frequencies. The reflected wave can be seen once or twice when the frequency is relatively low, here only the longitudinal ultrasonic guided waves whose mode is L (0,1) are produced in the bolt. However, when the frequency is comparatively high (100 kHz), a relative noise is generated in the test to result in the other modes of guided waves. Consequently, there appear multiple reflected waves, in the end it is difficult to accurately identify which one is the real bottom reflection. Therefore, when the grouted rock bolt is in excitation waves with different frequencies, the reflection waveform would greatly vary. It can be seen from Figure 5 that, generally speaking, the wave velocity increases as the frequency of excited wave gradually increases, nevertheless, when the frequency is less than 50 kHz, the increase of velocity is relatively small with the increase of frequency, while when the frequency is more than 50 kHz, the wave speed increases significantly with the increase of frequency, which, however, stands well below the velocity of longitudinal wave. This is due to the inconsistency of the degree of frequency dispersion when different guided waves propagate in grouted rock bolt. This shows that characteristics of the propagation of guided wave in grouted rock bolt are very different from that of the propagation of guided wave in a free rock bolt, in that the velocity of propagating in a free bolt gradually decreases with the increase of excitation frequency[4], and the wave attenuates slightly. The numerical simulations and experimental studies by Zhang[10] have suggested that the velocity of guided wave in grouted rock bolt changes tremendously with the variation of the strength of the grouting media; they proposed to measure the anchorage quality with the variation of wave velocity. In fact, however, there appears a great difference in wave velocities when the guided waves under the excitation waves with different frequencies propagate in the grouted rock bolt. At the same time, in the actual engineering excitation waves are generally produced by artificial means as hammering, and the frequencies of excitation wave thus generated by hammering are inconsistent with each other and also hard to control, therefore, both the strength of grouting medium and

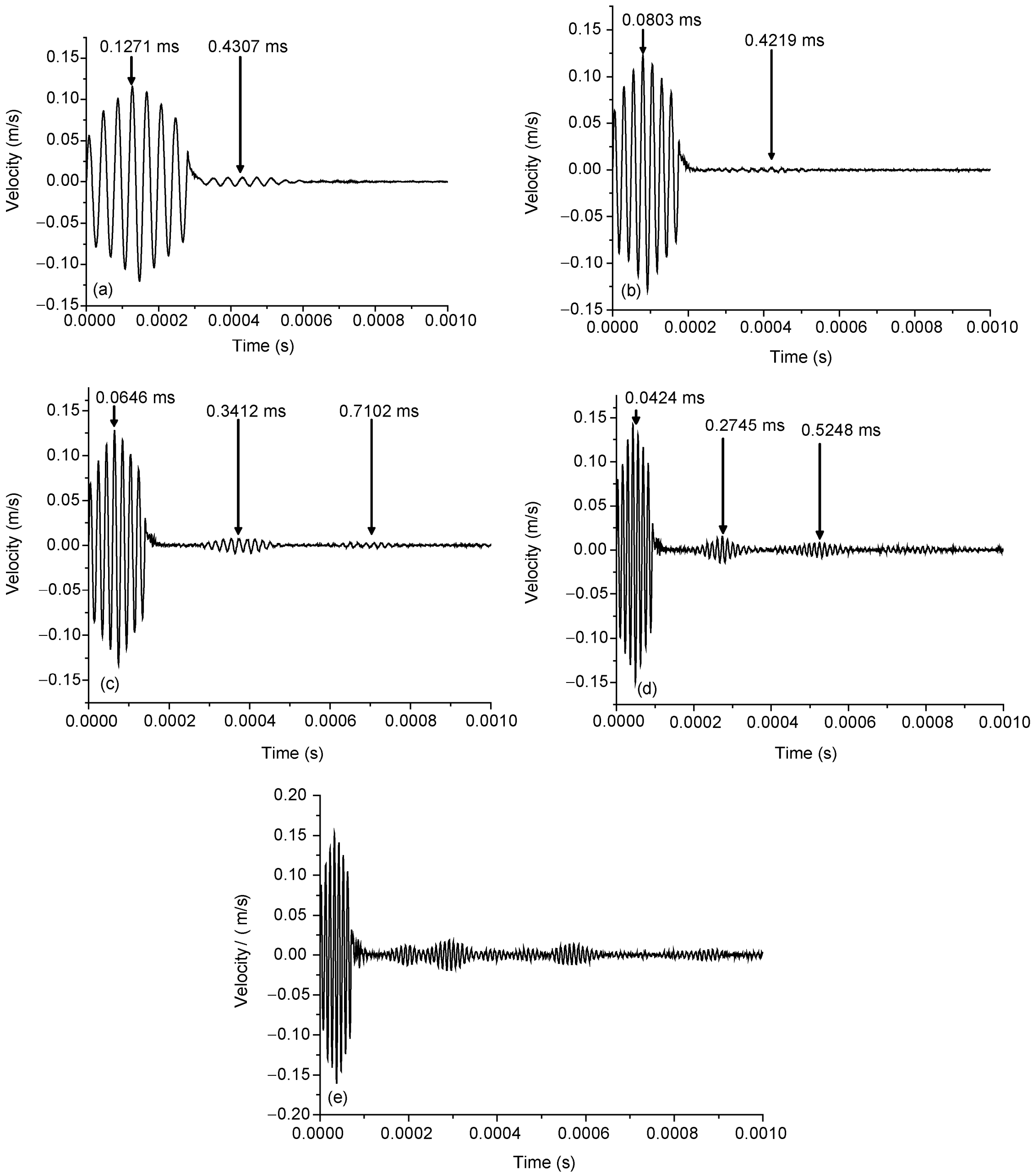

Figure 4 Velocity-time curves of bolt. (a) 25 kHz; (b) 40 kHz; (c) 50 kHz; (d) 75 kHz; (e) 100 kHz.

Table 2 Wave velocities with different excitation wave frequencies

Frequency (kHz)	Diameter (mm)	Monitoring point	Second wave velocity (m/s)	Third wave velocity (m/s)
25	20	Top	3294	—
40	20	Top	3470	—
50	20	Top	3628	2809
75	20	Top	4308	3995
100	20	Top	—	—

the frequency of excitation wave should be taken into consideration to calculate the velocity of guided wave propagating in the grouted rock bolt. Moreover, if the parameters obtained from guided wave propagating in free bolt or from longitudinal wave are used to determine the propagation characteristics of guided wave in grouted rock bolt, there would appear considerable inaccuracy.

From Figures 4 (c) and (d) we can clearly see that there are two reflected waves. The velocities of these two reflected waves were calculated, and the results were shown in Table 2. The Table reveals that the velocity based on the first reflected wave is less than that calculated according to the second reflected wave. In order to be sure about the findings of the research, a numerical simulation has been made on the influence of excitation waves with frequencies of 50 and 75 kHz in the grouted rock bolt with a diameter of 36mm, and the results are shown in Table 3. The changing

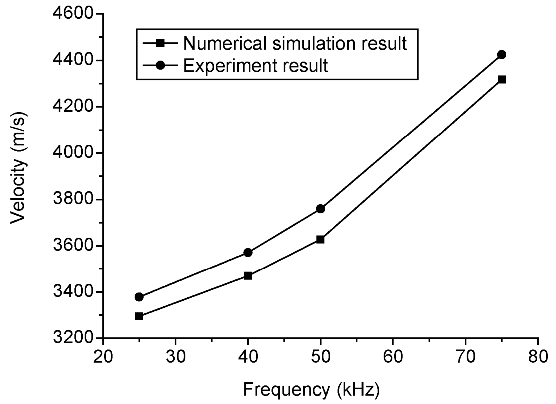

Figure 5 Comparison between numerical simulation and model experiment results.

regularity is consistent with the above analysis. When propagating in the grouted rock bolt, guided wave would be repeatedly reflected back and forth between the discontinuous interface (anchor interface) in the medium, and further produce a complex interference and geometric dispersion, yet the reflection, interference and geometric dispersion taken place on the interface are different with regard to the fact that waves with different frequencies which have different wavelengths, and so the velocity of guided waves with different frequencies propagating in the grouted rock bolt would be different, which is commonly referred to as frequency dispersion. Guided wave is propagating in the form of wave packet composed of a group of waves with different frequencies. The phase velocity of the harmonics with different frequencies in the wave packet is in great diversity, and the velocity of the wave packet is lower than that of any of the harmonics. The guided wave first spreads from the top to the bottom of the grouted rock bolts and then is reflected from the bottom to the top, and every time such wave group is reflected back and forth once, the guided wave will once again produce dispersion, and the velocity will accordingly decrease.

4.2 Analysis of the wave component and attenuation characteristics in grouted rock bolts

The velocity-time curves of the grouted rock bolt with a diameter of 36 mm in the excited wave frequencies of 50 and 75 kHz, respectively were illustrated as in Figure 6, in which (a) and (c) show the results monitored from at the top of the bolt, while (b) was from bottom of the bolt. It can be seen from the figure that the waveforms at the top and bottom of the bolt are largely different from each other, yet there has one thing in common: Compared with the first wave, the second wave (the first reflected wave amplitude) attenuates tremendously, whereas compared with the amplitude of the second wave, that of the third wave (second reflected wave) shows a relatively small attenuation. Such phenomenon also can be seen from Figure 4 as well as previous research results [5, 6, 8, 9]. Table 3 shows the results of the attenuation calculated according to the results of Figure 4, and the attenuation magnitudes are shown as follows

$$\alpha = \left(1 - \frac{A_2}{A_1}\right) \times 100\% , \tag{3}$$

where α indicates attenuation magnitude, A_2 the magnitude of reflected wave, and A_1 the amplitude of the first wave. The research results of Zhang[9] showed that as the strength of grouting medium intensifies in the grouted rock bolt, the interface wave generates while P-wave gradually attenuates to disappearance and the original dominance of P-wave transmitting in the bolt changes into that of the interface wave, meanwhile the wave velocity transmitting in the bolt part changes from the longitudinal wave speed to the interface wave speed. Interface waves are those waves that can spread along the interface between the media. Therefore, the first wave seen from Figure 6 (a) should be a coupling of longitudinal and interfacial waves, and the reflected wave should be the interface wave for the longitudinal wave vanishes as a result of a sharp attenuation (Noteworthy is that the attenuation magnitude amounts as high as 94% when the longitudinal wave spreads from the top to the bottom of the bolt, and then reflected from the bottom to the top). The curve of velocity monitored at the bottom of bolt is illustrated in Figure 6 (b). Compared with the first wave's amplitude in Figure 6 (a), the amplitude of the first wave appears a little smaller, while the second wave and third wave amplitude in Figure 6 (b) and the second wave amplitude in Figure 6 (a) stand on the same level, so it is safe to say that both the second and third waves in Figure 6 (b) and the second wave in Figure 6 (a) belong to the interfacial wave. In addition, it is noted that the time of the first vibration recorded in Figure 6 (b) is 0.1 ms, the wave velocity thus is around 5000 m/s, which is the P wave velocity in the bolt, so there exists a straight P wave in the first wave. This is because the P-wave does not completely attenuate when propagating from the top to the bottom of the bolt, and the

Table 3 Wave velocities and attenuation magnitude with different excitation wave frequencies

Frequency (kHz)	Diameter (mm)	Monitoring point	First wave magnitude	Second wave			Third wave		
				Magnitude	Attenuation magnitude	Velocity (m/s)	Magnitude	Attenuation magnitude	Velocity (m/s)
50	36	Top	1.306×10^{-1}	7.261×10^{-3}	94.4%	3318	—	—	—
50	36	Bottom	1.292×10^{-2}	6.359×10^{-3}	50.7%	3291	4.420×10^{-3}	30.5%	3030
75	36	Top	1.121×10^{-1}	1.420×10^{-2}	87.3%	4268	1.288×10^{-2}	9.3%	4186

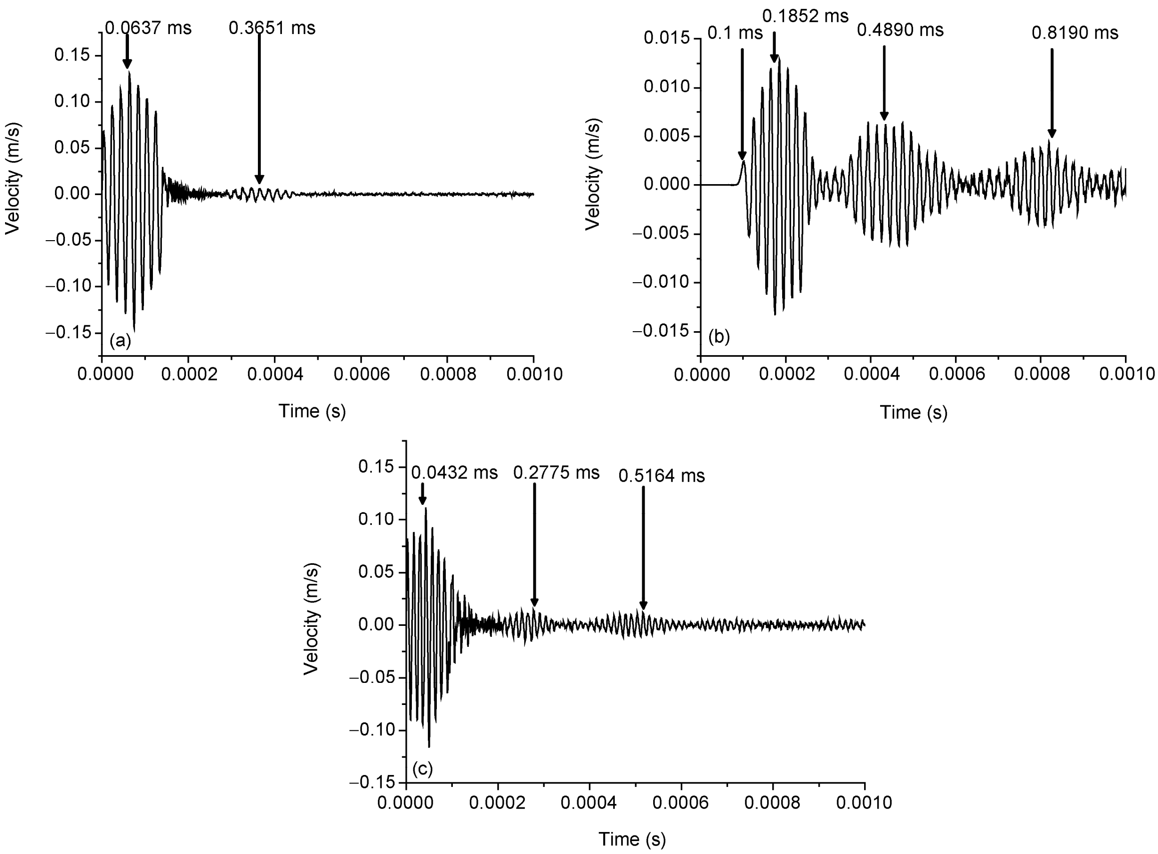

Figure 6 Velocity-time curves of bolt. (a) 50 kHz-top; (b) 50 kHz-bottom; (c) 75 kHz- top.

first wave in Figure 6(b) should be a coupling of the P-wave and interface wave, but the P-wave is very weak. Precisely because of this coupling, the amplitude of the first wave in Figure 6(b) is a bit less than those (interface wave and a strong P wave) in Figure 6(a), and is twice as much as the second wave in Figure 6(a) and (complete interface wave) Figure 6(b). Similarly, it can be analyzed that in Figure 6(c) the first wave is a coupling of the P-wave and interface wave, and the reflected waves (the second wave and the third wave) are interface waves resulted from the sharp attenuation of the P-wave when propagating from the top to the bottom of the bolt and then reflected from the bottom to the top. Table 3 tells that in the case of the two reflected waves (Figures 6 (b) and (c)), the attenuation magnitude (compared with the second wave) of the third wave is far smaller than that of the second wave (compared with the first wave). Theoretically speaking, the interface waves exist only on the interface of semi-infinite body in spite of the fact that there does not actually exist any semi-infinite space, nevertheless, as far as the model studied in present paper is concerned, the guided wave only has a wavelength of a few millimeters, while the model is as long as 0.5 m, and it can be entirely seen as an approximation of a semi-infinite space. Stoneley wave is a typical interface wave, a kind of non-uniform wave, which can only exist on the interface of material combinations, and the energy is concentrated on the interface within a distance of one wavelength. Stoneley wave propagates along the direction parallel to the interface (along the direction of bolt axis), and there is no attenuation along this propagation direction; although the wave does not travel along the perpendicular direction, it does present an $r^{-1/2}$ exponential decay in this direction[14, 15]. The P wave transmits and decays in the three-dimensional space with the rate of r^{-1}. Precisely owing to the fact that the P wave decays along with the (bolt axial) direction of propagation and the decay rate is high and the interface wave does not attenuate along the direction of propagation, compared with the magnitude of the first wave, that attenuation of the second wave appears quite large, and in the waveform only one P wave can be observed; at the same time because the interface waves do not decay along the propagation direction and they can spread far away, resulting in multiple interface wave reflections which can be observed in the waveform. The attenuation magnitude of the second wave in Figure 6(b) looks much less than those of the second wave in Figures 6(a) and (c), which is because the second wave in Figure 6(b) is a coupling of weak P wave and the interface wave, and this also indirectly verifies the correctness of the results of the above analysis. Therefore, when guided wave propagates in the grouted rock bolt, after the body wave decays, there is still the interface wave–Stonely wave that does not decay in the axial direction of the bolt.

Therefore, in the engineering testing of the quality of grouted rock bolt, the reflected wave received by the bolt detector is the interface wave, and the wave velocity should be the velocity of interface wave.

5 Conclusion

In this paper, a three-dimensional finite difference model of the grouted rock bolt system has been constructed based on the dynamics of finite difference software $FLAC^{3D}$4.0, and numerical simulations have been conducted to analyze the wave velocity, wave component and attenuation characteristics of the anchoring system under excitation waves with different frequencies. Conclusions are drawn mainly as follows.

1) The stress waves propagate in the form of guided wave in the routed rock bolt, and the velocity of guided waves are different when affected with excitation waves in different frequencies. As far as the model in this article is concerned, the velocity increases as the frequency of excitation wave gradually increases; when the frequency is less than 50 kHz, the increase of velocity with frequency is relatively small, but when the frequency is more than 50 kHz, the increase of velocity is rather significant; but both of these velocities keep lower than the velocity of P wave. Besides, the velocity of guided waves would gradually reduce each time they are reflected back and forth. Furthermore, both the strength of grouting medium and the frequency of excitation wave should be taken into consideration to calculate the velocity in the grouted bolt. In the actual testing, the excitation wave whose excitation mode is *L* (0,1) ,with a relatively low frequency, should be selected, because the selection of high-frequency wave would provoke guided waves with other complex modes, and it is not conducive to the extraction and recognition of reflected waves.

2) In the waveform received from bolt testing, generally the first waves are longitudinal wave and interface waves, and the reflected waves are interface waves. Compared with the first wave, the second wave in the waveform (the amplitude of the first reflected wave) decays sharply, while the amplitude of the third wave (the second reflected wave) attenuates relatively smaller than that of the second wave. Such phenomena have been discussed in this paper. This is because when guided wave propagates in the grouted rock bolt, after the body wave decays, there is still the interface wave–Stonely wave that does not decay in the axial direction of the bolt.

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1 Li Y, Wang C. Experiment study on bolt bonding integrity with stress reflected wave method (in Chinese). J China Coal Society, 2000, 25: 160–164
2 Beard M D, Lowe M J S, Cawley P. Ultrasonic guided waves for inspection of grouted tendons and bolts. Non-Destr Testing Condition Monitoring, 2002, 44: 19–24
3 Beaed M D, Lowe M J S. Non-destructive testing of rock bolt using guided ultrasonic waves. Int J Rock Mech Mining Sci, 2003, 40: 527–536
4 Wang C, Ning J G. Numerical simulation of guided wave propagation in anchored bolts (in Chinese). Chin J Rock Mech Eng, 2007, 28: 3946–3953
5 Zhang C S, Li Y, Zhao Y S, et al. Study on optimum excitation wave in grout quality nondestructive testing of rock bolt (in Chinese). Chin J Rock Mech Eng, 2006, 25: 1240–1245
6 Zhang C S, Zhou D H, Madenga V. Numerical simulation of wave propagation in grouted rock bolts and the effects of mesh density and wave frequency. Int J Rock Mech Mining Sci, 2006, (43): 634–639
7 He W, Wang C, Ning J G, et al. Guided wave determination method of service load in partially grouted bolt (in Chinese). Chin J Rock Mech Eng, 2009, 28: 1767–1772
8 Zhang C S, Li Y, Zou S. Numerical simulation of guided wave propagation in grouted rockbolt (in Chinese). J Taiyuan Univ Tech, 2009, 40: 274–278
9 Zhang S P, Zhang C S, Wang C, et al. Study on wave propagation characteristics of rock bolt grouted structure (in Chinese). Rock Soil Mech, 2007, 28: 2570–2578
10 Zhang C S, Li Y, Zou S. Research on consolidation wave in grouted rock bolt structure system (in Chinese). Chin J Rock Mech Eng, 2009, 28: 3604–3608
11 He W, Wang C, Ning J G, et al. Research on the guided wave reflecting from the upper interface of partially grouted rock bolt (in Chinese). J China Coal Society, 2009, 34: 1451–1455
12 Lin H C, Wang C, Ning J G, et al. Dynamic response of metal bar bonding system under transient excitation(in Chinese). Mech Eng, 2005, 27: 39–42
13 Yang F Z, Zhang C S, Zhu J S. The transmission characteristics of ultrasonic guided wave in rock grouted bolts(in Chinese). Shanxi Coal, 2008, 28: 27–29
14 Han Q B, Qian M L, Zhu C P. Study on solid-solid interface waves with laser ultrasonic. Acta Physica Sinica, 2007, 56: 313–320
15 Rose J L. Ultrasonic Wave in Solids. Beijing: Science Press, 2005